Formation and Evaporation of Regular Black Holes in New 2d Gravity

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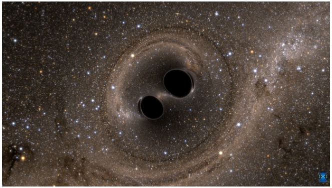
Motivation

http://www.black-holes.org



- Black holes exist (LIGO '16)
- Einstein's theory predicts singular core
- Black holes evaporate (Hawking '76) $\rightarrow$ Loss of predictability
- Quantum gravity hard

Can process be described by self-consistent non-singular effective theory?

Executive Summary: new improved 2D gravity

- Generalizes spher. symm. Einstein-Lanczos-Lovelock gravity:
  - higher order Lagrangian,
  - second order equations,
  - mass function,
  - Birkhoff's theorem.
- Has as sub-class $D \to \infty$ Einstein-Lanczos-Lovelock gravity.
- Can be designed to produce as unique solution any spherical black hole

Provides consistent model for studying effective dynamics of regular black hole formation and evaporation.

Outline:

1. Regular black holes
2. New (improved) 2D gravity
3. Adding radiation
4. Conclusions

Regular black holes *(Sakharov,'65, Bardeen, '68, Frolov-Vilkovisky, '88)*

Example: derived by Poisson and Israel '88 using semi-classical arguments

$$ds^2 = -\left(1 - \frac{2MR^2}{(R^3 + l_{pl}^3)}\right) dt^2 + \left(1 - \frac{2MR^2}{(R^3 + l_{pl}^3)}\right)^{-1} dR^2 + R^2 d\Omega^{(2)}$$

Ricci Scalar
$$\mathcal{R}^{(n)} = 12 l_{pl}^3 \frac{M(-R^3 + 2l_{pl}^3)}{(R^3 + l_{pl}^3)^3}$$

- deSitter core with curvature $\mathcal{R}^{(n)}_{core} = 12M/l_{pl}^3$
- Two horizons $R_+ \to 2M$ and $R_- \to \sqrt{\frac{l_{pl}}{2M}} l_{pl} \to_{M\to\infty} 0$
- mass gap $2M_{min} \sim l_{pl}$.
- $\mathcal{R}^{(n)}_{core} \to \infty$; $\quad R_- \to_{M\to\infty} 0$ as $M \to \infty$
  Semi-classical approximation doomed from beginning!
  Thanks to V. Frolov for stressing this.

A better class of regular black holes Hayward('06)

$$
\begin{aligned}
ds^2 &= -\left(1-\frac{2MR^2}{R^3+Ml_{pl}^2}\right)dt^2+\left(1-\frac{2MR^2}{R^3+Ml_{pl}^2}\right)^{-1}dR^2+R^2d\Omega^{(2)} \\
\mathcal{R}^{(n)} &= 24\frac{M^2l_{pl}^2(-R^3+2Ml_{pl}^2)}{(R^3+Ml_{pl}^2)^3} \qquad (1)
\end{aligned}
$$

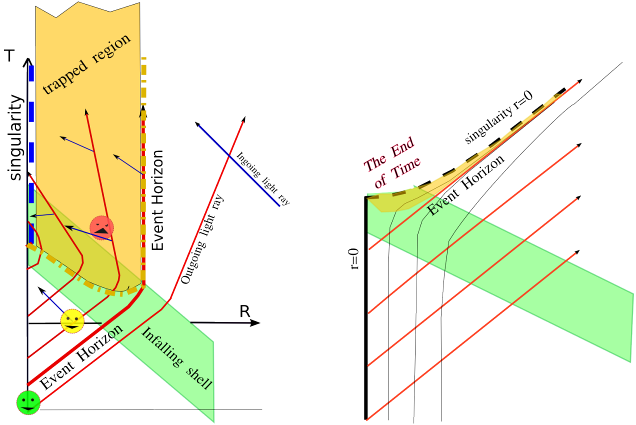
- deSitter core: $\mathcal{R}^{(n)}_{core} = 12/l_{pl}^2$
- Two horizons: $R_+ \sim 2M$ and $R_- \sim l_{pl}$
- Mass gap $2M_{min} = l_{pl}$
- Curvature bounded above, R_- bounded below.

Basis for consistent semi-classical model?

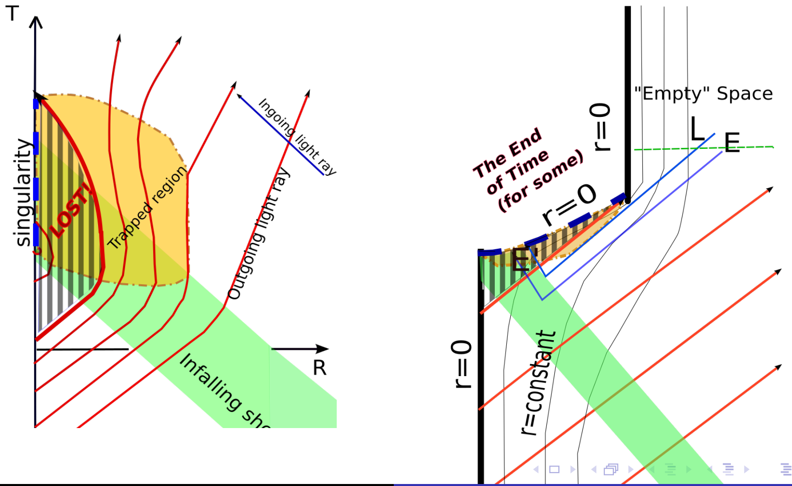
Why are we interested in regular black hole formation and evaporation?

- Expect qualitative changes to structure of complete semi-classical spacetime.

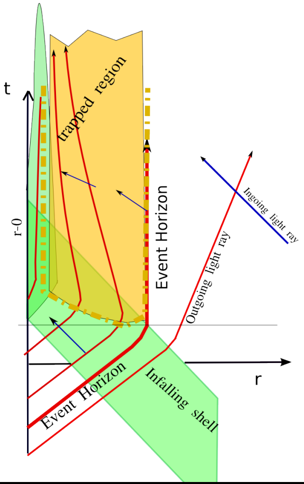
Recall: Singular Black Hole Formation



Singular Black Hole Formation and Evaporation



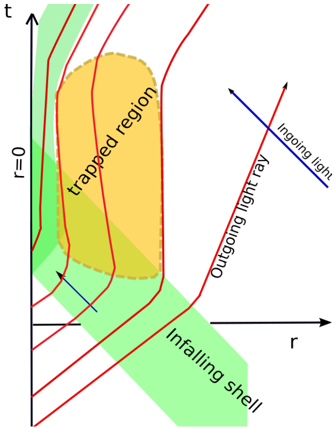
Regular black hole formation (Ziprick, GK '09; Maeda, Taves, GK '16)



- $r = 0$ timelike, regular.
- Expect mass inflation as matter piles up on inner horizon.
- As mass inflates, inner horizon generally shrink. **Stabilizes at l_{pl} for Hayward black holes.**

... and evaporation

Hayward '06: explicit construction by patching Vaidya spacetimes



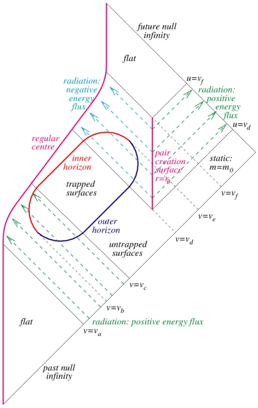


FIG. 5 (color online). Penrose diagram of formation and

...but can we find consistent dynamical equations that yield this spacetime?

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Recall: Generic 2D Dilaton Gravity (1990's)

Einstein action in n dimensions:

$$I_E = \frac{1}{16\pi G}\int d^n x\sqrt{-g^{(n)}}\mathcal{R}(g^{(n)}) \tag{2}$$

Dimensionally Reduced Action ($ds^2 = -g_{\mu\nu}dx^\mu dx^\nu + R^2 d\Omega^{(n-2)}$):

$$I_{(2)} = \frac{1}{l^{n-2}}\int d^2x\sqrt{-g}\Big[R^{n-2}\mathcal{R} + (n-2)(n-3)R^{(n-4)}(\nabla R)^2 + (n-2)(n-3)R^{(n-4)}\Big] \tag{3}$$

Generalization (Generic 2D Dilaton Gravity):

$$I_G = \frac{1}{l^{n-2}}\int d^2x\sqrt{-g}\Big\{\phi(R)\mathcal{R} + h(R)(DR)^2 + V(R)\Big\}. \tag{4}$$

Can't get bounded curvature metrics from this class of theories.

Extended Spherical Einstein-Lovelock-Lanczos Gravity

Recall Lovelock Action:

$$I = \frac{1}{2\kappa_n^2}\int \mathrm{d}^n x\sqrt{-g}\sum_{p=0}^{[n/2]}\alpha_{(p)}\left(\frac{1}{2^p}\delta^{\mu_1\cdots\mu_p\nu_1\cdots\nu_p}_{\rho_1\cdots\rho_p\sigma_1\cdots\sigma_p}R_{\mu_1\nu_1}{}^{\rho_1\sigma_1}\cdots R_{\mu_p\nu_p}{}^{\rho_p\sigma_p}\right), \quad (5)$$

where $\kappa_n := \sqrt{8\pi G_n}$ and $\delta^{\mu_1\cdots\mu_p}_{\rho_1\cdots\rho_p} := p!\delta^{\mu_1}_{[\rho_1}\cdots\delta^{\mu_p}_{\rho_p]}$..

Eg: Einstein-Gauss-Bonnet Gravity ($n \geq 5$):

$$I = \int d^n x\sqrt{-g}\left\{\mathcal{R} + \frac{\tilde{\alpha}}{2}\left[\mathcal{R}^2 - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma}\right]\right\}. \quad (6)$$

Generic spherically symmetric Lovelock terms:

Shown by G.K., Maeda and Taves '13, generalizing an identity used for Einstein-Gauss-Bonnet by Taves, Leonard, GK and Mann '11:

$$I = \frac{1}{2\kappa_n^2}\int \mathrm{d}^n x\sqrt{-g}\sum_{p=0}^{[n/2]}\alpha_{(p)}\mathcal{L}_{(p)}$$

$$\mathcal{L}_{(p)} = \frac{(n-2)!}{(n-2p)!}\left[pR^{2-2p}\overset{(2)}{\mathcal{R}} + (n-2p)(n-2p-1)\left\{(1-Z)^p + 2pZ\right\}R^{-2p}\right] + p(n-2p)R^{1-2p}\left\{1-(1-Z)^{p-1}\right\}\nabla R\cdot\frac{\nabla Z}{Z}.$$

where: $Z := |\nabla R|^2$

New 2D Gravity

Natural extension:

$$I = \frac{1}{l^{n-2}} \int d^2x \sqrt{-g} \Big\{ \phi(R)\mathcal{R} + H(R,Z) + \chi(R,Z) \nabla R \cdot \nabla Z \Big\}$$

$$Z := |\nabla R|^2 \tag{7}$$

- Second order equations for $g_{\mu\nu}$ and R
- Birkhoff's theorem
- Mass function: $\mathcal{M}$ such that $D_A \mathcal{M} = 0$ in vacuum on shell

Conjecture: Most general 2D with above properties?

Mass Function

Integrability Condition on Lagrangian Functions:

$$\phi_{,RR} = \eta_{,Z} - \chi_{,R}.$$

Guarantees existence of the mass function:

$$\mathcal{M}(R,Z) := -\phi_{,R}Z + \int^{Z} \chi(R,\bar{Z})\mathrm{d}\bar{Z}.$$

which gives

$$\begin{aligned} D_A\mathcal{M} =& (\chi - \phi_{,R})D_A Z - 2\left(\phi_{,RR}Z - \frac{1}{2}\eta(R,Z)\right)D_A R \\ =& 0 \qquad \text{in vacuum} \end{aligned}$$

Birkhoff Theorem

- Most general solution (Schwarzschild coords):

$$\mathrm{d}s^2 = -f(R; M)\mathrm{d}t^2 + f(R; M)^{-1}\mathrm{d}R^2.$$

where

$$\mathcal{M}(R, Z) = M = \text{constant}. \tag{8}$$

- $f(R; M)$ is determined by inverting (8) to solve for Z:

$$Z = f(R; \mathcal{M})$$

How to design your personalized black hole spacetime

1. Choose your favourite metric function $f(R, M)$
2. In Schwarzchild coords $Z := |\nabla R|^2 = f(R; M)$: Solve for $\mathcal{M} = \mathcal{M}(R, Z)$
3. Obtain lagrangian functions:

$$\frac{\partial \mathcal{M}}{\partial Z} = \chi - \phi_{,R},$$
$$\frac{\partial \mathcal{M}}{\partial R} = -2\left(\phi_{,RR} Z - \frac{1}{2}\eta(R, Z)\right).$$

4. Just add matter and solve collapse equations.

Example: Hayward black hole in $4D$

metric function : $$f(R) = 1 - \frac{2MR^2}{R^3 + l_{pl}^2 M} = Z\,.$$

mass function: $$2\mathcal{M} = \frac{(1-Z)R^3}{R^2 - l_{pl}^2(1-Z)}$$

Adding matter

Massless scalar field:

$$I_{matter} = -\frac{1}{2}\int d^2xR^2|\nabla\psi|^2$$

- Straightforward to derive Hamiltonian equation.

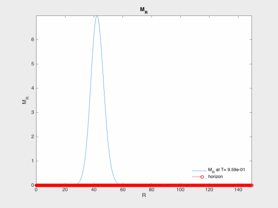
$$ds^2 = -N^2dt^2 + \Lambda^2(dx + N_r dt)^2 R^2(x) d\Omega^{(2)}$$

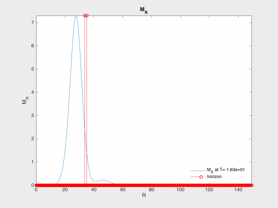
- We work in flat slice coordinates (mostly due to inertia).

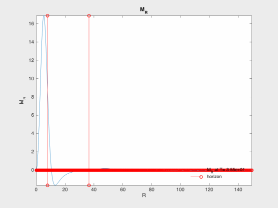
$$R = x \qquad\qquad \Lambda = 1$$

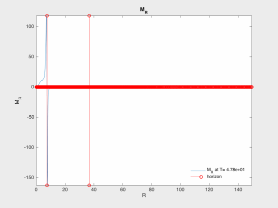
Numerical Results: l=5, $M_{ADM} = 78.3195$, 150,000 iterations to T=47.7636.

Mass Density $M_{,R}$:

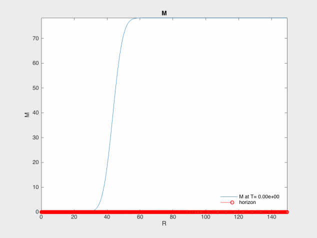


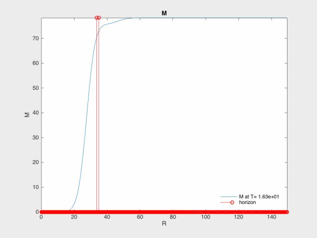


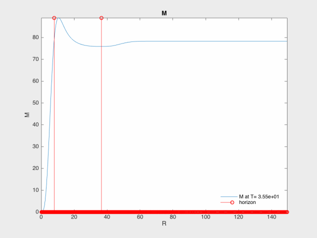


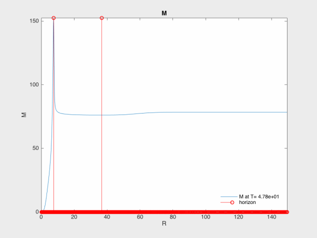


Mass Function:









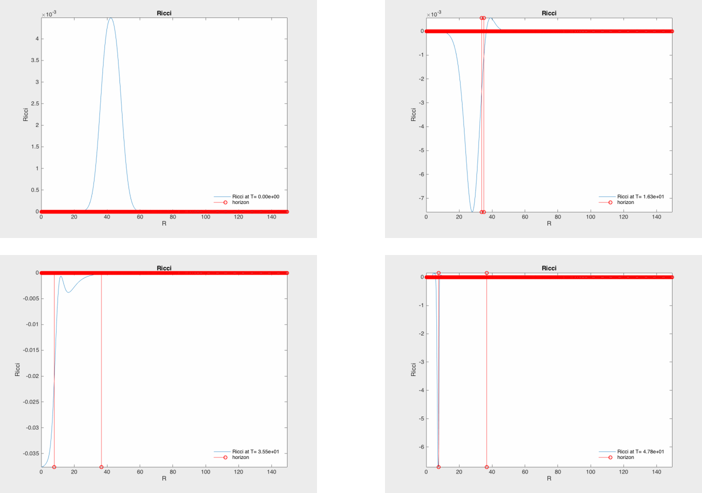
Ricci Scalar in terms of data on a slice

$$\mathcal{R}^{(n)} = -\frac{B'(R)}{\mathcal{M}_{,Z}}|\nabla\psi|^2 - \frac{\mathcal{M}_{,ZZ}}{\mathcal{M}^3_{,Z}}B^2|\nabla\psi|^4 + \frac{\mathcal{M}_{RR}}{\mathcal{M}_{,Z}} - 2\mathcal{M}_{,ZR}\left(\frac{\mathcal{M}_{,R}}{\mathcal{M}^2_{,Z}}\right) + \frac{\mathcal{M}_{,ZZ}}{\mathcal{M}_{,Z}}\left(\frac{\mathcal{M}_{,R}}{\mathcal{M}_{,Z}}\right)^2 + 2(n-2)\frac{\mathcal{M}_{,R}}{\mathcal{M}_{,Z}R} + (n-2)(n-3)\frac{1-Z}{R^2}$$

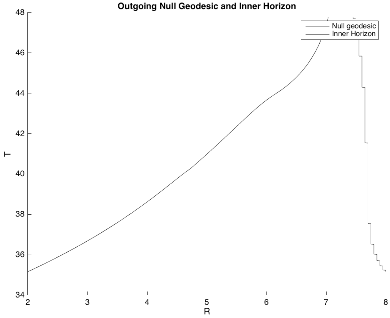
$$= \frac{-B'(R)R^5}{(R^3+\mathcal{M}l^2)^2}|\nabla\psi|^2 - \left(\frac{8l^2R^5B^2(R)}{(R^3+\mathcal{M}l^2)^3}\right)|\nabla\psi|^4 - \frac{3\mathcal{M}^2l^2(R^3-2^2)}{(R^3+\mathcal{M}l^2)^3}$$

$$|\nabla\psi|^2 = -\frac{P^2_\psi}{\Lambda^2B^2(R)} + \frac{\psi^2_{,x}}{\Lambda^2}$$

Ricci Scalar:



Outgoing null geodesic that asymptotes to inner horizon, and plots of Mass and Ricci scalar along it.



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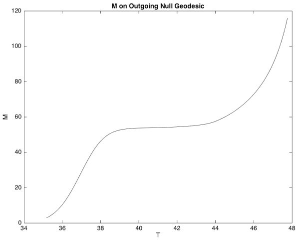

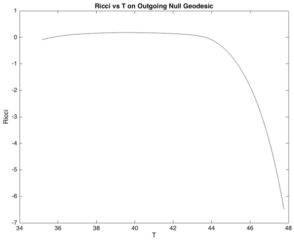

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Radiation Terms: Polyakov Action

$$I_{Poly} \sim -\int d^4x \sqrt{-g^{(4)}} \mathcal{R}^{(4)} \frac{1}{D^2_{(4)}} \mathcal{R}^{(4)} \qquad (9)$$

Local Form: auxilliary field z

$$I_{Poly} \sim -\int \sqrt{-g} R^2 \left[z\mathcal{R}(g) + D_A z D^A z \right] \qquad (10)$$

So far:

- Eqs. obtained and code running for Bardeen-type black hole: stability (and other?) issues
- Working on equations and code for new 2D gravity.

Summary

Presented new class of 2D theories to model the formation and evaporation of physical regular spherically symmetric black holes.

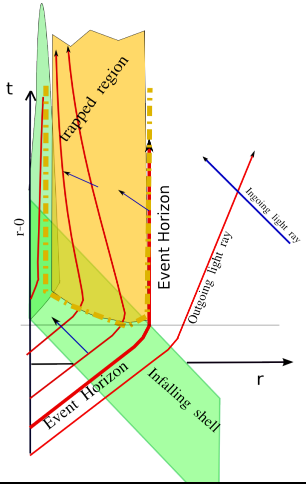
- Can design lagrangian to produce any 2D black hole, including Hayward.
- Sub-class (eg Hayward BH) have physical interpretation as infinite dimensional Lovelock: lagrangian functions need to have Taylor expansion in $1 - Z$.

Outstanding Questions:

- Can any of the new 2D gravity theories be obtained via dimensional reduction (Cf. Meyers, Robinson '10; Oliva, Ray '11)?
- Does Hayward black hole suffer (a lot) from mass inflation? (Stable inner horizon, curvature bounded.)
- Radiation of regular black holes should produce spacetime with compact trapping horizon. Consequences for information loss?

Thanks for listening!

Regular black hole formation (Ziprick, GK '09; Maeda, Taves, GK '16)



Penrose Diagram