Schwinger Effect in Curved Spacetimes

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Black Holes' New Horizons

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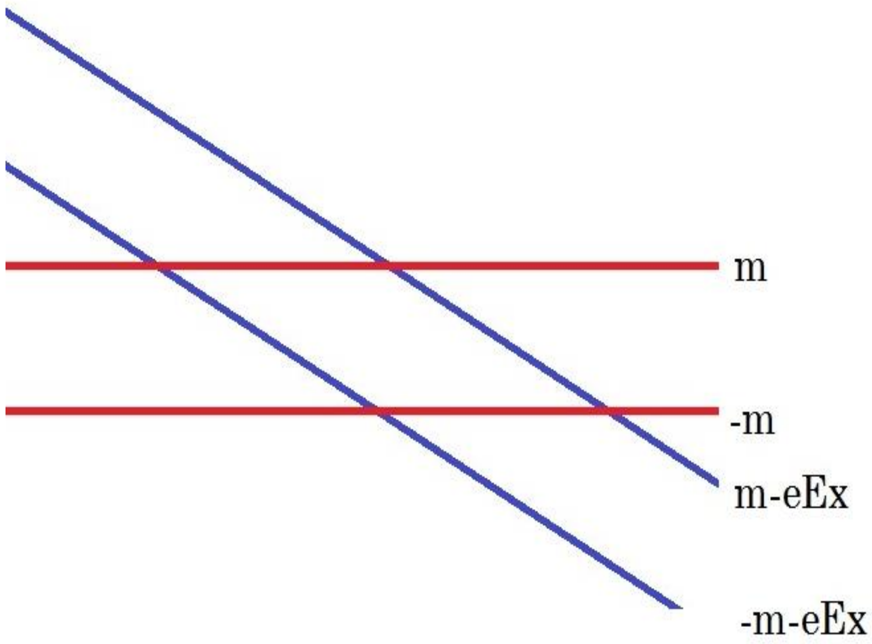
Outline

- Why Schwinger Effect in Curved Spacetimes?
- Perturbation Theory & Borel Summation & Vacuum Persistence Amplitude
- Effective Actions in In-Out Formalism
- Reconstructing Effective Action
- QED in (anti-) de Sitter Space
- Schwinger Effect in Near-extremal RN BHs
- Schwinger Effect in Near-extremal KN BHs
- Conclusion

Why Schwinger Effect in Curved Spacetimes?

What is Schwinger Effect?



- Constant E-field changes energy spectra in Minkowski spacetime:

$$\varepsilon_{\pm} = |eE|x \pm \sqrt{\vec{p}^{\,2} + m^2}$$

- Spontaneous creation of a particle-antiparticle pair from the Dirac sea (quantum mechanical tunneling)

$$N_S = \exp\left(-\frac{m}{2T_S}\right), \quad T_S = \frac{1}{2\pi}\left(\frac{qE}{m}\right)$$

- Critical (Schwinger) field to energetically separate the pair

$$eE_c \times \left(\frac{\hbar}{mc}\right) = mc^2$$

Schwinger Effect in Charged Black Holes

Zaumen ('74)

Carter ('74)

Gibbons ('75)

Damour, Ruffini ('76)

⋮

Khriplovich ('99)

Gabriel ('01)

SPK, Page ('04), ('05), ('08)

Ruffini, Vereshchagin, Xue ('10)

Chen, SPK, Lin, Sun, Wu ('12); Chen, Sun, Tang, Tsai ('15)

Ruffini, Wu, Xue ('13)

SPK ('13)

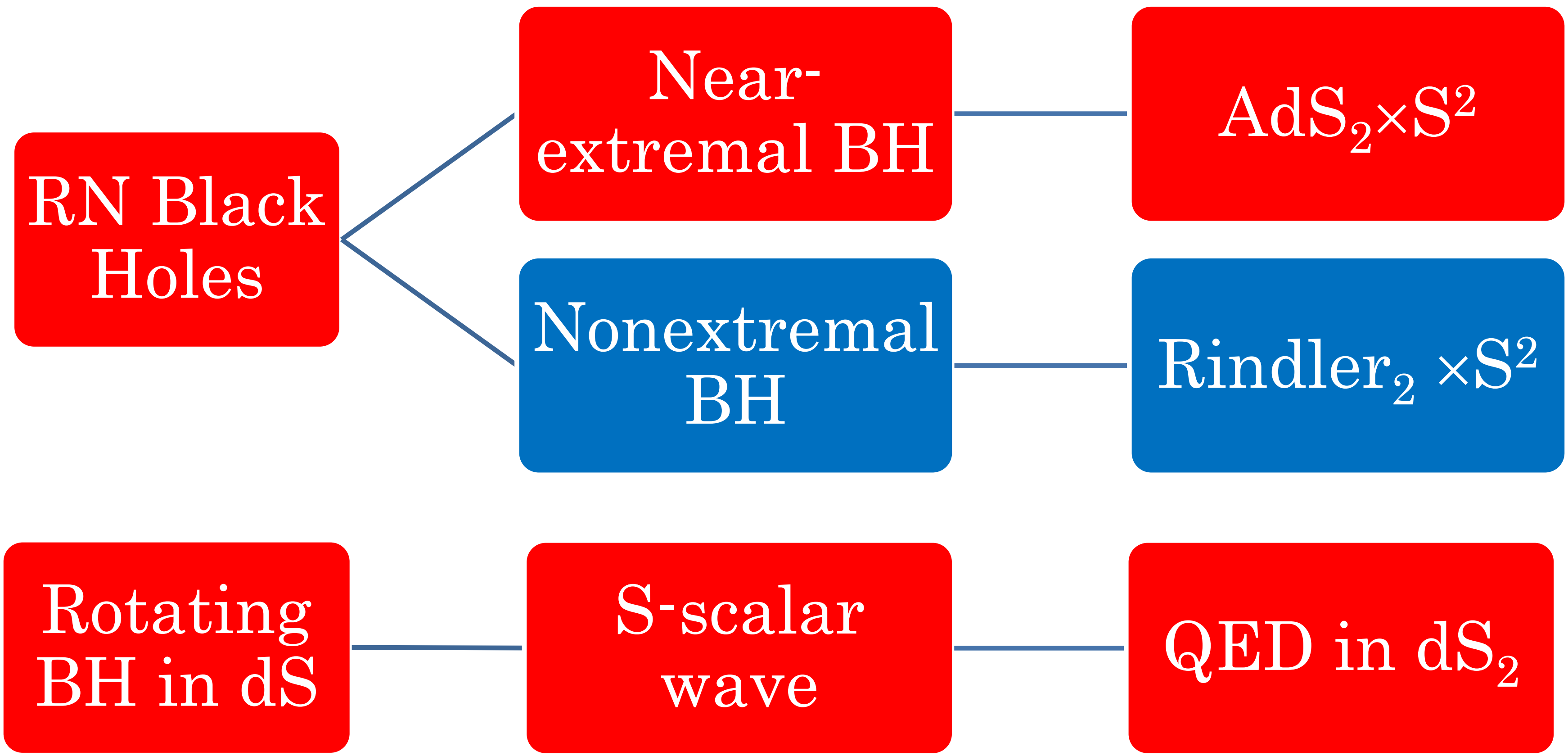
Cai, SPK ('14)

SPK, Lee, Yoon ('15); SPK ('15)

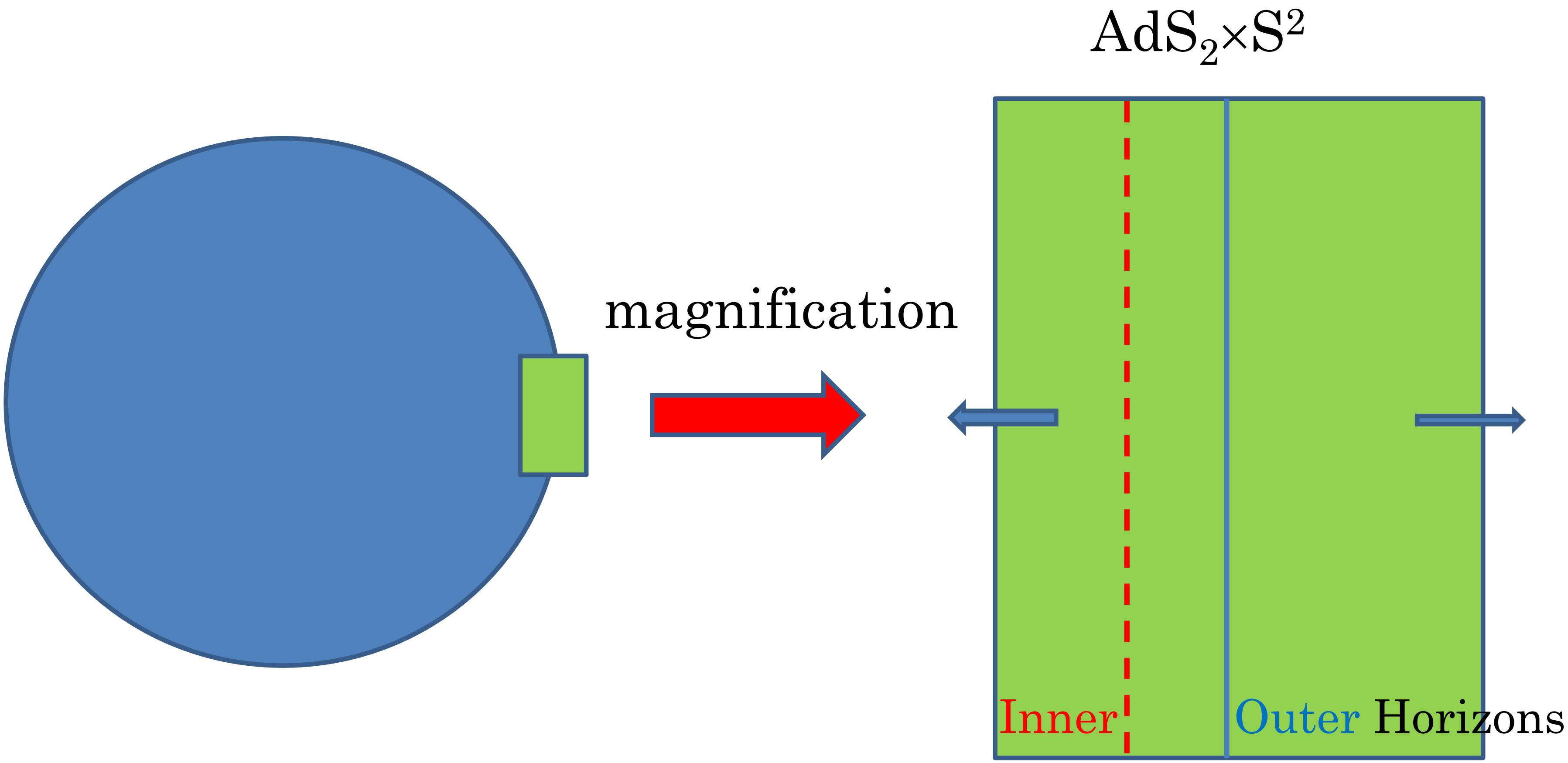
Chen, SPK, Tang, Kerr-Newman BH, in preparation ('16)

Why Schwinger Effect in (A)dS$_2$?

Near-Horizon Geometry of RN BHs



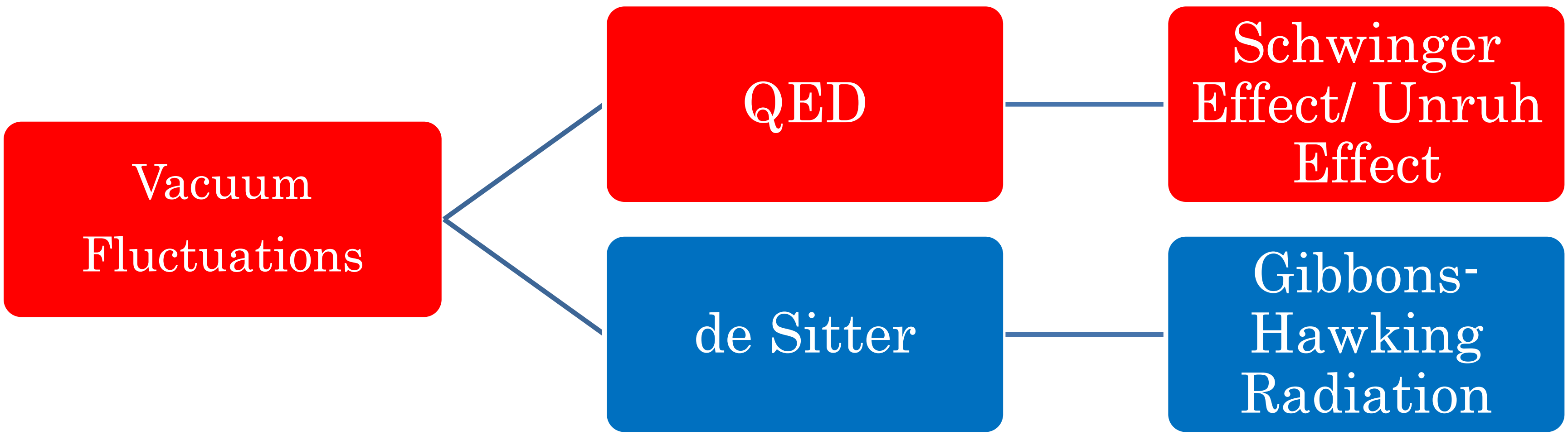
Near-horizon Geometry of Near-extremal RN BH



G. t'Hooft & A. Strominger, "conformal symmetry near the horizon of BH," MG14, July 2015.

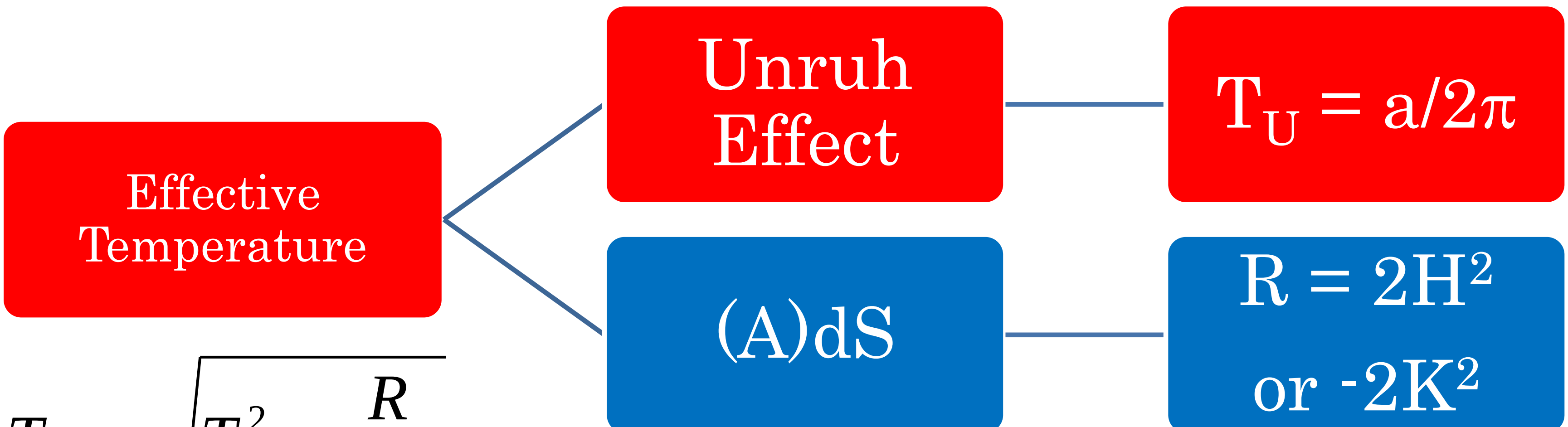
Schwinger Effect in (A)dS

[Cai, SPK (‘14)]



Effective Temperature for Unruh Effect in (A)dS

[Narnhofer, Peter, Thirring ('96); Deser, Levin ('97)]



$$T_{\mathrm{eff}} = \sqrt{T_{\mathrm{U}}^2 + \frac{R}{8\pi^2}}$$

Perturbation Theory & Borel Summation & Vacuum Persistence

Borel Summation

- Large-order perturbation theory may have a divergent power series with the asymptotic form with three real constant ρ, μ>0 and ν [Le Guillou, Zinn-Justin ('90)]

$$f(g) = \sum_{n=0}^{\infty} a_n g^n = \sum_{n=0}^{\infty} (-1)^n \rho^n \Gamma(\mu n + \nu) g^n, \quad (n \to \infty)$$

- Leading Borel approximation for alternating case (+ sign)/ nonalternating case (– sign) and vacuum persistence

$$f(g) = \frac{1}{\mu} \int_0^{\infty} \frac{ds}{s} \left(\frac{1}{1 \pm s} \right) \left(\frac{s}{\rho g} \right)^{\nu/\mu} \exp\left[- \left(\frac{s}{\rho g} \right)^{1/\mu} \right]$$

$$\mathrm{Im}\, f(-g) = \frac{\pi}{\mu} \left(\frac{1}{\rho g} \right)^{\nu/\mu} \exp\left[- \left(\frac{1}{\rho g} \right)^{1/\mu} \right]$$

Borel Summation

- Heisenberg-Euler-Schwinger QED action in a constant electric field E

$$a_n^{(1)} = (-1)^n \frac{m^4 g^2}{4\pi^6} \frac{\Gamma(2n+2)}{\pi^{2n}} \left(1 + \frac{1}{2^{2n+4}} + \frac{1}{3^{2n+4}}\right), \quad g = -\left(\frac{eE}{m^2}\right)^2$$

- Borel summation leads to the vacuum persistence in a constant electric field E [one-loop by Dunne, Hall ('99); two-loop by Dunne, Schubert ('00)]

$$2\,\mathrm{Im}\,L_{eff}(E) = \frac{m^4}{4\pi^3}\left(\frac{eE}{m^2}\right)^2 \sum_{k=1}^{\infty} \frac{1}{k^2} \exp\left[-\frac{m^2 \pi k}{eE}\right]$$

- Borel summation of real effective action for dS (AdS) and vacuum persistence for Gibbons-Hawking radiation [Dunne, Das ('06)]

Perturbative QED Action in Curved Spacetime

[Davila, Schubert ('10)]

$$
\begin{aligned}
\mathcal{L}_{\rm spin}^{R(4)} = -\frac{1}{8\pi^2}\frac{1}{m^2}\Big[& -\frac{1}{72}R(F_{\mu\nu})^2 + \frac{1}{180}R_{\mu\nu}F^{\mu\alpha}F^{\nu}{}_{\alpha} \\
& + \frac{1}{36}R_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} - \frac{1}{180}(\nabla_\alpha F_{\mu\nu})^2 + \frac{1}{36}F_{\mu\nu}\Box F^{\mu\nu}\Big] \\
& - \frac{1}{8\pi^2}\frac{1}{m^6}\Big[-\frac{1}{432}R(F_{\mu\nu})^4 + \frac{7}{1080}R\,\mathrm{tr}[F^4] \\
& - \frac{1}{945}R_{\alpha\beta}(F^4)^{\alpha\beta} - \frac{1}{540}R_{\alpha\beta}(F^2)^{\alpha\beta}(F_{\gamma\delta})^2 + \frac{1}{540}R_{\alpha\mu\beta\nu}(F^2)^{\alpha\beta}(F^2)^{\mu\nu} \\
& + \frac{11}{360}R_{\alpha\mu\beta\nu}(F^3)^{\alpha\mu}F^{\beta\nu} + \frac{1}{108}R_{\alpha\mu\beta\nu}F^{\alpha\mu}F^{\beta\nu}(F_{\gamma\delta})^2 \\
& - \frac{11}{945}F_{\alpha\beta:\gamma}F_{\mu}{}^{\beta:\gamma}(F^2)^{\alpha\mu} + \frac{2}{945}F_{\alpha\beta;}{}^{\mu}F_{\mu}{}^{\alpha}{}_{;\delta}(F^2)^{\beta\delta} \\
& + \frac{7}{270}(F^3)^{\mu\nu}\Box F_{\mu\nu} + \frac{1}{108}F^{\mu\nu}\Box F_{\mu\nu}(F_{\gamma\delta})^2 + \frac{1}{216}F_{\mu\nu:\alpha\beta}(F^2)^{\alpha\beta}F^{\mu\nu} \\
& + \frac{1}{540}F_{\mu\nu:\alpha\beta}(F^2)^{\alpha\nu}F^{\beta\mu} - \frac{1}{540}(F_{\alpha\beta:\gamma})^2(F_{\mu\nu})^2 \\
& - \frac{2}{189}F_{\alpha\beta:\gamma}F_{\mu\nu;}{}^{\gamma}F^{\alpha\mu}F^{\beta\nu} - \frac{2}{189}F_{\alpha\beta:\gamma}F_{\mu}{}^{\alpha}{}_{;\delta}F^{\beta\mu}F^{\gamma\delta}\Big].
\end{aligned}
\tag{3.2}
$$

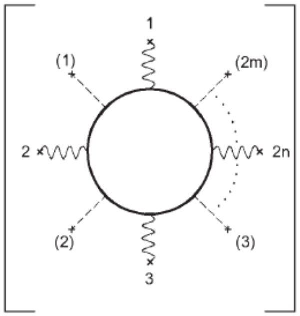
Borel summation? Find large-order perturbation for vacuum persistence!

Effective Actions in In-Out Formalism

In-Out Formalism for QED Actions

- In the in-out formalism, the vacuum persistence amplitude gives the effective action [Schwinger ('51); DeWitt ('75), ('03)] and is equivalent to the Feynman integral

$$e^{iW} = e^{i\int (-g)^{1/2} d^D x L_{\text{eff}}} = \langle 0, \text{out} \,|\, 0, \text{in} \rangle \quad = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty}$$



- The complex effective action and the vacuum persistence for particle production

$$\left| \langle 0, \text{out} \,|\, 0, \text{in} \rangle \right|^2 = e^{-2\,\text{Im}W} \;, \quad 2\,\text{Im}W = \pm VT \sum_{\text{k}} \ln(1 \pm N_k)$$

Effective Actions at T=0 & T

- Zero-temperature effective actions in proper-time integral via the gamma-function regularization [SPK, Lee, Yoon (‘08), (‘10); SPK (‘11)]; gamma-function & zeta-function regularization [SPK, Lee (‘14)]; quantum kinematic approach [Bastianelli, SPK, Schubert, in preparation (’16)]

$$W = \pm i \sum_{\mathrm{k}} \ln \alpha_{\mathrm{k}}^{*} = \pm i \sum_{l} \sum_{\mathrm{k}} \ln \Gamma\left(a_l + i b_l(\mathrm{k})\right)$$

- finite-temperature effective action [SPK, Lee, Yoon (‘09), (‘10)]

$$\exp\left[i \int d^3 x dt L_{\mathrm{eff}}\right] = \langle 0, \beta, \mathrm{in} | U^{+} | 0, \beta, \mathrm{in} \rangle = \frac{\mathrm{Tr}(U^{+} \rho_{\mathrm{in}})}{\mathrm{Tr}(\rho_{\mathrm{in}})}$$

Γ-Regularization

- Assumption: Bogoliubov coefficients of the form

$$\alpha_k = \prod \frac{\Gamma(a \pm ib)}{\Gamma(c \pm id)}; \quad \beta_k = \prod \frac{\Gamma(f \pm ig)}{\Gamma(h \pm ik)}$$

- The effective action from Schwinger variational principle

$$W_{\text{eff}} = -i \ln \langle 0, \text{out} \mid 0, \text{in} \rangle = \pm i \sum_{\text{k}} \ln \alpha_{\text{k}}^*$$

- The integral representation for gamma-function

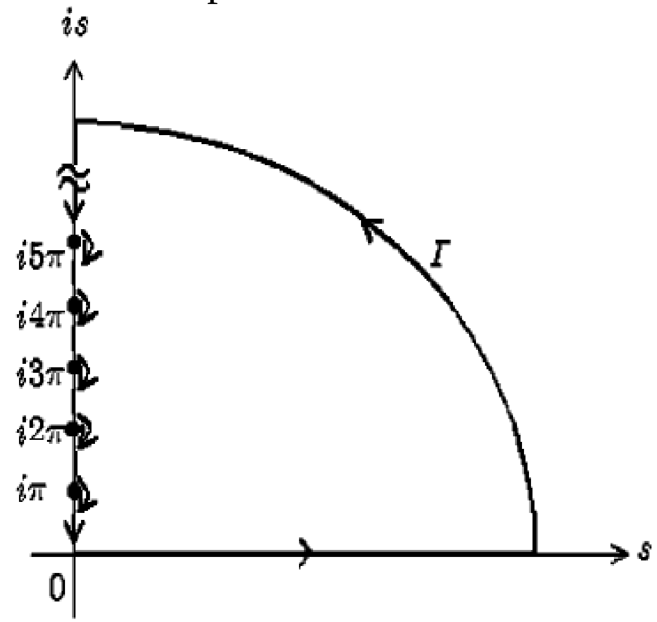
$$\ln \Gamma(a \pm ib) = \int_0^z \frac{dz}{z} \left[\underbrace{\frac{e^{-(a \pm ib)z}}{1 - e^{-z}}}_{\text{term to be renormalized}} \underbrace{- \frac{e^{-z}}{1 - e^{-z}} + (a \pm ib - 1) e^{-z}}_{\text{terms to be regulated away}} \right]$$

Γ-Regularization

- Γ-regularization [SPK, Lee, Yoon (‘08), (‘10); SPK (10), (‘11)]

$$\int_0^\infty \frac{dz}{z}\frac{e^{-(a\pm ib)z}}{1-e^{-z}} = \underbrace{P\int_0^\infty \frac{ds}{s}\frac{e^{-(a\pm ib)(\mp is)}}{1-e^{\pm is}}}_{\text{vacuum polarization}} \underbrace{\mp \pi i \sum_{n=1}^{\infty} \frac{e^{-(a\pm ib)(\mp 2n\pi i)}}{\mp 2n\pi i}}_{\text{vacuum persistence}}$$

- The Cauchy residue theorem



Reconstructing Effective Action

Conjecture

- Can one find the effective action from the pair-production rate? inverse procedure of Borel summation (Gies, SPK, Schubert)
- If the imaginary part (vacuum persistence) of the effective action can be factorized into a product of one plus or one minus exponential factors, then the structure of simple poles and their residues of these factors uniquely determine the analytical structure of the proper-time integrand of the effective action (vacuum polarization) (modulo entire function independent of renormalization via Mittag-Leffler theorem):

$$2\,\mathrm{Im}(L_{\mathrm{eff}}) = \pm \sum_{\mathrm{states}} \ln(1 \pm N) = \sum_{\mathrm{states}\{I\}} \sum (\pm) \ln(1 \pm e^{-\pi S^{\{I\}}})$$

Reconstructing Effective Action from Pair-Production Rate

- Scalar/Spinor effective action (Real part) vs Imaginary part (Cauchy theorem vs Mittag-Leffler theorem/Borel summation)

$$P\int_0^\infty ds \frac{e^{-Ss}}{s^2}\left[\frac{1}{\sin s} - \frac{1}{s} - \frac{s}{6}\right] \Leftrightarrow i\ln\left(1+e^{-\pi S}\right) = i\sum_{n=1} \frac{(-1)^{n+1}}{n} e^{-n\pi S}$$

$$-P\int_0^\infty ds \frac{e^{-Ss}}{s^2}\left[\frac{\cos s}{\sin s} - \frac{1}{s} + \frac{s}{3}\right] \Leftrightarrow -i\ln\left(1-e^{-\pi S}\right) = i\sum_{n=1} \frac{1}{n} e^{-n\pi S}$$

- Most of imaginary parts from the pair-production rate (Schwinger formula in E & B, Bose-Einstein or Fermi-Dirac distribution) can be written as a sum of the form

$$2\,\mathrm{Im}(L_{\mathrm{eff}}) = \sum_{\mathrm{states}} \left[\sum_{\{I\}} \ln(1 \pm e^{-\pi S^{\{I\}}}) - \sum_{\{II\}} \ln(1 \pm e^{-\pi R^{\{II\}}})\right]$$

QED in (A)dS

Schwinger formula in (A)dS

- (A)dS metric and the gauge potential for E

$$ds^2 = -dt^2 + e^{2Ht} dx^2 \,, \quad A_1 = -(E/H)(e^{Ht} - 1)$$

$$ds^2 = -e^{2Kx} dt^2 + dx^2 \,, \quad A_0 = -(E/K)(e^{Kx} - 1)$$

- Schwinger formula (mean number) for scalars in dS_2 [Garriga ('94); SPK, Page ('08)] and in AdS_2 [Pioline, Troost ('05); SPK, Page ('08)]

$$N = e^{-S} \,, \quad S = \frac{\pi m^2}{qE} \left(\frac{2 - \frac{R}{4m^2}}{1 + \sqrt{1 + \frac{m^2 R}{2(qE)^2} - \frac{R^2}{16(qE)^2}}} \right)$$

Effective Temperature for Schwinger formula

- Effective temperature for accelerating observer in (A)dS [Narnhofer, Peter, Thirring ('96); Deser, Levin ('97)]

$$N = e^{-m/T_{\rm eff}}\ ,\ T_{\rm eff} = \sqrt{T_{\rm U}^2 + \frac{R}{8\pi^2}},\ R = 2H^2, (-2K^2)$$

- Effective temperature for Schwinger formula in (A)dS [Cai, SPK ('14)]

$$N = e^{-\overline{m}/T_{\rm eff}}\ ,\ \overline{m} = \sqrt{m^2 - \frac{R}{8}},\ T_{\rm U} = \frac{qE/\overline{m}}{2\pi},\ T_{\rm GH} = \frac{H}{2\pi}$$

$$T_{\rm dS} = \sqrt{T_{\rm U}^2 + T_{\rm GH}^2} + T_{\rm U}\ ;\ T_{\rm AdS} = \sqrt{T_{\rm U}^2 + \frac{R}{8\pi^2}} + T_{\rm U}$$

Scalar QED Action in dS_2

- Mean number for pair production and vacuum polarization from the in-out formalism [Cai, SPK ('14)]

$$N_{\mathrm{dS}} = \frac{e^{-(S_\mu - S_\lambda)} + e^{-2S_\mu}}{1 - e^{-2S_\mu}}, \quad 2\,\mathrm{Im}\,W^{(1)}_{\mathrm{dS}} = \ln\left(1 + N_{\mathrm{dS}}\right)$$

$$L^{(1)}_{\mathrm{dS}} = \frac{H^2 S_\mu}{2(2\pi)} P\int_0^\infty \frac{ds}{s}\left[e^{-(S_\mu - S_\lambda)s/2\pi}\left(\frac{1}{\sin(s/2)} - \overbrace{\left(\frac{2}{s} + \frac{s}{12}\right)}^{\text{Schwinger subtraction}}\right) - e^{-S_\mu s/\pi}\left(\frac{\cos(s/2)}{\sin(s/2)} - \left(\frac{2}{s} - \frac{s}{6}\right)\right)\right]$$

$$S_\mu = 2\pi\sqrt{\left(\frac{qE}{H^2}\right)^2 + \left(\frac{m}{H}\right)^2 - \frac{1}{4}}, \quad S_\lambda = 2\pi\frac{qE}{H^2}$$

Scalar QED Action in AdS_2

- Mean number for pair production and vacuum polarization

$$N_{\mathrm{AdS}} = \frac{e^{-(S_\kappa - S_\nu)} - e^{-(S_\kappa + S_\nu)}}{1 + e^{-(S_\kappa + S_\nu)}}, \quad 2\,\mathrm{Im}\,W_{\mathrm{AdS}}^{(1)} = \ln\left(1 + N_{\mathrm{AdS}}\right)$$

$$L_{\mathrm{AdS}}^{(1)} = -\frac{K^2 S_\nu}{2(2\pi)} P\int_0^\infty \frac{ds}{s} e^{-S_\kappa s/2\pi} \cosh(S_\nu s/2\pi)\left[\frac{1}{\sin(s/2)} - \frac{2}{s} - \frac{s}{12}\right]$$

$$S_\nu = 2\pi\sqrt{\left(\frac{qE}{K^2}\right)^2 - \left(\frac{m}{K}\right)^2 - \frac{1}{4}}, \quad S_\kappa = 2\pi\frac{qE}{K^2}$$

Spinor QED Action in dS$_2$

- Mean number for pairs and vacuum polarization [SPK ('15)]

$$N_{\rm ds}^{\rm sp} = \frac{e^{-(S_\mu - S_\lambda)} - e^{-2S_\mu}}{1 - e^{-2S_\mu}}, \quad 2\,{\rm Im}W_{\rm dS}^{(1)} = -\ln\left(1 - N_{\rm ds}^{\rm sp}\right)$$

$$L_{\rm dS}^{\rm sp} = -\frac{H^2 S_\mu}{2\pi} P\int_0^\infty \frac{ds}{s}\left(e^{-(S_\mu - S_\lambda)s/2\pi} - e^{-S_\mu s/\pi}\right)\left(\cot(\frac{s}{2}) - \frac{2}{s} + \frac{s}{6}\right)$$

$$S_\mu = 2\pi\sqrt{\left(\frac{qE}{H^2}\right)^2 + \left(\frac{m}{H}\right)^2}, \quad S_\lambda = 2\pi\frac{qE}{H^2}$$

Spinor QED Action in AdS_2

- Mean number for pairs and vacuum polarization

$$N_{\mathrm{AdS}}^{\mathrm{sp}} = \frac{e^{-(S_\kappa - S_\nu)} - e^{-(S_\kappa + S_\nu)}}{1 - e^{-(S_\kappa + S_\nu)}}, \quad 2\,\mathrm{Im}\,W_{\mathrm{AdS}}^{\mathrm{sp}} = -\ln\left(1 - N_{\mathrm{AdS}}^{\mathrm{sp}}\right)$$

$$L_{\mathrm{AdS}}^{\mathrm{sp}} = -\frac{K^2 S_\nu}{2\pi} P\int_0^\infty \frac{ds}{s}\left(e^{-(S_\kappa - S_\nu)s/2\pi} - e^{-(S_\kappa + S_\nu)s/2\pi}\right)\left(\cot(\frac{s}{2}) - \frac{2}{s} + \frac{s}{6}\right)$$

$$S_\nu = 2\pi\sqrt{\left(\frac{qE}{K^2}\right)^2 - \left(\frac{m}{K}\right)^2}, \quad S_\kappa = 2\pi\frac{qE}{K^2}$$

Schwinger Effect in D-dimensional dS

- The Schwinger effect in a constant E in a D-dimensional dS should be independent of t and $x_{\|}$ due to the symmetry of spacetime and the field, and the integration of $k_{\|}$ gives the density of states D.
- dS radiation in E=0 limit and Schwinger effect in H=0 limit

$$\frac{d^D N_{\mathrm{dS}}}{dt d^{D-1}x} = \frac{(2|\sigma|+1)H^2 S_\mu}{2(2\pi)} \int \frac{d^{D-2}k_\perp}{(2\pi)^{D-2}} \left(\frac{e^{-(S_\mu - S_\lambda)} \pm e^{-2S_\mu}}{1-e^{-2S_\mu}} \right)$$

$$S_\mu = 2\pi \sqrt{\left(\frac{qE}{H^2}\right)^2 + \left(\frac{m}{H}\right)^2 - \left[\left(\frac{D-1}{2}\right)^2\right]}, \quad S_\lambda = 2\pi \frac{qE}{H^2} \left(\frac{qE/H}{\sqrt{(qE/H)^2 + \vec{k}_\perp^2}} \right)$$

Schwinger Effect in Near-extremal RN Black Hole

Interpretation of Schwinger Effect

- Thermal interpretation of Schwinger formula for charged scalars (upper signs) and fermions (lower signs) in spherical harmonics [SPK, Lee, Yoon ('15)]

$$N_{NBH} = \left(\frac{e^{-\frac{\bar{m}}{T_{RN}}} - e^{-\frac{\bar{m}}{\bar{T}_{RN}}}}{1 \pm e^{-\frac{\bar{m}}{\bar{T}_{RN}}}} \right) \times \left(\frac{1 \mp e^{-\frac{\omega - qA_0}{T_H}}}{1 + e^{-(\frac{\omega - qA_0}{T_H} + \frac{\bar{m}}{T_{RN}})}} \right),$$

$$T_{RN} = T_U + \sqrt{T_U^2 - (\frac{1}{2\pi Q})^2}, \quad \bar{T}_{RN} = T_U - \sqrt{T_U^2 - (\frac{1}{2\pi Q})^2}$$

$$T_U = \frac{qE_H / \bar{m}}{2\pi} = \frac{q}{2\pi \bar{m} Q}$$

Schwinger Effect and Hawking Radiation

- Thermal interpretation of Schwinger formula for charged scalars and fermions [SPK, Lee, Yoon ('15); SPK ('15)]

$$N_{NBH} = e^{\frac{\bar{m}}{T_{RN}}} \times \underbrace{\left(\frac{e^{-\frac{\bar{m}}{T_{RN}}} - e^{-\frac{\bar{m}}{\bar{T}_{RN}}}}{1 \pm e^{-\frac{\bar{m}}{\bar{T}_{RN}}}} \right)}_{\substack{\text{Schwinger Effect in } AdS_2 \\ \text{Cai \& SPK JHEP ('14)}}} \times \underbrace{\left(\frac{e^{-\frac{\bar{m}}{T_{RN}}} \left(1 \mp e^{-\frac{\omega - qA_0}{T_H}}\right)}{1 + e^{-\frac{\omega - qA_0}{T_H}} e^{-\frac{\bar{m}}{T_{RN}}}} \right)}_{\substack{\text{Schwinger Effect in Rindler Space} \\ \text{Gabriel \& Spindel AP ('00)} \\ \text{Hawking Radiation of charges}}}$$

Schwinger Effect in Near-extremal Kerr-Newman BH

Chen, SPK, Tang, in preparation ('16)

Near-Horizon Geometry

- Kerr-Newman (KN) black hole: $M, Q, a = \frac{J}{M}; r_0^2 \equiv Q^2 + a^2$
- Near-horizon geometry warped AdS_2 of near-extremal KN BH

$$r \to r_0 + \varepsilon r\,,\;\; \varphi \to \varphi + \frac{a}{r_0^2 + a^2} t\,,\;\; t \to \frac{r_0^2 + a^2}{\varepsilon} t\,,\;\; M \to r_0 + \frac{(\varepsilon B)^2}{2r_0}$$

$$ds^2 = \left(r_0^2 + a^2 \cos^2\theta\right)\left(-\left(r^2 - B^2\right)dt^2 + \frac{dr^2}{r^2 - B^2} + d\theta^2\right)$$

$$+ \frac{\left(r_0^2 + a^2\right)\sin^2\theta}{r_0^2 + a^2\cos^2\theta}\left(d\varphi + \frac{2ar_0}{r_0^2 + a^2} r dt\right)^2,$$

$$A = -Q\left(\frac{r_0^2 - a^2\cos^2\theta}{r_0^2 + a^2\cos^2\theta} r dt + \frac{r_0 a \sin^2\theta}{r_0^2 + a^2\cos^2\theta} d\varphi\right)$$

Schwinger Effect for Charged Scalars

- Schwinger formula for charged scalars in spheroidal harmonics in near-extremal KN BH

$$N_{NKN} = \left(\frac{e^{-(S_a - S_b)} - e^{-(S_a + S_b)}}{1 + e^{-(S_a + S_b)}} \right) \times \left(\frac{1 - e^{-(S_c - S_a)}}{1 + e^{-(S_c - S_b)}} \right),$$

$$S_a = 2\pi \frac{qQ^3 - 2nar_0}{r_0^2 + a^2}, \quad S_b = 2\pi \sqrt{\left(\frac{S_a}{2\pi} \right)^2 - m^2 \left(r_0^2 + a^2 \right) - \lambda - \frac{1}{4}},$$

$$S_c = 2\pi \frac{\omega}{B}$$

Interpretation of Schwinger Effect

- Thermal interpretation of Schwinger formula for charged scalars in spheroidal harmonics in near-extremal KN BH

$$N_{NKN}=\underbrace{\left(\frac{e^{-\frac{\overline{m}}{T_{KN}}}-e^{-\frac{\overline{m}}{\overline{T}_{KN}}}}{1+e^{-\frac{\overline{m}}{\overline{T}_{KN}}}}\right)}_{\text{Extremal KN BH}}\times\underbrace{\left(\frac{1-e^{-\frac{\omega_t}{T_H}+\frac{\overline{m}}{T_M}}}{1+e^{-\frac{\omega_t}{T_H}+\frac{\overline{m}}{\overline{T}_M}}}\right)}_{\text{Factor for Near-extremal KN BH}}$$

$$T_{KN}=T_U+\sqrt{T_U^2+\frac{R}{8\pi^2}},\quad \overline{T}_{KN}=T_U-\sqrt{T_U^2+\frac{R}{8\pi^2}}$$

$$\frac{2}{T_M}=\frac{1}{\overline{T}_{KN}}+\frac{1}{T_{KN}},\quad \frac{2}{\overline{T}_M}=\frac{1}{\overline{T}_{KN}}-\frac{1}{T_{KN}}$$

$$T_U=\frac{qQ^3-2nar_0}{2\pi\overline{m}\left(r_0^2+a^2\right)^2},\quad R=-\frac{2}{r_0^2+a^2},\quad \overline{m}=\sqrt{1+\frac{\lambda+1/4}{m^2\left(r_0^2+a^2\right)}}$$

Conclusion

- The in-out formalism is consistent and systematic QFT method for vacuum polarization and vacuum persistence in backgrounds (gauge and/or curved spacetimes) [cf. worldline formalism and instanton in progress with Schubert.]
- The vacuum polarization of QED in (A)dS and near-extremal RN black hole exhibits the gravity-gauge relation (or AdS/CFT).
- The production of charged particles from an near-extremal RN and Kerr-Newman black hole shows a strong interplay of the Schwinger effect and the Hawking radiation and may have a thermal interpretation.