Hamiltonian Systems and Celestial Mechanics (15w5010)
September 6-11, 2015 CMO-BIRS Workshop

MEALS

*Breakfast: 7:30 9:00 am, Restaurant Hotel Hacienda Los Laureles, Monday–Friday
*Lunch: 13:30 15:00 pm, Restaurant Hotel Hacienda Los Laureles, Monday–Friday
*Dinner: 19:00 21:00 pm, Restaurant Hotel Hacienda Los Laureles, Monday–Thursday
*Dinner: 19:30 22:00 pm, Restaurant Hotel Hacienda Los Laureles, Sunday only
*Continuous Coffee Breaks: Conference Room San Felipe, Hotel Hacienda Los Laureles

MEETING ROOMS

All lectures will be held in the Conference Room San Felipe at Hotel Hacienda Los Laureles. An LCD projector, laptop, document camera and blackboards are available for presentations.

SCHEDULE

Sunday

14:00	Check-in begins (front desk at your assigned hotel - open 24 hours)
19:30–22:00	Dinner, Restaurant Hotel Hacienda Los Laureles
20:30	Informal gathering Hotel Hacienda Los Laureles A welcome drink will be served by the hotel.

Monday

7:30–8:45	Breakfast
8:45–9:00	Introduction and Welcome
9:00–9:45	Y. Long
9:45–10:30	G. Contreras
10:30–11:00	Coffee Break
11:00–11:45	R. Montgomery
11:45–12:30	C. Stoica
12:30–13:00	Informal discussion
13:00–15:00	Lunch
15:00–15:45	E. Maderna
15:45–16:30	J. Muciño
16:30–17:00	Coffee Break
17:00–17:45	J. Galán-Vioque
17:45–18:30	C. García-Azpeitia
19:00-21:00	Dinner

Tuesday

7:30–8:45	Breakfast
9:00–9:45	D. Offin
9:45–10:30	A. Portaluri
10:30–11:00	Coffee Break
11:00–11:45	D. Schmidt
11:45–12:30	K. Meyer
12:30–13:15	L. Benet
13:00–15:00	Lunch
15:00–15:45	A. Ureña
15:45–16:30	S. Boatto
16:30–17:00	Coffee Break
17:00–17:45	J. Burgos
17:45–18:30	H. Sánchez
18:30–19:0	Informal discussion
19:00-21:00	Dinner

Wednesday

7:30–8:45	Breakfast
9:00–9:45	A. Delshams
9:45–10:30	T. Martínez-Seara
10:30–11:00	Coffee Break
11:00–11:45	M. Gidea
11:45–12:30	P. Roldán
12:30–13:00	Group Photo
13:00–15:00	Lunch
15:00–19:00	Free afternoon
19:00–21:00	Dinner

Thursday

7:30–8:45	Breakfast
9:00–9:45	M. Levi
9:45–10:30	R. Calleja
10:30–11:00	Coffee Break
11:00–11:45	Z. Xie
11:45–12:30	S. Rybicki
12:30–13:15	J. Palacián
13:00–15:00	Lunch
15:00–15:45	A. Bengochea
15:45–16:30	T. Schmah
16:30–17:00	Coffee Break
17:00–17:45	A. Hernández
17:45–18:30	J. A. Arredondo
18:30–19:00	Open problems, perspectives
19:00-21:00	Dinner

Friday

7:30–8:45	Breakfast
9:00–9:45	G. Pinzari
9:45–10:30	Z. Wang
10:30–11:00	Coffee Break
11:00–11:45	M. Corbera
11:45–12:30	C. Robinson
12:30–13:00	Open problems, perspectives. Closing

Checkout
(by 12 noon)

** 5-day workshop participants are welcome to use Hotel Hacienda Los Laureles facilities until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

Hamiltonian Systems and Celestial Mechanics (15w5010)
September 6-11, 2015

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Arredondo, John** (Universidad Konrad Lorenz, Colombia)
Title: *Some Aspects in Symplectic Integrators*
Abstract: In this talk we present a summary in the principal aspects of symplectic integrators, the geometric frame behind this objects and some of the principal numerical integrators developed in the last year. As a new ingredient, we present a new kind of integrator developed by Hugo Jimenez, and examples of one degree Hamiltonians that we are investigating in a joint work with Hugo Jiménez.

Speaker: **Benet, Luis** (Instituto de Física, UNAM-Cuernavaca, México)
Title: *A simple model for the location of Saturn's F ring*
Abstract: In this talk I will discuss a simplified model to understand the location of Saturn's F ring and address some of its structural properties. The model is a planar restricted five-body problem defined by the gravitational field produced by Saturn, including its second zonal harmonic J_2, the shepherd moons Prometheus and Pandora, Titan, and a ring particle, and considers an ensemble of non-interacting ring particles. We compute precise long-time numerical integrations for ring particles initially located in the region between the orbits of Prometheus and Pandora, and address whether the test particles escape or remain trapped in this region. We obtain a wide region of initial conditions of the test particles that remain confined. We define a stability indicator based on a frequency-like analysis. Using this indicator in the set of trapped initial conditions, we select the subset of the most stable test particles, and obtain a narrow eccentric ring with sharp edges. Comparison of our results with the nominal location of the F ring shows a remarkable agreement.

This is joint work with Angel Jorba.

Speaker: **Bengochea, Abimael** (UAM-Iztapalapa, México)
Title: *Exchange orbits for the 1+4 body problem*
Abstract: We introduce exchange orbits in the 5 body problem with four equal masses. This what done in analogy with the horseshoe orbits of the three-body problem, for instance the Saturn-Janus-Epimetheus system. We determine numerically, by means of solving a boundary value problem with the software AUTO, a mono-parametric family of exchange orbits, for a mass ratio equals $3.5\times^{-4}$, which is 10^5 times the Janus-Saturn mass ratio. We focus on the most symmetric orbits with the aim of reduce the dimension of the phase space.

Speaker: **Boatto, Stefanella** (Universidade Federal de Rio de Janeiro, Brazil)
Title: *N-body dynamics on surfaces of revolution and the axioms of mechanics*
Abstract: One of the today's challenges is the formulation of the N-body and N-vortex dynamics on Riemann surfaces. In this talk we show how the two problems are strongly related one another when looking at them from the point of view of the intrinsic geometry of the surface where the dynamics takes place. Given a surface M of metric g, the distribution of matter S on M, we deduce the dynamics of the masses and some of its properties. Among other things, we find that in the plane the two masses problem does not obeys to the known Kepler laws. Moreover, Newton's Laws are not longer verified on closed surfaces with variable curvature.

Speaker: **Burgos, Jaime** (Instituto Tecnológico Autónomo de México (ITAM), México)
Title: *A mechanism of diffusion in the elliptic Hill's four body problem*

Abstract: The Hill's four body problem was introduced as an alternative approach to study the dynamics of a massless particle under the gravitational force of a small body, e.g., an asteroid, and the gravitational influence of two massive and distant bodies, e.g., a planet and the sun, the three main bodies are in an equilateral configuration which is a solution of the general three body problem. The Hamiltonian of this system can be written as periodic perturbation of a system that possesses a normal hyperbolic invariant manifold and transversal intersections between its invariant manifolds, in such a case, the parameter of the perturbation is the eccentricity of the orbits of the main bodies. In this talk we will show the dynamics of the unperturbed system and some heuristics regarding the mechanism of diffusion for this problem that is a joint work with Marian Gidea.

Speaker: **Calleja, Renato** (IIMAS, UNAM, México)
Title: *Domains of analyticity of KAM tori in mechanical systems with friction*
Abstract: Many problems in Celestial Mechanics are described by conformally symplectic systems (e.g. mechanical systems with a friction proportional to the velocity). I will present a study of the limit of small dissipation in conformally symplectic systems. The Lindstedt series of the parametrization of quasi-periodic orbits with Diophantine frequency depend on a small dissipative parameter ϵ for which $\epsilon = 0$ corresponds to the symplectic case. We study the domains of analyticity of the parametrization of invariant tori close to the $\epsilon = 0$ limit. This is joint work with Alessandra Celletti and Rafael de la Llave.

Speaker: **Contreras, Gonzalo** (CIMAT, México)
Title: *Generic Mañé Sets.*
Abstract: We prove that for any compact surface M and any Tonelli lagrangian L(x,v) on TM there is an open and dense set of C2 real functions f on M such that the lagrangian L(x,v) + f(x) has a unique minimizing measure and it is supported on a hyperbolic periodic orbit.

Speaker: **Corbera, Montserrat** (Universitat de Vic, Barcelona, España)
Title: *Central configurations of the spatial* 5*–body problem with four equal masses*
Abstract:We analyze the families of central configurations of the spatial 5–body problem with four equal masses for arbitrary values of the fifth mass m coming from central configurations of the $(4 + 1)$–body problem with four equal masses and a fifth infinitesimal mass. First we continue numerically, taking m as a parameter, the central configurations of the spatial $(4 + 1)$–body problem with four equal masses and $m = 0$ to the spatial 5–body problem with five equal masses (i.e. $m = 1$) and we find the bifurcation values of m where the number of central configurations changes. Then we revisit the work of M. Alvarez et. al

in which the central configurations of the spatial 5–body problem with equal masses are continued to the spatial $(1 + 4)$–body problem with four equal infinitesimal masses and we complete the study of the bifurcation values. This last study will provide the families of central configurations of the 5–body problem with four equal masses when $m > 1$.

This is a joint work with Martha Alvarez–Ramírez (UAM-Iztapalapa) and Jaume Llibre (UAB).

Reference

M. Alvarez–Ramírez, J. Delgado, and J. Llibre, *On the spatial central configurations of the 5–body problem and their bifurcations*, Discrete Contin. Dyn. Syst. Ser. S **1**, 505–518 (2008).

Speaker: **Delshams, Amadeu** (Universidad Politécnica de Cataluña, Spain)
Title: *Global Instability through non-transverse heteroclinic chains, with an application to the periodic cubic defocusing NLS equation*
Abstract: We introduce a new mechanism for global instability in dynamical systems, based on the shadowing of a sequence of invariant tori connected along non-transverse heteroclinic orbits, under some geometric restrictions. This mechanism can be readily applied to systems of large dimensions, like infinite-dimensional Hamiltonian systems, particularly the periodic cubic defocusing nonlinear Schrödinger (NLS) equation.

This is a joint work with Adrià Simon and Piotr Zgliczyński.

Speaker: **Galán-Vioque, Jorge** (Universidad de Sevilla, Spain)
Title: *Continuation of periodic orbits in the three body problem*
Abstract: We present continuation results for periodic solutions in two paradigmatic examples of three body problem: the symmetric solutions of the Sitnikov problem as the eccentricity of the elliptical solutions of the primaries is changed and the figure 8 solution and its close relatives for three equal masses as one of the masses varies. We concentrate on stabilities, variational aspects and global connections as the parameters are varied.

In collaboration with Abimael Bengochea (UAM-I), Ernesto Pérez-Chavela (ITAM) and D. Nuñez and A. Rivera (Javierana Universidad de Cali, Colombia).

Speaker: **García-Azpeitia, Carlos** (Fac. de Ciencias, UNAM, México)
Title: *Global bifurcation in the (n+1)-body problem*
Abstract: We investigate the bifurcation of periodic solutions from the Maxwell relative equilibrium, which consists of n bodies at the vertices of a polygon turning around a massive body at the center. The global bifurcation of periodic solutions is proved using equivariant degree theory. The bifurcating branches have symmetries that let us establish connections with other particular solutions of the (n+1)-body problem known as choreographies and Hip-Hop.

Speaker: **Gidea, Marian** (Yeshiva University, USA)
Title: *A geometric mechanism for Arnold diffusion in the a priori stable case*
Abstract: : We prove the existence of diffusion orbits drifting along heteroclinic chains of normally hyperbolic 3-dimensional cylinders, under suitable assumptions on the dynamics on the cylinders and on their homoclinic/heteroclinic connections. These assumptions are satisfied in the a priori stable case of the Arnold diffusion problem. We provide a geometric argument that extends Birkhoff's procedure for constructing connecting orbits inside a zone of instability for a twist map on the annuls. This is joint work with J.-P. Marco.

Speaker: **Hernández, Antonio** (UAM-Iztapalapa, México)
Title: *Kite configurations in the four body problem*
Abstract: The kite problem is a four-body problem, consisting of a pair of masses on the perpendicular bisector of the segment joining two equal masses. I will present some advances related to the study of the stability of relative equilibria in the kite problem using slice coordinates.

Speaker: **Levi, Mark** (Pennsylvania State University, USA)
Title: *Traveling waves and equilibrium states in lattices, and particles in magnetic fields.*
Abstract: I will discuss some results, old and new, on dynamics and statics of chains of coupled particles in periodic potentials (such as the Frenkel-Kontorova model), and will pose some open questions.

Speaker: **Long, Yiming** (Chern Institute of Mathematics, Nankai University)
Title: *Periodic solutions of Hamiltonian systems with prescribed energies.*
Abstract: Studies about periodic solutions of Hamiltonian systems on given energy hypersurfaces in R^{2n} can be traced back to Lyapunov. The global study on such solution orbits started from the famous works of Rabinowitz and Weinstein in 1978, and so far many interesting and deep results on this topic have appeared. In this talk, I shall give a brief survey about the existence, multiplicity and stability problem on such solution orbits obtained in recent years and briefly introduce some ideas in these studies.

Speaker: **Maderna, Ezequiel** (Centro de Matemática, Uruguay)
Title: *Generic uniqueness of the minimal moulton central configuration*
Abstract: We will prove that for generic (open and dense) values of the masses, the Newtonian potential function of the collinear N-body problem has exactly N!/2 critical values when restricted to a fixed inertia level. In particular, we prove that for generic values of the masses, there is only one global minimal Moulton

configuration. One step of the proof will consist in a slightly different version of the well-known proof of Moulton's theorem using the Gershgorin circle theorem. Finally, we will discuss the importance of this question in the non collinear wider context.

Speaker: **Martínez-Seara, Tere** (Universidad Politécnica de Cataluña, Spain)
Title: *Oscillatory orbits in the restricted planar elliptic three body problem*
Abstract: The restricted planar elliptic three body problem models the motion of a massless body under the Newtonian gravitational force of two bodies evolving in Keplerian ellipses.

In 1922 Chazy gave a complete classification of all possible final motions that the body $q(t)$ can approach. In the restricted three body problem, the possible final states are reduced to four:

- Hyperbolic: $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to c > 0$ as $t \to \pm\infty$.
- Parabolic: $\|q(t)\| \to \infty$ and $\|\dot{q}(t)\| \to 0$ as $t \to \pm\infty$.
- Bounded: $\limsup_{t\to\pm\infty} \|q\| < +\infty$.
- Oscillatory: $\limsup_{t\to\pm\infty} \|q\| = +\infty$ and $\liminf_{t\to\pm\infty} \|q\| < +\infty$.

Examples of all these types of motion, except the oscillatory ones, were already known by Chazy.

Using a previous results by the authors, in this talk we prove the existence of oscillatory motions for any value of the masses of the primaries assuming they move in ellipses with small enough eccentricity.

The key idea is to proof that the so called invariant manifolds of infinity intersect transversaly because this allows us to define and study a scattering map which gives the existence of transition chains of periodic orbits. A suitable shadowing lemma for parabolic periodic orbits provides the existence of oscillatory motions.

This is a joint work with M. Guàrdia and P. Martín.

Speaker: **Meyer, Kenneth** (Department of Mathematical Sciences, University of Cincinnati, USA)
Title: *Remarks on Bifurcation and Stability in Limiting Cases*
Abstract: Over the years I have encountered the first elliptic function known as the *sin lemniscate* function several times. Sometime it is uncovered after a series of transformations and other times it just pops up. I will discuss my encounters with it in the stability of a nonlinear oscillator, bifurcation and stability of fixed points of an area preserving map, and the stability and evolution of $\mathcal{L}_4$ in the restricted problem.

Speaker: **Montgomery, Richard** (University of California at Santa Cruz, USA)
Title: *Can the "12" syzygy class be realized?*
Abstract: Question: can this simplest syzygy sequence, the sequence of a "tight binary", be realized as the primitive sequence of a reduced-periodic solution to the Newtonian zero-angular momentum equal mass planar three-body problem? I will explain terms. I will motivate the question. Motivation includes (A) my recent work with Rick Moeckel where we proved every (reduced) syzygy sequence is realized by a (relatively) periodic solution in the equal (or near-equal mass) case, under the hypothesis that the angular momentum be non-zero (but arbitrarily small), (B) variational reasoning which holds only in the zero-angular momentum case, and (C) common sense in the face of repeated failures. I will provide partial evidence for a "no" answer.

Speaker: **Muciño Raymundo, Jesús** (Centro de Ciencias Matemáticas, UNAM, México)
Title: *Lie commutativity of polynomial vector fields and configurations of zeros*
Abstract: Let X, Y be polynomial vector fields on the plane having degree at most d. We study the relation between; the integrability condition $[X, Y] \equiv 0$, and the configurations of points $zeros(X)$, $zeros(Y)$. Joint work with J. A. Arredondo.

Speaker: **Offin, Dan** (Queens University, Canada)
Title: *Maslov index and some questions of dynamic stability*

Abstract: Several examples from stability theory of dynamical systems, have common symplectic and geometric features connected with the calculation and interpretation of the Maslov index. We will consider two well known global families of periodic orbits in Hamiltonian dynamics, and use an interpretation of the Maslov index to discuss the nature of the stability type of these families. The family of homographic periodic solutions in the parallelogram four body problem which keep a rhombus configuration, are shown to be minimum distance lines after reduction by a rotational symmetry. The instability of this family is thereby expressed in geometric terms. We consider the notion of minimum distance lines, in Jacobi metric for Hamiltonian systems of kinetic plus potential type. All such examples are generically hyperbolic on their energy surface. Other well known examples into this category. Finally we consider an interesting family of mountain pass critical (not minimum distance) curves in Henon-Hieles Hamiltonian, with two degrees of freedom. We discuss the evolution of wave front sets along brake orbits in this example, and relate the singularities of these wavefront sets, to the determination of hyperboloc stability type for an important family of mountain pass lines.

Speaker: **Palacián, Jesús** (Universidad Pública de Navarra, Spain)
Title: *Singular Reduction in Resonant Hamiltonian Systems with N Degrees of Freedom*
Abstract: We deal with the problem of analysing Hamiltonian systems from the point of view of establishing the existence, stability and bifurcations of periodic solutions as well as invariant tori. Our purpose is to illustrate the use of singular reduction on this problem. Singular reduction lowers the dimension of the problem under study; so, given that our first test problem is a two-degree-of-freedom Hamiltonian system in $\mathbf{R}^4$, it will be reduced to a Hamiltonian system of one degree of freedom on a two-dimensional surface called orbifold. The planar restricted three-body problem is considered as a benchmark for the last 85 years, and in particular there has been a bunch of works to obtain periodic solutions and related invariant sets around the equilibrium points L_4 and L_5. We shall illustrate how reduction theory is used in this context to establish the existence of these solutions rigorously.

Then we shall handle the n degrees of freedom case where the polynomial invariants needed in the reduction theory are determined using an algorithm based on integer programming to obtain a Hilbert basis for a given resonance. The cardinality of the Hilbert basis is not known a priori but it is lower-bounded by n^2. After computing the Hilbert basis we use Gröbner bases and the division algorithm for multivariate polynomials to deal with the equations of motion in terms of the invariants. Besides we build the orbifold from the invariants and the constraints among them. Our aim is to reconstruct the periodic solutions of the full system from the critical points in the orbifold. To achieve it we use local symplectic coordinates around these points. We apply the theory in order to find some periodic solutions in resonant Hamiltonian systems of n degrees of freedom with semisimple linear part.

This is a joint work with Ken R. Meyer and Patricia Yanguas.

Speaker: **Pinzari, Gabriella** (University of Roma 3, Italy)
Title: *Perihelia reduction for the planetary problem, with applications*
Abstract: In the XIX century, Jacobi discovered that the motions of three bodies in gravitational interaction may be described by a system of the eighth order, rather than eighteenth. Some degrees of freedom might be neglected by the invariance of the problem by translations and rotations. Next, Radau wrote these equations in Hamiltonian form: he introduced a Hamilton function and four couples of canonical coordinates. The reduction of order by Jacobi and Radau has been extensively applied in the literature, and, for about one century and one half, it appeared as the only available one. In 1963, V.I.Arnold wrote that the lack of a generalization of it to the case of more bodies was an obstacle to the extension of his "Theorem of stability of planetary motions". In 1982-1983 F. Boigey and A. Deprit extended Jacobi-Radau's reduction to the general case. The coordinates by Boigey and Deprit were next rediscovered by the author and applied to the problem, allowing for a direct proof of Arnold's statement. Important feature of Jacobi-Radau-Boigey-Deprit reduction are: (i) they are not defined for the problem constrained in the plane and (ii) no symmetry in the Hamiltonian appears, relatively to the invariance of the problem by reflections. We shall present an alternative reduction based on the perihelia of instantaneous orbits that

takes into account items (i) and (ii). Next, we shall show how this new set of coordinates can be used to obtain a more global formulation of Arnold's statement and how we project to use them to in order to study the stability of the planetary problem.

Speaker: **Portaluri, Alessandro** (University of Torino, Italy)
Title: *An index theory for colliding motions in Celestial Mechanics*
Abstract: In the last decades a zoo of new symmetric periodic orbits for the n-body problem appeared in the literature as critical points of the Lagrangian action functional. However in order to penetrate the intricate dynamics of this problem a key role is represented by the collision orbits. Certainly for a deeper investigation of the dynamics of this singular problem one of the most important feature of such orbits is the knowledge of their Morse index main related to the role played in the study of the linear (in)stability properties. An essential tool for computing this topological invariant is an index theorem based on a spectral flow formula and on a refined computation of a Maslov-type index. In this talk, after a presentation of a new variational regularisation of the Lagrangian action functional, we show how to construct a suitable index theory for an important class of colliding trajectories. This is a joint work with, V. Barutello, X. Hu and S. Terracini

Speaker: **Robinson, Clark** (Department of Mathematics, Northwestern University, USA)
Title: *Reparametrization in Flow Equivalence*
Abstract: In the statement and proof of structural stability of flows with hyperbolicity and transversality, a reparametrization is needed away from the fixed point and is not needed near the fixed points. One of the technical differences from the case for diffeomorphisms is the need to make a transition between these two situations. The Pugh-Shub 'linearization' of a flow near a normally hyperbolic submanifold did not need to use a reparametrization because the trajectories come in-phase to those on the submanifold. For orbits near the partially parabolic three body problem (the separation of one body from the other two is parabolic), the extended flow has a weakly hyperbolic three sphere of orbits with infinite separation. Easton-Robinson were still able to proof the existence of a manifold of parabolic orbits. These orbits do not come in-phase to those on the invariant submanifold. In order to define a decoupled flow near the partially parabolic orbits, a reparametrization is needed. This reparametrization is also needed to obtain the conjugacy of the decoupled flow to the original motion.

Speaker: **Roldán, Pablo** (Instituto Tecnológico Autónomo de México)
Title: *Arnold's mechanism of diffusion in the spatial circular restricted three-body problem*
Abstract: We consider the spatial circular restricted three-body problem, on the motion of an infinitesimal body in the Sun–Earth system. This can be described by a three-degree of freedom Hamiltonian system. We fix an energy level close to that of the collinear libration point L_1 located between Sun and Earth. Near L_1 there exists a normally hyperbolic invariant manifold, diffeomorphic to a three-sphere. Trajectories lying on this three-sphere are characterized by an out-of-plane amplitude that can vary only slightly. Nevertheless, we show that we can obtain trajectories whose out-of-plane amplitude changes significantly by alternatively following trajectories of the flow restricted to the three-sphere and homoclinic trajectories that turn around the Earth. We provide an abstract theorem for the existence of diffusing trajectories, and numerical evidence that the premises of the theorem are satisfied in the three-body problem considered here. The geometric mechanism underlying this construction is reminiscent of the Arnold diffusion problem for Hamiltonian systems. Our arguments, however, do not involve transition chains of tori as in the classical example of Arnold. We exploit mostly the 'outer dynamics' along homoclinic trajectories, and use very little information on the 'inner dynamics' restricted to the three-sphere. We explore a few energy levels and estimate the largest orbital inclination that can be achieved with our construction.

Speaker: **Rybicki, Slawek** (Nicolaus Copernicus University. Torun, Poland)
Title: *Symmetric Liapunov center theorem*
Abstract: The aim of may talk is to present a sketch of prove of the Liapunov center theorem for symmetric

potentials. Let $\Omega \subset \mathbb{R}^n$ be an open and invariant subset of an orthogonal representation $\mathbb{R}^n$ of a compact Lie group Γ with free Γ-action. If $\Gamma(q_0) \subset \Omega \cap (\nabla U)^{-1}(0)$ is a non-degenerate (i.e. $\dim \nabla^2 U(q_0) = \dim \Gamma(q_0)$) orbit of critical points of Γ-invariant C^2-potential $U : \Omega \to \mathbb{R}$ and there is at least one positive eigenvalue of the Hessian $\nabla^2 U(q_0)$ then in any neighborhood of the orbit $\Gamma(q_0)$ there is a periodic orbit of solutions of equation $\ddot{q}(t) = -\nabla U(q(t))$. Moreover, we estimate the minimal period of these solutions. The basic idea of the proof is to apply the infinite-dimensional version of the equivariant Conley index theory.

Speaker: **Sánchez, Hector** (Instituto de Matemáticas, UNAM, México)
Title: *Solutions of the Hamilton Jacobi equation for the N body problem*
Abstract: We study viscosity solutions of the Hamilton Jacobi equation, more precisely Busemann functions defined by the parabolic homotetic motion for a minimal central configuration.

Speaker: **Schmah, Tanya** (University of Toronto, Canada)
Title: *Controlling rigid body attitude via shape change*
Abstract: Satellite attitude control is typically achieved via reaction wheels (i.e. rotors) or magnets, which leave the moment of inertia fixed. We investigate an alternative control mechanism: sliding point masses, which change the moment of inertia and thus the angular velocity, while leaving angular momentum fixed.
Joint work with Cristina Stoica

Speaker: **Schmidt, Dieter** (University of Cincinnati, USA)
Title: *Nonlinear Stability of Stationary Points in the Problem of Robe*
Abstract: In 1977 Robe considered a modification of the Restricted Three Body Problem, where one of the primaries is a shell filled with an incompressible liquid. The motion of the small body of negligible mass takes place inside this sphere and is therefore affected by the buoyancy force of the liquid. Robe suggested that "Such a model may be useful to study the small oscillations of the Earth's inner core taking into account the Moon attraction." The physical significance of the problem might be overstated. On the other hand it provides an interesting mathematical problem, since there are now two parameters, which come into play when studying the stability of the equilibrium points. In recent years the problem has again received some attention and different modifications have been proposed. Robe himself had considered the case where the smaller primary moves on an elliptic orbit around the shell, but we will restrict ourselves to the planar circular case. We investigate the existence and stability of the equilibrium points and discuss the range of the parameters for which the problem has a physical meaning, something often ignored by others. Our main contribution is to establish the Lyapunov stability of these equilibrium points. We achieve this by putting the Hamiltonian function of Robe's problem into its normal form and then use the theorems of Arnol'd, Markeev and Sokol'skii. As usual a few resonance cases and some exceptional cases will require special treatment and it is included in our discussion.

Speaker: **Stoica, Cristina** (Wilfred Laurier University, Waterloo, Canada)
Title: *About symmetry and point dynamics on a sphere*
Abstract: In the presence of symmetry, reduction methods contribute greatly to the understanding of dynamics. In this talk I will review some of these methods as applied to point dynamics on a sphere.

Specifically, I will apply the energy momentum method to the analysis of stability of relative equilibria of equally weighted points in gravitational interaction and amend some established results. I will also discuss the use of discrete symmetries and representation theory in determining invariant manifolds and their bifurcation and stability. As applications, I will present some findings related to homographic motions and (relative) equilibria.

Speaker: **Ureña, Antonio** (Universidad de Granada, Spain)
Title: *The higher-dimensional Poincaré-Birkhoff theorem for Hamiltonial systems*
Abstract: We propose a higher dimensional generalization of the Poincaré-Birkhoff Theorem which applies to Poincaré time maps of Hamiltonian systems. The maps under consideration are neither required to

be close to the identity nor to have a monotone twist. The annulus is replaced by the product of an N-dimensional torus and the interior of an embedded sphere in the N-dimensional euclidean space; on the other hand, the classical boundary twist condition is replaced by an avoiding rays condition. This is a joint work with Alessandro Fonda (University of Trieste).

Speaker: **Wang, Zhiqiang** (Chongqing University, China)
Title: *Periodic Solutions for Newtonian n-Body Problems with Dihedral Group Symmetry and Topological Constraints*
Abstract: In this talk, we prove the existence of a family of new non-collision periodic solutions for the classical Newtonian n-body problems. In our assumption, the $n = 2l \geq 4$ particles are invariant under the dihedral rotation group D_l in $\mathbb{R}^3$ such that, at each instant, the n particles form two twisted l-regular polygons. Our approach is variational minimizing method and we show that the minimizers are collision-free by level estimates and local deformations.

Speaker: **Xie, Zhifu** (Virginia State University, USA)
Title: *Variational method with SPBC and the existence of Henon solutions of three-body problem*
Abstract: Henon numerically found a one-parameter family of periodic orbits in a rotating system with a constant angular velocity in the planar problem of three bodies with equal masses. But there is no existence proof of such solutions by variational approach. In this talk, we will discuss how to use the variational method with SPBC to numerically find some of Henons solutions and to theoretically prove the existence of those solutions. In order to obtain the family of solutions, we choose appropriate structural prescribed boundary conditions (SPBC) in the variational frame. Numerical simulations will also be presented.