

GEOMETRICAL TOOLS FOR STRING COSMOLOGY

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1 Overview of the Field

In the past decades, string theory has evolved to provide a robust basis for phenomenological studies of physics. String theory is the leading candidate for a theory of quantum gravity, and reduces at low energy to a supersymmetric version of Einstein’s general relativity (supergravity). Mathematical consistency, however, requires string theory to have extra dimensions, which may be hidden from view by an appropriate compactification process. In these string compactifications, the geometry and topology of the compact manifold determine the laws of physics in the four-dimensional world. Conversely, four-dimensional physics requirements, such as long-lived protons and three generations of elementary particles, can be reformulated as topological constraints on the compact geometry. Furthermore, through compactifications, string theory has acted as an experimental branch of mathematics and led to surprising predictions in algebraic geometry and number theory.

In the last years, string theory model builders have taken a strong interest in the cosmological relevance of the effective four-dimensional models. At the same time, cosmology has emerged as a precision science, where observations of the Cosmic Microwave Background and Large Scale Structure pinpoints our Universe as expanding at an accelerated rate, being dominated by dark energy and having a large fraction of dark matter [1, 2, 3, 4], and very likely having undergone a period of inflation [5, 6, 7, 8, 9] some 14 billion years ago. String theory models describing these cosmological features have been widely investigated over the last years. Advances include both model independent conclusions, such as the presence of a large number of compactification moduli and axions, and explicit constructions of inflationary models with specific observable signals (e.g. non-Gaussianity, large primordial gravity waves). In particular, flux compactifications of the type IIB string on conformal Calabi–Yau (CY) manifolds have led to many interesting inflationary models, such as Dirac–Born–Infeld (DBI) inflation, axion monodromy and fibre inflation [10]. Another important development is the construction of de Sitter solutions in such type IIB compactifications [11], which may describe a spacetime dominated by dark energy.

A recurrent theme in these developments is that moduli challenge string model building. Classically, the moduli lead to massless scalar fields in four dimensions, which pose an immediate problem in cosmology; these fields would mediate new forces, and, for example, be detectable through their effect on planetary orbits. Moreover, moduli are prone to interfere with string realisations of cosmological accelerated expansion, and may also affect e.g. the reheating phase after inflation. Thus, realistic string compactifications require a mechanism that “stabilises” moduli, giving the corresponding four dimensional scalars a mass. In the models

described above, such mechanisms include effects from so-called flux (p -form analogues of electromagnetic field strengths), curvature, and torsion, which can all be described in the ten dimensional low-energy supergravity description of string theory. All models also include non-classical effects that are described on the level of the four dimensional effective field theory, such as gaugino condensation and instanton corrections.

2 Recent Developments and Open Problems

A major theme in recent string theory research [13, 14, 15] has been the scrutiny of the validity of the phenomenological models presented above as low-energy approximations of a consistent UV complete theory. Various conjectures have been put forward to determine the properties of four dimensional effective field theories, which can be consistently derived from string theory, thus belonging to a “landscape” of string vacua. On the other hand, those 4D theories which do not have a UV completion would belong to a “swampland” [12] of theories inconsistent with quantum gravity. Of particular interest for cosmology is the proposed “no de Sitter”-conjecture which claims that the models found in [11] are not proper string theory constructions. Moreover, it claims that it will in fact be impossible to achieve de Sitter vacua from string compactifications [14]. This conjecture has clear implications for early universe acceleration, inflation, as well as late time acceleration, dark energy, and has therefore become an important area of research in the last months.

Consequently, it is fair to say that to date, rigorous embeddings of dark energy, dark matter and inflationary models into string theory remain a challenging problem. Indeed, there are important formal requirements that must be resolved before robust predictions can be made in string cosmology, and this issue was the main motivation for this workshop. With these aspects in mind, the workshop schedule revolved around

1. The existence of de Sitter vacua in string theory and supergravity. This outstanding problem is connected to supersymmetry breaking, and also to the so-called moduli stabilisation, which is related to the problem of finding the mathematical moduli space of the theory. It is a central problem of the landscape/swampland discussion.
2. The effect of compactification moduli on string cosmology. In particular: the use of moduli as inflatons, how moduli affect inflationary physics through, for example, their effect on the couplings of the low-energy effective field theory, and reheating at the end of inflation.
3. Geometrical/mathematical tools of use in building concrete Calabi-Yau compactifications in type IIB string theory; their relevance in mathematics and their potential applications to early universe acceleration.
4. Recent developments in the mathematical foundations of other branches of string theory, such as F-theory and heterotic string theory, and potential applications to string cosmology.
5. Recent developments of computational techniques that are of use for the analysis of vacua in the string theory landscape.
6. Recent constraints on cosmological inflation from experiments and potential use of gravitational waves to probe the early universe.

The workshop was designed to promote developments on these outstanding questions. A key objective of the workshop was to bring two communities together which do not regularly meet at regular conferences. Namely, researchers in (string) cosmology on the one hand and on the other hand, string theorists focusing on mathematical and/or geometrical developments.

With the same motivation, the theme of the workshop followed two interconnected directions. In the more mathematical part, the focus was on recent formal developments of the geometry underlying supergravity, string theory and F-theory. Of particular relevance were recent advances regarding the construction of compactification geometries, geometries admitting string compactifications with flux, the duality between string compactifications, and the so-called non-geometric compactifications. The workshop also covered

mathematical tools of relevance for string compactifications (such as algebraic and toric geometry) and computational techniques of use for the analysis of string vacua (such as computational algebraic geometry and machine learning). Complementing this, the second strand of the workshop focused on phenomenological discussions of the progress in cosmology and in related string theory models, including the possibility of making predictions for future observations.

Investigating these questions in both theoretical and mathematical physics, with the aim to connect string theory to cosmological phenomenology, has led in the past to several rich and unexpected developments in mathematics, such as mirror symmetry and quantum cohomology. In this workshop, it became evident that the search for phenomenologically realistic string vacua still serves as a driving force for development in various branches of geometry, deformation theory and even number theory. Moreover, given the observational success of the inflationary paradigm, it is of utmost importance to develop a solid mathematical and theoretical framework to support it and which may guide future experiments searches. This workshop was organised to further promote such developments, by providing string theorists with the opportunity to assess the status of string cosmology, and identify the most promising avenues of future research. As well as deepening our basic understanding of string theory and its mathematics, in working towards concrete realisations of inflation and dark energy in string theory, we will be led to new predictions, which future experiments and observations can test. Thus string cosmology provides a unique opportunity to relate the fundamental physics and mathematics of quantum gravity to observations in the sky. This philosophy permeated many discussions during the workshop.

3 Presentation Highlights

The highlights of the workshop fall naturally into three main areas: developments in cosmology, formal studies of string theory and talks at the interface of cosmology and formal aspects.

3.1 Cosmology: observational constraints and model building

Inflation

The workshop kicked off with a review by Gianmassimo Tasinato of recent and future prospects to probe the physics of the early universe, inflation with current and future gravitational interferometers. It is well known that cosmological inflation predicts the existence of a stochastic background of gravitational waves, whose features depend on the specific model of inflation. Some existing well motivated frameworks predict an enhancement of the primordial gravity wave spectrum at frequency scales testable with gravitational wave experiments, with distinctive features such as parity violation and non-Gaussianity. We learned about the properties of such scenarios, and their distinctive gravity wave power spectrum and bispectrum and how to quantitatively test these predictions with current and future interferometers.

The inflationary scenario and its predictions for large classes of models were further reviewed by Vincent Vennin in view of the most recent cosmological observations, highlighting those models for which the data show the strongest preference. Novel aspects of multifield models of inflation, which have recently acquired a renewed interest due to recent theoretical developments discussed above, were discussed by Spyros Sypsas and Diederik Roest.

An interesting novel alternative to the vanilla models of inflation where the potential energy of a scalar (or scalars) is responsible to drive the accelerated expansion in the early universe was presented by Luisa Jaime. In this framework, the presence of an inflationary epoch is a natural, almost unavoidable, consequence of the existence of a sensible effective action involving an infinite tower of higher-curvature corrections to the Einstein-Hilbert action.

A further modification of gravity, the cuscuton scenario, was presented by Ghazal Geshnizjani. In this modification of gravity, the cuscuton action modifies the equations of motion in infra red limit allowing the background to go through a regular bounce phase, contrary to the case in general relativity. At the same time, since it does not contain any dynamical degrees of freedom, it does not lead to ghosts or other instabilities.

Recent progress on effective field theory methods in the presence of gravity and consequences for the inflationary paradigm was discussed by Cliff Burgess.

Axions

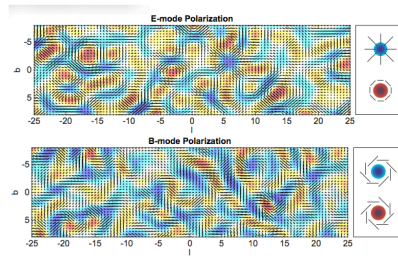
One of the most interesting predictions in string theory compactifications, is the existence of a large number of light axion-like fields with masses in a wide range of values. These arise as zero modes of antisymmetric gauge fields on the compactified dimensions and their number is determined by the topology and of the extra dimensions. These fields can therefore have relevant cosmological implications, depending on the value of their masses. This has been referred to as the axiverse [16].

Cody Long discussed the newest developments on the distribution of the couplings of these stringy axions to the photon, which is an important question given the current and future experiments exploring these couplings. Furthermore Thomas Bachlechner suggested that the distinct meta-stable minima which arise from theories with several axions may be able to account for the apparently tuned features of our universe, including its current enormous size, age, and tiny energy densities compared to the scales of fundamental physics.

But axions have also been shown recently to be a prominent source of gravitational waves. Yuko Urakawa presented recent developments on this interesting topic. She explain how string axions in various mass ranges generate gravitational waves with peaks at various frequencies determined by the mass scales, dubbed the gravity wave forest and how this may help us explore the string axiverse through future multi-frequency gravity wave observations.

How to detect GWs from inflation?

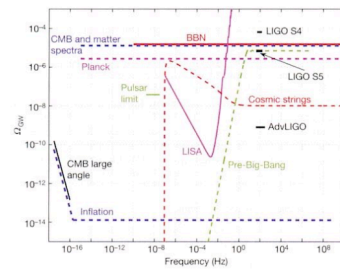
CMB Polarization



Credit: Kamionkowski -Kovetz, Annu. Rev. Astron. Astrophys. 2016.

But what if $r \ll 10^{-3}$ at CMB scales?

Direct detection?



Amplitude of GW's from vanilla models of inflation is too small to be detected \Rightarrow go beyond

Figure 1: *Cosmological inflation predicts the existence of a stochastic background of gravitational waves, whose features depend on the model of inflation under consideration. There exist well motivated frameworks predicting an enhancement of the primordial gravity wave spectrum at frequency scales testable with gravitational wave experiments, with distinctive features as parity violation and non-Gaussianity. During the review, Tasinato explained the properties of such scenarios, and their distinctive predictions for what respect the gravity wave power spectrum and bispectrum. He then discussed how to quantitatively test these predictions with current and future interferometers. (Picture taken from Tasinato's talk available at the event's web page).*

3.2 Interface of cosmology and formal aspects

Dark Energy and de Sitter in string theory

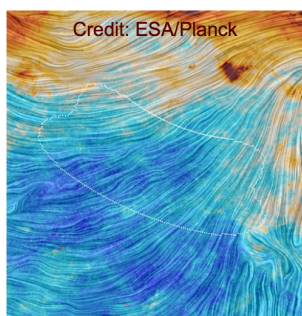
Progress at the interface of cosmology and formal aspects included recent review on dark energy in string theory by Susha Parameswaran. She discussed the current situation to explain present day acceleration put in context with the most recent developments on the formal theory. In particular, if the recent “de Sitter swampland conjectures” are taken seriously, one finds with a challenging situation to explain dark energy. Mariana Graña on the other hand review the recent developments in understanding pure de Sitter solutions in different string theory constructions, in particular the various non-classical corrections needed to provide potentials for moduli with the required properties.

Swampland conjectures

In the focus talks, Ander Retolaza presented very recent developments on the question of moduli stabilization in type II string compactifications on generalised Calabi–Yau manifolds. Giuseppe Dibitetto also addressed this topic, in a talk about the implications of the swampland conjecture within a specific model in massive type IIA compactifications, and discussed the status of the search for classical de Sitter solutions arising from reductions on SU(3) structure manifolds. Finally Oscar Loaiza-Brito discussed the de Sitter vacua and moduli in the context of the so called non-geometric fluxes, reviewed also by Erik Plauschinn.

Irene Valenzuela on the other hand, presented the recent progress on understanding the validity of the so called swampland distance conjecture for which infinite distances in field space imply an infinite tower of states becoming exponentially light. She present new evidence for this conjecture in Calabi-Yau manifolds, by studying the monodromy transformations associated to the infinite distance singularities in the moduli space. She then showed how one can apply these results to the complex structure and Kähler moduli spaces of Type II and M/F-theory Calabi-Yau compactifications.

Developments on string cosmology



What can we learn by detecting a Gravitational Wave signal in the CMB?

$$\frac{\Delta\phi}{M_{Pl}} \gtrsim \left(\frac{r}{0.01}\right)^{1/2}$$

- Inflation occurred near the GUT scale
- Inflation displacement is super-Planckian

Need understanding of UV completion of inflation

Figure 2: *What would happen if primordial gravity waves are detected by future experiments? According to the Lyth bound [17] for the simplest scenarios, it will indicate that inflation happen at scales close to the GUT scale and that the field displacement was superplanckian. In such situation a proper understanding of inflations’ embedding into a consistent quantum gravity theory, such as string theory is essential as discussed by McAllister in his review of inflation in string theory. (Picture taken from McAllister’s talk available at the event’s web page).*

String inflation

Finally, the current status of models of inflation in string theory was reviewed by Liam McAllister and Michele Cicoli. Cicoli presented the model of fibre inflation in the type IIB Large Volume Scenario, and how the moduli affect inflation and reheating in this model. This topic was also touched upon in McAllister's review talk. In more detail, McAllister argued that string theory provides a solid framework for addressing cosmological questions, and puzzles related to the dark sector. Conversely, these cosmological challenges provide important guidance in the study and development of string theory, highlighting, in particular, the need for controlled supersymmetry breaking. The talk included a discussion of the benefits and shortcomings of string models of inflation, which sparked off a discussion of how the recent landscape/swampland conjectures could be used to guide string phenomenologists to construct better models for inflation. This discussion continued throughout the workshop.

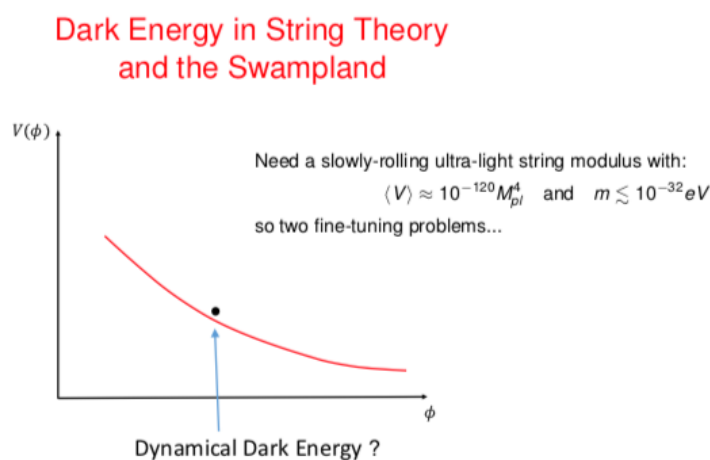


Figure 3: In the review by Parameswaran on dark energy models in string theory, she explained how quintessence [18] models in string theory need a slowly-rolling ultra-light string modulus with the correct vacuum energy and (very light) mass. Moreover, many of the same ingredients and challenges as in de Sitter constructions arise in this case as well. She then gave us an overview the recent swampland conjectures and string theory candidates for quintessence. (Picture taken from Parameswaran's talk available at the event's web page).

3.3 Formal aspects: geometry, duality and non-geometry

The last theme of the workshop revolved around geometrical and mathematical developments of relevance for compactifications of type II string theory, F-theory and the heterotic string.

F-theory

Progress in F-theory model building and toric geometry was reported in talks by Paul Oehlmann and Damian Mayorga. Mayorga discussed models with U(1) gauge group and charged matter, and their relation to type IIB compactifications via the Sen weak coupling limit. Oehlmann focused on F-theory on smooth torus fibered quotient Calabi-Yau threefolds which do not admit a section of the fibration. We learned about the features of these theories: how monodromy breaks the gauge group of the covering theory to a discrete remnant and in addition gauges the strongly coupled sector at the fixed points. This talk also included comments on realisations and bounds of higher order discrete symmetries within the context of the swampland program, which resonated well with the cosmological focus of the workshop.

Calabi–Yau geometry

There were two talks on Calabi–Yau geometry. Philip Candelas’ talk *Attractor points and the arithmetic of Calabi–Yau manifolds* discussed the arithmetic properties of Calabi–Yau spaces defined over finite fields, which reveals a deep relationship between number theory and properties of string vacua. On a more practical note, Candelas showed that these arithmetic methods can be used to determine topological quantities of Calabi–Yau manifolds, which are of relevance for model building. There was also a discussion of mirror symmetry by Nana Cabo-Bizet, who reported on the connection between mirror symmetry in non-complete intersection Calabi–Yau varieties and newly developed non-Abelian dualities in their gauged linear sigma model descriptions.

Double field theory and non-geometric fluxes

The talks by Erik Plauschinn and Eric Bergshoeff were also related to string duality. Bergshoeff showed how Double Field Theory can be used to analyse branes in string theory. We learned that the Wess–Zumino terms of different branes can be embedded within Double Field Theory. Crucial ingredients in the construction are the identification of the correct brane charge tensors and using the Double Field Theory potentials that arise from dualizing the standard Double Field Theory fields. This leads to a picture where under T-duality the brane does not change its world-volume directions but where instead it shows different faces depending on whether some of the world-volume and/or transverse directions invade the winding space, thus creating a non-geometric object. Further aspects of non-geometry were discussed by Plauschinn, in a brief review of compactifications of string theory with geometric and non-geometric fluxes. Such theories can be argued to give rise to four-dimensional effective field theories with non-trivial scalar potentials, which can be used for moduli stabilization and cosmology. The talk emphasises various consistency conditions, and reported on preliminary new results on the distribution of flux vacua. There were clear connections between these more formal talks, and, for example, Loaiza-Brito’s non-geometrical model building talk.

Machine learning in string theory

Finally, Fabian Ruehle presented very recent applications of machine learning techniques in string theory. It is well-known that string theory compactifications lead to a gigantic number of backgrounds, each of which comes with different implications for particle physics and cosmology. On top of this, every background has a huge number of possible vacua or near-vacua. Ruehle described the computational complexity of the challenges associated with both finding a viable background and finding vacua for this background, and how machine learning techniques may be used to analyse some of these problems. The methods were illustrated by a study of intersecting brane models in type IIA string theory, where it has been shown that a machine learning agent can develop genuinely new strategies for the construction of phenomenologically relevant vacua.

4 Scientific Progress

During the review talks (see figures 1-4), we were presented with the bigger picture and the status of the field of string cosmology, as well as the future prospects to make progress in both the formal as well as observational sides in the forthcoming years. The focus talks, on the other hand, were an excellent source for the dissemination of very recent (or unpublished) research results in the areas covered by the workshop. Since the workshop brought together two communities that do not often meet at regular conferences (namely cosmologists and string theorists), this mixture of overview and detail had the good result of informing both communities about these results.

The format of the workshop with a small number of participants, as well as the excellent professional, but still relaxed, environment, allowed for both formal and informal discussions. These discussions were often initiated during the talks, but they often continued throughout the day. Also the informal discussions included members from the two communities, inspiring new avenues for further developments in the subject of string cosmology. The workshop also allowed to establish and/or reinforce scientific collaborations among

de Sitter vacua in string theory where do we stand?

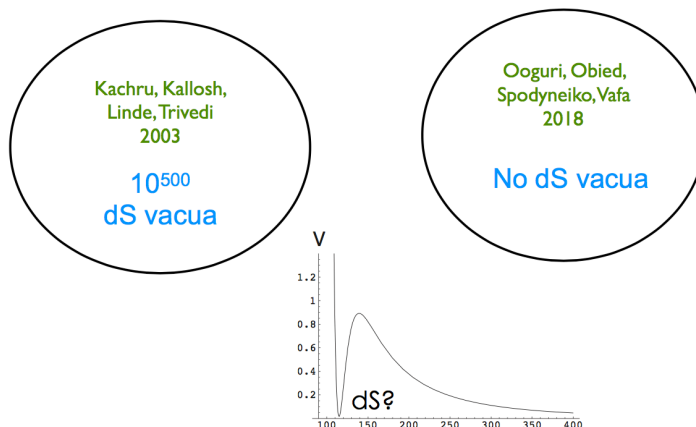


Figure 4: *The recent swampland conjectures have revived the long-standing debate about whether string theory supports a landscape of de Sitter vacua. Graña reviewed the KKLТ construction of de Sitter solutions, which is at the core of the landscape paradigm and showed the recent and not so recent ten-dimensional calculations pointing out problems in the string theory construction. (Picture taken from Graña’s talk available at the event’s web page).*

the participants, as indeed made clear in the testimonials of some of the participants. It is likely, that the outcome of these collaborations will result in future publications.

5 Outcome of the Meeting

In addition to the scientific development discussed above, an important objective of this workshop was to bring together international experts working in cosmology, supergravity and string theory to provide a solid mathematical and theoretical foundation for the description of the early Universe. The participants included scientists at all levels: research leaders in mathematical physics and theoretical cosmology, as well as more junior researchers in these fields. Participants came from all over the world with participants from Europe, Asia and the Americas. Mexico was naturally the country with most participants, and an important outcome of the workshop was an increased visibility of the Mexican theoretical physics community. Several of the overseas participants also took the opportunity to combine the workshop with other scientific activities in Mexico, which further strengthened this aspect.

The workshop schedule was organised in order to promote discussions, exchange of ideas and new collaborations. To cater for the broad research interests of the participants, there were one-hour survey talks presenting the status of the respective fields. This was complimented by 30-minutes focus talks on recent developments, and ample time for discussion. This organisation, in combination with the excellent CMO facilities with multiple discussion points, led to an exceptionally interactive workshop, where participants from different communities discussed throughout the day. Indeed, several participants commented on how successful this aspect of the workshop was.

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