A Multi-Resolution Scheme for Analysis of Brain Connectivity Networks

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Joint work with Nagesh Adluru, Emily Balczewski, Barb Bendlin, Moo Chung, SeongJae Hwang, Sterling Johnson, WonHwa Kim and Ozioma Okonkwo

Motivation

Group analysis of image derived representations



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Workflow

- Lots of QC and corrections
- Figure out how to warp one image to another
- Perform atom-wise statistical analysis after a laundry list of pre-processing



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Finally obtain heat maps of discriminative voxels (after p-value corrections)

Figure 7a



Figure 7b



Voxel wise analysis on the Image grid



Voxel wise analysis on the Image grid



The multiple testing problem is significant especially when testing at a million (independent?) voxels

Type 1 and Type 2 Errors in Neuroimaging

	H_0 is true	H_0 is false
	voxel not discriminative	voxel is discriminative
Reject H_0	Type 1 Error (false positive)	Correct
\neg Reject H_0	Correct	Type 2 Error (false negative)

To be safe on Type 1 errors, a super conservative strategy may lead to many false negatives

Analysis of cortical meshes







Analysis of connectivity networks





DTI Tractography

White matter orientational information from DTI



DTI Tractography

Streamline fiber tracking from orientation field



DTI Tractography

Brain atlas and connectivity matrix



- Statistical analysis on connectivity matrix involves $\mathcal{O}(n^2)$ terms
- Sample sizes are small in Neuroimaging studies: large p, small n
- The "connectivity" between different "nodes" of the graph is arbitrary

Challenges

- Statistical analysis on connectivity matrix involves $\mathcal{O}(n^2)$ terms
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Need for methods which are <u>sensitive</u> to small signal differences

<u>Multi-resolution</u> analysis of shapes and connectivity networks

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- The "connectivity" between different "nodes" of the graph is arbitrary
- Identifying "differences" is related to finding "similarity".
- Comparison of signals in multiple resolutions
- E.g., SIFT feature
- An end-to-end statistical explanation as well (with some disclaimer)

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Fourier Transform

- Superposition of sinusodial functions $e^{i\omega t}$ in different frequencies
- Fourier Transform of f(x):
 (From native space to the frequency space)

$$\hat{f}(\omega) = \int f(x) \mathrm{e}^{-j\omega x} \mathrm{d}x$$

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• Inverse Fourier Transform: (Reconstruct my signal)

$$f(x) = \frac{1}{2\pi} \int \hat{f}(\omega) \mathrm{e}^{j\omega x} \mathrm{d}\omega$$

Weather in Madison affects forecast in Banff

 Ringing Artifact (e.g., Gibbs phenomenon) (caused by infinite support of Fourier bases)



Figure: Ringing artifact using Fourier bases.

Wavelet bases

Unlike Fourier bases, Wavelets are localized in both time and frequency

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• Mother wavelet ψ : function of dilation s and translation a

$$\psi_{s,a}(x) = \frac{1}{s}\psi\left(\frac{x-a}{s}\right)$$

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Wavelets
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Inverse Fourier
$$f(x) = \frac{1}{2\pi} \int \hat{f}(\omega) e^{j\omega x} d\omega$$

Inverse Wavelets
$$f(x) = \frac{1}{C_{\psi}} \iint W_f(s, a) \psi_{s,a}(x) da ds$$



Wavelet in the Frequency Domain

- ψ (blue) in the frequency domain: band-pass filters
- ϕ (red) in the frequency domain: low-pass filter



Figure: A scaling function (red) and band-pass filters (blue) in the frequency domain.

Classical Wavelet example



Figure: Example of multi-resolutional characterization (from SGWT toolbox)
• Wavelets in Euclidean Space



• Wavelets in Euclidean Space



• Wavelets on Graphs (Scale? Translation?)



Key Idea

- Native domain: $G = \{V, E, \omega\}$
- First step: define analogue of Fourier transform on graphs (Maggioni, 2006), (Hammond 2012)
- Construct orthonormal bases defined on structure of G
- Construction of filters in the frequency domain to get band pass effect

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Ingredients

- Adjacency A: square matrix, $a_{m,n}$ for connectivity information
- Degree Matrix: D: Diagonals are the sum of weights
- Graph Laplacian: L = D A
- Orthonormal bases χ_l and eigenvalues λ_l , $l \in \{0, \dots, N-1\}$ of L

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Fourier
$$\hat{f}(\omega) = \langle f, e^{j\omega x} \rangle = \int f(x) e^{-j\omega x} dx$$

Graph Fourier

$$\hat{f}(l) = \langle f, \chi_l \rangle = \sum_{n=1}^{N} f(n) \chi_l^*(n)$$

For graph Fourier transform, the orthonormal bases come from spectral graph theory: from a self-adjoint operator (the graph Laplacian)

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Construct original signal using graph Fourier bases and coefficients f̂
L ≥ 0, so χ and χ* are the same.

Wavelet function on node m, localized on node n

$$\psi_{s,n}(m) = \sum_{l=0}^{N-1} g(s\lambda_l) \chi_l^*(n) \chi_l(m)$$

Choose a kernel function g (band-pass filter)

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Choose a kernel function g (band-pass filter)

Spectral Graph Wavelet Transform on f(n) on node n

$$W_f(s,n) = \sum_{l=0}^{N-1} g(s\lambda_l) \hat{f}(l) \chi_l(n)$$

An Example of Wavelet Functions on Graphs

Example of wavelets on graphs (mesh surface)



An Example of Wavelet Transform on Graphs

Forward and inverse wavelet transform of f(n) (on a brain mesh)

$$W_f(s,n) = \langle f, \psi_{s,n} \rangle = \sum_{l=0}^{N-1} g(s\lambda_l) \hat{f}(l) \chi_l(n)$$

$$f(n) = \frac{1}{C_g} \sum_{n=1}^N \int_0^\infty W_f(s,n) \psi_{s,n}(m) \frac{ds}{s}$$



Figure: Forward and Inverse wavelet transform.

- Group analysis: Identify regions showing differences between disparate groups
 - Alzheimer's disease versus Healthy controls
- Statistical Parametric Mapping
 - Hypothesis test at vertex level
 - Multiple comparison correction (Bonferroni, etc.)
 - Check which regions survive

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Domain: Brain surface mesh;

Signal: Cortical thickness (distance between inner/outer cortical surfaces)



- Wavelet multi-scale descriptor (WMD):
 - A set of wavelet coefficients at each vertex n for each scale s

$$WMD_f(n) = \{W_f(s, n) | s \in S\}$$
(1)



WMD on Lenna at scale 0, 1, 2.

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(1)



WMD on cortical thickness at scale 0, 1, 2.

• Group analysis: Alzheimer's disease (AD) subjects versus healthy controls

ADNI data				
Category	AD (mean)	AD (s.d.)	Ctrl (mean)	Ctrl (s.d.)
# of Subjects	160	-	196	-
Age	75.53	7.41	76.09	5.13
Gender (M/F)	86 / 74	-	101 / 95	-
MMSE at Baseline	21.83	5.98	28.87	3.09
Years of Education	13.81	4.61	15.87	3.23

Table: ADNI data details

• *p*-values from Hotelling's T^2 . Then, multiple comparison correction



Figure: p-values (in $-\log_{10}$ scale) after FDR correction at $q = 10^{-5}$. Row 1: Cortical thickness, Row 2: SPHARM, Row 3: WMD

- *p*-values and False discovery rate (FDR)
- Group difference: the vertices below the FDR threshold



Figure: Sorted *p*-values from group analysis on cortical thickness (cyan), SPHARM (green), and WMD (red), and the FDR thresholds are represented by dotted lines.

• Effect of changes of q in FDR



Figure: p-values (in $-\log_{10}$ scale and normalized) showing the effect of FDR correction on a left hemisphere using WMD with FDR $q = 10^{-3}$ (left column), $q = 10^{-5}$ (middle column) and $q = 10^{-7}$ (right column) respectively.

- Group analysis on ADRC (local) dataset
- The dataset consists of 42 AD and 50 Controls subjects
- Expect to find weaker signal but the same regions found from ADNI (small *n*)

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Figure: Group analysis on AD vs Controls. *p*-values in $-\log_{10}$ scale after FDR correction at q = 0.05 are shown on a smoothed brain surface. Top row: Result using smoothed cortical thickness, Bottom row: Result using WMD

Line Graph Transform

- We would like to detect group differences in brain connectivity.
- Here, the information lies on the edges of a graph, not on the vertices.
- We need to transform the graph G to apply our framework.



Line Graph Transform

- Line graph L(G) is a dual form of graph G.
- Interchange of the roles of $\mathcal V$ and $\mathcal E$ in $\mathcal G$.
- Let g_{ij} be the elements in the adjacency matrix A_L of L(G),

$$g_{ij} = egin{cases} 1 & ext{if} \quad v \in \mathcal{V}, \; v \backsim e_i, e_j \ 0 & ext{otherwise} \end{cases}$$

where v is a vertex in \mathcal{V} and e is an edge in \mathcal{E} .



Figure: Examples of graphs and the corresponding line graphs. Original graphs with vertices (red) and edges (yellow) with edge weights (thickness), and corresponding line graphs with vertices (yellow) with function (vertex size) and edges (red).

Filtering Process on Line Graphs

- Once we obtain L(G), we have a function defined on vertices.
- Now, we can apply filtering operations.
- Recall that wavelet transform is a band-pass filtering operation.
- An illustration of smoothing operation by line graph transformation.



Figure: A toy example of graph structure filtering. The top panel shows the graph filtering steps: (1) Construction of the line graph, (2) filtering the signal on the line graph vertices, (3) reconstructing the filtered graph. The bottom panel shows the corresponding adjacency matrices.

Multi-scale Descriptor for Brain Connectivity

• Derive multi-scale descripter on L(G) and transform back to G

Wavelet Connectivity Signature (WaCS) $WaCS_f(e) = \{W_f(s, e) | s \in S\}$ (2)



Figure: An example of multi-resolution on graph edges at various resolutions.

ADRC and WRAP Dataset

- ADRC: 58 healthy vs. 44 AD subjects
- WRAP: 93 Family history (FH) positive vs. 250 negative subjects
- 162 parcellated brain regions as region of interest (ROI)
- Mean fractional anisotropy (FA) of tracts between ROIs
- $\bullet~162\times162$ connectivity matrix for each subject



Figure: Left: ROIs and track bundles, Right: connectivity matrices.

ADRC Study









Application 2: Brain Connectome Discrimination - ADRC

- GLM on FA (control for age/gender) for *p*-values
- Bonferroni at $\alpha = 0.01$ for multiple comparisons correction
- The edge color represents the direction of difference
 - Red: stronger connection in Controls group
 - Blue: stronger connection in AD group



Figure: Significant connection difference between AD vs control group with direction of significance. Edge thickness corresponds to *p*-value, and the color denotes to the direction of strength.

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Figure: 81 significant connection difference between AD vs control group with direction of significance. Edge thickness corresponds to *p*-value, and the color denotes to the direction of strength.

- \bullet Hub regions: ROIs with connected edges ≥ 5
 - Left: superior and transverse occipital sulcus, superior parietal lobule,
 - Right: hippocampus, transverse occipital sulcus, precuneus, medial occipito-temporal gyrus



Figure: Illustration of the hub ROIs with connections identified as showing significant group difference between AD and control groups.

Further Analysis on Preclinical AD (small effect size)

- 615 Connections of interest (COIs) selected from ADRC study
- COI selection: FDR at 0.001 on AD vs. CN analysis
- Family History analysis on the COIs using WRAP data
Application 2: Brain Connectivity Discrimination - WRAP

Further Analysis on Preclinical AD (small effect size)

- Applying standard GLM analysis on FA revealed no connection
- MGLM (controlling for age/gender) for WaCS on COIs for *p*-values
- FDR at $\alpha = 0.05$ for multiple comparisons correction (less conservative than Bonferroni) to detect subtle variations

Application: Brain Connectivity Discrimination - WRAP

• 7 connections identified

- Left: 5 ROIs (orbital gray matter, calcarine sulcus, lateral orbital sulcus, postro ventral cingular gyrus and pericallosal sulcus)
- Right: 4 ROIs (precuneus, superior parietal lobule, posterior sylvian fissure, calcarine sulcus, pericallosal sulcus)



Figure: Significant group difference between FH+ and FH- groups. Color gives sign of strength: red (and blue) are stronger in FH- (and FH+ group).

Longitudinal Analysis of PIB Images

- W. H. Kim, B. B. Bendlin, M. K. Chung, S. C. Johnson, V. Singh, Statistical Inference Models for Image Datasets with Systematic Variations, CVPR, 2015.
- Longitudinal SUVR images from PIB scans from two time points



Figure: Changes of longitudinal SUVR images from a single subject. Decreases of the intensities are shown in various regions.

Analysis of Images with Imperfect Registration

- W. H. Kim, S. Ravi, S. C. Johnson, O. Okonkwo, V. Singh, On Statistical Analysis of Neuroimages with Imperfect Registration, ICCV, 2015
- Analysis with incorrectly registered brain images

(b)

(a)



Figure: Registered FDG-PET scans of a subject. a) Using the original deformation field, b), c) Using deformation field with 5% and 10% noise level, d) A slice of GRF for spatially correlated noise, e) Using deformation field with d) as noise.

(d)

(c)

(e)

The BEMMA hammer (Dahl and Newton, 2006)



The BEMMA hammer (Dahl and Newton, 2006)



The BEMMA hammer (Dahl and Newton, 2006)



Graph restricted Chinese Restaurant Processes



Graph restricted Chinese Restaurant Processes



- The CRP is explicitly restricted by the image lattice
- Sampling must become significantly easier (relative to distance dependent CRPs)
- For cluster C_i , improvements in statistical power $\propto |C_i|$?





Thanks to the organizers!

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