Intermittency in Integrable Turbulence

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Intermittency in integrable turbulence

Previous talk by Pierre Suret:
- Focusing 1D-NLSE
- Changes in the statistics of the GLOBAL field (→ Optical rogue waves)

In this talk:
- Defocusing 1D-NLSE
- Changes in the statistics of the GLOBAL field
- Separation of large scales from small scales → Intermittency phenomenon
Integrable Turbulence

- Statistical properties of incoherent nonlinear waves propagating in a wave system ruled by an integrable equation.

“Nonlinear wave systems integrable by the Inverse Scattering Method could demonstrate a complex behavior that demands the statistical description. The theory of this description composes a new chapter in the theory of wave turbulence: Turbulence in Integrable Systems.”


Our experiment is dimensioned in order to keep light fluctuations within the bandwidth of a fast oscilloscope (36 GHz)

- Bandwidth of input/output incoherent light: 14 GHz → 34 GHz
- Bandwidth of photodiodes and oscilloscope: 36 GHz

Incoherent light source with \textbf{gaussian} statistics

\[|\psi(z=0, t)|^2 \sim 100 \text{ ps}\]

Single-mode fiber

Fast direct detection in time domain

\[|\psi(z=L, t)|^2\]

Statistics of output light ???

\[i \partial_z \psi(z, t) = -\frac{\beta_2}{2} \partial_z^2 \psi(z, t) + \gamma |\psi(z, t)|^2 \psi(z, t)\]

\[\beta_2 > 0, \gamma > 0\]

Nonlinear propagation in \textit{defocusing} regime
Optical fiber experiments

Partially-coherent light source

- Ytterbium fiber laser 1064nm
- Optical filters
- Ytterbium fiber amplifier
- Tunable bandpass optical filter
- 1.5km-long PM fiber
- Propagation
- PC

Detection

- OSA
- PDs + Oscilloscope

Δν_{Detection} = 36 GHz

Δν_{in} = 14 GHz
Δν_{out} = 34 GHz
The input optical field: Measurement of a Gaussian Statistics

Gaussian statistics for $\text{Re}(\psi(z = 0, t)) \rightarrow$ Exponential decay of $PDF(P(t)/<P(t)>)$

$$P(t) = |\psi(z = 0, t)|^2$$

$$PDF[P(t)] = PDF[|\psi(z = 0, t)|^2] = \frac{1}{2\sigma^2} \exp\left(-\frac{P}{<P>}\right)$$

$$PDF[\text{Re}(\psi(z = 0, t))] = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{\text{Re}[\psi(z=0,t)^2]}{2\sigma^2}\right)$$

Oscilloscope Bandwidth : 36 GHz
Sampling frequency : 80 Gsa/s (one point every 12.5 ps)

$40.10^6$ points recorded in 500 $\mu$s
The field at the output of the nonlinear fiber: low-tail PDF

Experiments described by the defocusing and integrable 1D-NLSE

\[
i \frac{\partial \psi(z, t)}{\partial z} = -\frac{\beta_2}{2} \frac{\partial^2 \psi(z, t)}{\partial t^2} + \gamma |\psi(z, t)|^2 A(z, t)
\]

Coherent Structures (dark solitons, DSWs) \(\rightarrow\) embedded inside random nonlinear wave
Optical fiber experiments: The tunable bandpass optical filter

Partially-coherent light source

- Ytterbium fiber laser 1064nm
- Optical filters
- Ytterbium fiber amplifier

Propagation

- 1.5km-long PM fiber

Detection

- Tunable bandpass optical filter
- PDs + Oscilloscope
- OSA

\[ \Delta \nu_{\text{in}} = 14 \text{ GHz} \]
\[ \Delta \nu_{\text{out}} = 32 \text{ GHz} \]
Optical fiber experiments: The tunable bandpass optical filter

Bandwidth of the optical power spectrum: 34 GHz
Bandwidth of the optical filter: 6 GHz
Statistics and Dynamics at the output of the tunable bandpass optical filter

Centered Bandpass Filter

Off-Centered Bandpass Filter

Optical Power

Frequency (GHz)

Optical Power

Frequency (GHz)

P(t)/<P(t)>

Time (ns)

P(t)/<P(t)>

Time (ns)

PDF[P(t)/<P(t)>]

P(t)/<P(t)>

PDF[P(t)/<P(t)>]

P(t)/<P(t)>
Intermittency phenomenon (Turbulence)

Intermittency in an integrable optical wave system


Intermittency in Wave Turbulence

PDF of 2nd order differences of the water elevation

Intermittency in an integrable optical wave system

Statistical properties
Defocusing and integrable 1D-NLSE

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“Intermittency in integrable turbulence”
Numerical Simulations of the Experiment

\[ i \frac{\partial \psi(z, t)}{\partial z} = -\frac{\beta_2}{2} \frac{\partial^2 \psi(z, t)}{\partial t^2} + \gamma |\psi(z, t)|^2 \psi(z, t) - i \frac{\alpha}{2} \psi(z, t) \]  

(2)

\( \beta_2 = +20 \text{ ps}^2 \text{ km}^{-1}, \gamma = 6.2 \text{ W}^{-1} \text{ km}^{-1}, \alpha = 0.23 \text{ km}^{-1} \)

Initial Condition:

\[ \psi(z = 0, t) = \sum_{n=-N}^{N} \tilde{\psi}(z = 0, \nu)e^{in2\pi \nu t}. \]  

(3)

\( \text{Re}[\tilde{\psi}(z = 0, \nu)] \) and \( \text{Im}[\tilde{\psi}(z = 0, \nu)] \) are \( \delta \)-correlated independent random variables with gaussian statistics \( \rightarrow \) Random Phase and Amplitude (RPA) model

\( \text{see e. g. S. Nazarenko, Wave Turbulence, Lecture Notes in Physics, Springer (2011)} \)

Our numerical simulations of the experiment take into account (i) fiber losses, (ii) finite detection bandwidth, (iii) exact shape of the input power spectrum, (iv) spectral shape of the tunable optical filter
\[ i \frac{\partial \psi(z, t)}{\partial z} = -\frac{\beta_2}{2} \frac{\partial^2 \psi(z, t)}{\partial t^2} + \gamma |\psi(z, t)|^2 \psi(z, t) - i \frac{\alpha}{2} \psi(z, t) \]
Numerical Simulations of the Experiment

\[ i \frac{\partial \psi(z, t)}{\partial z} = -\frac{\beta_2}{2} \frac{\partial^2 \psi(z, t)}{\partial t^2} + \gamma |\psi(z, t)|^2 \psi(z, t) - i \frac{\alpha}{2} \psi(z, t) \]
Numerical Simulations of the 1D-NLSE (defocusing regime)

\[ i \frac{\partial \psi(z, t)}{\partial z} = -\frac{\beta_2}{2} \frac{\partial^2 \psi(z, t)}{\partial t^2} + \gamma |\psi(z, t)|^2 \psi(z, t) \]

\[ H_L = \frac{\beta_2}{2} \int dt |\partial_t \psi(z, t)|^2 \]

\[ H_{NL} = \gamma \int dt |\psi(z, t)|^4 \]

![Graph showing propagation distance vs. energy for experiment and stationary statistical state.](image-url)
Numerical Simulations of the 1D-NLSE (defocusing regime)

\[ i \frac{\partial \psi(z, t)}{\partial z} = -\frac{\beta_2}{2} \frac{\partial^2 \psi(z, t)}{\partial t^2} + \gamma |\psi(z, t)|^2 \psi(z, t) \]

Stationary Statistical State

- (a) Power Spectrum (W/GHz)
- (b) PDF $P(t)/\langle P(t) \rangle$
- (c) PDF $P_F(t)/\langle P_F(t) \rangle$
- (d) Kurtosis
Numerical Simulations of the 1D-NLSE: Focusing regime

\[ i \frac{\partial \psi(z, t)}{\partial z} = -\frac{\beta_2}{2} \frac{\partial^2 \psi(z, t)}{\partial t^2} + \gamma |\psi(z, t)|^2 \psi(z, t) \]

Optical Power Spectrum (Focusing)

PDFs of the global field

PDFs of the filtered field
Nonlinear propagation in a wave system described by the 1D-NLSE

Changes of the statistics of the global field

Separation of small scales from large scales: intermittency phenomenon both in focusing and in defocusing regimes
The End

Thank you for your attention