Bridges Between Noncommutative Algebra and Algebraic Geometry: 16w5088

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1 Overview of the Field

Noncommutative algebra is a rich and diverse field that has influences rooted in algebraic and differential geometry, representation theory, algebraic combinatorics, mathematical physics, and other areas. Rapid developments in noncommutative algebra, especially in noncommutative algebraic geometry, influence many other mathematical disciplines. We briefly give an overview of some of the main trends that shape our field at this time, with an emphasis on the areas represented during the workshop.

1. **Noncommutative surfaces.** Since curves, having one parameter, are close to being commutative, (or finitely generated modules over their centers), as shown in [5, 6], it is natural to study noncommutative surfaces next. In algebraic geometry there is a classification of surfaces with the most crucial results being Castelnouvo's and Enriques' which characterize rational and ruled surfaces respectively. Artin's conjecture predicts that noncommutative surfaces are either rational, ruled or module finite over their centers [2]. The study of noncommutative rational surfaces produced seminal work in this area [7, 8], as well as new techniques and objects such as twisted homogeneous coordinate rings. Recently, there have been remarkable developments in the theory of noncommutative surfaces (and 3-folds) that are finite modules over their centers and these results extend several commutative programs to the noncommutative setting.

2. **Birationally commutative surfaces and birationally Sklyanin surfaces** In [31, 48, 49, 50] Keeler, Rogalski, Sierra and Stafford have defined and studied a family of noncommutative surfaces that are birationally commutative, some of which are naïve blowups of commutative surfaces. These surfaces behave pathologically in some ways; for example, while they are often noetherian, they do not retain the noetherian property when one tensors with certain commutative noetherian rings. They have shown that all birationally commutative surfaces that are generated in degree 1 are either part of this new family of algebras or are twisted homogeneous coordinate rings. Recently, Rogalski-Sierra-Stafford [46, 47] have started studying noncommutative surfaces that are birationally equivalent to Sklyanin algebras of dimension three. By using noncommutative blowups, Rogalski-Sierra-Stafford have classified all connected graded subalgebras of Sklyanin algebras of dimension three which are birationally equivalent to these Sklyanin algebras and are maximal with respect to inclusion. These results are crucial to understanding general noncommutative surfaces. Rogalski's talk was based on his joint work with Sierra and Stafford.

3. **Noncommutative Crepant Resolutions.** The study of noncommutative crepant resolutions represents a fundamental connection between resolutions of (commutative) singularities and noncommutative algebra. Noncommutative algebras can be used to build resolutions as moduli spaces and exhibit derived equivalences between them. They are also important in applications of algebraic geometry to string theory. The area began with Van den Bergh's definition [52, 53] and his new approach to Bridgeland's proof of Orlov's conjecture that crepant resolutions of terminal Gorenstein singularities are derived equivalent. Recent results in this area include work of Iyama-Wemyss [29, 30] and Buchweitz-Leuschke-Van den Bergh [12, 13] which build resolutions of singularities from noncommutative algebras. This area was touched upon in some of the talks, including the talk of Faber, which we describe in detail below.

4. **(Twisted or skew) Calabi-Yau algebras.** Calabi-Yau triangulated categories were introduced by Kontsevich in 1998 and Calabi-Yau algebras were introduced by Ginzburg in 2006 as a noncommutative version of the coordinate rings of Calabi-Yau varieties. Since, the study of Calabi-Yau algebras/categories has been related to a large number of research areas, including quivers with superpotentials, differential graded algebras, cluster algebras/categories, string theory and conformal field theory, noncommutative crepant resolutions, and so forth. Twisted or skew Calabi-Yau algebras are a generalization and a companion of Calabi-Yau algebras. It is well-known that all connected graded Artin-Schelter regular algebras are twisted Calabi-Yau and that only a subclass of Artin-Schelter regular algebras are Calabi-Yau. One invariant attached to a twisted Calabi-Yau algebra is its Nakayama automorphism. Several homological identities that relate the Nakayama automorphism with other invariants have been proved. These identities helps to understand several crucial questions concerning group/Hopf actions and smash products, quantum groups, noncommutative resolutions, twisted superpotentials, and noncommutative McKay correspondence. Calabi-Yau algebras and their twisted counterparts were discussed in the talks of

Mori, Reyes, Sierra and others.

5. **Hopf Algebra Actions.** The invariant theory of finite group/finite dimensional Hopf algebra actions on noetherian Artin-Schelter regular algebras has been developed, see [33, 34, 35] and a nice survey by Kirkman [32], which includes noncommutative versions of several classical results such as the Watanabe theorem (a criterion for the fixed subrings being Gorenstein), the Shephard-Todd-Chevalley theorem (a criterion for the fixed subring being regular) and the Kac-Watanabe theorem (a criterion for the fixed subrings being a complete intersection). The classification of finite quantum subgroups of quantum $\mathrm{SL}_2(\mathbb{C})$, namely, so-called quantum binary polyhedral groups, has been completed in [17], leading to the study of the Kleinian or Du Val singularities of Hopf actions on noncommutative surfaces and noncommutative McKay correspondence [18, 19]. Hopf actions on the Weyl algebras and commutative domains have recently been studied by Cuadra, Etingof, Walton and others [21, 22, 24, 25, 26] with several surprising results. These studies are closely connected to the representation theory of finite groups/Hopf algebras, homological aspects of noncommutative algebra and noncommutative algebraic geometry. These topics were discussed in the talks of Walton and Kirkman.

2 Recent Developments and Open Problems

We summarize a few of the main open problems in noncommutative algebra, and in particular, noncommutative algebraic geometry, today. We place emphasis on the problems that were viewed as most important by speakers and participants at the conference and we do not claim to give a complete list of all of the most important open problems in the area.

1. *Artin's Conjecture*

   Artin [2] conjectured that every division algebra of transcendence degree two that arises from a noncommutative projective surface is either finite-dimensional over its center or is isomorphic to a Sklyanin division ring or to a division ring of the form $K(x;\sigma,\delta)$, where K is the function field of a smooth projective curve, σ is an automorphism of K, and δ is a σ-derivation of K. This has been one of the major open problems in noncommutative algebraic geometry for more than twenty years. By looking at rank one discrete valuations of the division algebras in Artin's list, it can be shown that they are non-isomorphic. This conjecture is very natural after the landmark work of Artin-Stafford [5, 6] that classified all noncommutative projective curves (equivalently, noncommutative graded domains of Gelfand-Kirillov dimension two). It is generally believed that the conjecture is currently beyond available methods, but there has been a large amount of work devoted to understanding this conjecture. As some examples, we note the work of Chan, Hacking, Ingalls, Kulkarni [14, 15, 16], the manuscript by Artin and de Jong, and the series of papers by Keeler, Rogalski, Sierra and Stafford [31, 46, 47, 48, 49, 50], some of which look at classifying projective surfaces in a given birational class on Artin's list.

2. *Noncommutative McKay Correspondence*

One influential program in noncommutative algebra is the noncommutative McKay correspondence. This program is highly interdisciplinary, involving the study of (1) Group/Hopf algebra actions on noncommutative algebras/schemes, (2) quotient singularities of noncommutative schemes, (3) representation theory of Hopf algebras and fixed subrings, (4) homological algebraic aspects (such that various derived categories are related), (5) combinatorial information coming from algebra, geometry and representation theory (such as McKay quivers and ADE quivers), and most importantly, the connections between these fields. Some work in dimension 2 has been finished [17, 18, 19]. As the classification of Artin-Schelter regular algebras of dimension 3 is now known, thanks to the work of Artin, Schelter, Tate and Van den Bergh [3, 7, 8], this question could be tractable in dimension 3, and would lead to significant new mathematics. In the commutative case, the Auslander theorem is a key step in establishing the McKay Correspondence. One immediate open question in this program is the noncommutative version of the Auslander theorem [18, Conjecture 0.2]: Let H be a semisimple Hopf algebra and suppose that A is a noetherian Artin-Schelter regular graded H-module algebra such that the homological determinant of the H-action is trivial. Then is there a graded algebra isomorphism $A\#H \cong \mathrm{End}(A_{A^H})$?

3. *Noncommutative discriminants*

The noncommutative discriminant is a powerful invariant that has long been used to study orders and lattices in central simple algebras [45]. Recently, the noncommutative discriminant has effectively been used to tackle (1) the automorphism group of noncommutative algebras and the Tits alternative, (2) the isomorphism problem, (3) the Azumaya locus of algebras that are finite modules over their center, and (4) the Zariski cancellation problem in the noncommutative setting. Nguyen-Trampel-Yakimov introduced a new method of using Poisson primes to calculate successfully the discriminant for quantum matrices, the negative part of the quantized universal enveloping algebras, and quantum Schubert cell algebras at roots of unity [44]. Levitt and Yakimov then computed the discriminant for quantized Weyl algebras at roots of unity [37]. Gaddis-Kirkman-Moore applied other methods to deal with discriminants for Ore extensions and for skew group rings [27]. Despite these successes, it is still extremely difficult to compute the discriminant in the general noncommutative case, partly due to the fact that it is not easy to understand the Poisson structure in a general situation. One challenging question is to understand and calculate more effectively the discriminant from different viewpoints. One should relate this question to the classification of group actions on noncommutative algebras. Another interesting question is how to use discriminants to study singularities of the center of noetherian PI Artin-Schelter regular algebras. Further, it is also essential to understand how the discriminant is related to other significant invariants such as superpotential and Hochschild cohomology.

3 Presentation Highlights

A common theme throughout the workshop was the high quality of the lectures at this conference; indeed, there were many excellent lectures delivered over the course of the week. We include here some of the highlights.

Walton spoke on her work with Chirvasitu and Wang [20], in which they defined a universal quantum group that preserves a pair of Hopf comodule maps whose underlying vector space maps are preregular forms defined on dual vector spaces. This construction generalizes a construction of Bichon-Dubois-Violette [10], where the target of these comodule maps is the base field. Using this new construction, she showed how one can recover various earlier constructions of quantum groups, including those introduced by Dubois-Violette-Launer [23], Takeuchi [51], Artin-Schelter-Tate [4], and Mrozinski [41]. Particularly their work gives an explicit presentation of a universal quantum group that coacts simultaneously on a pair of N-Koszul Artin-Schelter regular algebras with arbitrary quantum determinant.

Kirkman gave a wonderful talk on extending a classical result of Auslander, who proved that when a finite linear group contains no reflections and acts naturally on a polynomial ring with fixed subring B, then the skew group algebra generated by the polynomial ring and the group is isomorphic to the ring of B-linear endomorphisms of the polynomial ring. A common theme in the field of Hopf algebra actions is to extend results of this flavor to the setting of non(co)commutative Hopf algebra actions on Artin-Schelter regular algebras, where one must define what one means by containing no "reflections". Bao-He-Zhang [9] developed the notion of pertinency, and used this notion to prove the Auslander Theorem for certain group/Hopf actions on various Artin-Schelter regular algebras. Kirkman touched upon work in progress with Gaddis, Moore and Won that proves the Auslander Theorem for the permutation action of S_n on a class of noetherian PI Artin-Schelter regular algebras and her work with Chan, Walton and Zhang [17] that proves the Auslander theorem for Artin-Schelter regular algebras of dimension 2 with the action of a semisimple Hopf algebra H so that A is a graded H-module algebra under an action that is inner faithful and has trivial homological determinant.

Coinciding somewhat with Kirkman's talk, Faber spoke on joint work with Buchweitz and Ingalls [11] also regarding the Auslander theorem, albeit from a completely different point of view. Here we have G, a finite subgroup of $\mathrm{GL}(n, K)$ for a field K whose characteristic does not divide the order of G and which acts naturally on the polynomial ring in n variables. Unlike Kirkman's talk, which dealt with extending the result of Auslander to Hopf algebra actions in which there are no "reflections", this took an orthogonal point of view and looked at what happens when G has reflections. By studying the geometry of the discriminant of the group action and using the theory of noncommutative resolutions of singularities, the authors were able to extend the Auslander theorem on the algebraic version of the McKay correspondence to reflection groups.

For many, one of the highlights of the workshop was the work of Yakimov [37, 44] on noncommutative discriminants. Discriminants play a key role in the study of PI algebras, orders in central simple algebras, Azumaya loci of PI algebras, and the study of the isomorphism and automorphism problems for PI algebras. Previously, they were computed only for very few PI algebras and with great difficulty. Yakimov presented a general method for computing discriminants of PI algebras coming from Poisson geometry. From a different perspective the technique builds a bridge to the theory of discriminants of number fields, where factorizations into primes are replaced by factorizations into Poisson primes.

There were several interesting talks concerning the structure and classification of Artin-Schelter regular algebras. Mori's talk was based on his joint work with Smith and Ueyama on Artin-Schelter regular algebras of global dimension three [38, 39, 40]. It is well-known that every N-Koszul Artin-Schelter regular algebra is determined by a twisted superpotential. They proved that, in dimension three, the correspondence twisted superpotential is uniquely determined by the algebra. Using the twisted superpotential associated to an Artin-Schelter regular algebra A, they computed (1) the Nakayama automorphism of A, (2) the group of graded algebra automorphisms of A, and (3) the homological determinant of a graded algebra automorphism of A.

Another Artin-Schelter regular algebra talk was given by Reyes. As mentioned before, a connected graded algebra is Artin-Schelter regular if and only if it is twisted Calabi-Yau. In joint work with Rogalski, Reyes showed that when A is graded but not connected graded, the twisted Calabi-Yau property is also equivalent to a suitable analogue of the Artin-Schelter regularity in the sense of Martinez-Villa and Minamoto-Mori. Reyes also described the structure and properties of graded twisted Calabi-Yau algebras in dimension at most 2.

Sierra gave an overview of her interesting joint work with Lecoutre [36]. In this work, they generalize a construction of Pym and construct a family of Calabi-Yau Artin-Schelter algebras, $R(n)$, which gives a Poisson deformation of a polynomial ring in n variables for each n. She then went on to give an overview of this new Poisson bracket and showed that the Poisson spectra of the associated Poisson algebras are homeomorphic to the prime spectra of Zhang twists of her original algebras $R(n)$.

Hochschild cohomology is another topic that underwent extensive discussion. Negron gave a survey on his joint work with Schedler and Witherspoon on the Hochschild cohomology of global quotient orbifolds (and other work in [42, 43]). A global quotient orbifold is the stack quotient of a quasi-projective scheme by a finite group in characteristic 0. The vector space structure on the cohomology of such an object was given in a recent paper of Arinkin-Căldăraru-Hablicsek [1]. One of Negron's main results is to describe the algebraic structure of the Hochschild cohomology in terms of the geometries of the fixed spaces under the actions of individual group elements.

A complete list of talks can be found at the BIRS website.

4 Scientific Progress Made

As a result of this meeting, many people working in different areas of noncommutative algebra were able to engage in fruitful discussions and start projects with people working on related areas (see the section on testimonials of participants for examples). In fact, there are many examples of projects that were begun at this meeting. We note that this meeting had many younger researchers, postdocs, and graduate students and this meeting was particularly useful in allowing them to network and create joint projects with more senior researchers. One of the often repeated remarks made by different participants was an interest in the new work of Yakimov, which brought together many new and exciting ideas.

5 Outcome of the Meeting

This workshop brought together 41 researchers, which included a mix of Ph.D. students, postdocs, and senior researchers from Europe, China, and North America, working in different areas of noncommutative algebra and geometry, including ring theory, noncommutative algebraic geometry, representation theory, the study of Hopf algebra actions, and quantum groups.

Many of the participants noted that this was a great conference with many interesting talks. The program was full of lectures highlighting recent progress on difficult problems and we include here some testimonials from participants about research programs that they started at Banff and thoughts on results disseminated during the lectures.

Chelsea Walton said: "I enjoyed Richard Ng's talk on "Traces of Powers of Antipodes," joint with Cris Negron. He gave a nice introduction to invariants of representation categories of Hopf algebras ("gauge invariants") including various examples. Much as (generalized) Frobenius-Schur indicators are used to tell apart representation categories, knowing that the sequence of traces of powers of antipodes (under the Chevalley property condition) is a gauge invariant is a super useful result. It was only a 30 minute talk, but I could have listened for another 60 minutes—I like this material and the way in which Ng gives talks. He's super clear and I always learn a lot from him. I'm currently thinking about the dual problem, invariants of co-representation categories of Hopf algebras, in a joint project with Adriana Mejia, Susan Montgomery, Sonia Natale, and Maria Vega (which also began at BIRS this past March). So, Negron and Ng's work is definitely on my radar—it's a great result."

Ragnar-Olaf Buchweitz said: "Concerning mathematical content, I list the four talks I liked best (refraining from any comments on Eleonore, as that is joint work). For me, the best talk was the one by Yakimov. This should be a game changer, as they say. Beautiful mathematics delivered impeccably. I greatly appreciated the talks by Sarah Witherspoon, Ellen Kirkman, and Susan Sierra. Nice surveys with interesting results in the first two cases, beautiful results nicely presented in Sue's case.

I commend you for giving so many young researchers an opportunity to present their work. I had heard of some of them, but hadn't met several of them before (e.g. Manuel Reyes, Matt Satriano who also gave nice talks.) Colin, Eleonore and I appreciated the opportunity to continue working together, we used every lunch break to do so."

Alexandru Chirvasitu said: "This was my first time visiting BIRS, and it has been every bit as impressive as conversations with my colleagues had led me to believe. I'd like to echo one of them in saying that I feel very lucky to have been able to be there.

The conference was very conducive to collaboration (there seem to be a couple brewing as far as I am concerned) and very intense, and the Station and its program seem tailor made to foster this type of atmosphere (no distractions, beautiful place, availability of any amenity one might think of, research facilities / tools of all types, etc. etc.).

All in all it was a terrific week, and I hope to be able to attend one of these meetings again sometime."

Paul Smith said: "It was good to see some fresh faces, and hear some fresh voices, like Negron, Woods, Belmans, Chirvasitu, Satriano, Faber, Reyes.

It was good to have some experts in algebraic geometry there. I learned a bit more about deformation theory and low-dimensional Hochschild cohomology (for rings and varieties and stacks) from Cris Negron, Max Lieblich, Ragnar Buchweitz, and Pieter Belmans. I now feel more confident about the precision and worth of some "deformation-theoretic" questions I have been thinking about recently and, related to that, how to formalize questions about the behavior in families of algebras and abelian categories that are important to noncommutative algebraic geometry. Belmans, Chirvasitu, and I, and Mori, had some interesting conversations about Poisson structures and deformation quantization (we also had some interesting email exchanges with Brent Pym about these things)."

Louis Rowen wrote: "I think it was an excellent meeting. It had an excellent balance between geometry and algebra, showing how algebra still impacts meaningfully in related areas. I was very interested to see the diverse work in AS regular algebras, which has motivated me to take a closer look at them.

Also there was an impressive mix of generations, which gave me the opportunity to meet some sharp younger mathematicians."

Lutz Hille wrote: "One highlight have been the talks of Ellen Kirkman and Eleonore Faber, which I had preferred to see more closely. Both deal with fundamental open questions: the action of a group, respectively a Hopf algebra, on a commutative (or even noncommutative) ring. Whereas group actions are studied since a long time, the action of a Hopf algebra is more recent research. Both are, in my opinion, fundamental questions in noncommutative algebraic geometry. The case Faber considered is certainly one of the main open questions, the case Kirkman has considered seem to be new and hopefully the beginning of a new subject with many applications.

The talk of Ken Goodearl was the one most closest to my own research, I work myself on problems about irreducible components. It was nice to see somebody else working on this and the case (he considered mainly) of all relations of a fixed length is certainly a nice first step. Even if I know, the general problem is hopeless, since any algebraic variety essentially occurs as a representation space, even for a very small dimension vector, there might be good classes where one can solve the problem. Thus, one has to concentrate on particular relations.

The most exciting talk was certainly Cris Negron, I was deeply impressed about his power and the deepness of his results, I certainly keep in contact with him and I got ideas how this is related to my own work."

Martin Lorenz said the following: "This meeting gave me the opportunity to catch up on current developments, especially in the area of Hopf algebras and their actions. I found the talks by Walton, Witherspoon, Ng, Kirkman and Chirvasitu to be very useful in that regard, but I enjoyed the lectures by Yakimov, Goodearl and Launois very much as well."

Milen Yakimov said: "I have learned a great deal of math both in terms of new directions of research and methods that I can incorporate in my own research. I was particularly excited that the workshop represented a very wide range of topics and not a single concrete direction: from Hopf symmetries of regular algebras, structure of Jacobian algebras for superpotentials and twisted CY algebras, noncommutative blow ups, to structure of division algebras, the McKay correspondence to reflection groups, and birational automorphisms of projective varieties. The breadth of the workshop represents the dynamics of the area and the numerous connections with other fields that emerged in the last years. During the workshop I had the opportunity to initiate 2 research projects with Tim Hodges. The first is on establishing a Levi type decomposition theorem for all Belavin-Drinfeld quantum groups. The second is on describing a large class of noncommutative projective spaces via quantum Hamiltonian reduction."

Following this workshop, there will be a closely related workshop at ICMS, Edinburgh in the summer of 2017, entitled "Linking noncommutative rings and algebraic geometry". This BIRS conference will inevitably play a major role in shaping the research done from now until this conference.

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