Orthogonal and Multiple Orthogonal Polynomials

August 9–13, 2015

Schedule of Presentations

	Monday	**Tuesday**	**Wednesday**	**Thursday**
9:00 – 9:40	Kuiljaars	G-López	Grunbaum	Ismail
9:45 – 10:25	Miki	A-López	Iliev	Stylianopoulos
10:25 – 11:00	Break	Break	Break	Break
11:00 – 11:40	Van Assche	Lubinsky	Vinet	Baratchart
11:45 – 12:25	Arvesu	Derevyagin	Lapointe	Atakishiyev
12:25 – 2:00	Lunch	Lunch	Lunch	Lunch
2:00 – 2:40	Berg	Wang	Roman	Geronimo
2:45 – 3:25	Aptekarev	Duits	van Diejen	Henegan
3:25 – 4:00	Break	Break	Break	Break
4:00 – 4:40	Simanek	De-la-Inglesia		Garza
4:45 – 5:25	Wolfe	Duran		Kozlowska (4:45–5:05)

Abstracts

Approximation of Algebraic Functions by Rational Functions

Alexander I. Aptekarev (Keldysh Institute Applied Mathematics)

Let f be a germ of an analytic function with finite number of branch points in the complex plane. For example, f be a power series of an algebraic function. We discuss properties of rational approximants of function f. Convergents of a continued fraction for f (other name is Pade approximants) serves as the rational approximants. There is an important relation between the maximal domain, where f has a single-valued branch and the domain of convergence of the rational approximants for f. Indeed, the approximants, which are rational functions and thus single-valued, approximate a holomorphic branch of f in the domain of their convergence. At the same time most of their poles tend to the boundary of the domain of convergence and the support of their limiting distribution models the system of cuts that makes the function f single-valued. J. Nuttall has conjectured that this system of cuts has minimal logarithmic capacity among all other systems converting the function f to a single-valued branch. The complete proof of Nuttall's conjecture was obtained by H. Stahl. In our joint work with Maxim Yattselev [1], we derived strong or Bernshtein-Szego type) asymptotics for the denominators of the rational approximants for this problem. We discuss various applications of this result: a functional analog of Thue-Siegel-Roth theorem, classification of spurious or wandering poles.

[1] A. I. Aptekarev and M. L. Yattselev, Pade approximants for functions with branch points – strong asymptotics of Nuttall-Stahl polynomials, 1–47, (submitted) ArXiv 1109.0332

nth Root Asymptotics for Multiple Meixner Polynomials

Jorge Arvesu Carballo (Universidad Carlos III de Madrid)

The nth root asymptotic behavior of multiple Meixner polynomials is presented. Two main ingredients of the proposed approach for the study of the aforementioned asymptotic behavior are used and discussed; namely, an algebraic function formulation for the solution of the equilibrium problem with constraint to describe their zero distribution and the limiting behavior of the coefficients of the recurrence relations.

On a Discrete Number Operator and its Eigenvectors Associated with the 5D Discrete Fourier Transform

Natig M. Atakishiyev (U.N.A.M.)

We construct an explicit form of a dirence analogue of the quantum number operator in terms of the raising and lowering operators that govern eigenvectors of the 5D discrete (finite) Fourier transform. Eigenvalues of this dirence operator are represented by distinct nonnegative numbers so that it can be used to sistematically classify, in complete analogy with the case of the continuous classical Fourier transform, eigenvectors of the 5D discrete Fourier transform, thus resolving the ambiguity caused by the well-known degeneracy of the eigenvalues of the discrete Fourier transform.

Joint work with M. K. Atakishiyeva and J. Méndez Franco.

Exterior Asymptotics of weighted Bergman Polynomials

Laurent Baratchart (INRIA Sophia Antipolis)

For Ω the unit disk and $w \in L^1(\Omega)$ a non-negative weight, we consider the orthonormal polynomials P_n in $L^2(\Omega, w)$. We prove exterior asymptotics for P_n under mild regularity assumptions for the weight. Namely, it should have radial limit in $L^p(\partial\Omega$ for some $p > 1$ and its $\log\log^+$ should satisfy a Hardy condition of order 1. We discuss generalization to analytic simply connected domains, and also cases where the assumptions do not hold and the result fails. This generalizes some of the results by Korovkin, Suetin, Mina-Diaz and Simanek. Part of the talk is joint work with N. Stylianopoulos.

Gegenbauer Polynomials and Positive Definiteness

Christian Berg (University of Copenhagen)

We recall that the Gegenbauer polynomials $C_n^{(\lambda)}$ are given by the generating function ($\lambda > 0$ with modifications if $\lambda = 0$)

$$(1 - 2xr + r^2)^{-\lambda} = \sum_{n=0}^{\infty} C_n^{(\lambda)}(x) r^n, \quad |r| < 1, x \in \mathbb{C}$$

and we define the normalized ultraspherical polynomials by

$$c_n(d, x) = C_n^{(\lambda)}(x)/C_n^{(\lambda)}(1), \quad \text{for } \lambda = (d-1)/2, \quad d = 1, 2, \ldots$$

In a famous paper from 1942 Schoenberg characterized the functions

$$f(x) = \sum_{n=0}^{\infty} b_{n,d} c_n(d, x), \quad b_{n,d} \geq 0, \quad \sum_{n=0}^{\infty} b_{n,d} < \infty$$

$$[f(\xi_k \cdot \xi_l)]_{k,l=1}^{n}$$

is positive semidefinite. In the talk I shall report on a recent paper [1] which contains a far reaching generalization of Schoenberg's result involving arbitrary locally compact groups G (written multiplicatively and with neutral element e). The class denoted $\Psi(\mathbb{S}^d, G)$ consists of the continuous functions $f : [-1, 1] \times G \to \mathbb{C}$ which are positive definite in the sense that for any $n \in \mathbb{N}$ and any $(\xi_1, u_1), \ldots, (\xi_n, u_n) \in \mathbb{S}^d \times G$ the $n \times n$-matrix

$$\left[f(\xi_k \cdot \xi_l, u_k^{-1} u_l)\right]_{k,l=1}^{n}$$

is hermitian and positive.

These functions are precisely those with expansions

$$f(x, u) = \sum_{n=0}^{\infty} \varphi_{n,d}(u) c_n(d, x),$$

where $\varphi_{n,d}$ is a sequence of continuous positive definite functions on G satisfying $\sum_{n=0}^{\infty} \varphi_{n,d}(e) < \infty$. The result has applications in geostatistics in the special case where the group G is the real line because it characterizes completely space-time isotropic and stationary covariance functions of Gaussian fields $Z(\xi, u)$ defined on $\mathbb{S}^d \times \mathbb{R}$.

[1] C. Berg and E. Porcu, *From Schoenberg coefficients to Schoenberg functions.* ArXiv1505.05682

On a Higher-Order Analogue of the Discrete Time Toda Equation

Maxim Derevyagin (University of Mississippi)

Nowadays, it is well understood that the discrete time Toda equation appears as a relation between Hankel determinants associated with a measure that generates a normal Pade table. In my talk, I am going to present a generalization of the discrete time Toda equation to a higher dimension. This generalization is based on the theory of multiple orthogonal polynomials and the associated discrete integrable systems.

Orthogonal Polynomials, Non-Colliding Processes and the Gaussian Free Field

Maurice Duits (Royal Inst of Technology)

This talk will address the global multi-time fluctuations of certain non-colliding process. First, a standard construction in integrable probability will be recalled how to define such non-colliding processes starting from a single Markov process associated to orthogonal polynomials. We will then show how the global fluctuations can be analyzed by using the recurrence coefficients for the polynomials and the eigenvalues of the generator. The main result is that the fluctuations are governed by the Gaussian Free Field.

Some Conjectures on Wronskian Determinants of Orthogonal Polynomials

Antonio Duran (Universidad de Sevilla)

In this talk we consider some conjectures on regularity properties for the zeros of Wronskian determinants whose entries are orthogonal polynomials. These determinants are formed by choosing orthogonal polynomials whose degrees run on a finite set F of nonnegative integers. The case when F is formed by consecutive integers was studied by S. Karlin and G. Szegö in 1961.

On a Matrix Approach for Semiclassical Orthogonal Polynomials

Luis E. Garza Gaona (Universidad de Colima)

I will show some recent results on characterizations of semiclassical orthogonal polynomials based on lower semi-infinite matrices containing the coefficients of the polynomials. Some matrix relations involving the Jacobi matrices associated with such sequences of orthogonal polynomials will be discussed.

The Fourier continuation method and strong asymptotics of a class of discrete orthogonal polynomials on the unit circle

J. Geronimo (Georgia Tech)

The Fourier continuation method is a method used in numerical analysis to extend a discrete set of interpolation points to a periodic function which can be analyzed using Fourier techniques. Useful in this technique are sequences of orthogonal polynomials on discrete sets where the sets in the limit of an infinite number of interpolation points become an arc of the circle. We discuss the Fourier continuation method and the strong asymptotics of associated orthgonal polynomials using the Riemann-Hilbert technique.

Joint work with K. Liechty

The Bispectral Problem in a Matrix Valued Setup

Alberto Grunbaum (University of California-Berkeley)

I plan to discuss a general matrix valued formulation of the bispectral problem, show some good examples, and then returnto the origin of this story: the time-band limiting questionof Shannon, Slepian, Landau and Pollack at Bell Labs circa 1960in a few matrix valued cases.

Asymptotics of Polynomials Orthogonal Over Multiply Connected Domains

James Henegan (University of Mississippi)

We shall present some new results on the asymptotic behavior of polynomials that are orthogonal with respect to area measure over a bounded domain that is conformally equivalent to a disk with finitely many subdisks removed. The work is in collaboration with Erwin Miña-Díaz.

Krall-Hahn Orthogonal Polynomials

Manuel Domínguez de la Iglesia (Universidad Nactional Autónoma de México)

From the classical discrete family of Hahn polynomials $h_n^{a,b,N}$ and an arbitrary set of polynomials Y_j we construct a new family of polynomials q_n using certain Casorati determinant. Under some conditions these polynomials q_n will be eigenfunctions of a higuer-order difference operator. Finally we explain how to choose appropriately the arbitrary polynomials Y_j in terms of dual Hahn polynomials, such that the polynomials q_n are also orthogonal with respect to a measure. Therefore they will be bispectral. This is joint work with A. Duran

Connection Coefficients for Classical Orthogonal Polynomials of Several Variables

Plamen Iliev (Georgia Institute of Technology)

Connection coefficients between two bases of orthonormal polynomials satisfy two orthogonal relations. Especially interesting are cases when they can be expressed in terms of discrete orthogonal polynomials and their weights. I will discuss such examples related to the Jacobi polynomials on the simplex, for which different bases can be generated by the action of the symmetric group. The corresponding connection coefficients can be written in terms of Racah polynomials. These results allow also to compute connection coefficients for Hahn polynomials and Krawtchouk polynomials of several variables, and for orthogonal polynomials on balls and spheres. The talk is based on joint work with Yuan Xu.

2-D orthogonal polynomials

Mourad Ismail (University of Central Florida and King Saud University)

We discuss the analytic and combinatorial properties of a class of 2-D orthogonal polynomials.

Transition Asymptotics for Toeplitz Determinants

Katarzyna Kozlowska (University of Reading)

Each Toeplitz matrix has a symbol - a function associated with it. The symbols can be smooth or they could have various singularities. In this talk I will describe the change in asymptotics of Toeplitz determinants as we go from the case of a smooth symbol to one that possess a Fisher-Hartwig singularity. In addition, the case in which we see emergence of additional singularities will be discussed. These types of results model phase transitions in numerous problems arising in statistical mechanics.

Products of Random Matrices and Multiple Orthogonal Polynomials
Arno Kuijlaars (Katholieke Universiteit Leuven)

Orthogonal polynomials are basic to the analysis of unitarily invariant random matrix models. Multiple orthogonal polynomials play a similar role in matrix models with external source and two matrix models. I will discuss the application of multiple orthogonal polynomials to yet another model: products of complex Ginibre matrices.

Macdonald Polynomials in Superspace and the 6 Vertex Model
Luc Lapointe (Universidad de Talca)

The Macdonald polynomials in superspace are symmetric polynomials involving commuting and anticommuting variables that generalize the Macdonald polynomials. We will describe how the combinatorics of the Macdonald polynomials extends to superspace. We will focus in particular on how the partition function of the 6 vertex model arises in the Pieri rules for the Macdonald polynomials in superspace.

High Order Recurrence Relations, Hermite-Padé Approximation, and Nikishin Systems
G. López Lagomasino (Universidad Carlos III de Madrid)

The study of sequences of polynomials satisfying high order recurrence relations is connected with the asymptotic behavior of multiple orthogonal polynomials, the convergence properties of type II Hermite-Padé approximation, and eigenvalue distribution of banded Toeplitz matrices. We present some results for the case of recurrences with constant coefficients which match what is known for the Chebyshev polynomials of the first kind. In particular, under appropriate assumptions, we show that the sequence of polynomials satisfies multiple orthogonality relations with respect to a Nikishin system of measures.

Multiple Orthogonal Polynomials for a Nikishin System on a Star-Like Set
Abey Lopez-Garcia (University of South Alabama)

In this talk we describe asymptotic and non-asymptotic properties of multiple orthogonal polynomials associated with a Nikishin system of measures supported on a star-like set centered at the origin with $p+1$ equidistant rays. Of particular importance is the fact that these polynomials satisfy a three-term recurrence relation of the form $zQ_n = Q_{n+1} + a_n Q_{n-p}$, with positive coefficients a_n. We discuss ratio asymptotics for the polynomials Q_n and the corresponding asymptotic behavior of the coefficients a_n. This is a joint work with E. Miña-Díaz.

Polynomials that Scale into Entire Functions of Exponential Type
Doron Lubinsky (Georgia Institute of Technology)

We discuss some problems where we scale a sequence of polynomials and obtain entire functions of exponential type. This might include: (i) Bernstein's unsolved problem on approximation of $|x|$ by polynomials (ii) Universality limits for random matrices (iii) Marcinkiewicz-Zygmund inequalities (iv) Lp Christoffel functions (v) Nikolskii Inequalities And yes, several of these do have relevance to orthogonal polynomials.

Multiple Orthogonal Polynomials and Toda-Type Integrable System

Hiroshi Miki (Doshisha University)

It is a well-known fact that orthogonal polynomials appear as eigenfunctions of the Lax-pair of Toda lattice. From this viewpoint, several integrable systems have been found and refound by using the familiy of orthogonal functions. In this talk, I will show the generalization of Toda lattice can be derived from multiple orthgonal polynomials both in continuous and discrete time case. Furthermore, the bilinear forms of these systems are also discussed.

Matrix-Valued Orthogonal Polynomials Related to Compact Gelfand Pairs and Quantum Groups

Pablo Román (Universidad Nacional De Cordoba)

In this talk, we will be concerned with matrix-valued orthogonal polynomials. In order to derive new examples and study their properties, a suitable group theoretic interpretation has shown to be fruitful. This has been carried out for Gelfand pairs of rank one, leading to families of matrix-valued orthogonal polynomials that can be considered as analogues of Gegenbauer and Jacobi polynomials. We will briefly discuss the construction of these families.

We will focus on the analogue construction for quantum groups, in particular for the quantum analogue of the pair $(G, K) = (\mathrm{SU}(2) \times \mathrm{SU}(2), \mathrm{diag})$. We will introduce matrix-valued orthogonal polynomials in one variable $\{P_n\}_{n \geq 0}$ by studying the spherical functions of any type on the quantum group and show that they are orthogonal with respect to a positive definite weight matrix W. We will derive q-difference operators that have the polynomials as eigenfunctions from the study of certain central elements in the quantum group.

We will also discuss how to extend this construction to other quantum groups. In particular we are interested in the case of the quantized universal enveloping algebra of sl_3.

This is a joint work with N. Aldenhoven and E. Koelink.

Orthogonal Polynomials and the Bergman Shift Matrix

Brian Simanek (Vanderbilt University)

We will discuss the relationship between the Bergman Shift matrix and the asymptotic behavior of orthogonal polynomials. In particular, we will show that certain asymptotic behaviors such as ratio asymptotics can be related to the asymptotic behavior of the entries of the Bergman Shift matrix as we move along the diagonals. This relationship will allow us to show that relative ratio asymptotic behavior occurs in some surprising examples. We will also explore some consequences for random orthogonal polynomials on the unit circle.

Estimates for Bergman Polynomials in Domains with Corners

Nikos Stylianopoulos (University of Cyprus)

Let G be a bounded simply-connected domain in the complex plane $\mathbb{C}$, whose boundary $\Gamma := \partial G$ is a Jordan curve, and let $\{p_n(z)\}_{n=0}^{\infty}$ denote the sequence of Bergman polynomials of G. This is defined as the unique sequence of polynomials

$$p_n(z) = \lambda_n z^n + \cdots, \quad \lambda_n > 0, \quad n = 0, 1, \ldots,$$

that are orthonormal with respect to the area measure on G.

The strong asymptotics of $p_n(z)$ in the *exterior* of Γ and of the leading coefficients λ_n, in cases when Γ is piecewise analytic, have been recently established in [1]. The purpose of this talk is to present, for the same class of curves:

- Some definite results and a conjecture for the asymptotics of $p_n(z)$ on Γ.
- A relation between the strong asymptotics for the leading coefficients λ_n and the *Grunsky coefficients* of the normalized exterior conformal map ψ from the exterior of the unit circle to the exterior of Γ.

[1] N. Stylianopoulos, *Strong asymptotics for Bergman polynomials over domains with corners and applications*, Constr. Approx. **38** (2013), no. 1, 59–100.

Multiple Orthogonal Polynomials on Overlapping Intervals

Walter Van Assche (KU Leuven)

We investigate multiple orthogonal polynomials for which the first measure is supported on $[-1, a]$ and the second measure on $[-a, 1]$, where $0 < a < 1$, i.e.,

$$\int_{-1}^{a} P_{n,n}(x) x^k \, d\mu_1(x) = 0, \qquad 0 \leq k \leq n-1,$$

$$\int_{-a}^{1} P_{n,n}(x) x^k \, d\mu_2(x) = 0, \qquad 0 \leq k \leq n-1,$$

where we assume the measures μ_1 and μ_2 to be absolutely continuous and that the Radon-Nikodym derivative $d\mu_1/d\mu_2$ extends from $[-a, a]$ to an analytic function on $\mathbb{C}$.

The interesting new aspect for such multiple orthogonal polynomials is that both intervals intersect on $[-a, a]$. We investigate these multiple orthogonal polynomials using a Riemann-Hilbert problem and the analysis requires a Riemann surface with three sheets for an algebraic function of order three. The genus of this Riemann surface is two for $0 < a < 1/\sqrt{2}$ and the genus is one for $a \geq 1/\sqrt{2}$. This is joint work with Alexander Aptekarev and Maxim Yattselev.

Multivariate Orthogonal Polynomials and Quantum Integrable Particle Systems on Lattices

Jan Felipe van Diejen (Universidad de Talca)

Recurrence relations for multivariate orthogonal polynomials can be interpreted as eigenvalue equations for a lattice quantum integrable particle system. We exploit this correspondence to diagonalize a lattice version of the hyperbolic quantum Ruijsenaars-Schneider system in an exponential Morse potential. As an application, this allows to compute the n-particle scattering operator and to identify the bispectral dual particle model. (Based on joint work with Erdal Emsiz.)

The Relation Between the Bannai-Ito Polynomials and the Non-Symmetric Wilson Polynomials

Luc Vinet (University of Montreal)

The non-symmetric Wilson polynomials will be introduced together with their connection to the degenerate double affine Hecke algebra of type $(\check{C}_1, C_1)$. The Bannai-Ito polynomials will also be presented as well as the Bannai-Ito algebra $\{K_i, K_j\} = K_k + \omega_k$ where (ijk) is a cyclic permutation of $\{1, 2, 3\}$ which encodes their bispectrality. It will be shown that the two families of orthogonal polynomials coincide and that the two algebraic structures are isomorphic. Based on work in collaboration with V. X. Genest (Montréal) and A. Zhedanov (Donetsk).

Gaussian Discrete Orthogonal Polynomials and Nonintersecting Brownian Motions with Different Boundary Conditions

Dong Wang (National University of Singapore)

We consider nonintersecting Brownian motions starting from a common point and ending at a common point. We study the model on the domain of a finite interval, and assume the boundary condition on the two end points is one of the follows: (1) periodic, (2) absorptive, and (3) reflective. This probabilistic model has been studied in physics literature, because it can be interpreted as a 2D Yang-Mills model with (1) unitary, (2) orthogonal, and (3) symplectic symmetry. In this talk we discuss the rigorous analysis of this model with the help of the Gaussian discrete orthogonal polynomials. We derive the double scaling limit of the model as the number of particles tends to infinity. The main technique is solving the interpolation problem (IP) satisfied by the discrete orthogonal polynomials, which in turn is expressed in a Riemann-Hilbert problem (RHP).

This is joint work with Karl Liechty.

Raising and lowering differential operators for the eigenfunctions of a canonical fractional 2D Fourier transform

Kurt Bernardo Wolf (Universidad Nacional Autnoma de Mxico)

We define a fractional two-dimensional (2D) Fourier transform that self-reproduces a one-parameter family of bivariate Hermite functions, which are eigenfunctions of a differential operator of second order, whose exponential generates this transform subgroup of linear canonical transforms, and stands for its Hamiltonian. Raising and lowering partial differential operators of first degree are given for these functions. This is joint work with Ivan Area, Natig Atakishiyev, Eduardo Godoy