On the Role of Interaction in Network Information Theory

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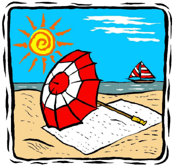
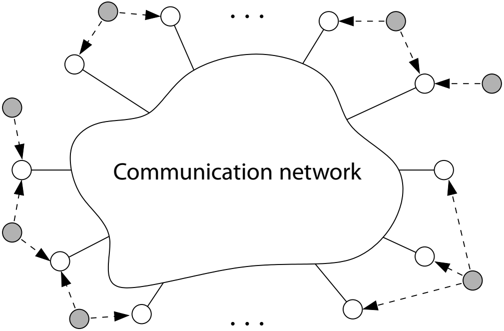


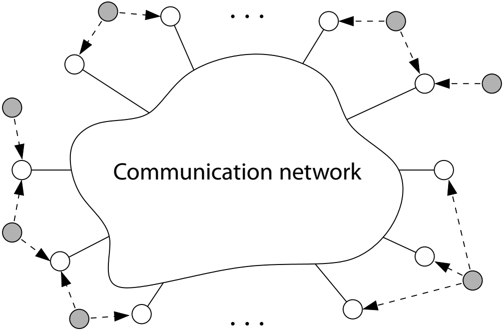
Banff Workshop on Interactive Information Theory
January 2012

Networked Information Processing System



- System: Internet, peer-to-peer network, sensor network, …
- Sources: Data, speech, music, images, video, sensor data
- Nodes: Handsets, base stations, processors, servers, sensor nodes, …
- Network: Wired, wireless, or a hybrid of the two
- Task: Communicate the sources, or compute/make decision based on them

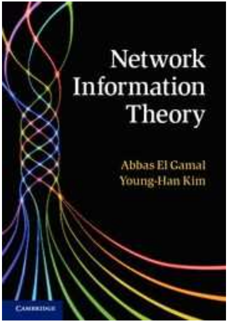
Network Information Theory



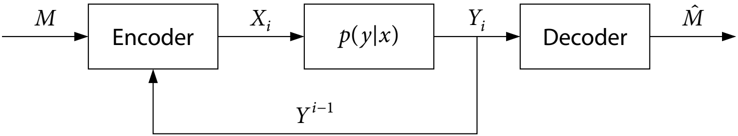
- Network information flow questions:
  - What is the limit on the amount of communication needed?
  - What are the coding schemes/techniques that achieve this limit?
- Challenges:
  - Many networks inherently allow for two-way interactions
  - Most coding schemes are limited to one-way communications

Objectives of the Talk

- Review coding schemes that utilizes two-way interactions
- Focus on the channel coding side of the story (given yesterday's talks)
- Draw mostly from a few classical examples and open problems (El Gamal–K 2011)



Discrete Memoryless Channel (DMC) with Feedback



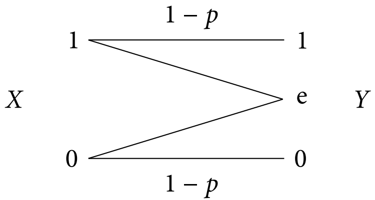
- Feedback does not increase the capacity of a DMC (Shannon 1956):

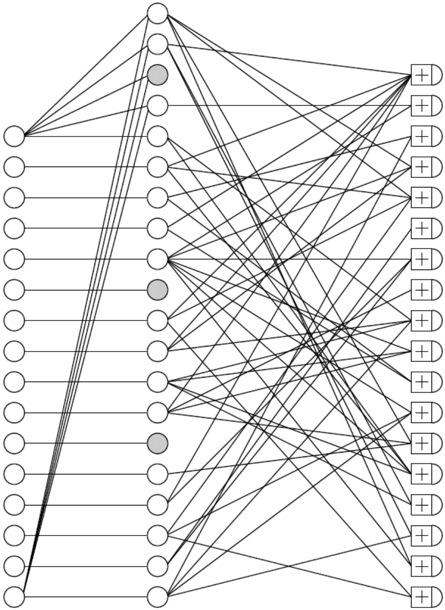
$$C_{\text{FB}} = \max_{p(x)} I(X;Y) = C$$

- Nonetheless, feedback can help communication in several important ways
  - Feedback can simplify coding and improve reliability (Schalkwijk–Kailath 1966)
  - Feedback can increase the capacity of channels with memory (Butman 1969)
  - Feedback can enlarge the capacity region of DM multiuser channels (Gaarder–Wolf 1975)
- Insights on the fundamental limit of two-way interactive communication

Iterative Refinement

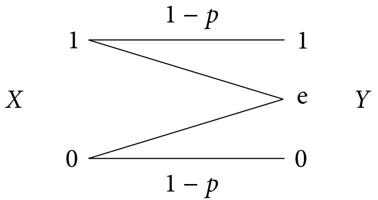
- Binary erasure channel:

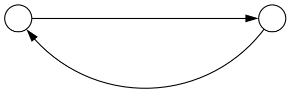




Iterative Refinement

- Binary erasure channel:

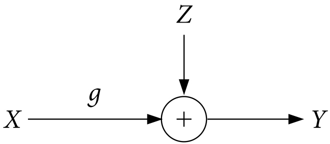




- Basic idea:
  - First send a message at a rate higher than the channel capacity (without coding)
  - Then iteratively refine the receiver's knowledge about the message

- Examples:
  - Schalkwijk–Kailath coding scheme (1966)
  - Horstein's coding scheme (1963)
  - Posterior matching scheme (Shayevitz–Feder 2011)
  - Block feedback coding scheme (Weldon 1963, Ahlswede 1973, Ooi–Wornell 1998)

Gaussian Channel with Feedback



- Expected average transmitted power constraint

$$\sum_{i=1}^{n} \mathsf{E}(x_i^2(m, Y^{i-1})) \le nP, \quad m \in [1:2^{nR}]$$

- Schalkwijk–Kailath Coding Scheme (Schalkwijk–Kailath 1966, Schalkwijk 1966):

$$X_1 \propto \theta,$$
$$X_i \propto \theta - \hat{\theta}_{i-1}(Y^{i-1})$$

- Doubly exponentially small probability of error

Posterior Matching Scheme (Shayevitz–Feder 2011)

- Recall the Schalkwijk–Kailath coding scheme:

$$X_1 \propto \Theta \sim \mathrm{N}(0,1),$$
$$X_i \propto \Theta - \hat{\Theta}_{i-1}(Y^{i-1}) \propto X_{i-1} - \mathsf{E}(X_{i-1}|Y^{i-1}) \perp Y^{i-1}$$

  - $Y_1, Y_2, \ldots$ are i.i.d.

- Consider a general DMC $p(y|x)$ with a capacity-achieving input pmf $p(x)$:

$$X_1 = F_X^{-1}(F_\Theta(\Theta)), \qquad \Theta \sim \mathrm{Unif}[0,1)$$
$$X_i = F_X^{-1}(F_{\Theta|Y^{i-1}}(\Theta|Y^{i-1})) \perp Y^{i-1}$$

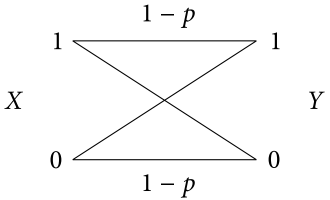
  - $Y_1, Y_2, \ldots$ are i.i.d.

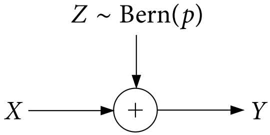
- Generalizes repetition for BEC, S–K for Gaussian, and Horstein for BSC

- Actual proof involves properties of iterated random functions

- Question: Elementary proof (say, for BSC)?

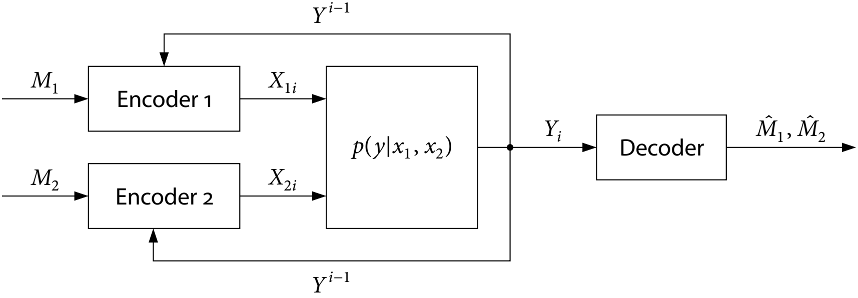
Block Feedback Coding Scheme





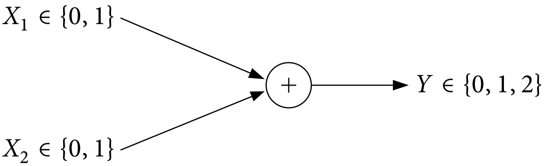
- Implementation of iterative refinement at the block level (Weldon 1963):
  - Initially, transmit k bits uncoded
  - Learn the error (via feedback), compress it using $kH(p)$ bits, and transmit the compression index uncoded
  - Communicate the error about the error ($kH^2(p)$ bits)
  - Communicate the error about the error about the error
- Achievable rate: $k/(k + kH(p) + kH^2(p) + kH^3(p) + \cdots) = 1 - H(p)$
- Extensions (Ahlswede 1973, Ooi–Wornell 1998)

Multiple Access Channel (MAC) with Feedback



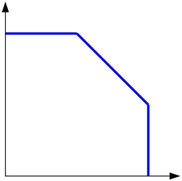
- Transmission cooperation: $x_{1i}(M_1, Y^{i-1})$, $x_2^n(M_2, Y^{i-1})$
- Capacity region $\mathscr{C}$ is not known in general

Example: Binary Erasure MAC



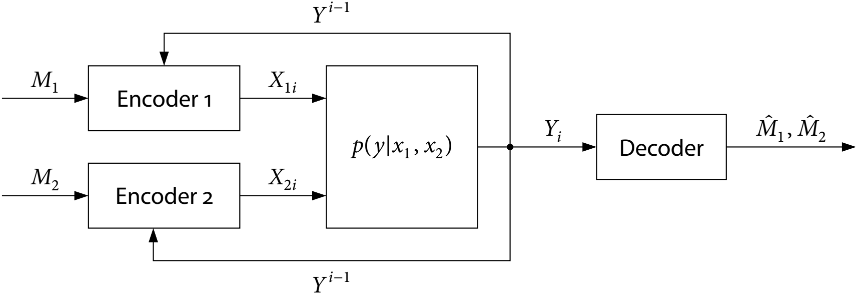
- Capacity region without feedback:

$$
\begin{aligned}
R_1 &\le 1, \\
R_2 &\le 1, \\
R_1 + R_2 &\le 3/2
\end{aligned}
$$

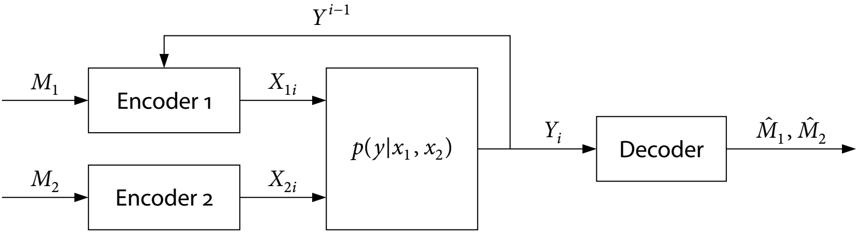


- Block feedback coding scheme (Gaarder–Wolf 1975):
  - $R_{\text{sym}} = 2/3$: k uncoded transmissions + $k/2$ one-sided retransmissions
  - $R_{\text{sym}} = 3/4$: k uncoded transmissions + $k/4$ two-sided retransmissions + $k/16 + \cdots$
  - $R_{\text{sym}} = 0.7602$: k uncoded transmissions + $k/(2 \log 3)$ cooperative retransmissions
- $R^*_{\text{sym}} = 0.7911$ (Cover–Leung 1981, Willems 1982)

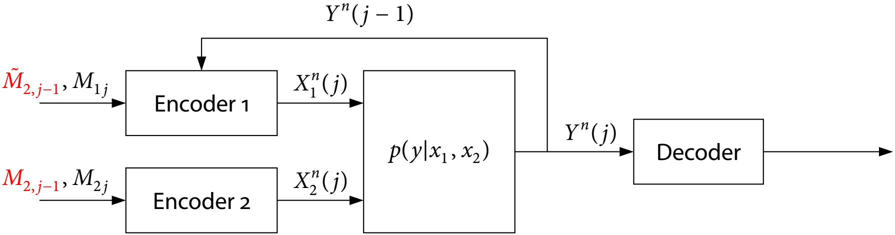
Cover–Leung Coding Scheme



Cover–Leung Coding Scheme

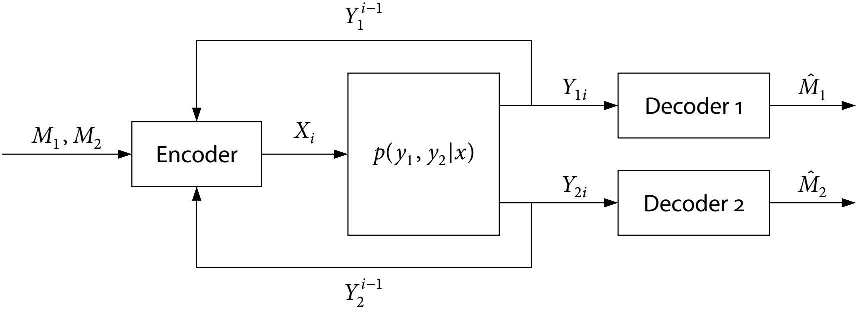


Cover–Leung Coding Scheme



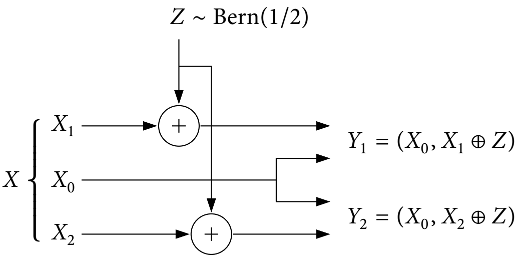
- Block Markov coding
- Backward decoding (Willems–van der Meulen 1985, Zeng–Kuhlmann–Buzo 1989)
- Willems condition (1982): Optimal when X_1 is a function of (X_2, Y)
- Not optimal for the Gaussian MAC (Ozarow 1984)
- Question: Posterior matching for MAC?
- Question: Optimality of Cover–Leung for one-sided feedback?

Broadcast Channel (BC) with Feedback



- Receivers operate separately (regardless of feedback)
- Physically degraded BC $p(y_1|x)p(y_2|y_1)$:
  - Feedback does not enlarge the capacity region (El Gamal 1978)
- How can feedback help?

Dueck's Example

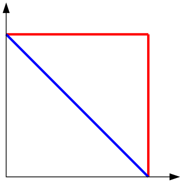


- Capacity region without feedback:

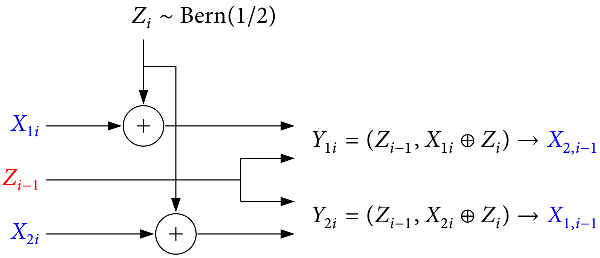
$$\{(R_1, R_2) : R_1 + R_2 \leq 1\}$$

- Capacity region with feedback (Dueck 1980):

$$\{(R_1, R_2) : R_1 \leq 1,\ R_2 \leq 1\}$$



Dueck's Example

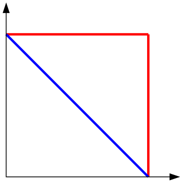


- Capacity region without feedback:

$$\{(R_1, R_2) : \; R_1 + R_2 \le 1\}$$

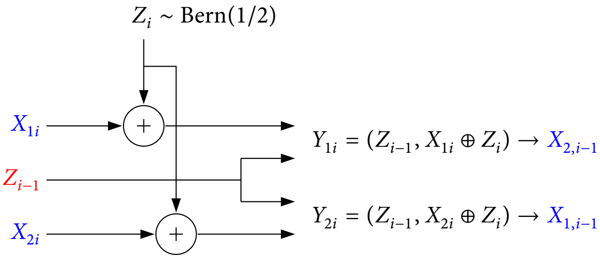
- Capacity region with feedback (Dueck 1980):

$$\{(R_1, R_2) : \; R_1 \le 1, \; R_2 \le 1\}$$



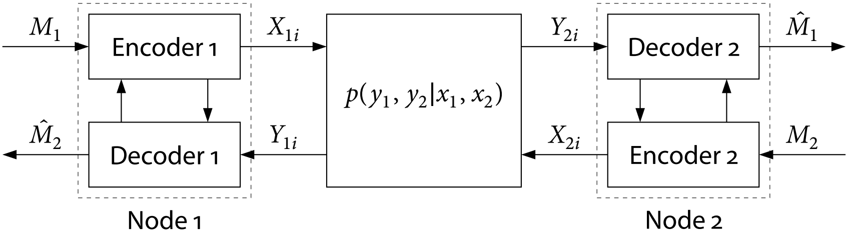
- Feedback helps by letting the encoder broadcast common channel information

Dueck's Example



- Extension to general BC (Shayevitz–Wigger 2010)
- "Learn from the past, don't predict the future" (Tse 2011)
- Gaussian BC: Schalkwijk–Kailath coding scheme to LQG control (Ozarow–Leung 1984, Elia 2004, Ardestanizadeh–Minero–Franceschetti 2011)
- Question: What's going on with Gaussian? (Exactly why feedback helps?)

Two-Way Channel



- The first multiuser channel model (Shannon 1961)
- Capacity region $\mathscr{C}$ is not known in general
- Main difficulties:
  - Two information flows share the same channel, inflicting interference to each other
  - Each node has to play two competing roles of communicating its own message and providing feedback to help the other node
- Two-way channel with common output: $Y_1 = Y_2 = Y$

Bounds on the Capacity Region

- Simple inner bound (Shannon 1961): A rate pair (R_1, R_2) is achievable if

$$R_1 < I(X_1; Y | X_2),$$
$$R_2 < I(X_2; Y | X_1),$$

  for some $p(x_1)p(x_2)$

  - One-way communication

Bounds on the Capacity Region

- Simple inner bound (Shannon 1961): A rate pair (R_1, R_2) is achievable if

$$R_1 < I(X_1; Y | X_2, Q),$$
$$R_2 < I(X_2; Y | X_1, Q)$$

  for some $p(q)p(x_1|q)p(x_2|q)$

  - One-way communication
  - Can be improved using time sharing
  - Not tight in general (Dueck 1979, Schalkwijk 1982)

Bounds on the Capacity Region

- Simple inner bound (Shannon 1961): A rate pair (R_1, R_2) is achievable if

$$R_1 < I(X_1; Y | X_2, Q),$$
$$R_2 < I(X_2; Y | X_1, Q)$$

  for some $p(q)p(x_1|q)p(x_2|q)$

- Simple outer bound (Shannon 1956): If a rate pair (R_1, R_2) is achievable,

$$R_1 \le I(X_1; Y | X_2),$$
$$R_2 \le I(X_2; Y | X_1)$$

  for some $p(x_1, x_2)$

- Dependence balance bound (Hekstra–Willems 1989):

$$R_1 \le I(X_1; Y | X_2, U),$$
$$R_2 \le I(X_2; Y | X_1, U)$$

  for some $p(u, x_1, x_2)$ such that $I(X_1; X_2|U) \le I(X_1; X_2|Y, U)$

Multiletter Characterization of the Capacity Region

- Causally conditional pmf: $p(x^k||y^{k-1}) = \prod_{i=1}^n p(x_i|x^{i-1}, y^{i-1})$
- Causally conditional directed information (Marko 1973, Massey 1990):

$$I(X^n \to Y^n \| Z^n) = \sum_{i=1}^n I(X^i; Y_i | Y^{i-1}, Z^i)$$

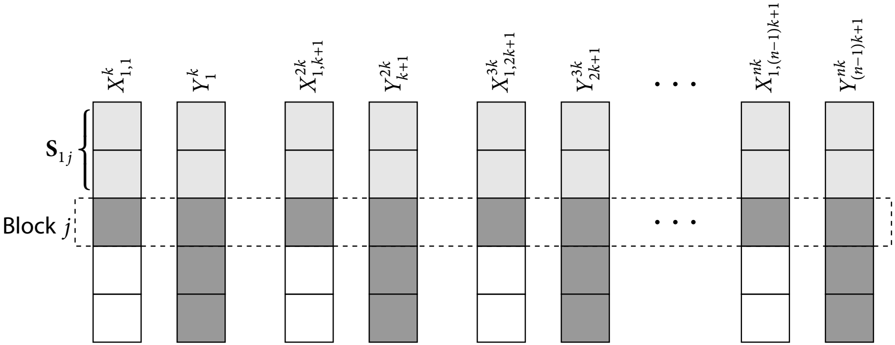
- Capacity region (Kramer 2003): Let $\mathscr{C}_k$ be the set of rate pairs (R_1, R_2) such that

$$R_1 \le \frac{1}{k} I(X_1^k \to Y^k || X_2^k),$$
$$R_2 \le \frac{1}{k} I(X_2^k \to Y^k || X_1^k)$$

  for some $p(x_1^k||y^{k-1})p(x_2^k||y^{k-1})$. Then $\mathscr{C} = \cup_k \mathscr{C}_k$

  - Similar characterizations can be found for general TWC and MAC with feedback
  - Each choice of k and $p(x_1^k||y^{k-1})p(x_2^k||y^{k-1})$ leads to an inner bound
  - Not computable

Interactive Coding Scheme

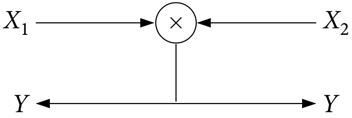


- Code over interleaved blocks (block j = times $j, k+j, 2k+j, \ldots, (n-1)k+j$)
- Block j: input $\mathbf{X}_{1j}$, output $(\mathbf{X}_2^k, \mathbf{Y}_j^k)$, causal channel state $(\mathbf{X}_1^{j-1}, \mathbf{Y}^{j-1})$

$$R_{1j} < I(X_{1j}; X_2^k, Y_j^k \,|\, X_1^{j-1}, Y^{j-1}) \quad \text{is achievable}$$

- Summing over blocks shows that $\sum_{j=1}^{k} R_{1j} < I(X_1^k \to Y^k \| X_2^k)$ is achievable

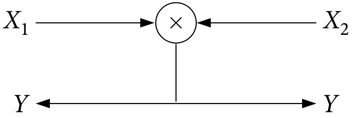
Example: Shannon–Blackwell Binary Multiplying Channel



- Simple bounds on the symmetric capacity (Shannon 1961):

$$\max_{p(x_1)p(x_2)} \frac{1}{2}(I(X_1;Y|X_2)+I(X_2;Y|X_1)) \le C_{\text{sym}} \le \max_{p(x_1,x_2)} \frac{1}{2}(I(X_1;Y|X_2)+I(X_2;Y|X_1))$$

Example: Shannon–Blackwell Binary Multiplying Channel

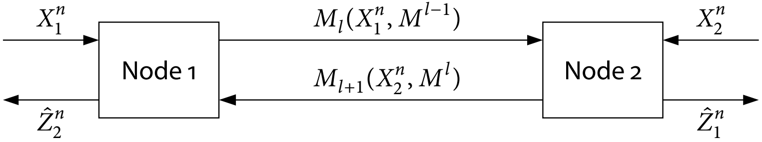


- Simple bounds on the symmetric capacity (Shannon 1961):

$$0.6170 \le C_{\text{sym}} \le 0.6942$$

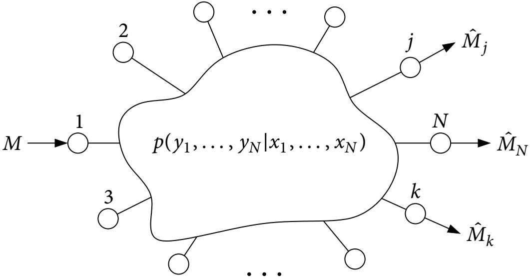
- DB bound + channel augmentation (Hekstra–Willems 1989): $C_{\text{sym}} \le 0.6463$
- Schalkwijk's lower bounds:
  - Iterative refinement coding scheme (Schalkwijk 1982): $0.6191 \le C_{\text{sym}}$
  - + Slepian–Wolf (Schalkwijk 1983): $0.6306 \le C_{\text{sym}}$
  - Further extension (Meeuwissen–Schalkwijk–Bloemen 1995): $0.6307 \le C_{\text{sym}}$
- Directed information inner bound: $\frac{1}{2k}(I(X_1^k \to Y^k \| X_2^k) + I(X_2^k \to Y^k \| X_1^k))$
  - Ardestanizadeh (2010): $0.6191 \le C_{\text{sym}}$
- Question: Can we outperform Schalkwijk (via directed information expression)?

Intermission: Interactive Source Coding and Computing



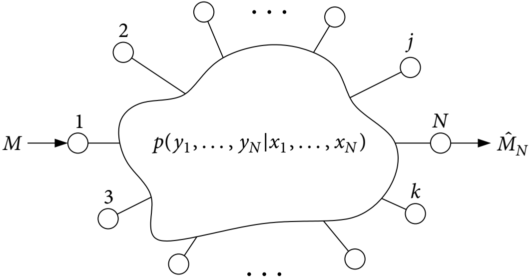
- Two-way lossless source coding:
  - Interaction does not enlarge the optimal rate region
  - One-way Slepian–Wolf coding is optimal (Csiszár–Narayan 2004)
- Two-way lossy source coding:
  - Interaction enlarges the rate–distortion region for correlated sources
  - q-round interactions (Kaspi 1985)
- Two-way lossless computing:
  - Interaction enlarges the optimal rate region even for independent sources
  - Infinite-round interactions (Ma–Ishwar 2008, 2009)

Relay Network



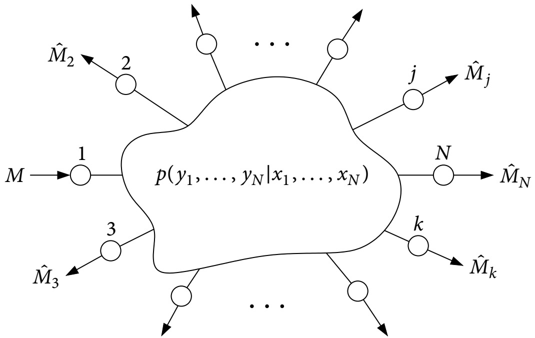
- Topology of the network is defined through $p(y^N | x^N)$

Relay Network



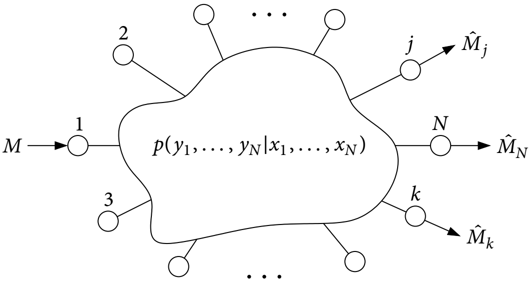
- Topology of the network is defined through $p(y^N | x^N)$
- Unicast

Relay Network



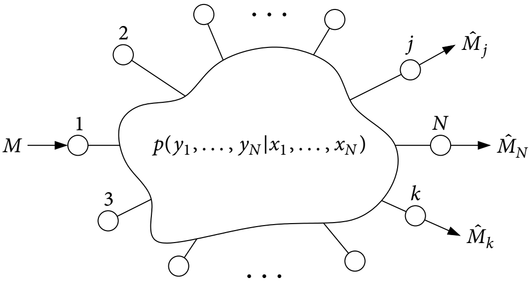
- Topology of the network is defined through $p(y^N | x^N)$
- Unicast vs. broadcast

Relay Network



- Topology of the network is defined through $p(y^N | x^N)$
- Unicast vs. broadcast vs. multicast

Relay Network



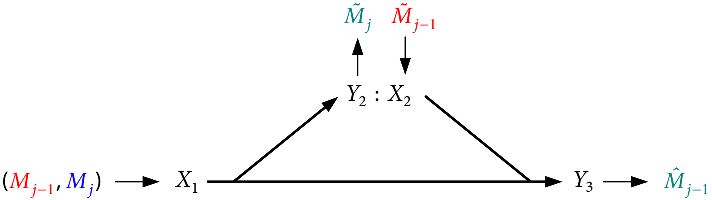
- Topology of the network is defined through $p(y^N | x^N)$
- Unicast vs. broadcast vs. multicast
- Capacity is not known in general
- Many coding schemes have been proposed

Dictionary of Coding Schemes

- Standard parlance: decode–forward, compress–forward, amplify–forward
- Extended vocabulary: partial decode–forward, noncoherent decode–forward, coherent compress–forward, generalized amplify–forward
- Recent coinages: hash–forward, compute–forward, quantize–map–forward, rematch–forward
- Loanwords: analog network coding, noisy network coding, hybrid coding
- Dialects: calculate–forward, clean–forward, combine–forward, demodulate–forward, denoise–forward, detect–forward, estimate–forward, flip–forward, mix–forward, quantize–forward, rotate–forward, scale–forward, (randomly) select–forward, sum–forward, truncate–forward

Basic Coding Schemes

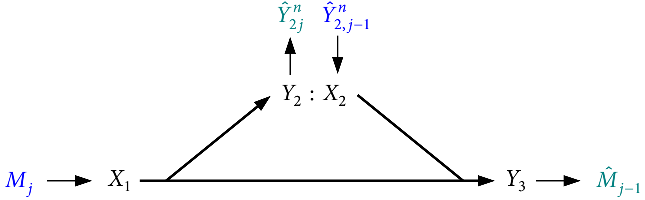
- Decode–forward (Cover–El Gamal 1979)



Basic Coding Schemes

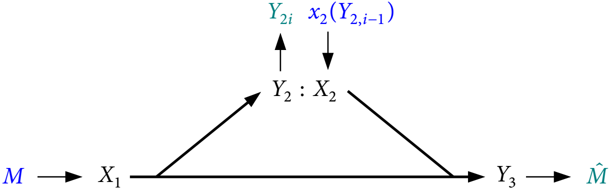
- Decode–forward (Cover–El Gamal 1979)

- Compress–forward (Cover–El Gamal 1979)



Basic Coding Schemes

- Decode–forward (Cover–El Gamal 1979)
- Compress–forward (Cover–El Gamal 1979)
- Amplify–forward (Schein–Gallager 2000)



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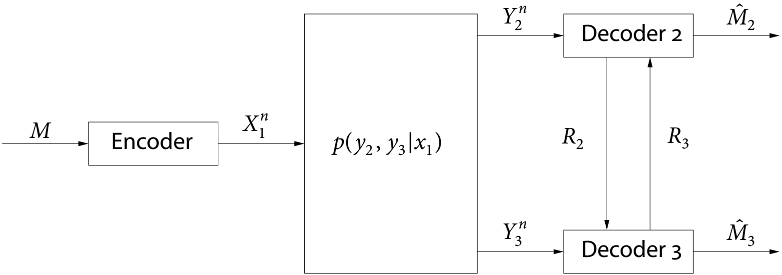
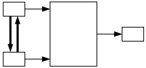
- Compress–forward (Cover–El Gamal 1979)
- Amplify–forward (Schein–Gallager 2000)
- *–forward and extensions (Ahlswede–Cai–Li–Yeung 2000, Kramer–Gastpar–Gupta 2005, Avestimehr–Diggavi–Tse 2011, Lim–Kim–El Gamal–Chung 2011): no/limited interaction

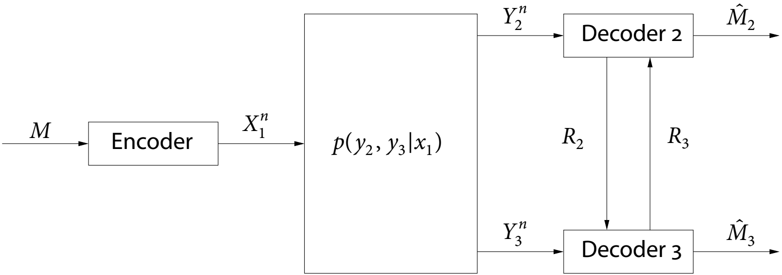
Broadcast Relay Channel (BRC)



- A common message M is to be broadcast to both receivers (Draper–Frey–Kschischang 2003)
- Dual to MAC with partially cooperating encoders (Willems 1983)

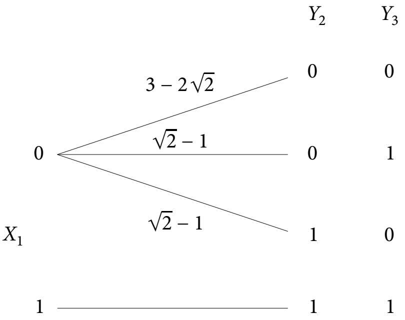


Broadcast Relay Channel (BRC)

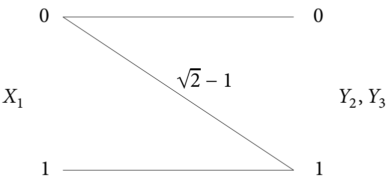


- A common message M is to be broadcast to both receivers (Draper–Frey–Kschischang 2003)
- Dual to MAC with partially cooperating encoders (Willems 1983)
- Capacity $C(R_2 + R_3)$ is not known in general

Example: Binary BRC (Xiang–Wang–K 2011)

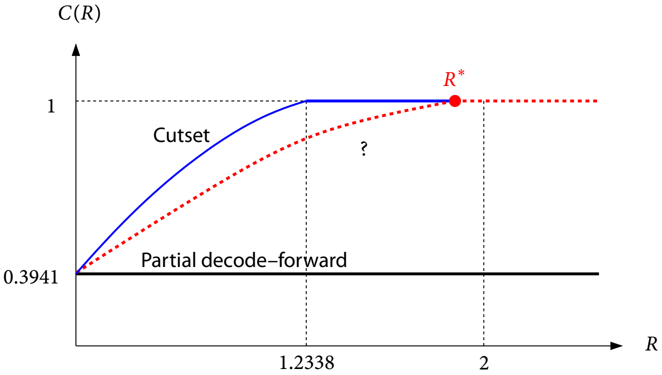


$X_1 = Y_2 \cdot Y_3$



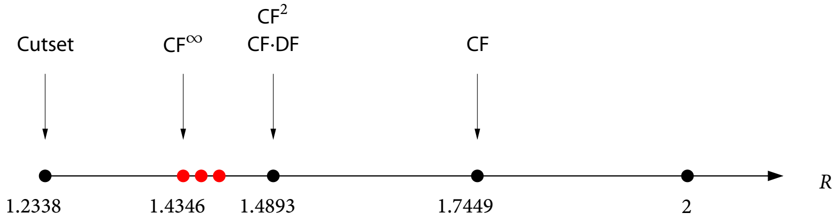
- $C(0) = 0.3941$ (Z channel capacity)
- $C(2) = 1$
- $C(R) = ?$

Example: Binary BRC (Xiang–Wang–K 2011)



- Cutset: $\max_{p(x_1)} \min\{I(X_1; Y_2) + R/2,\ I(X_1; Y_2, Y_3)\}$ $\quad$ ($C(R) = 1$ for $R \geq 1.2338$)
- Partial decode–forward: $C(0)$
- R^*: Interactive computing of $X_1 = Y_2 \cdot Y_3$

Example: Binary BRC (Xiang–Wang–K 2011)



- Compress–forward (Orlitsky–Roche 2001): $H_{\mathcal{G}}(Y_2|Y_3) + H_{\mathcal{G}}(Y_3|Y_2) = 1.7449$
- Interactive relaying:
  - Compress–forward and decode–forward (Draper–Frey–Kschischang 2003):
    $1 - I(X_1; Y_2) + H_{\mathcal{G}}(Y_2|Y_3) = H(Y_2) + H(X_1|Y_3) = 1.4893$
  - Two-round compress–forward: $H(Y_2) + H(X_1|Y_3) = 1.4893$
  - Three-round compress–forward: 1.4488
  - Four-round compress–forward: 1.4427
- Infinite-round compress–forward (Ma–Ishwar 2008, 2009):
  $(1+p)H(p) + p\log(pe^{1-p})\big|_{p=1/\sqrt{2}} = 1.4346 < \mathrm{CF}^{q-1}\cdot\mathrm{DF} = \mathrm{CF}^q$
- Questions: Optimality? Generalizations? Implications?

Concluding Remarks

- Interaction enables richer cooperation among network users
  - Coherent transmission (MAC with feedback)
  - Channel information broadcasting (BC with feedback)
  - Sequential coding (two-way channel)
  - Cooperative decoding (broadcast relay channel)
- Theoretical challenges:
  - Capacity still open for many basic problems
  - Inherently multiletter solutions
    (Permuter–Cuff–Van Roy–Weissman 2008, Ma–Ishwar 2008, 2009, K 2010)
- Practical relevance:
  - How to use feedback (beyond channel estimation, ARQ)
  - Coordinated multipoint (CoMP) transmission/reception

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