Holographic non-relativistic transport in strongly correlated systems from Horava gravity

Gauge/Gravity Duality & Condensed Matter Physics, Banff, Alberta

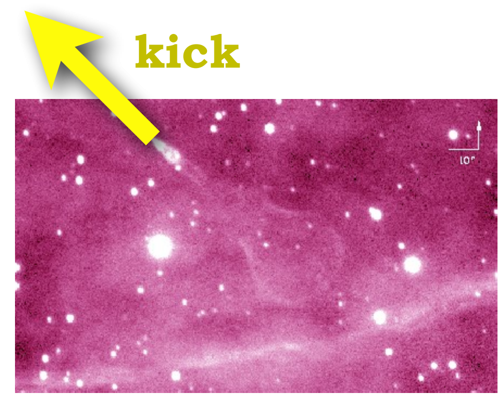
04.Mar.2016

Matthias Kaminski
University of Alabama

in collaboration with
Richard Davison, Saso Grozdanov, Stefan Janiszewski, Steffen Klug

Things I will not talk about …

Not today: Neutron star kicks



observation: neutron stars undergo a large momentum change (a kick)
[Chatterjee et al.; Astrophys. J (2005)]

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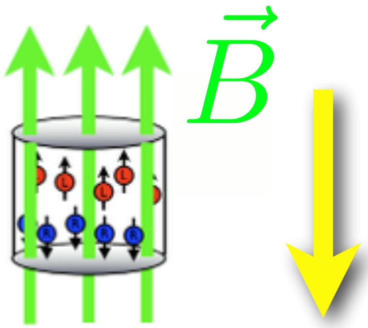

hydrodynamics: fluids with left-handed and right-handed particles produce a **current** along magnetic field
[Son,Surowka; PRL (2009)]
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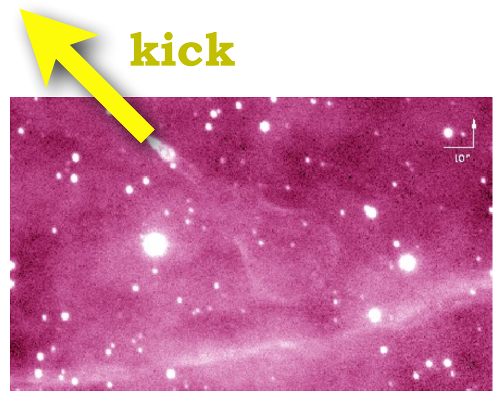

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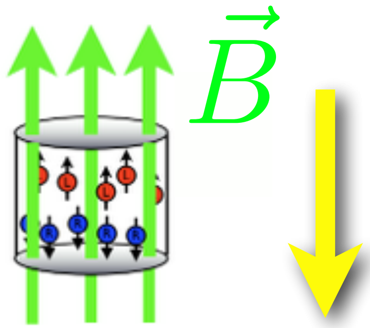

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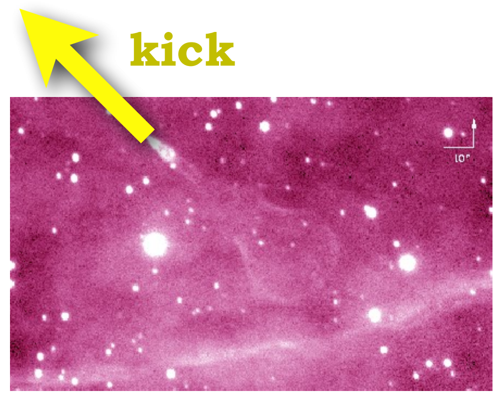

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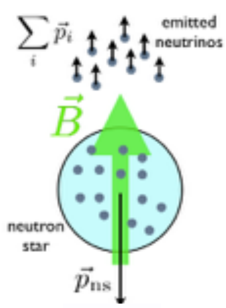


Anomalous hydrodynamics leads to neutron star kicks

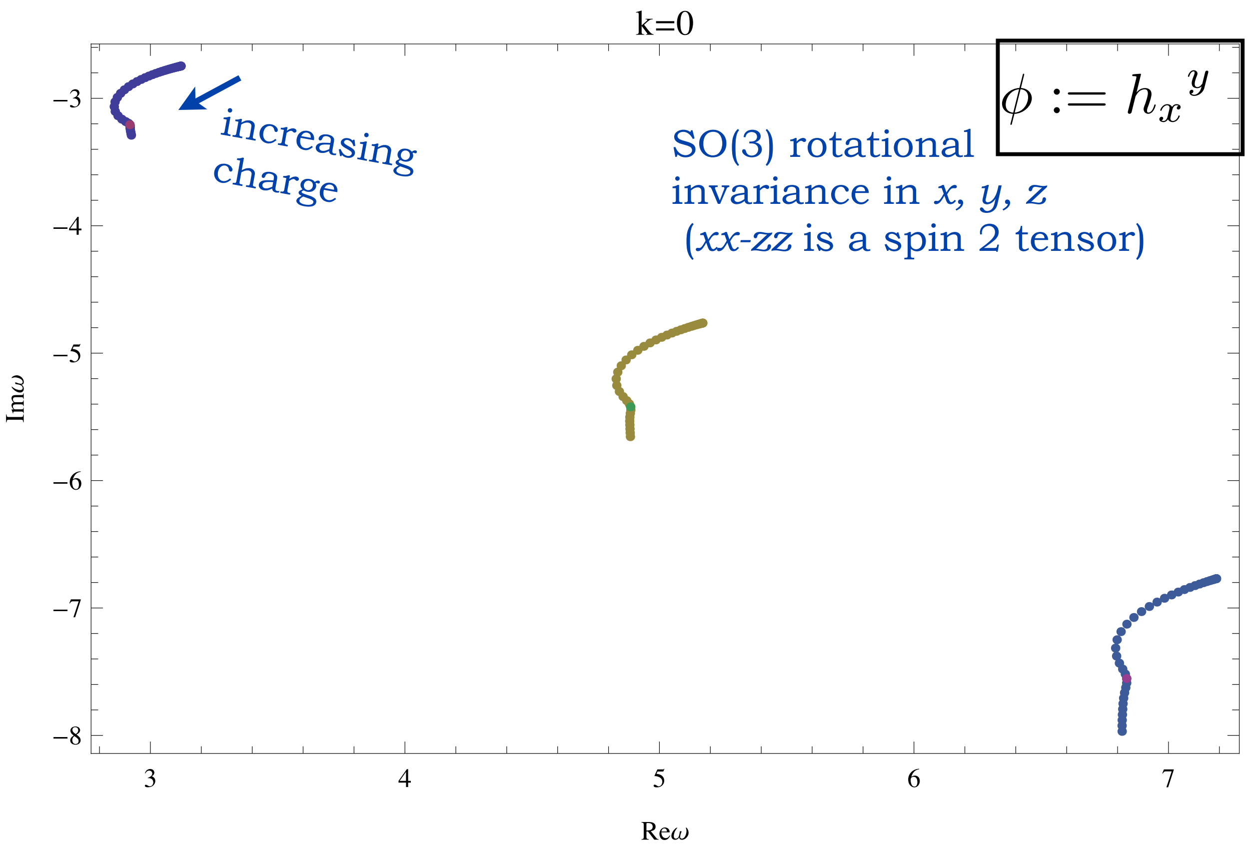
[Kaminski, Uhlemann, Schaffner-Bielich, Bleicher; (2014)]

see also [Shaverin, Yarom;(2014)]

Also not today: quasi normal modes

Equilibrium solution *[Janiszewski, Kaminski; PRD (2016)]*

Reissner-Nordstrom (charged) black branes in 5-dim AdS



Agreement with far from equilibrium setup at late times.

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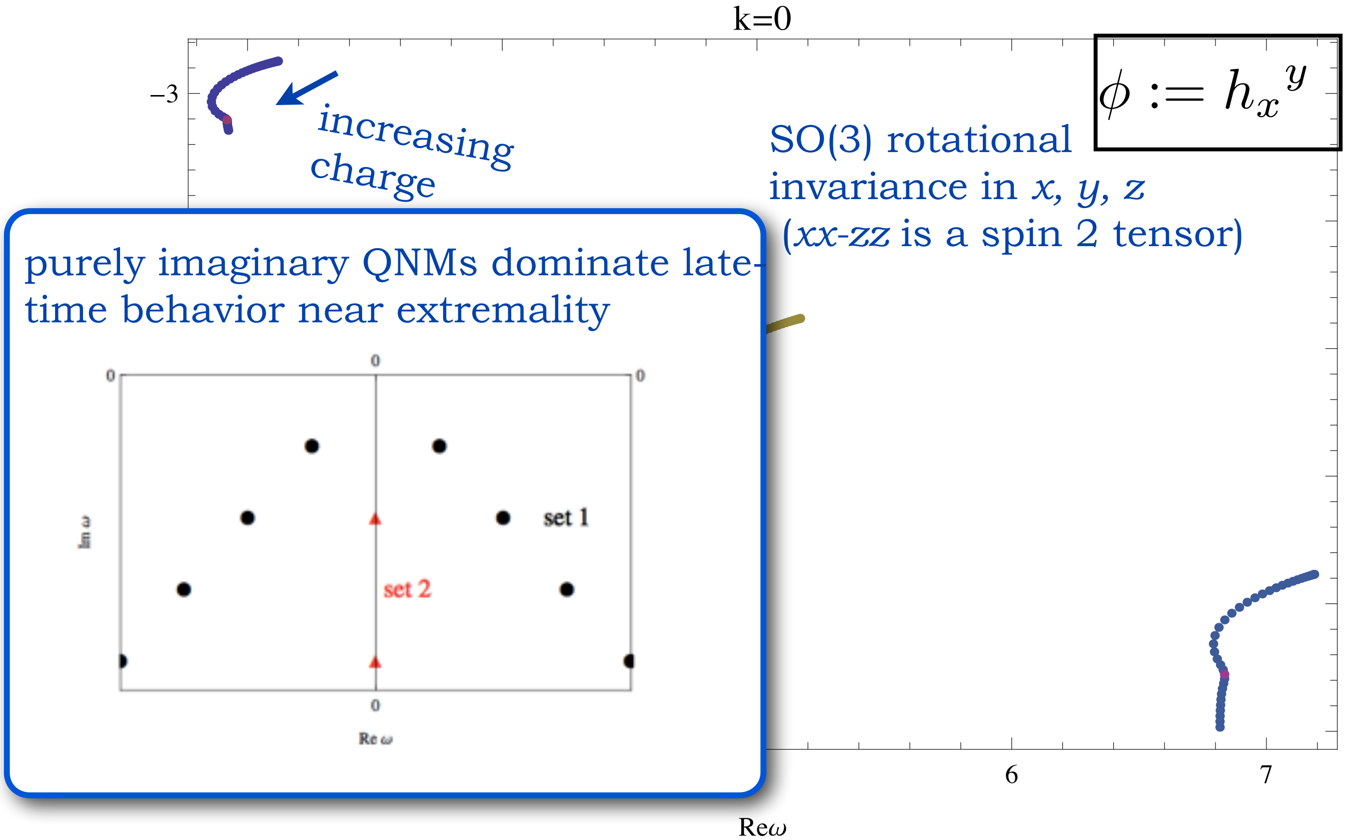

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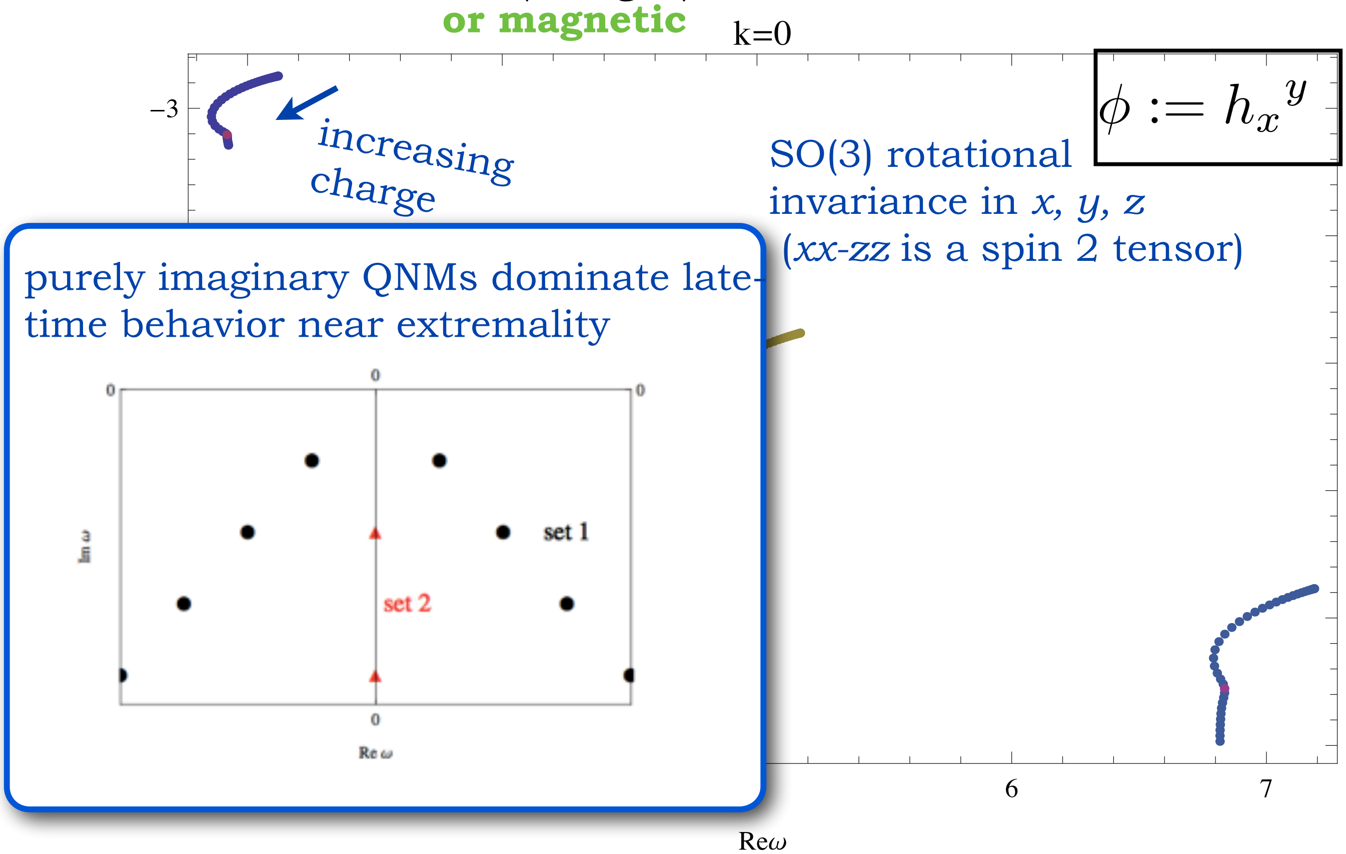
or magnetic



$\boxed{\phi := {h_x}^y}$

SO(3) rotational invariance in *x, y, z* (*xx-zz* is a spin 2 tensor)

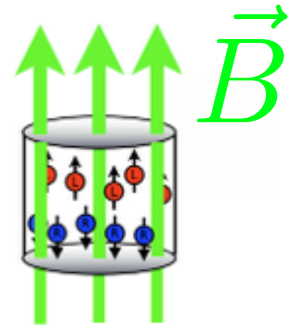
purely imaginary QNMs dominate late-time behavior near extremality

Agreement with far from equilibrium setup at late times.

Outline

1. Motivation: limits of relativistic theories
2. Horava gravity
3. Non-relativistic hydrodynamics from Horava
4. Lessons learned

1. Motivation: limits of relativistic theories



Hydrodynamic transport with anomalies

- chiral transport in 3+1 dimensions!?
- chiral transport present in 2+1 dimensions?
- measurable in table-top experiment?
- non-relativistic experiments *c.f. Sachdev's talk: "all holographic models are relativistic at some level"*

Claim: Horava gravity is a "generic" holographic model with "non-relativistic" symmetries, allowing computation of shear viscosity over entropy density and conductivities.

1. Motivation: limits of relativistic theories

Example:
- chiral fluids in 2+1 dimensions
- start with relativistic hydrodynamics
- “send speed of light to infinity”

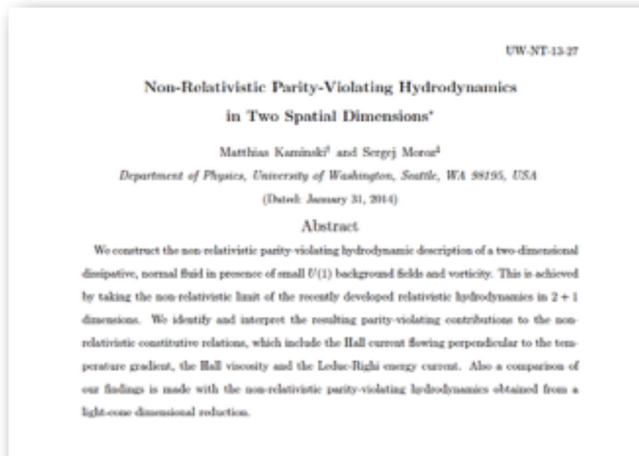
arXiv:1112.4498v1 [hep-th] 19 Dec 2011

Parity-Violating Hydrodynamics in 2+1 Dimensions

ABSTRACT: We study relativistic hydrodynamics of normal fluids in two spatial dimensions. When the microscopic theory breaks parity, extra transport coefficients appear in the hydrodynamic regime, including the Hall viscosity, and the anomalous Hall conductivity. In this work we classify all the transport coefficients in first order hydrodynamics. We then use properties of response functions and the positivity of entropy production to restrict the possible coefficients in the constitutive relations. All the parity-breaking transport coefficients are dissipationless, and some of them are related to the thermodynamic response to an external magnetic field and to vorticity. In addition, we give a holographic example of a strongly interacting relativistic fluid where the parity-violating transport coefficients are computable.

$$c \to \infty$$

$$\Longrightarrow$$



UW-NT-13-27

Non-Relativistic Parity-Violating Hydrodynamics in Two Spatial Dimensions

Matthias Kaminski and Sergej Moroz

Department of Physics, University of Washington, Seattle, WA 98195, USA

(Dated: January 31, 2014)

Abstract

We construct the non-relativistic parity-violating hydrodynamic description of a two-dimensional dissipative, normal fluid in presence of small $U(1)$ background fields and vorticity. This is achieved by taking the non-relativistic limit of the recently developed relativistic hydrodynamics in $2+1$ dimensions. We identify and interpret the resulting parity-violating contributions to the non-relativistic constitutive relations, which include the Hall current flowing perpendicular to the temperature gradient, the Hall viscosity and the Leduc-Righi energy current. Also a comparison of our findings is made with the non-relativistic parity-violating hydrodynamics obtained from a light-cone dimensional reduction.

relativistic, parity-odd transport included
[Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom; JHEP (2012)]

non-relativistic, parity-odd transport included
[Kaminski, Moroz; PRB, (2014)]

Successfully reproduces (parity-preserving) Navier-Stokes equation

continuity: $\partial_t \rho + \partial_i(\rho v^i) = 0,$

momentum: $\partial_t(\rho v^i) + \partial_j \Pi^{ij} = \frac{\rho}{m}\left(\mathcal{E}^i + \mathcal{B}\epsilon^{ij} v_j\right),$

energy: $\partial_t\left(\epsilon_{nr} + \frac{1}{2}\rho v^2\right) + \partial_i j^i_\epsilon = \frac{\rho}{m}\mathcal{E}^i v_i,$

[Landau, Lifshitz]

Parity-violating "Navier-Stokes"

[Kaminski, Moroz; PRB, (2014)]

continuity: $\partial_t \rho + \partial_i(\rho v^i) = 0,$

momentum: $\partial_t(\rho v^i) + \underline{\partial_j \Pi^{ij}_{tot}} = \mathcal{E}^i j^0 + \mathcal{B}\epsilon^{ij} \underline{j_j},$

energy: $\partial_t \underline{j^0_{\epsilon,tot}} + \partial_i \underline{j^i_{\epsilon,tot}} = \mathcal{E}^i \underline{j_i},$

solutions: [Lucas, Surowka; (2014)]

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Working in "magnetovortical frame" in the relativistic parent theory, we obtain a suspicious term:

charge current: $$j^i = \frac{1}{m}\left[\rho v^i + \boxed{\frac{\partial \Pi}{\partial \Omega_{nr}}} \epsilon^{ij} \partial_j \ln T\right]$$

transport coefficient restricted by hand to be function of T only in order to comply with Galilean boost invariance

1. Motivation: limits of relativistic theories

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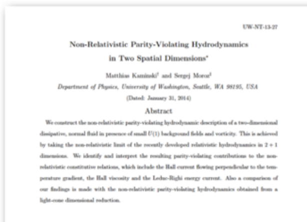

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- Newton-Cartan data relevant
- **Milne-invariance important**

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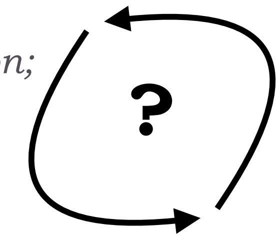

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Milne-invariant if “weird“ transport coefficient $\frac{\partial \Pi}{\partial \Omega_{nr}}$ vanishes.

“Only a subset of relativistic hydrodynamic frames has a regular large c limit. Magnetovortical frame spoils Milne invariance.“
[Jensen, Karch; (Dec. 2014)]

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[Jensen; (Nov. 2014)]

2. Horava gravity

c.f. discussion session Banff 2013

- well-defined setting
- non-relativistic symmetries
- sub-sector can be mapped to General Relativity
- can be obtained as a large c limit (if desired)

2. Horava gravity

[Horava; PRD (2009)]

conjectured holography: [Janiszewski, Karch; PRL (2012), JHEP (2012)]

Philosophies:

1. Obtain Horava as large c limit / embed it in string theory

$$c \to \infty$$
$$\epsilon \to nmc^2 + \epsilon_{nr}$$
$$\mu \to mc^2 + \mu_{nr}$$

2. Assume Horava is all there is.

2. Horava gravity

[Horava; PRD (2009)]
[Janiszewski, Karch; PRL (2012), JHEP (2012)]

Horava gravity in khronon formulation:

$$S_K = \frac{1}{16\pi G_K}\int\sqrt{-g}\left(\mathcal{R} - 2\Lambda + c_4 u^M \nabla_M u^N u^P \nabla_P u_N - c_2\left(\nabla_M u^M\right)^2 - c_3 \nabla_M u^N \nabla_N u^M\right)$$

Einstein gravity coupled to scalar,
providing time-foliation via $\langle\phi\rangle = t$

$$u_M \equiv \frac{-\partial_M \phi}{\sqrt{-g^{NP}\partial_N\phi\partial_P\phi}}$$

time-like foliation vector

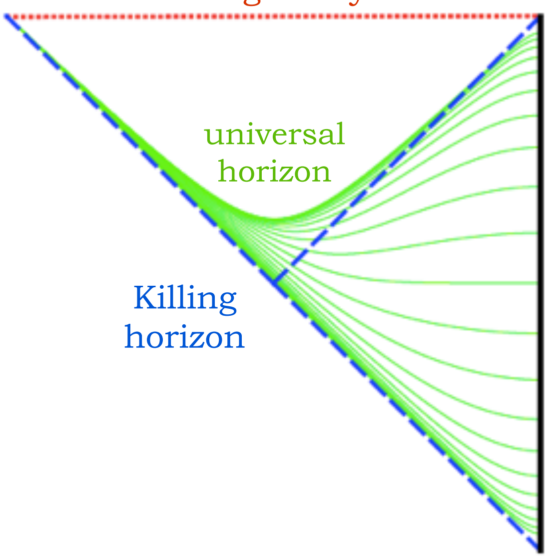
Mode velocities : $$s_2^2 = \frac{1}{1-c_3}, \quad s_0^2 = \frac{(c_2+c_3)(D-1-c_4)}{c_4(1-c_3)(D-1+Dc_2+c_3)}$$

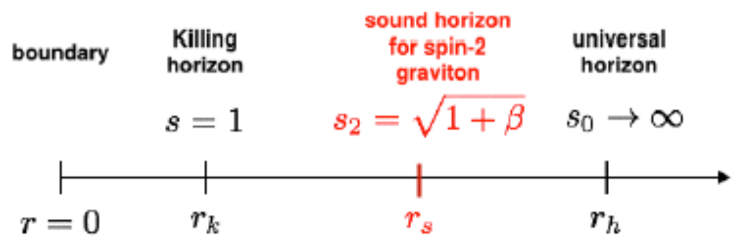
spin 2 and spin 0 modes travel with distinct velocities

Analytic Horava black brane solution

[Janiszewski; JHEP (2014)]

Black brane solution





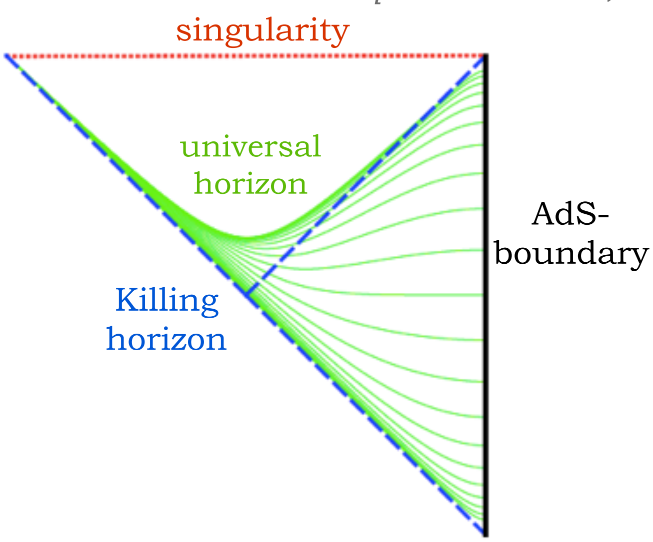
- thermodynamics *[Janiszewski; (2014)]* *[Bhattacharyya, Mattingly; (2014)]*
- fluid/gravity for z=1 (perturbative in coupling) *[Eling, Oz; (2014)]*
- parity-odd transport for QHE $B \sim \mathcal{O}(\partial^0)$ *[Wu, Wu; (2014)]*
- hydrodynamics *[Davison, Grozdanov, Janiszewski, Kaminski, Klug; in preparation]*

This Horava black brane solution is remarkable as it is perturbatively stable against switching on all possible Horava couplings.
[Janiszewski; JHEP (2014)]

Symmetries of Horava black brane

Black brane solution

[Janiszewski; JHEP (2014)]



Lifshitz symmetry *c.f. Andrade's talk*

c.f. [Griffin, Horava, Melby-Thompson; PRL (2013)]

- Weyl scalings: $t \to \lambda^z t; \quad z = 1$
  $x^i \to \lambda x^i; \quad i$ spatial
- translations: $t \to t + a^0$
  $x^i \to x^i + a^i$
- spatial rotations: $x^i \to R_{ij}\, x^i$
- no boost symmetry,

no particle number conservation

Bulk symmetries: foliation-preserving diffeomorphisms

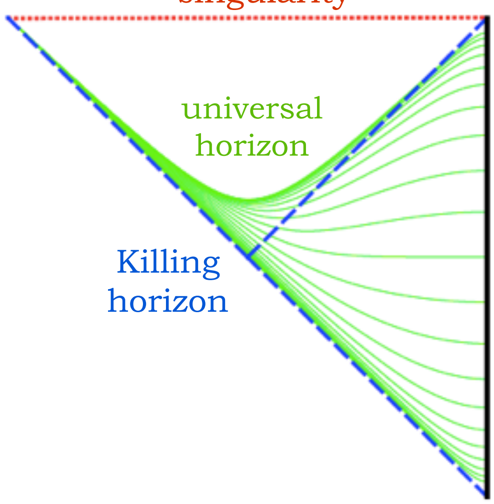
These are not quite the non-relativistic symmetries we had in mind previously, but we have analytic solutions here.

Details of Horava black brane

More convenient formulation of Horava action (invariant under foliation-preserving diffeomorphisms)

$$S^H = \frac{1}{16\pi G_H}\int d^4x\, N\sqrt{G}\left(K_{IJ}K^{IJ} - (1+\lambda)K^2 + (1+\beta)(R-2\Lambda) + \alpha\frac{\nabla_I N \nabla^I N}{N^2}\right)$$

Black brane solution *[Janiszewski; JHEP (2014)]*



$$g_{XY}dx^X dx^Y = -N^2dt^2 + G_{IJ}\left(dx^I + N^I dt\right)\left(dx^J + N^J dt\right)$$

$$\alpha = 0 \qquad z = 1$$

$$N = \frac{r_h^3 - r^3}{r r_h^3},$$

$$N_I = \left(\frac{r\sqrt{1+\beta}}{r_h^3 - r^3}, 0, 0\right),$$

$$G_{IJ} = \begin{pmatrix} \frac{r_h^6}{r^2(r_h^3-r^3)^2} & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & \frac{1}{r^2} \end{pmatrix},$$

Universal horizon: r_h

Killing horizon: $r_k \equiv r_h/(1+\sqrt{1+\beta})^{1/3}$

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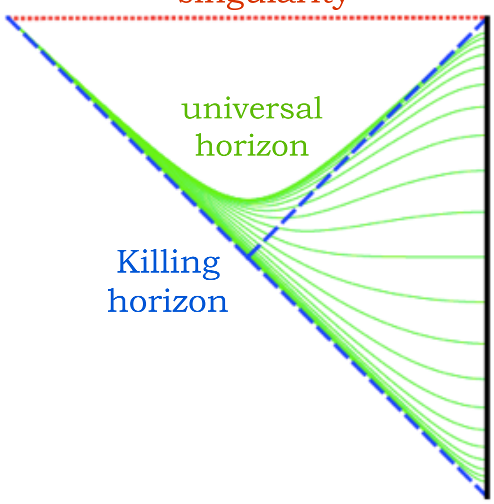
Black brane solution *[Janiszewski; JHEP (2014)]*



$$g_{XY}dx^X dx^Y = \underline{-N^2 dt^2} + \underline{G_{IJ}}\left(dx^I + N^I dt\right)\left(dx^J + \underline{N^J dt}\right)$$

$$N = \frac{r_h^3 - r^3}{r r_h^3},$$

$$\alpha = 0 \qquad N_I = \left(\frac{r\sqrt{1+\beta}}{r_h^3 - r^3}, 0, 0\right),$$

$$z = 1$$

$$G_{IJ} = \begin{pmatrix} \frac{r_h^6}{r^2(r_h^3 - r^3)^2} & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & \frac{1}{r^2} \end{pmatrix},$$

Universal horizon: r_h

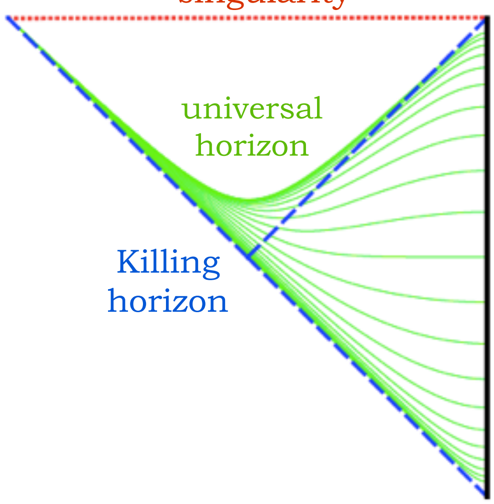
Killing horizon: $r_k \equiv r_h/(1+\sqrt{1+\beta})^{1/3}$

Details of Horava black brane

More convenient formulation of Horava action (invariant under foliation-preserving diffeomorphisms)

$$S^H = \frac{1}{16\pi G_H}\int d^4x\, N\sqrt{G}\left(K_{IJ}K^{IJ} - (1+\lambda)K^2 + (1+\beta)(R-2\Lambda) + \alpha\frac{\nabla_I N \nabla^I N}{N^2}\right)$$

Black brane solution *[Janiszewski; JHEP (2014)]*



$$g_{XY}dx^X dx^Y = \underline{-N^2 dt^2} + \underline{G_{IJ}}\left(dx^I + N^I dt\right)\left(dx^J + \underline{N^J dt}\right)$$

$\alpha = 0$

$z = 1$

$$\underline{N = \frac{r_h^3 - r^3}{r r_h^3}},$$

$$N_I = \left(\frac{r\sqrt{1+\beta}}{r_h^3 - r^3}, 0, 0\right),$$

$$G_{IJ} = \begin{pmatrix} \frac{r_h^6}{r^2(r_h^3 - r^3)^2} & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & \frac{1}{r^2} \end{pmatrix},$$

Universal horizon: r_h

Killing horizon: $r_k \equiv r_h/(1+\sqrt{1+\beta})^{1/3}$

Checking tool at *z*=1: field redefinitions

Horava gravity at *z*=1

Field redefinitions

$$\hat{g}_{XY} \equiv g_{XY} - (\sigma - 1) u_X u_Y$$

$$\hat{u}^X \equiv u^X / \sqrt{\sigma},$$

General relativity

3. Non-relativistic hydrodynamics from Horava gravity

preliminary results

3. Non-relativistic hydro from Horava gravity

Strategy: *repeat [Policastro, Son, Starinets; JHEP (2002)]*

• **Solve gravitational problem:**

» derive fluctuation equations for gravity fields

» expand in powers of frequency and momentum (hydrodynamic approximation)

» solve fluctuation equations analytically

• **use gauge/gravity to translate gravity result into field theory result:**

correlation functions are holographically dual to second variation of quadratic part of the gravitational on-shell action

Fluctuations of gravity fields

Example: consider spin 2 fluctuation of the spatial metric

$$\delta G_{xy} \equiv \Phi(t,r)/r^2$$

this gravity field sources the *xy*-component of the energy-momentum tensor in the dual field theory, allowing to compute $\langle \Pi_{xy} \Pi_{xy} \rangle$

Fourier transform

$$\Phi(t,r) \equiv \int d\omega e^{-i\omega t} \Phi_0(p) F_p(r)$$

Fluctuation equation of motion to be solved:

$$\boxed{\begin{aligned}&\sqrt{1+\beta}\left(r_h^3 - r^3\right)\left[r\left(r_h^3 - r^3\right)\left(r_h^3 - 2r^3\right)F_p''(r) + 2\left(r^6 - r_h^6 - i\omega r_h^3 r^4/\sqrt{1+\beta}\right)F_p'(r)\right] \\ &+ \omega r_h^3 r\left(-2ir^5 - ir_h^3 r^2 + \omega r_h^6/\sqrt{1+\beta}\right)F_p(r) = 0.\end{aligned}}$$

Boundary conditions for gravity fields

Fluctuation equation of motion to be solved

$$\sqrt{1+\beta}\left(r_h^3-r^3\right)\left[r\left(r_h^3-r^3\right)\left(r_h^3-2r^3\right)F_p''(r)+2\left(r^6-r_h^6-i\omega r_h^3 r^4/\sqrt{1+\beta}\right)F_p'(r)\right]$$
$$+\,\omega r_h^3 r\left(-2ir^5-ir_h^3r^2+\omega r_h^6/\sqrt{1+\beta}\right)F_p(r)=0.$$

has regular singular points at

Boundary conditions for gravity fields

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has regular singular points at

AdS boundary $r=0$

universal horizon r_h

spin 2 sound horizon $r_s = r_h/2^{1/3}$

Boundary conditions for gravity fields

Fluctuation equation of motion to be solved

$$\sqrt{1+\beta}\left(r_h^3-r^3\right)\left[r\left(r_h^3-r^3\right)\left(r_h^3-2r^3\right)F_p''(r)+2\left(r^6-r_h^6-i\omega r_h^3r^4/\sqrt{1+\beta}\right)F_p'(r)\right]$$
$$+\omega r_h^3 r\left(-2ir^5-ir_h^3r^2+\omega r_h^6/\sqrt{1+\beta}\right)F_p(r)=0.$$

has regular singular points at

	position	indicial exponents
AdS boundary	$r=0$	
universal horizon	r_h	$\propto (r-r_h)^{\frac{ir_h\omega}{3\sqrt{1+\beta}}},\quad (r-r_h)^{1+\frac{ir_h\omega}{3\sqrt{1+\beta}}}$
spin 2 sound horizon	$r_s=r_h/2^{1/3}$	$\propto (r-r_s)^0,\quad \boxed{(r-r_s)^{-\frac{2i\omega r_s}{3\sqrt{1+\beta}}}}$

both out-going

regular

in-going

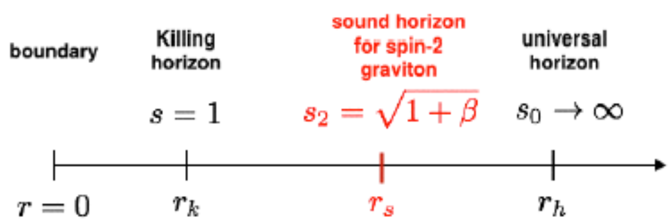
Impose infalling boundary condition at sound horizon.

Boundary conditions for gravity fields

Fluctuation equation of motion to be solved

$$\sqrt{1+\beta}\left(r_h^3-r^3\right)\left[r\left(r_h^3-r^3\right)\left(r_h^3-2r^3\right)F_p''(r)+2\left(r^6-r_h^6-i\omega r_h^3 r^4/\sqrt{1+\beta}\right)F_p'(r)\right]$$
$$+\,\omega r_h^3 r\left(-2ir^5-ir_h^3r^2+\omega r_h^6/\sqrt{1+\beta}\right)F_p(r)=0.$$

has regular singular points at



	position	indicial exponents
AdS boundary	$r=0$	
universal horizon	r_h	$\propto(r-r_h)^{\frac{ir_h\omega}{3\sqrt{1+\beta}}},\quad (r-r_h)^{1+\frac{ir_h\omega}{3\sqrt{1+\beta}}}$ both out-going
spin 2 sound horizon	$r_s=r_h/2^{1/3}$	$\propto(r-r_s)^0,\quad \boxed{(r-r_s)^{-\frac{2i\omega r_s}{3\sqrt{1+\beta}}}}$

regular in-going

Impose infalling boundary condition at sound horizon.

Find fluctuation solutions in hydro limit

Hydrodynamic expansion:

$$F_p(r) = \left(1 - \frac{r}{r_h/2^{1/3}}\right)^{-\frac{i2^{2/3} r_h \omega}{3\sqrt{1+\beta}}} \left(F_p^0(r) + \omega F_p^1(r) + \omega^2 F_p^2(r) + \cdots\right)$$

General solution near spin 2 horizon:

$$F_p^0(r) = C_1 + C_2 \log(r^3 - r_h^3/2)$$

$F_p(r=0) = 1$ normalization

$C_2 = 0$ regularity

Correlator from fluctuations:

$$S_2^H\Big|_{on-shell} \equiv \int d\omega \Phi_0(-p)\mathcal{H}(p,r)\Phi_0(p)\big|_{\text{boundaries}},$$

$$G^R_{xy,xy}(p) = 2\mathcal{H}(p, r=0).$$

$$\boxed{G^R_{xy,xy}(p) = -\frac{i\sqrt{1+\beta}\,\omega}{8\pi G_H 2^{1/3} r_h^2} + \mathcal{O}(\omega^2)}$$

encodes shear viscosity

(Non)relativistic shear viscosity

Kubo formula applies to both cases

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\left[\lim_{k\to 0} G^R_{xy,xy}(p)\right]$$

non-relativistic (Horava)		**relativistic**
$\eta^H = \frac{\sqrt{1+\beta}}{8\pi G_H 2^{1/3} r_h^2}$		$\eta^{GR} = \frac{1}{16\pi G_N r_k^2}$
$s^H = \frac{\sqrt{1+\beta}}{4 G_H r_h^2}$		$s^{GR} = 1/\left(4 G_N r_k^2\right)$
$\frac{\eta^H}{s^H} = \frac{2^{2/3}}{4\pi}$	*confirms perturbative result [Eling, Oz; JHEP (2014)]*	$\frac{\eta^{GR}}{s^{GR}} = \frac{1}{4\pi}$

Non-relativistic entropy production over entropy density is larger.

Charge vs. momentum diffusion

Add electromagnetism:

$$S_{HEM} = \frac{-1}{4\mu_0} \int d^4x N \sqrt{G} \left(F^{IJ} F_{IJ} - \frac{2}{c^2 N^2} \left(E_I - F_{JI} N^J\right) \left(E^I - F^{JI} N_J\right) \right)$$

Momentum diffusion:

$$G^R_{x,x}(p) \equiv 2\mathcal{H}_{h,h}(p, r=0) = \frac{\sqrt{1+\beta}\, k^2}{8\pi G_H 2^{1/3} r_h^2 \left(i\omega - \frac{\sqrt{1+\beta} r_h k^2}{2^{1/3} 3} \right)},$$

$$G^R_{xy,x}(p) \equiv 2\mathcal{H}_{f,h}(p, r=0) = \frac{-\sqrt{1+\beta}\, k\omega}{8\pi G_H 2^{1/3} r_h^2 \left(i\omega - \frac{\sqrt{1+\beta} r_h k^2}{2^{1/3} 3} \right)},$$

$$G^R_{xy,xy}(p) \equiv 2\mathcal{H}_{f,f}(p, r=0) = \frac{\sqrt{1+\beta}\, \omega^2}{8\pi G_H 2^{1/3} r_h^2 \left(i\omega - \frac{\sqrt{1+\beta} r_h k^2}{2^{1/3} 3} \right)},$$

Spin 1 sound horizon:

$$r = r_s^c \equiv \frac{r_h}{\left(1 + \frac{\sqrt{1+\beta}}{c}\right)^{1/3}}$$

Charge diffusion:

$$G^R_{q,q}(p) = \frac{k^2}{\mu_0 c \left(i\omega - c k^2 r_s^c\right)},$$

$$G^R_{q,j_y}(p) = -\frac{\omega k}{\mu_0 c \left(i\omega - c k^2 r_s^c\right)},$$

$$G^R_{j_y,j_y}(p) = \frac{\omega^2}{\mu_0 c \left(i\omega - c k^2 r_s^c\right)}.$$

Diffusion coefficients

$$D^H = \frac{\sqrt{1+\beta}\, r_h}{3 \cdot 2^{1/3}}$$

$$D^H_{EM} = c\, r_s^c$$

General structure of diffusion coefficients

Diffusion coefficients

momentum

$$D^H = \frac{\sqrt{1+\beta}\, r_h}{3 \cdot 2^{1/3}}$$

charge

$$D^H_{EM} = c\, r^c_s$$

$$D^H = \frac{1}{3}(\text{speed}) \times (\text{horizon}),$$

$$D^H_{EM} = (\text{speed}) \times (\text{horizon}).$$

with

$$s_2^2 = 1 + \beta, \qquad c^2 = \frac{2}{2+\kappa},$$

$$r_s = \frac{r_h}{2^{1/3}}, \qquad r^c_s = \frac{r_h}{\left(1 + \frac{\sqrt{1+\beta}}{c}\right)^{1/3}}$$

4. Lessons learned

Summary

• non-relativistic hydrodynamics more involved than one may think

• Horava gravity provides set of non-relativistic models yielding transport coefficients

• entropy production per entropy density increases non-relativistically

• technicality: geometries with various horizons ask to impose in-falling condition at appropriate sound horizon for each fluctuation

Outlook

• Horava black branes with other values of z
• relations of heat/charge/other conductivities

c.f. talks by Gouteraux, Lucas, Sachdev

• add Chern-Simons terms, study anomalous transport
• Lifshitz hydrodynamics

c.f. [Hoyos, Oz et al.]

4. Lessons learned

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• add Chern-Simons terms, study anomalous transport
• Lifshitz hydrodynamics *[Jensen; (Nov. 2014)]*

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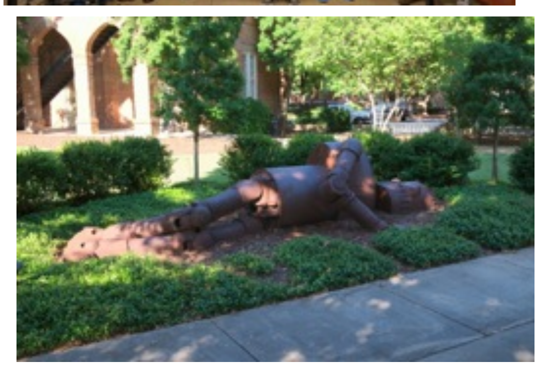
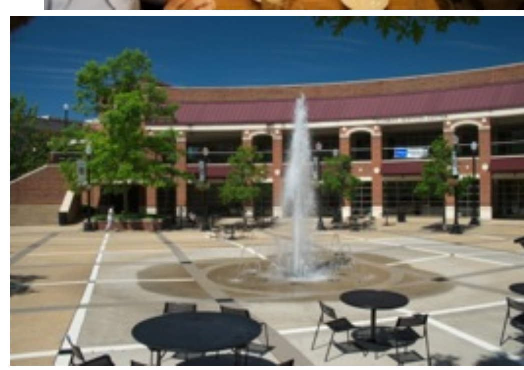




APPENDIX

Checking tool at *z*=1: field redefinitions

Horava action and *z*=1 black brane solution with spin 2 fluctuation

$$S^H = \frac{1}{16\pi G_H}\int d^4x\, N\sqrt{G}\left(K_{IJ}K^{IJ} - (1+\lambda)K^2 + (1+\beta)(R-2\Lambda) + \alpha\frac{\nabla_I N \nabla^I N}{N^2}\right)$$

Field redefinitions

$$\hat{g}_{XY} \equiv g_{XY} - (\sigma - 1)u_X u_Y$$

$$\hat{u}^X \equiv u^X/\sqrt{\sigma},$$

Einstein-Hilbert action with spin 2 metric fluctuation,
black brane metric with Killing horizon,
sound horizon of spin 2 metric fluctuation mapped to
Killing horizon,

Reminder: relativistic hydrodynamics

Universal **effective field theory** for microscopic QFTs, expansion in gradients of temperature, chemical potential, velocity; gauge field

- fields $T(x),\ \mu(x),\ u^\nu(x);\ A_\mu$
- conservation equations

Energy & momentum $\nabla_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$

Charge U(1) $\nabla_\nu j^\nu = 0$

- constitutive equations

Energy momentum tensor $T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}(g^{\mu\nu} + u^\mu u^\nu) + q^{(\mu} u^{\nu)} + t^{\mu\nu}$

Conserved current $J^\mu = \mathcal{N} u^\mu + j^\mu$

*functions of (derrivatives of) fields; depend on **frame choice***

2+1 dimensions, relativistic, parity-violating

Constitutive equations

$\boxed{c = 1}$

$$T^{\mu\nu} = (\epsilon - \tilde{x}_\Omega \Omega) u^\mu u^\nu + (P - \zeta \nabla_\alpha u^\alpha - \tilde{x}_B B - \tilde{x}_\Omega \Omega) \Delta^{\mu\nu} - \eta \sigma^{\mu\nu} - \tilde{\eta} \tilde{\sigma}^{\mu\nu}$$

$$J^\mu = (n - \tilde{x}_B \Omega) u^\mu + \sigma V^\mu + \tilde{\sigma} \tilde{V}^\mu + \tilde{\chi}_E \tilde{E}^\mu + \tilde{\chi}_T \epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T$$

$$\Omega = -\epsilon^{\mu\nu\rho} u_\mu \nabla_\nu u_\rho, \qquad B = -\frac{1}{2} \epsilon^{\mu\nu\rho} u_\mu F_{\nu\rho},$$

Features

$$dP = s\,dT + n\,d\mu + \frac{\partial P}{\partial B} dB + \frac{\partial P}{\partial \Omega} d\Omega$$

thermodynamic interpretation of $\tilde{\chi}_E$, $\tilde{\chi}_\Omega$, $\tilde{\chi}_B$

$\tilde{\eta}$ Hall viscosity

$\tilde{\chi}_E, \tilde{\sigma}$ off-diagonal conductivity (anomalous Hall)

$\tilde{\chi}_T$ "thermal Hall conductivity"

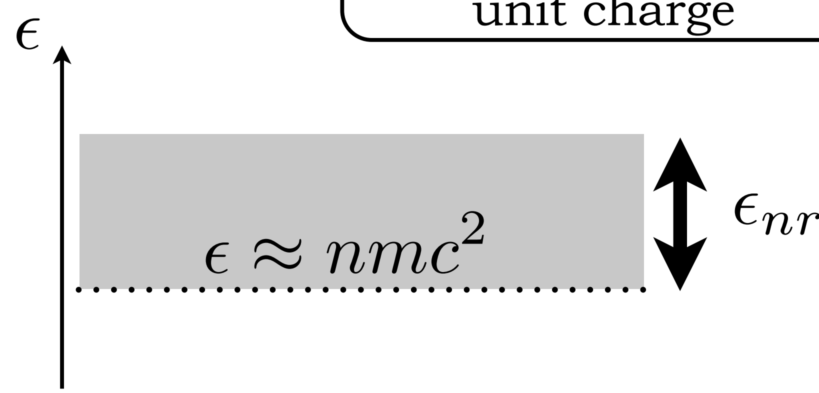
Non-relativistic (large *c*) limit

single particle species
unit charge

Mass m sets scale

$$\epsilon \to nmc^2 + \epsilon_{nr}$$

$$\mu \to mc^2 + \mu_{nr}$$



Rest mass energy density

$$\rho c^2 = \Gamma n m c^2$$

$$\Gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

How to scale transport coefficients?
Guided by physical intuition/prejudice.

Some thoughts on symmetries

- relativistic theories allow particle production / annihilation

- non-relativistic theories (generally) require particle number conservation

Where does this *U(1)* symmetry associated with particle number conservation come from?

Our answer for now: consider gapped relativistic theories which already have a *U(1)* symmetry

Scaling of transport coefficients with *c*

Relativistic "susceptibilities"

$$\begin{aligned}
\tilde{x}_B &= \frac{\partial P}{\partial B},\\
\tilde{x}_\Omega &= \frac{\partial P}{\partial \Omega},\\
\tilde{\chi}_E &= \frac{\partial n}{\partial B} + \frac{n_0}{\epsilon_0 + P_0}\left[\frac{\partial P}{\partial B} - c\frac{\partial n}{\partial \Omega}\right],\\
T\tilde{\chi}_T &= \frac{1}{c}\frac{\partial \epsilon}{\partial B} + \frac{n_0}{\epsilon_0 + P_0}\left[\frac{\partial P}{\partial \Omega} - \frac{\partial \epsilon}{\partial \Omega}\right],
\end{aligned}$$

Re-instate *c*, yields non-relativistic "susceptibilities" in terms of relativistic ones

uniquely determined

$$\begin{aligned}
\tilde{x}_B &\to \frac{\partial P}{\partial \mathcal{B}},\\
\tilde{x}_\Omega &\to c\frac{\partial P}{\partial \Omega_{nr}},\\
\tilde{\chi}_E &\to \frac{1}{m}\left[\frac{\partial}{\partial \mathcal{B}} - \frac{1}{m}\frac{\partial}{\partial \Omega_{nr}}\right](\rho - \frac{1}{2}\frac{\rho v^2}{c^2}) + \frac{1}{mc^2}\left[\frac{\partial P}{\partial \mathcal{B}} + \frac{w_{nr,0}}{m\rho_0}\frac{\partial \rho}{\partial \Omega_{nr}}\right] = \frac{1}{mc^2}\frac{\partial \Pi}{\partial \mathcal{B}},\\
T\tilde{\chi}_T &\to \frac{1}{c}\left[\frac{\partial}{\partial \mathcal{B}} - \frac{1}{m}\frac{\partial}{\partial \Omega_{nr}}\right](\rho c^2 + \epsilon_{nr} - \frac{1}{2}\rho v^2) + \frac{1}{mc}\left[\frac{\partial P}{\partial \Omega_{nr}} + \frac{w_{nr,0}}{\rho_0}\frac{\partial \rho}{\partial \Omega_{nr}}\right]\\
&= \frac{1}{mc}\boxed{\frac{\partial \Pi}{\partial \Omega_{nr}}},
\end{aligned}$$

should be set to zero

Parity-violating "Navier-Stokes"

continuity: $\partial_t \rho + \partial_i(\rho v^i) = 0,$

momentum: $\partial_t(\rho v^i) + \partial_j \Pi^{ij}_{tot} = \mathcal{E}^i j^0 + \mathcal{B}\epsilon^{ij} j_j,$

energy: $\partial_t j^0_{\epsilon,tot} + \partial_i j^i_{\epsilon,tot} = \mathcal{E}^i j_i,$

Recall: parity-preserving

$$\partial_t \rho + \partial_i(\rho v^i) = 0,$$
$$\partial_t(\rho v^i) + \partial_j \Pi^{ij} = \frac{\rho}{m}\left(\mathcal{E}^i + \mathcal{B}\epsilon^{ij} v_j\right),$$
$$\partial_t\left(\epsilon_{nr} + \frac{1}{2}\rho v^2\right) + \partial_i j^i_\epsilon = \frac{\rho}{m}\mathcal{E}^i v_i,$$

$$\Pi^{ij}_{tot} = \Pi^{ij} \underbrace{-\tilde{\eta}_{nr}(\epsilon^{ik}\delta^{jl} + i \leftrightarrow j)V_{kl}}_{T^{ij}_{\text{Hall}}} + P_{\not{p}}\delta^{ij},$$

$$j^0_{\epsilon,tot} = \epsilon_{nr} + \frac{1}{2}\rho v^2 - \boxed{\frac{\partial \Pi}{\partial \Omega_{nr}}}\,\epsilon^{ij} v_i \partial_j \ln T,$$

should be set to zero

$$j^i_{\epsilon,tot} = j^i_\epsilon + j^i_{\tilde{\epsilon}} + T^{ij}_{\text{Hall}} v_j + P_{\not{p}} v^i,$$

$$j^0 = \frac{\rho}{m}, \qquad P_{\not{p}} = -\frac{\partial P}{\partial \mathcal{B}}\mathcal{B} - \frac{\partial P}{\partial \Omega_{nr}}\Omega_{nr}$$

$$j^i = \frac{1}{m}\left[\rho v^i + \boxed{\frac{\partial \Pi}{\partial \Omega_{nr}}}\epsilon^{ij}\partial_j \ln T\right].$$

should be set to zero