Combinatorial Reconfiguration

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1 Overview of the Field

In the decade since the framework of combinatorial reconfiguration was proposed [13], the area has attracted the attention of researchers in a broadening range of related research areas. Unlike in conventional optimization problems, where the goal is to find a solution with the most desirable property or properties (such as minimum size or maximum weight), reconfiguration studies the relationship among feasible solutions. In a real-life setting, knowing the relationships among solutions makes it possible to determine whether and how it is possible to transform one solution step-by-step into another solution with minimal stoppage of a production line or minimal disruption to clients. Additional examples of reconfiguration range from classical puzzles and games (where a transformation might be the rotation of a *Rubik's cube* or the sliding of a tile) to well-studied classical optimization problems (where a transformation might change, add, or delete one element of a solution).

The study of reconfiguration problems brings together problems and techniques in a variety of fields in mathematics and computer science such as combinatorial game theory, enumeration, random sampling via Monte Carlo Markov chains, bioinformatics, discrete geometry, and statistical physics. Many algorithmic and combinatorial questions related to this framework have been introduced and studied in the last few years; in addition, many old conjectures can be reformulated in this setting. In particular, many classical combinatorial results, such as Vizing's theorem and the 5-color theorem, are based on reconfiguration arguments. References for the listed topics and further information on combinatorial reconfiguration can be found in two recent surveys on the topic [12, 16].

More formally, for a given classical problem, selected type of transformation step, and definition of feasibility, a *reconfiguration graph* for an instance of the problem is formed by creating a vertex in the graph for each feasible solution and an edge for each transformation from one solution to another by the application of a single step. The challenges in solving reconfiguration problems arise from the fact that the number of solutions is typically exponential in the size of the instance, requiring more nuanced approaches than simply generating and exploring the reconfiguration graph.

One natural reconfiguration problem is the *reachability problem*, where the goal is to determine whether there is a path connecting the vertices corresponding to two given solutions. Although it may not be surprising that the reachability problem is intractable (PSPACE-hard) for many classical problems, more surprising is the fact that there is not a clear correspondence between tractability of classical problems and tractability of reachability problems derived from those classical problems. When reachability proves to be intractable, further investigations are required to determine the dividing lines between classes of instances for which reachability is tractable and classes of instances for which it is not. In such cases, there is not always an obvious relationship between "easy" classes for classical problems and "easy" classes for reachability.

The idea of trying to solve the reachability problem simultaneously for all possible pairs of solutions results in the idea of determining whether or not a reconfiguration graph is connected. Once it has been determined that there exists a path between a pair of solutions, next steps include algorithms to find a path or to find a shortest such path. In turn, such endeavours raise the question of the diameter of the graph (the maximum length of the shortest path between any pair of vertices). Connectivity and diameter of the reconfiguration graphs are of particular interest in random sampling. Researchers have also considered the impact of varying the choice of the type of transformation step, and have studied additional properties of the reconfiguration graph, such as determining whether the reconfiguration graph is isomorphic to the instance [11, 1], and when the reconfiguration graph is Hamiltonian [4].

Questions that are now framed as reconfiguration problems have been asked about specific classical problems as far back as the 1800's [14] (for the 15-puzzle) by both computer scientists and mathematicians. Examples include Hirsch's conjecture in combinatorial optimization and the flip distance [15] between triangulations. Only recently has the reconfiguration framework been defined and become a focus of study; such work has brought together mathematicians and computer scientists who previously were working individually on reconfiguration of a variety of different classical problems. These collaborations have resulted in significant progress on the reconfiguration of classical problems, both individually and grouped in classes, and the understanding of the reconfiguration framework itself, including both restrictions and extensions.

2 Presentation Highlights

The presentations consisted of tutorials, invited talks, contributed talks, and mentoring sessions. The summaries of the presentations are lightly edited versions of material supplied by the speakers themselves.

2.1 Invited Tutorials

2.1.1 Takehiro Ito (Tohoku University): Invitation to Combinatorial Reconfiguration

I gave a broad introduction talk for combinatorial reconfiguration. Combinatorial reconfiguration studies reachability and related questions over the solution space formed by feasible solutions of an instance of a combinatorial search problem. The study of reconfiguration problems is motivated from a variety of fields such as puzzles, statistical physics, and industry. I briefly explained these backgrounds and applications. In particular, the application to power distribution networks was interesting also for researchers who study reconfiguration problems for a long time. I also explained trends of research on combinatorial reconfiguration, and tried to explain why reconfiguration problems are difficult (and hence interesting) to solve.

2.1.2 Catherine Greenhill (UNSW Sydney): Markov chains, mixing time and connections with reconfiguration

After outlining what I understood reconfiguration people to be interested in, I discussed some of the basic theory about Markov chains, the fact that irreducibility of a reversible Markov chain is equivalent to connectivity of the reconfiguration graph, and then discussed the canonical path method for analysing the mixing time of a Markov chain: this method uses paths through the reconfiguration graph in an intrinsic way.

2.1.3 Amer Mouawad (University of Bremen): Parameterized algorithms for reconfiguration problems

A *graph vertex-subset problem* defines which subsets of the vertices of an input graph are feasible solutions. We view a feasible solution as a set of tokens placed on the vertices of the graph. A *reconfiguration variant* of a vertex-subset problem asks, given two feasible solutions of size k, whether it is possible to transform one into the other by a sequence of token slides (along edges of the graph) or token jumps (between arbitrary vertices of the graph) such that each intermediate set remains a feasible solution of size k. Many algorithmic questions present themselves in the form of reconfiguration problems: Given the description of an initial system state and the description of a target state, is it possible to transform the system from its initial state into the target one while preserving certain properties of the system in the process? Such questions have received a substantial amount of attention under the so-called combinatorial reconfiguration framework. We consider reconfiguration variants of three fundamental graph vertex-subset problems, namely INDEPENDENT SET, DOMINATING SET, and CONNECTED DOMINATING SET [3]. We survey some older and more recent work on the parameterized complexity of all three problems when parameterized by the number of tokens k. The emphasis will be on positive results and the most common techniques for the design of fixed-parameter tractable algorithms.

2.2 Invited Talks

2.2.1 Jun Kawahara (Kyoto University): Invited talk: A ZDD-based solver for combinatorial reconfiguration problems

Joint work with Takehiro Ito, Yu Nakahata, Takehide Soh, Akira Suzuki, Junichi Teruyama, Takahisa Toda

In this talk, we introduce a solver for combinatorial reconfiguration problems based on a data structure called ZDD (Zero-suppressed binary Decision Diagram). A ZDD is a compact representation of a family of sets and provides various efficient operations such as taking the union and intersection of two families. It is known that given a graph, we can efficiently construct a ZDD representing the set of all the independent sets of the graph. Using this property, we propose a method for solving the independent set reconfiguration problem and other problems.

2.2.2 Jonathan Narboni (Labri): Vizing's conjecture holds

2.3 Contributed Talks

2.3.1 Daniel Cranston (Virginia Commonwealth University): Kempe Equivalent List Colorings

Joint work with Reem Mahmoud [6]

Mohar conjectured that if G is connected and k-regular, then each two of its k-colorings are k-Kempe equivalent. This was proved for $k = 3$ by Feghali, Johnson, and Paulusma [8] (with a single exception $K_2\square K_3$, also called the 3-prism) and for $k \geq 4$ by Bonamy, Bousquet, Feghali, and Johnson [2]. We prove an analogous result for list-coloring. For a list-assignment L and an L-coloring ϕ, a Kempe swap is called *L-valid* for ϕ if performing the Kempe swap yields another L-coloring. Two L-colorings are called *L-equivalent* if we can form one from the other by a sequence of L-valid Kempe swaps. Let G be a connected k-regular graph with $k \geq 3$. We prove that if L is a k-assignment, then all L-colorings are L-equivalent (again with a single exception $K_2\square K_3$). When $k \geq 4$, the proof is completely self-contained, so implies an alternate proof of the result of Bonamy et al.

2.3.2 Hiroshi Eto (Tohoku University): Reconfiguration of Regular Induced Subgraphs

Joint work with Takehiro Ito, Yasuaki Kabayashi, Yota Otachi, Kunihiro Wasa [7]

We introduced the problem of reconfiguring d-regular induced subgraphs in a graph, which generalizes the well-studied independent set reconfiguration problem. In the problem, we are given a graph G and d-regular induced subgraphs U_S and U_T of G, and we are asked to decide whether there is a reconfiguration sequence between U_S and U_T under the token jumping (TJ) or token sliding (TS) rule. Therefore, the problem is exactly the independent set reconfiguration problem when $d = 0$.

In the talk, we systematically study the complexity of the problem, in particular, on chordal graphs and bipartite graphs. Our results for $d \geq 1$ give interesting contrasts to known ones for $d = 0$. More specifically, on chordal graphs, both TS and TJ rules have the same complexity (i.e., PSPACE-complete) for $d \geq 1$, whereas they are different for $d = 0$: the problem under TS rule remains PSPACE-complete, while the problem under TJ rule can be solved polynomial time.

2.3.3 Henning Fernau (Universität Trier): Order Reconfiguration under Width Constraints

Joint work with Emmanuel Arrighi and Mateus de Oliveira Oliveira from Bergen, Norway and with Petra Wolf from Trier, Germany

We consider the following order reconfiguration problem: Given a graph G together with linear orders ω and ω' of the vertices of G, can one transform ω into ω' by a sequence of swaps of adjacent elements in such a way that at each time step the resulting linear order has cutwidth (pathwidth) at most ℓ? We show that this problem always has an affirmative answer when the input linear orders ω and ω' have cutwidth (pathwidth) of at most $\ell/2$. This result also holds in a weighted setting.

Using this result, we establish a connection between two apparently unrelated problems: the reachability problem for two-letter string rewriting systems and the graph isomorphism problem for graphs of bounded cutwidth. This opens an avenue for the study of the famous graph isomorphism problem using techniques from term rewriting theory.

As further avenues of research, we suggest to study this cutwidth (pathwidth) reconfiguration problem in more details. More generally, we propose to look into other reconfiguration problems based on swapping neighboring elements in a finite linear order. For instance, dynamic aspects of the One-Sided Crossing Minimization problem (rather famous in Graph Drawing) lead to another special case of this type of reconfiguration problems.

2.3.4 Sevag Gharibian (Paderborn University): Reconfiguration in the Quantum Setting

Based on joint work with Jamie Sikora [10], Johannes Bausch and James Watson [18], and Dorian Rudolph (draft in preparation)

In this talk, we give a gentle introduction to the quantum analogue of reconfiguration problems for SAT. This includes a brief primer on quantum computation and how one defines ”quantum SAT”, i.e. the local Hamiltonian problem. We then formalize the reconfiguration problem of Ground State Connectivity, which asks whether two ground states of a local Hamiltonian H are ”connected” through the ground space of H via a short sequence of 2-qubit quantum gates. We then cover various results on GSCON over the last years, including: (1) GSCON with poly-length sequences of gates is QCMA-complete, where QCMA is a quantum analogue of Merlin-Arthur (MA), (2) GSCON on 1D, translation-invariant chains is QMAexp-complete, where QMAexp is the quantum analogue of NEXP, and (3) GSCON with exponentially many gates and inverse polynomial promise gap is in P, since we show that any pair of ground states of a local Hamiltonian H are connected in this setting, even up to inverse exponential precision.

2.3.5 Guilherme Gomes (Google): Some results on Vertex Separator Reconfiguration

We present the first results on the complexity of the reconfiguration of vertex separators under the three most popular rules: token addition/removal, token jumping, and token sliding. We show that, aside from some trivially negative instances, the first two rules are equivalent to each other and that, even if only on a subclass of bipartite graphs, TJ is not equivalent to the other two unless NP = PSPACE; we do this by showing a relationship between separators and independent sets in this subclass of bipartite graphs. In terms of polynomial time algorithms, we show that every class with a polynomially bounded number of minimal vertex separators admits an efficient algorithm under token jumping, then turn our attention to two classes that do not meet this condition: $\{\text{3P1}, \text{diamond}\}$-free and series-parallel graphs. For the first, we describe a novel characterization, which we use to show that reconfiguring vertex separators under token jumping is always possible and that, under token sliding, it can be done in polynomial time; for series-parallel graphs, we also prove that reconfiguration is always possible under TJ and exhibit a polynomial time algorithm to construct the reconfiguration sequence.

2.3.6 Sajed Haque (University of Waterloo): Labelled Token Sliding Reconfiguration of Independent Sets on Forest

2.3.7 Arnott Kidner (Memorial University of Newfoundland): Gamma-Switchable Homomorphisms

Informally, a (m, n)-mixed graph is a mixed graph whose edges are assigned m colours and arcs are assigned n colours. For a permutation π that acts on the edge colours, arc colours, and arc orientations, we say switching at a vertex v with respect to π changes the edges/arcs incident with v with the action of π. We show that it is polynomial time decidable to determine whether; for a fixed permutation group, there admits a sequence of switches on a (m, n)-mixed graph such that the resulting graph admits a homomorphism to a simple target on 2 vertices. This is accomplished using the reconfiguration graph.

2.3.8 Kshitij Gajjar (NUS, Singapore) : Reconfiguring Shortest Paths in Graphs and Abhiruk Lahiri (Charles University): Revisiting shortest path reconfiguration

In the Shortest Path Reconfiguration (SPR) problem, we are given two shortest paths and the goal is to transform one shortest path to the other by changing one vertex at a time so that all the intermediate configurations are also shortest paths. SPR has several real-world applications like repaving roads in a systematic way and cargo container stowage on ships.

We presented our work in two separate talks. In the first talk, presented by Kshitij Gajjar, we showed that SPR can be solved in polynomial time for many graph classes, including circle graphs, bridged graphs, the Boolean hypercube and bounded diameter graphs. Most of our proofs are by providing a complete characterization of the shortest paths for the graph class. We also explored a generalization of SPR known as k-SPR and showed that k-SPR is PSPACE-complete, even for some graph classes (viz. line graphs) for which SPR is known to be solvable in polynomial time.

In the second talk, presented by Abhiruk Lahiri, we continued our discussion on shortest path reconfiguration. We presented an alternative proof for the shortest path reconfiguration of interval graphs. We also showed that there exists an intersection graph of triangles such that the reconfiguration sequence can have a size exponential in the number of vertices. These are the only few results we know of the shortest path reconfiguration problem on geometric graphs.

The talks are based on joint work with Agastya Vibhuti Jha, Manish Kumar and Abhiruk Lahiri [9].

2.3.9 Stephanie Maaz (University of Waterloo): Parameterized Complexity of Reconfiguration of Atoms

The work presented was motivated by the challenges arising when preparing arrays of atoms for use in quantum simulation. It tackles the problem of reconfiguring one arrangement of tokens (representing atoms) to another using as few moves as possible; because the problem is NP-complete on general graphs as well as on grids, it focuses on presenting the parameterized complexity for various parameters, considering both undirected and directed graphs, and tokens with and without labels. For unlabelled tokens, the presentation went over the fixed-parameter algorithms under the following parameters: the number of tokens, the number of moves, and the number of moves plus the number of vertices without tokens in either the source or target configuration. It also presented the proof of the problem's intractability under both labelled and unlabelled tokens when parameterizing by the difference between the number of moves and the number of differences in the placement of tokens in the source and target configurations.

2.3.10 Thomas Suzan (G-SCOP): Reconfiguration of digraph homomorphisms

For a fixed graph H, the H-Recoloring problem asks whether given two homomorphisms from a graph G to H one can be transformed into the other by changing the color of a single vertex in each step and maintaining a homomorphism to H throughout. This problem generalizes the reconfiguration of graph colorings by changing the color of a single vertex at a time and has been studied mainly in the context of undirected graphs. In general, the H-recoloring problem is PSPACE-complete, but there are some classes of undirected graphs for which a polynomial algorithm is known. The most general algorithmic result so far has been proposed by Wrochna in 2014, who introduced a topological algorithm that solves the H-Recoloring problem in polynomial time when H is square-free. We show that his topological approach can be generalized to the setting of digraph homomorphisms. In particular we show that

1. if H is loopless, the corresponding reconfiguration problem admits a polynomial-time algorithm if H does not contain a 4-cycle of algebraic girth 0 and that
2. if H is reflexive (that is, H has a loop on each vertex), the problem admits a polynomial-time algorithm if H contains no triangle of algebraic girth 1 and no 4-cycle of algebraic girth 0.

While the first result is based on the so-called monochromatic neighborhood property that also plays a crucial role in Wrochna's algorithm, for the second result we introduce the so-called *push-or-pull property*, which allows us to work with the same topological approach in the reflexive digraph setting.

2.4 Mentoring sessions

2.4.1 Hugo Akitaya (Tufts University):Reconfiguration of District Maps

Motivated by applications in gerrymandering detection, we study a reconfiguration problem on connected partitions of a connected graph G. A partition of $V(G)$ is connected if every part induces a connected subgraph. It is desirable to obtain parts of roughly the same size, possibly with some (additive) slack s. A Balanced Connected k-Partition with slack s, denoted (k, s)-BCP, is a partition of $V(G)$ into k nonempty subsets, of sizes $n_1, \ldots, n_k$ with $|n_i - n/k| \leq s$, each of which induces a connected subgraph. We present an overview of complexity and algorithmic results for the reachability and connectivity problems using flips (changing the membership of a single vertex) and recombinations (merging two adjacent districts and re-splitting them into connected parts).

2.4.2 Reza Bigdeli (University of Waterloo): Disconnecting the Triangulation Flip Graph of Points in the Plane by Forbidding Edges

The *flip graph* for a set P of points in the plane has a vertex for every triangulation of P, and an edge when two triangulations differ by one flip that replaces one triangulation edge by another.

The flip graph is known to have some connectivity properties:

(1) the flip graph is connected;

(2) connectivity still holds when restricted to triangulations containing some constrained edges between the points;

(3) for P in general position of size n, the flip graph is $\lceil \frac{n}{2} - 2 \rceil$-connected, a recent result of Wagner and Welzl [17].

We introduce the study of connectivity properties of the flip graph when some edges between points are forbidden. An edge e between two points is a flip cut edge if eliminating triangulations containing e results in a disconnected flip graph. More generally, a set X of edges between points of P is a flip cut set if eliminating all triangulations that contain edges of X results in a disconnected flip graph. The flip cut number of P is the minimum size of a flip cut set.

We give a characterization of flip cut edges that leads to an $O(n \log n)$ time algorithm to test if an edge is a flip cut edge and, with that as preprocessing, an $O(n)$ time algorithm to test if two triangulations are in the same connected component of the flip graph. For a set of n points in convex position (whose flip graph is the 1-skeleton of the associahedron), we prove that the flip cut number is $n - 3$.

2.4.3 Hany Ibrahim (University of applied science Mittweida): Edge Contraction and Forbidden Induced Graphs

I gave a talk about edge contraction and forbidden induced graphs, in which I raised some questions by the end of the talk. While being in the working session about "token sliding and minimum vertex separators", I think I was able to suggest a constructive idea which is reducing the order of the graph by deleting vertices which are irrelevant to the problem. That is, to label every vertex either essential or irrelevant (maybe a vertex is irrelevant if it don't belong to any minimum vertex separator). Removing such irrelevant vertices would enable a specific lattice structure to form and eventually an encoding for the lattice. The idea is without proof yet, but looks interesting.

2.4.4 Jeffrey Kam (University of Waterloo): Extension of subgraph reconfiguration

I gave a mentoring session talk on extending some results in subgraph reconfiguration. For most part of the talk, I describe how one could show the NP-hardness of reconfiguring trees with path-width k or less, for any fixed k. I also briefly mentioned the NP-hardness result for reconfiguring graphs with tree-width k or less. Towards the end, I suggested some future directions for the project, such as extending to the multi-token model and the idea of a global minimum running buffer (GMRB) with respect to a graph property. In various subgraph reconfiguration problems under the vertex-variant, the PSPACE-completeness results are derived from the Shortest Path Reconfiguration (SPR) problem. So, our extension towards the multi-token model under the vertex-variant will likely utilize results from the k-Shortest Path Reconfiguration problem.

2.4.5 Rin Saito (Tohoku University): Reconfiguration of vertex disjoint shortest paths on split graphs

I introduced a new reconfiguration problem, called the Vertex-Disjoint Shortest Paths reconfiguration problem, which is a combination of the well-studied vertex-disjoint paths problem (studied from the 1970's) and the shortest path reconfiguration problem (introduced by Bonsma in 2010). I presented that the problem is PSPACE-complete for bipartite graphs, while I proposed a polynomial-time algorithm for split graphs.

3 Scientific Progress Made

The presentations as well as the open problem sessions provided a variety of starting points for discussions and collaborations among participants. Although most of the groups were in-person only or virtual-only, there were also several hybrid groups. Not only was there progress during the workshop itself, but also in subsequent meetings as groups continued to work on problems together.

The groups listed here are only the ones that were reported to the organizers; they can be viewed as a mere sampling of all the activity that the workshop fostered.

Because work listed here includes work in progress, details may be omitted.

3.1 Colouring reconfiguration

Collaborators: Daniel Cranston, Jeffrey Kam, Arnott Kidner, Ben Moore, Mortiz Mühlenthaler, Thomas Suzan

Question: Is 3-mixing in P for graphs embedded in a fixed surface Σ.

We mostly considered the torus, as the planar case was already solved by Johnson, van den Heuvel, and Cereceda [5]. We developed a better understanding of the planar algorithm, as Dan noted that if a graph has a basis for the cycle space that consists of 4-cycles, then G is 3-mixing. In the planar case, this happens to be sufficient, so we can restate the planar structure theorem in the following way:

Theorem: A graph G is 3-mixing if and only if G is bipartite, and contains a basis for its cycle space that consists of 4-cycles.

Given that a graph embedded on some surface Σ has a cycle space of "faces + generators for non-contractible cycles", the rephrasing suggests we should be looking at some sort of conditions on the generators for the cycle space to be sufficient for 3-mixing when embedded on some fixed surface. It is even interesting just in the torus, but we don't quite know how to do the torus yet. In particular, it's not clear when two vertices x, y of a graph embedded in the torus where $xy \notin E(G)$, can be identified and preserve the embedding.

There are some possible easier questions that also popped up:

Question: Is it true a planar graph G is C_5-mixing if and only if it has a cycle basis of 4,6 and 8 cycles?

Question: can a similar theorem to the 3-mixing result hold for C_4-free graphs?

Question: Is H-reconfiguration in P if each edge lies in at most one 4-cycle?

Question: Is H-reconfiguration PSPACE-complete if H contains a K_4?

As partial progress, consider the graph $H = (7, 2)-$circular clique plus an apex vertex. Since $(7, 2)$-circular recolouring reduces to C_7 recolouring by subdividing each edge of the input graph twice, we start with this, and then follow the original reduction for showing the 8-vertex wheel reconfiguration is PSPACE-complete, to seemingly show that H-recolouring is PSPACE-complete.

3.2 Cutwidth

Collaborators: Faisal Abu-Khzam, Nicolas Bousquet, Henning Fernau, Amer Mouawad, Naomi Nishimura, Vinicius dos Santos

The workshop provided an opporunity to determine a group of researchers interested in working on open problems presented by Henning Fernau. Meetings will take place after the workshop, when there is more time to schedule them in.

3.3 Graph colourings

Collaborators: Matthew Johnson, Carl Feghali

Although the authors had collaborated in the past, the workshop gave them an opportunity to reconnect in the examination of a new area of research.

3.4 Minimum st-Separator Reconfiguration

Collaborators: Siddharth Gupta, Clment Legrand-Duchesne, Tom C. van der Zanden, Guilherme C. M. Gomes, Reem Mahmoud, Amer Mouawad, Yoshio Okamoto, Vinicius F. dos Santos

The group started meeting during the workshop, and has been meeting regularly since then. The authors have results relating to both the token sliding and jumping models, considering both decision variant of the problem as well as the problem of determining a minimum length reconfiguration sequence.

3.5 Reconfiguring trees

Collaborators: Hugo Akitaya, Therese Biedl, Daniel Cranston, Jeffrey Kam, Théo Pierron, Vinicius dos Santos, Ryuhei Uehara

The problem is trying to transform one (unlabeled) tree into another by repeatedly deleting an edge and adding an edge (staying a tree at each step).

We consider two variants of the problem: "edge-flipping" (no constraint on the deleted/added edge pair) and "edge-sliding" (a single edge can be "slid" along one edge; if wx is an edge, then wy can be replaced by xy).

We studied reconfiguration problems, especially under the aspect of wanting approximation algorithms for the shortest reconfiguration path between two instances (a topic that appears to have been barely studied). We focused on the problem of reconfiguring trees under edge flips. We proved that this is APX-hard (even for caterpillars), but also showed that there exists an O(1)-approximation algorithm for reconfiguring between two caterpillars.

3.6 Shortest path reconfiguration

Collaborators: Nicolas Bousquet, Kshitij Gajjar, Abhiruk Lahiri, Amer Mouawad

The collaborators started to work on the reconfiguration of shortest paths from a parameterized point of view during the workshop. They continued to meet on a weekly regular basis since the end of the workshop. They were able to prove that the problem is $W[1]$-hard parameterized by $k+\ell$ on bounded degeneracy graphs (and by k on general graphs), k being the length of the path and ℓ, the length of the desired transformation. On the positive side, they show that the problem is FPT parameterized by k on nowhere dense graphs. The work is still in progress and many questions still have to be solved, in particular the complexity on restricted graph classes such as bounded treewidth graphs.

3.7 Other new connections

Many participants reported having made new connections at the workshop; a few samples are listed below:

- Sevag Gharibian reported new connections with work related to the quantum setting, notably Alexandre Cooper, Stephanie Maaz, Amer E Mouawad, and Naomi Nishimura.
- Arnott Kidner received useful feedback on his talk, such as the suggestion of looking at 'lights out' and the idea of posing the problem of Γ-switch equivalence as a game. Work has now started on the game variant of the problem, which has resulted in progress towards a result on Γ-switch equivalence on hypergraphs.

- Rin Saito reported that after his mentoring session, Ryuhei Uehara advised that the problem is probably solvable in polynomial time also for Ptolemaic graphs. He and his co-authors are planning to discuss the details with Ryuhei Uehara after the workshop.
- Jeffrey Kam reports that Ryuhei Uehara suggested that we should consider outerplanar graphs for the minimum running buffer problem, which eventually leads to the following problem: *Is there a minor closed graph property where it is not closed under k-subdivision but has a valid and bounded global minimum running buffer?*

4 Outcomes of the Meeting

The meeting not only achieved the intended goals, but also helped to shape ideas on how to conduct future meetings. We first discuss how the hybrid format facilitated our goals, and then discuss topics adapted and expanded from those listed in the proposal.

4.1 Learning from the challenges and opportunities resulting from a hybrid format

Our community benefited greatly from the hybrid format, allowing participation by those for whom in-person participation would be not be possible, due to financial constraints, family obligations, health issues, and other reasons. Due to the backlog in visa processing, three in-person participants had to cancel their travel plans at the last minute, but were still able to participate virtually. In addition, the ability to selectively participate, rather than commit an entire week to the workshop, facilitated the participation of those new to the area, whose work is currently only tangential to reconfiguration. This broaded exposure resulted in 77 participants, helping our community to thrive and grow.

BIRS was instrumental in ensuring that the in-person and virtual participants were able to interact constructively. Many participants provided feedback on the excellence of the infrastructure and staff support and how, in contrast to their experiences in other venues, it was easy for participants to hear and see others both during talks and during question periods and discussions. Our experience was so positive that we have chosen to continue to have hybrid meetings in the future.

The tools that we used to be able to communicate, including a Discord server and a shared Overleaf document, allowed collaborations and communications to take place. We were so pleased with the results that we've decided to continue to use them in the long term. We also made use of Gather Town as a social space for virtual participants, such as during coffee breaks.

The overwhelming success of the hybrid model, and in particular the infrastructure and support provided by BIRS, has influenced plans for future meetings of the research community.

We intend to retain the following features of the program structure:

- Introductions: Each participant, in-person or virtual, is given a chance to speak briefly about their topics of interest. Such a session makes it possible for each participant to have a chance to address the community, even if not later giving a presentation. Several participants mentioned how useful it was to find out whom else to contact in the future to work on problems of mutual interest.
- Tutorials: In order to welcome newcomers and broaden the community, we intend to continue to ask for in-depth presentations on various topics. The tutorials are aimed at non-experts in a particular subarea, serving not only to level the playing field for all participants, but also to add to the archive of videos, enriching the resources available to those who might wish to teach graduate courses in the area.
- Invited talks: New breakthroughs in the area are highlighted in invited talks.
- Contributed talks: We would like to continue the policy of allowing everyone interested to present talks. In future workshops, we might need to consider changing the amount of time per talk or to find other ways to ensure that we leave adequate time for other types of sessions.
- Mentoring sessions: Instead of limiting presentations to those with completed work, we invited participants to present work in progress for the purposes of getting feedback from more experienced researchers in the area.

- Open problem sessions: As participants may be looking for collaborators on problems of interest, we dedicate time for presentation of open problems, which later serve as inspirations for working sessions.
- Working sessions: Problems identified in the open problem sessions, talks, and/or tutorials are discussed in small groups. Participants are welcome to join any of the groups and to move between different groups on different days.

4.2 Strengthening the community

As we noted in the proposal, researchers in the area of combinatorial reconfiguration do not have many opportunities to gather together in one place, as they are not only located on several different continents, but attend different conferences in different fields in mathematics and computer science. Since the community is still quite young and developments are occurring rapidly, it is important to be able to share ideas and help shape not only what research will be done but also how we will strengthen and build the community.

We gave all participants a chance to introduce their interests, giving others a chance to link a face to a name, and know whom to contact for future discussions (and well as ones that took place during the workshop).

As discussed in detail in Section 3, new collaborations were formed both among in-person participants, among virtual participants, and in at least one instance a mixture of in-person and virtual participants. In addition, several of the participants reported that even though they did not begin collaborations at the workshop, the workshop helped them to find whom to contact in the future, as well as new directions of interest.

4.3 Broadening the community

Our goals were to welcome newcomers to the community and to support underrepresented and early career participants as well as to build bridges to closely-related fields. The hybrid format made it possible to participate without making the financial and time commitment required to attend the entire in-person workshop.

Several talks, both formally designated as tutorials and otherwise, provided background information in related fields at a level suitable for newcomers to the areas. Catherine Greenhill provided an invited tutorial on connections between Markov chains, mixing time, and reconfiguration. Amer Mouawad's invited tutorial discussed connections between parameterized complexity and reconfiguration. In addition, Sevag Gharibian's contributed talk on reconfiguration in the quantum setting provided a gentle introduction to the area.

All participants were given chances to interact with others, whether through introductions, talks, mentoring sessions, or working sessions, as well as informally during coffee breaks. The workshop was, for several of our participants, their first opportunity to present their work to an international audience.

One of the ways that our community supports diversity is through diverse ideas and approaches. One example is the advent of a programming challenge (discussed and promoted in the invited talk by Jun Kawahara), complementing the theoretical work in the area.

To continue the connections in the community, participants were invited to join a mailing list in the area and a satellite workshop at ICALP taking place this year. A Discord channel continues to be used for communication and collaboration.

The recordings of the presentations provide additional resources to help others learn about the community and the work being done.

4.4 Providing suport for early-career participants

The workshop was designed to promote, showcase, connect, and encourage all participants, with particular attention to early-career participants. All participants were encouraged to give presentations, including in mentoring sessions, contribute to open problem sessions, and form collaborations in working sessions.

Although details have been omitted here due to the work being still in progress, various participants reported on new directions of research based on either questions or comments on their presentations or in collaborations that followed in the working sessions.

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