Spectral Analysis, Stability and bifurcations in Modern Nonlinear Physical Systems, BIRS, 2012

Effect of dissipation on local and global instabilities

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Joint work with :

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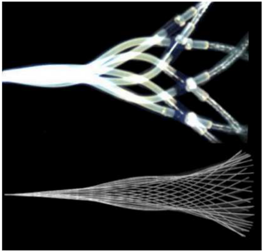
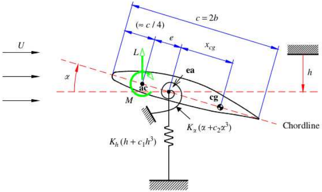
Introduction

Flutter instability

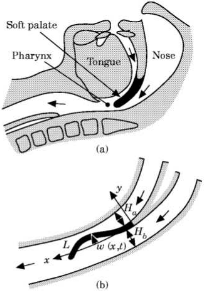
Pipe, flag, wing, soft palate ...



Doaré&de Langre
Eloy et al



Lee



Auégan&Dépollier

- But also vocal folds, paper in high speed printers, reeds in some musical instruments (eg. harmonica).
- Recent interest in energy harvesting applications

Fluid-conveying pipe: a model problem

The fluid-conveying pipe can be considered as a model problem for many physical systems where the dynamics of a slender structure is coupled to an axial flow.

Fluid-conveying pipe: a model problem

The fluid-conveying pipe can be considered as a model problem for many physical systems where the dynamics of a slender structure is coupled to an axial flow.

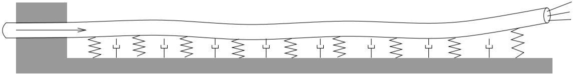
- Simplest model describing the linear dynamics of a fluid-conveying pipe $\equiv$ Euler-Bernoulli beam with an internal plug flow.

$$\underbrace{EI\frac{\partial^4 Y}{\partial X^4} + m\frac{\partial^2 Y}{\partial T^2}}_{\text{Euler-Bernoulli beam}} + \underbrace{M\frac{\partial^2 Y}{\partial T^2}}_{\text{Added mass}} + \underbrace{MU^2\frac{\partial^2 Y}{\partial X^2}}_{\text{Centrifugal force}} + \underbrace{2MU\frac{\partial^2 Y}{\partial X \partial T}}_{\text{Coriolis force}} = 0$$

(Bourrières 1939, Gregoy & Païdoussis 1966, Païdoussis 1998)

Flags, compliant walls : Similar physical effects, although differences in the expressions

Additionnal elastic and viscous forces



- Fluid-conveying pipe with elastic foundation, tension, viscous, and viscoelastic dissipations :

$$\underbrace{EI\frac{\partial^4 Y}{\partial X^4}+m\frac{\partial^2 Y}{\partial T^2}}_{\text{Euler-Bernoulli beam}} + \underbrace{M\frac{\partial^2 Y}{\partial T^2}}_{\text{Added mass}} + \underbrace{MU^2\frac{\partial^2 Y}{\partial X^2}}_{\text{Centrifugal force}} \underbrace{+2MU\frac{\partial^2 Y}{\partial X \partial T}}_{\text{Coriolis force}}$$

$$\underbrace{-N\frac{\partial^2 Y}{\partial X^2}}_{\text{tension}} \quad \underbrace{+SY}_{\text{Spring foundation}} \quad \underbrace{+E^\star I\frac{\partial^5 Y}{\partial X^4 \partial T}}_{\text{Structural dissipation}} \underbrace{+c\frac{\partial Y}{\partial T}}_{\text{Viscous dissipation}} \quad = 0$$

Boundary conditions : Clamped-free beam

$$Y(X=0) = \left.\frac{\partial Y}{\partial X}\right|_{X=0} = \left.\frac{\partial^2 Y}{\partial X^2}\right|_{X=L} = \left.\frac{\partial^3 Y}{\partial X^3}\right|_{X=L} = 0 \qquad (1)$$

Local/Global approaches

Local: wave equations in an infinite domain

$$\frac{\partial^2}{\partial t^2}[\mathcal{M}(y)] + \frac{\partial}{\partial t}[\mathcal{C}(y)] + \mathcal{K}(y) = 0 \quad \text{on } \Omega = [-\infty, +\infty] \tag{2}$$

- Solutions in the form of harmonic plane wave : $y = y_0 \mathrm{e}^{\mathrm{i}(kx-\omega t)}$
- Dispersion relation $D(k,\omega) = 0$
- Instability if $\exists k \in \mathbb{R} \setminus \mathrm{Im}[\omega(k)] > 0$

Global: wave equations, finite length, boundary conditions

$$\frac{\partial^2}{\partial t^2}[\mathcal{M}(y)] + \frac{\partial}{\partial t}[\mathcal{C}(y)] + \mathcal{K}(y) = 0 \quad \text{on } \Omega \tag{3}$$

$$\mathcal{B}_i(y) = 0 \,, \quad i = 1..N \quad \text{on } \partial\Omega \tag{4}$$

- Solutions of the form : $y = \phi(x)\mathrm{e}^{-\mathrm{i}\omega t} \rightsquigarrow$ Strum-Liouville eigenvalue problem
- Instability if $\mathrm{Im}(\omega) > 0$

Objectives

Litterature

- Large amount of litterature, on both local and global instabilities
- In many works, a destabilizing effect of damping has been evidenced

- **Pipes:** Bourrières (1939), Bolotin (1963), Gregory & Païdoussis (1964), Roth (1964), Païdoussis (1970,1998), Lottati & Kornecki (1986), Kulikovskii (1988), de Langre & Ouvrard (1999)
- **Compliant walls:** Landahl (1962), Benjamin (1963), Kornecki et al (1976), Brazier-Smith & Scott (1984), Carpenter & Garrad (1985), Crighton & Oswell (1991), Lucey & Carpenter (1992), Peake (1997,2001,2004), Wiplier & Ehrenstein (2000,2001)
- **Flags:** Datta & Gottenberg (1975), Shayo (1980), Aurégan & Dépollier (1995), Huang (1995), Shelley et al (2005), Lemaître et at (2005), Eloy et al (2007,2008), Michelin & Llewellyn Smith (2009), Tang & Païdoussis (2009)

Objectives

- Perform local and gobal stability analyses on a given system
- Focus on the effect of dissipation

Outline

1. “Simple” pipe 2. Pipe on elastic foundation 3. Energy harvesting

Local/global analysis of the simple fluid-conveying pipe

Infinite fluid-conveying pipe

- Non-dimensional equations of the problem:

$$\frac{\partial^2 y}{\partial t^2} + \frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial x^2} + 2\sqrt{\beta}\frac{\partial^2 y}{\partial x \partial t} + \mathcal{D}(y) = 0, \quad (5)$$

Only one or two parameters: $\beta = \frac{m+M}{M}$ + a damping parameter.

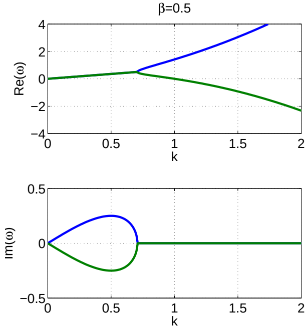
Local stability analysis (no damping)

- Solution in the form of a propagating wave, $y = y_0 \mathrm{e}^{\mathrm{i}(kx-\omega t)}$
- Dispersion relation:

$$k^4 - \omega^2 + k^2 + 2\sqrt{\beta}k\omega = 0. \qquad (6)$$

- Frequency associated to a real wavenumber k:

$$\omega_\pm = \sqrt{\beta}k \pm k\sqrt{\beta + k^2 - 1}. \qquad (7)$$



- For $\beta \in [0,1[$ and $k \in [0, \sqrt{1-\beta}]$, frequencies $\omega_\pm$ are complex conjugate
- $\Rightarrow$ Locally unstable $\forall \beta \in [0,1[$
- For $k > \sqrt{1-\beta}$, $\omega(k) \in \mathbb{R}$ and waves are neutral

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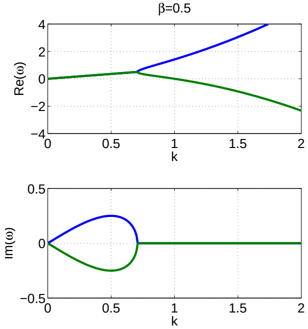

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- $\Rightarrow$ Locally unstable $\forall \beta \in [0, 1[$
- For $k > \sqrt{1-\beta}$, $\omega(k) \in \mathbb{R}$ and waves are neutral
- What happens when damping is added?

Effect of damping on neutral waves

- Dispersion relation without damping:

$$D(k,\omega) = 0. \tag{8}$$

- Dispersion relation with a small amount of viscous damping:

$$D_1(k,\omega+\delta\omega) = D(k,\omega+\delta\omega) - ic(\omega+\delta\omega) = 0 \tag{9}$$

- At first order, the perturbation of the frequency due to damping satisfies:

$$\delta\omega \left.\frac{\partial D}{\partial \omega}\right|_{(k,\omega)} - ic\omega = 0 \tag{10}$$

- Perturbation of the growth rate:

$$\delta\sigma = \frac{c\omega}{\partial D/\partial\omega}. \tag{11}$$

- In the context of the dynamics of the interface between two fluids, Cairns (1979) calculate the wave energy as the work to do on the system to generate a neutral wave from $t = -\infty$ to $t = 0$:

$$E = -\frac{\omega}{4}\frac{\partial D}{\partial \omega} y_0^2. \tag{12}$$

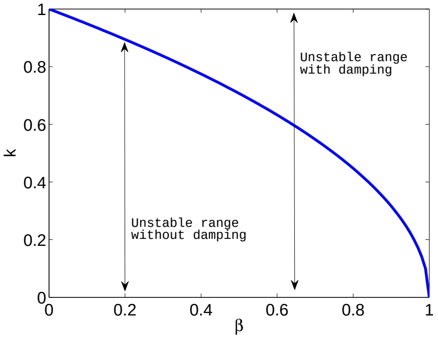
- $\rightsquigarrow$ $\delta\sigma$ has the opposite sign of the wave energy

Damping effect in the fluid conveying pipe

- Wave energy:

$$E_{\pm} = \frac{k^2\sqrt{k^2+\beta-1}\left(\sqrt{k^2+\beta-1} \pm \sqrt{\beta}\right)}{2} \quad (13)$$

- $E_- < 0$ for $k \in]\sqrt{1-\beta}, 1[$
- $\Rightarrow$ when damping is added, the range of unstable wavenumbers is extended from $[0, \sqrt{1-\beta}]$ to $[0, 1]$.



- The Coriolis term $\beta \partial^2 y / \partial x \partial t$ stabilizes waves in the range $[\sqrt{1-\beta}, 1]$
- Damping cancels this effect

Global stability

Equations

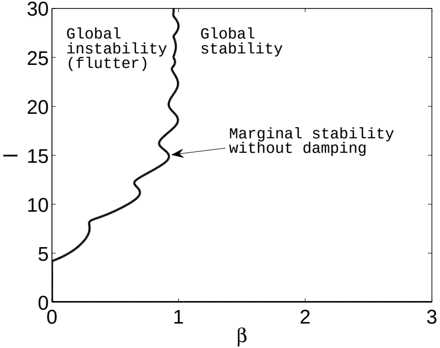
$$\frac{\partial^2 y}{\partial t^2} + \frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial x^2} + 2\sqrt{\beta}\frac{\partial^2 y}{\partial x \partial t} + \mathcal{D}(y) = 0, \tag{14}$$

(+ Boundary conditions)
Parameters: β (and l) + a damping parameter.

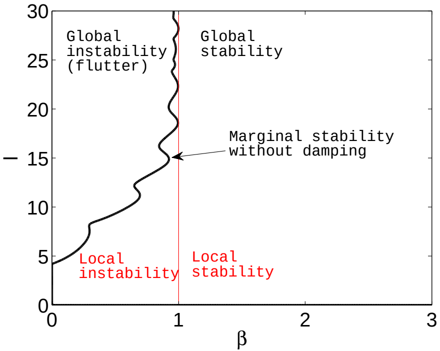
Method: Galerkin decomposition

- y decomposed on beam modes that satisfy boundary conditions
- Projection over beam modes, truncature $\rightsquigarrow$ discrete mechanical system
- Discrete eigenvalue problem $\rightsquigarrow$ eigenfrequencies ω_n

Global stability

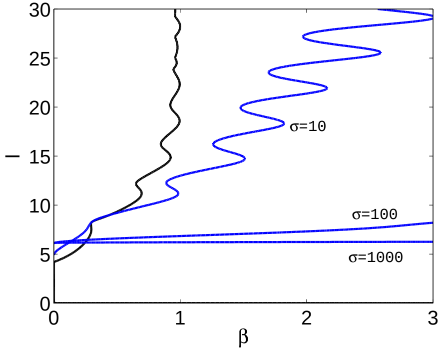


Global stability



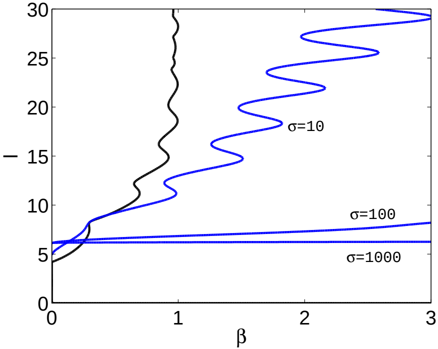
- Long system limit given by a local criterion: local stability criterion

Global stability



- When damping is increased, the marginal stability curve tends to a different limit

Global stability



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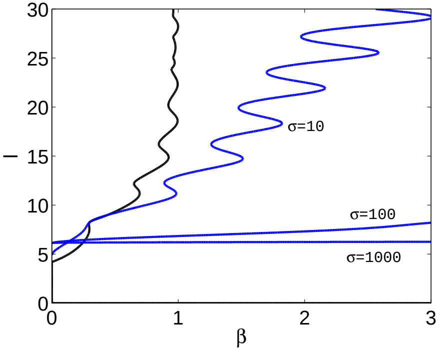
- As the damped medium is always locally unstable, no local criterion can predict the phenomenon

Global stability



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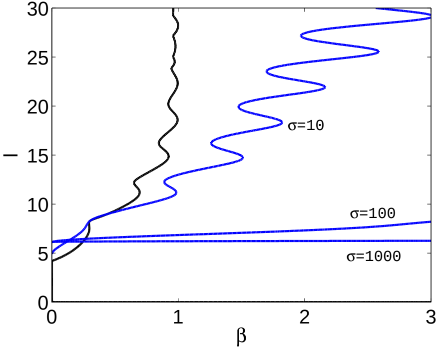
- Statement: when the medium is locally unstable, global stability is due to confinement

Global stability



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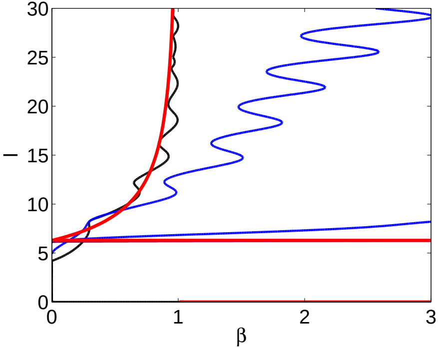
- Smallest unstable wavelength:

$$\lambda_1 = \frac{2\pi}{\sqrt{1-\beta}} \text{ (no damping)}$$

$$\lambda_2 = 2\pi \text{ (damping)}$$

Global stability



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Conclusions (1/3)

- Coriolis term stabilizes the range $k \in [\sqrt{1-\beta}, 1]$
- This range is destabilized by damping
- Destabilization is due to negative energy waves

- Finite length stability boundaries affected by confinement effects
- Destabilization by damping due to negative energy waves

- Except for $\beta = 0$...

(Lottati & Kornecki 1986)

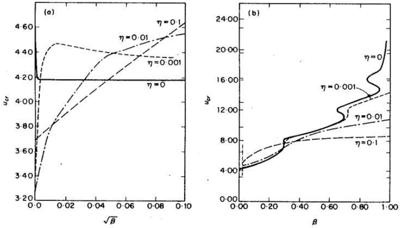

Figure 8. Critical flow velocity as function of the mass ratio β for different internal damping coefficients η ($\kappa = 0$, external damping $\delta = 0$), (a) $0 \leq \beta \leq 0{\cdot}01$; (b) $0{\cdot}01 \leq \beta \leq 1$.

Pipe on elastic foundation

Pipe on elastic foundation *without* dissipation

- Non-dimensional equation :

$$\frac{\partial^2 y}{\partial t^2} + \frac{\partial^4 y}{\partial x^4} + v^2 \frac{\partial^2 y}{\partial x^2} + 2\sqrt{\beta} v \frac{\partial^2 y}{\partial x \partial t} + y + \mathcal{D}(y) = 0 \qquad \text{with } \mathcal{D}(y) = 0 \qquad (15)$$

Absolute/convective instabilities: See Briggs (1964): Plasma physics, Brazier-Smith & Scott (1984): Compliant panels with flows, Huerre & Monkewitz (1990): Shear layer problems.

Pipe on elastic foundation ***without*** dissipation

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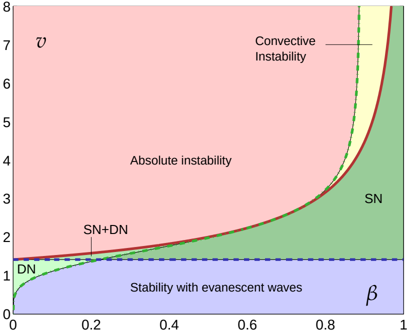
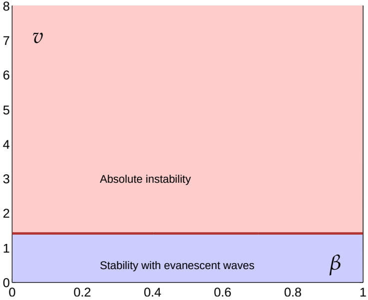


- Criterion for instability : $v > \left(\frac{2}{1-\beta}\right)^{1/2}$
- Criterion for absolute instability : $v > \left(\frac{12\beta}{8/9-\beta}\right)^{1/4}$
- Criterion for existence of neutral waves :
  - Static range : Stability and $v > \sqrt{2}$
  - Dynamic range : Stability and $v > \left(\frac{12\beta}{8/9-\beta}\right)^{1/4}$

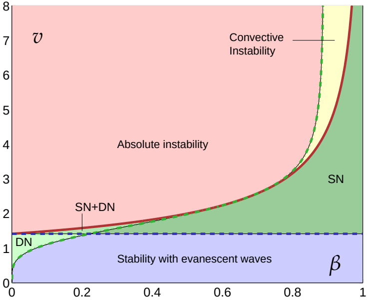
Absolute/convective instabilities: See Briggs (1964): Plasma physics, Brazier-Smith & Scott (1984): Compliant panels with flows, Huerre & Monkewitz (1990): Shear layer problems.

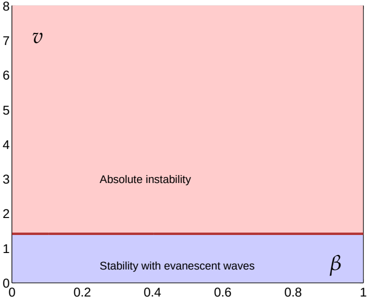
Pipe on elastic foundation *with* dissipation



- Stability properties depend neither on the type of dissipation nor its value
- Criterion for instability : $v > \sqrt{2}$
- Instability is always absolute
- When stable, no neutral range

Pipe on elastic foundation - Effect of dissipation

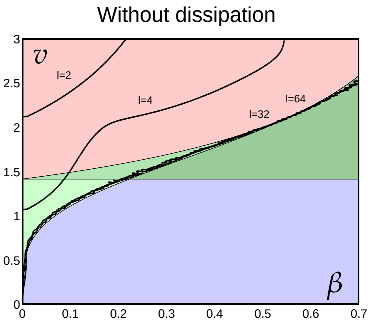




- Dynamic range : Positive energy waves
- Static range : Negative energy waves
- Destabilization by damping is due to negative energy waves in the static range.

Pipe on elastic foundation - Local/global comparison

- Projection of the equation over ~ 150 beam modes



- Long system limit : Criterion for existence of the dynamic range of neutral waves without damping, criterion of instability with damping
- Without damping, one can observe a system which is locally stable but globally unstable

Lengthscale criterion

The three lengthscales of the system

Length of the system L	Elastic rigidity lengthscale η	Dissipation lengthscale η_D
L	$\eta = \left(\frac{EI}{S}\right)^{1/4}$	$\eta_D = \left(\frac{EI(\mu_f+\mu)}{c^2}\right)^{1/4}$
$l = L/\eta$	1	$\rho = \eta_D/\eta$

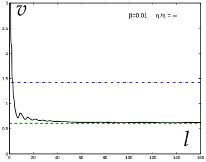
- If $L < \eta$ and $L < \eta_D \Rightarrow$ Confined system, no local criterion
- If $L > \eta$ but $L < \eta_D \Rightarrow$ Local criterion without dissipation
- If $L > \eta_D \Rightarrow$ Local criterion with dissipation

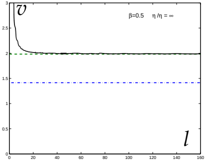
- If $l < 1$ and $l < \rho \Rightarrow$ Confined system, no local criterion
- If $l > 1$ but $L < \rho \Rightarrow$ Local criterion without dissipation
- If $l > \rho \Rightarrow$ Local criterion with dissipation

Global stability curves

$\beta = 0.01$ $\beta = 0.5$

$\rho = \eta_D / \eta = \infty$





— — — Local criterion w/o dissipation

· · · Local criterion w/ dissipation

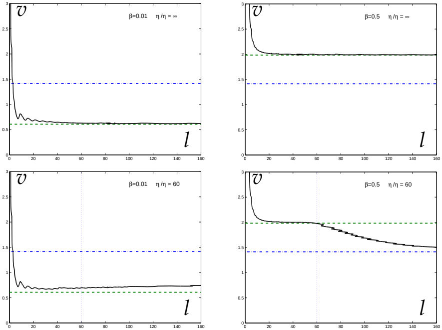
$l = \eta_\sigma / \eta$

Global stability curves

$\beta = 0.01$ $\beta = 0.5$

$\rho = \eta_D/\eta = \infty$

$\rho = \eta_D/\eta = 60$



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⋮ $l = \eta_\sigma/\eta$

Global stability curves

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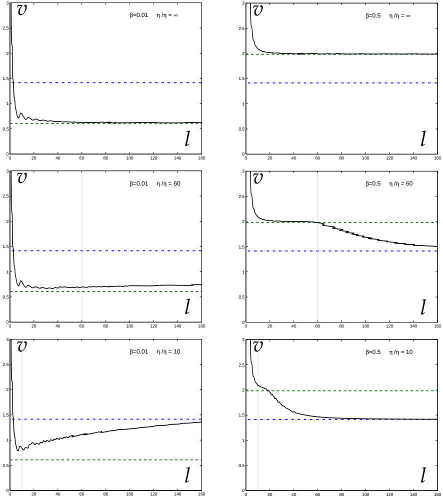


$\rho = \eta_D/\eta = \infty$

$\rho = \eta_D/\eta = 60$

$\rho = \eta_D/\eta = 10$

— — — Local criterion w/o dissipation

· · · Local criterion w/ dissipation

Conclusions (2/3)

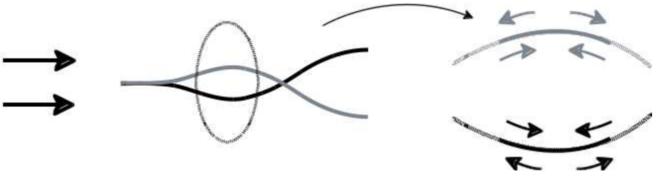
Main results

- Destabilization by damping observed in infinite media as well as finite systems
- Destabilization by dissipation in the finite system is also related to negative energy waves
- Pipe wthout damping: boundary conditions may destabilize the system. Condition: neutral (propagative) waves, positive or negative energy
- Lengthscale criteria to determine the long system limit

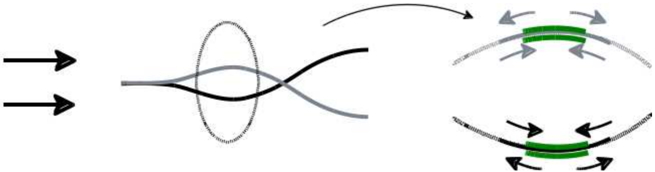
- O. Doaré & E. de Langre. Local and global stability of fluid-conveying pipes on elastic foundations. Journal of Fluids and Structures, 16(1):1–14, 2002.
- O. Doaré & E de Langre. The role of boundary conditions in the instability of one-dimensional systems. European Journal Of Mechanics B-Fluids, 25:948–959, 2006.
- O. Doaré. Dissipation effect on local and global stability of fluid-conveying pipes. Journal of Sound and Vibration, 329(1):72–83, 2010.

Application to energy harvesting

Energy harvesting from piezoelectric fluttering flags



Energy harvesting from piezoelectric fluttering flags



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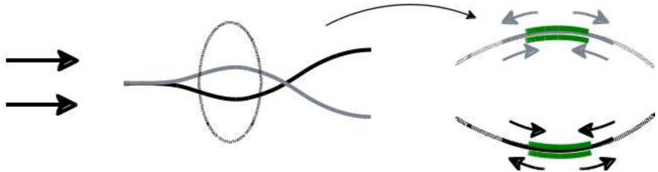
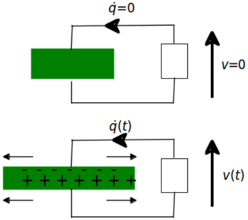




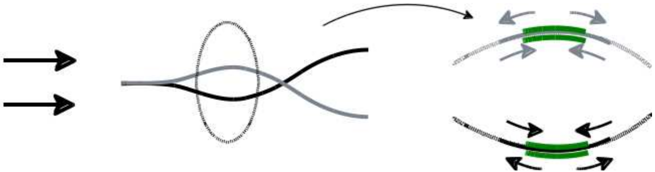
- Deformation ⇝ charge transfert between electrodes

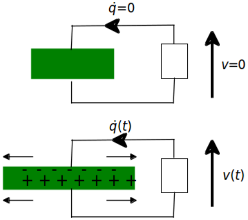
$$Q = CV + \chi \int_{x^-}^{x^+} F_p(x) w''(x) dx$$

- Voltage ⇝ momentum exerted on the plate

$$\mathcal{M}_{piezo}(x) = -\chi V F_p(x)$$

Energy harvesting from piezoelectric fluttering flags





- Deformation ⇝ charge transfert between electrodes

$$Q = CV + \chi \int_{x^-}^{x^+} F_p(x) w''(x) dx = CV + \chi [w']_{x^-}^{x^+}$$

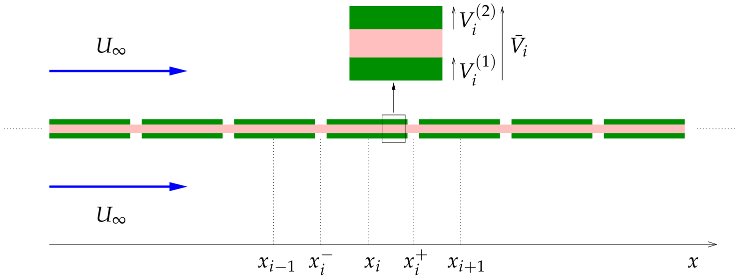
- Voltage ⇝ momentum exerted on the plate

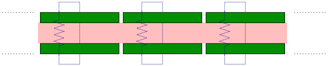
$$\mathcal{M}_{piezo}(x) = -\chi V F_p(x) = -CV[H(x-x^-) - H(x-x^+)]$$

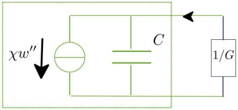
- Shape function of the piezo:

$$F_p(x) = H(x-x^-) - H(x-x^+)$$

Choice of a configuration







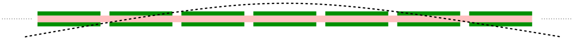
- Plate with a series of piezoelectric elements
- Harvesting circuit modelled by a shunted resistance
  $\rightsquigarrow$ Harvested energy $\equiv$ energy dissipated in the resistance

Coupled mechanical-electrical wave equation

- Large wavelengths (small piezos) limit:

$$\left[w'\right]_{x_i^-}^{x_i^+} \simeq w''(x_i)l, \quad (16)$$

$$\sum_i \bar{V}_i[H(x - x_i^-) - H(x - x_i^+)] \simeq v(x). \quad (17)$$



- Coupled wave equations:

$$\left(B + \frac{\chi^2}{c}\right) w'''' + \mu\ddot{w} - \frac{\chi}{c}q'' = -[P] \quad (18)$$

$$\frac{1}{g}\dot{q} + \frac{1}{c}q - \frac{\chi}{c}w'' = 0 \quad (19)$$

+ Potential flow theory $\rightsquigarrow$ pressure linear function of the displacement w.

Definition of an efficiency

- Windmill-type efficiency:

$$E = \frac{\text{Power harvested in the electrial circuits}}{\text{Fluid's kinetic energy flux through the surface occupied by oscillations}} \propto \frac{A^2}{A}$$

$\rightsquigarrow$ Scales as the amplitude of the mode $\Rightarrow$ diverges in the linear case

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- More precisely:

$$\boxed{r = \frac{\displaystyle\int_0^T \langle \mathscr{P}_{el} \rangle dt}{\displaystyle\frac{1}{T}\int_0^T \langle \mathscr{E} \rangle dt}} \tag{20}$$

with

$$\mathscr{P}_{el} = -v\dot{q}, \qquad \mathscr{E} = \frac{1}{2}\rho_s \dot{w}^2 + \frac{1}{2}Bw''^2 + \frac{1}{2}cv^2 \tag{21}$$

$$\langle . \rangle \equiv \text{Spatial average (on a wave or an eigenmode)} \tag{22}$$

Non-dimensional equation

$$\frac{1}{V^{*2}}(1+\alpha^2)\tilde{w}'''' + \ddot{\tilde{w}} - \frac{\alpha}{V^*}\tilde{q}'' = -[\tilde{p}], \quad (23)$$

$$\gamma\dot{\tilde{q}} + \tilde{q} - \frac{\alpha}{V^*}\tilde{w}'' = 0, \quad (24)$$

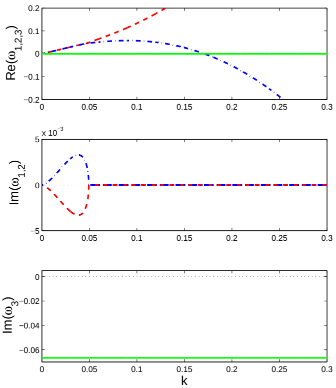
with

$$V^* = \sqrt{\frac{\mu^3 U_\infty^2}{B\rho_f^2}} \text{ (Non dimensional velocity)} \quad (25)$$

$$\alpha = \frac{\chi}{\sqrt{cB}} \text{ (Coupling coefficient)} \quad (26)$$

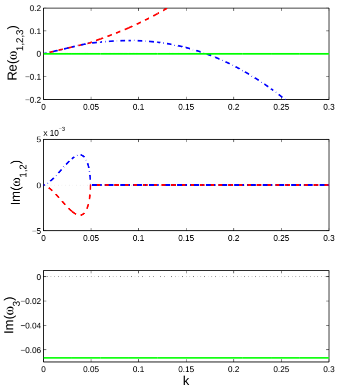
$$\gamma = \frac{\rho_f U_\infty c}{\mu g} \text{ (Timescales ratio)} \quad (27)$$

Stability analysis for $V^* = 0.05$, $\gamma = 15$



Without coupling, $\alpha = 0$

Stability analysis for $V^* = 0.05$, $\gamma = 15$



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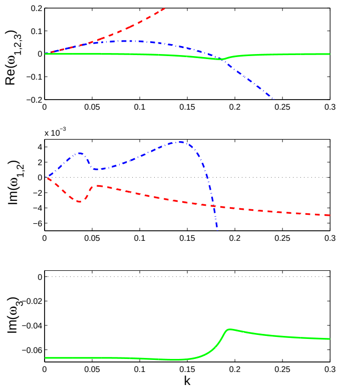
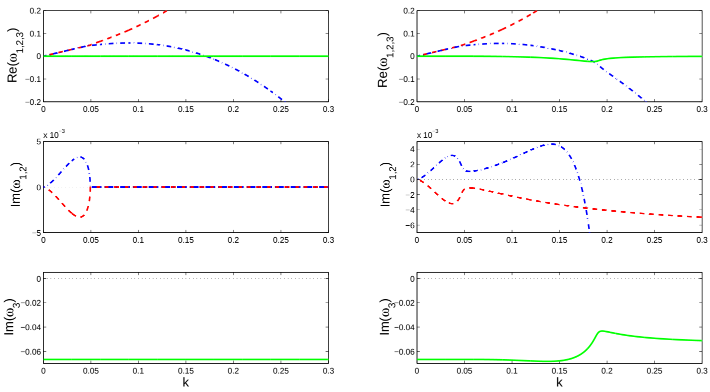


With coupling, $\alpha = 0.5$

Stability analysis for $V^* = 0.05$, $\gamma = 15$



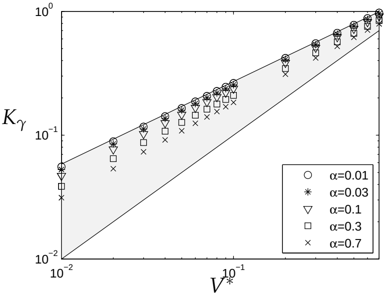
Without coupling, $\alpha = 0$ With coupling, $\alpha = 0.5$

Destabilized waves are again negative energy waves (NEW)

$$\delta\sigma \simeq \omega\alpha^2\gamma k^4 \Bigg/ \left(V^{*2}(1+\omega^2\gamma^2)\frac{\partial D_0}{\partial \omega} \right) \tag{28}$$

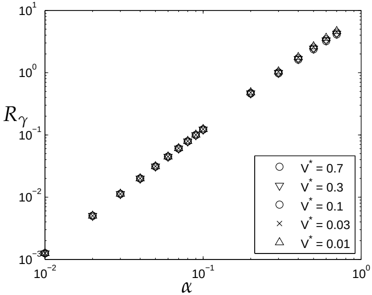
Conversion efficiency of waves

- Wavenumber K_γ that maximizes efficiency as function of velocity.
- Gray region = Negative energy waves



- Maximum efficiency is always for a wave destabilized by addition of coupling

- Maximum R_γ of efficiency among all parameters as function of coupling coefficient



- Efficiency scales as α^2

Finite length problem

- Non-dimensional equation:

$$\frac{1}{U^{*2}}(1+\alpha^2)\hat{w}'''' + \ddot{\hat{w}} - \frac{\alpha}{U^*}\hat{q}'' = -M^*\hat{p}, \tag{29}$$

$$\beta\dot{\hat{q}} + \hat{q} - \frac{\alpha}{U^*}\hat{w}'' = 0, \tag{30}$$

- Non-dimensional parameters:

$$M^* = \frac{\rho_f L}{\mu}, \quad U^* = UL\sqrt{\frac{\mu}{B}} = V^*M^*, \quad \beta = \frac{cU_\infty}{gL} = \frac{\gamma}{M^*}, \quad \alpha = \frac{\chi}{\sqrt{cB}}. \tag{31}$$

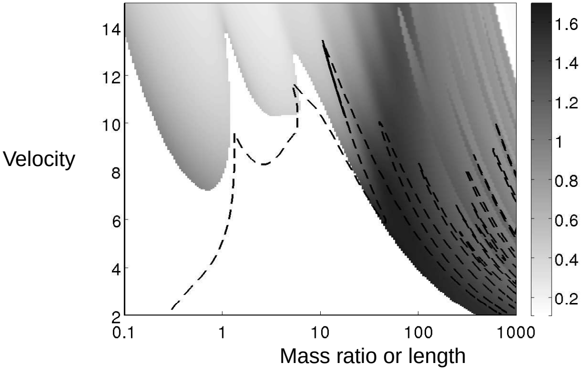
- Clamped-free boundary conditions:

$$\text{for } \hat{x} = 0 \qquad \begin{cases} \hat{w} = 0 \\ \hat{w}' = 0 \end{cases} \tag{32}$$

$$\text{for } \hat{x} = 1 \qquad \begin{cases} (1+\alpha^2)\hat{w}'' - \alpha U^*\hat{q} = 0 \\ (1+\alpha^2)\hat{w}''' - \alpha U^*\hat{q}' = 0 \end{cases}. \tag{33}$$

Conversion efficiency of the dominant unstable mode

Efficiency ($\alpha = 0.5$, $\beta = 0.25$)



Long systems $\equiv M^* \gg 1 \rightsquigarrow$ Behavior of the finite length system similar to that of the infinite one

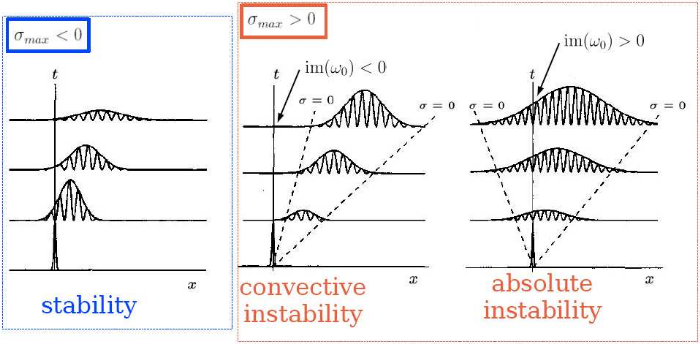
Conclusions (3/3)

- Energy harvesting destabilizes negative energy waves
- Destabilized negative energy waves maximizes the efficiency
- Finite length system properties are again influenced by wave properties

- O. Doaré & S. Michelin. Piezoelectric coupling in energy-harvesting fluttering flexible plates : linear stability analysis and conversion efficiency. Journal of Fluids and Structures, 27(8):1357–1375, 2011.
- S. Michelin & O. Doaré, Energy harvesting efficiency of piezoelectric flags in axial flows. Journal of Fluid Mechanics, in press, 2012.

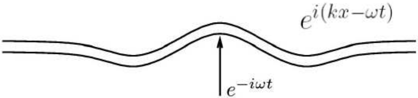
Absolute & convective instabilities

- Maximum growth rate : $\sigma_{\max} = \max_{k\in\mathrm{R}} \mathrm{Im}\omega(k)$
- Absolute frequency ω_0 : $\left.\frac{\partial\omega}{\partial k}\right|_{\omega=\omega_0} = 0$



See Briggs (1964): Plasma physics, Brazier-Smith & Scott (1984): Compliant panels with flows, Huerre & Monkewitz (1990): Shear layer problems.

The signaling problem



- A branch analysis in the complex $k-$ and $\omega-$ planes is necessary to to know the side $x > 0$ or $x < 0$ the waves propagate
- Different typical responses :
  - Evanescent waves at all frequencies
  - Only neutral (propagative) waves at some frequencies
  - In case of convective instability : Amplified waves at some frequencies
  - Absolute instability : Response dominated by the absolute frequency