

# **Workshop on Coupled Mathematical Models for Physical and Biological Nanoscale Systems and Their Applications**



**Banff International Research Station,  
The Banff Centre, Banff, Canada,  
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# Chaotic current self-oscillations in doped, weakly coupled semiconductor superlattices for true random number generation

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# Collaborators

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Michael Rosenbluh

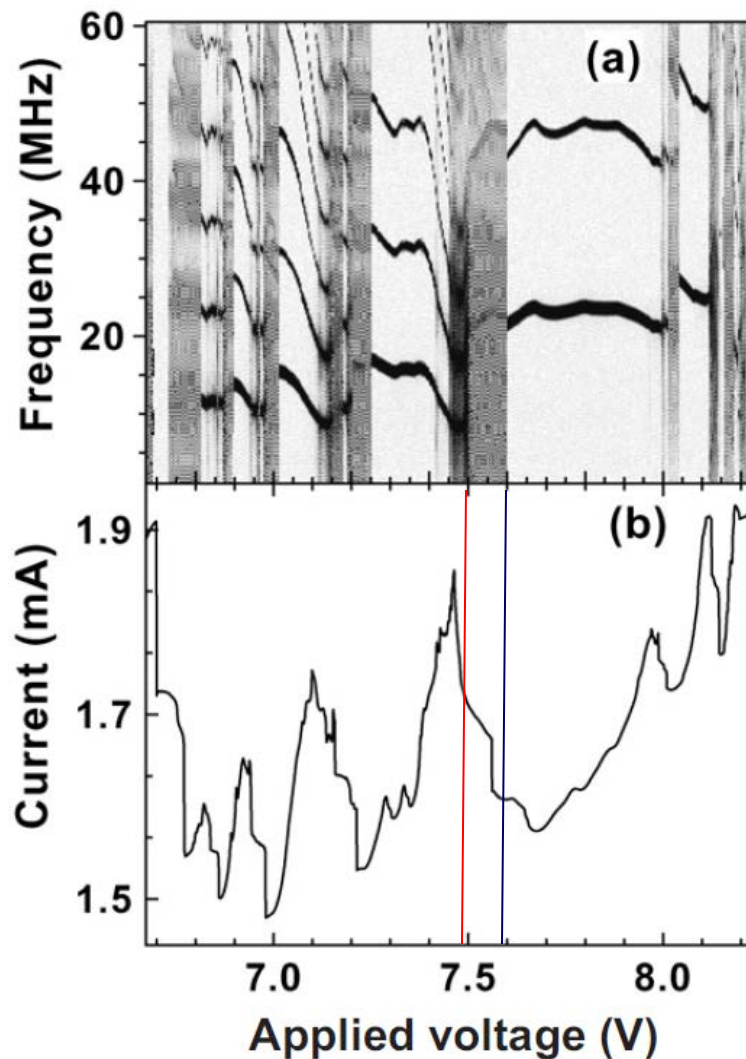
Universidad Carlos III  
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Luis L. Bonilla  
Manuel Carretero  
Miguel Ruiz-García

## Synchronization and Chaos Induced by Resonant Tunneling in GaAs/AlAs Superlattices

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(Received 19 March 1996)



**$T = 30$  K: 9.0 nm GaAs  
4.0 nm AlAs  
40 periods**

**(a) Frequency spectra of periodic and chaotic oscillations**

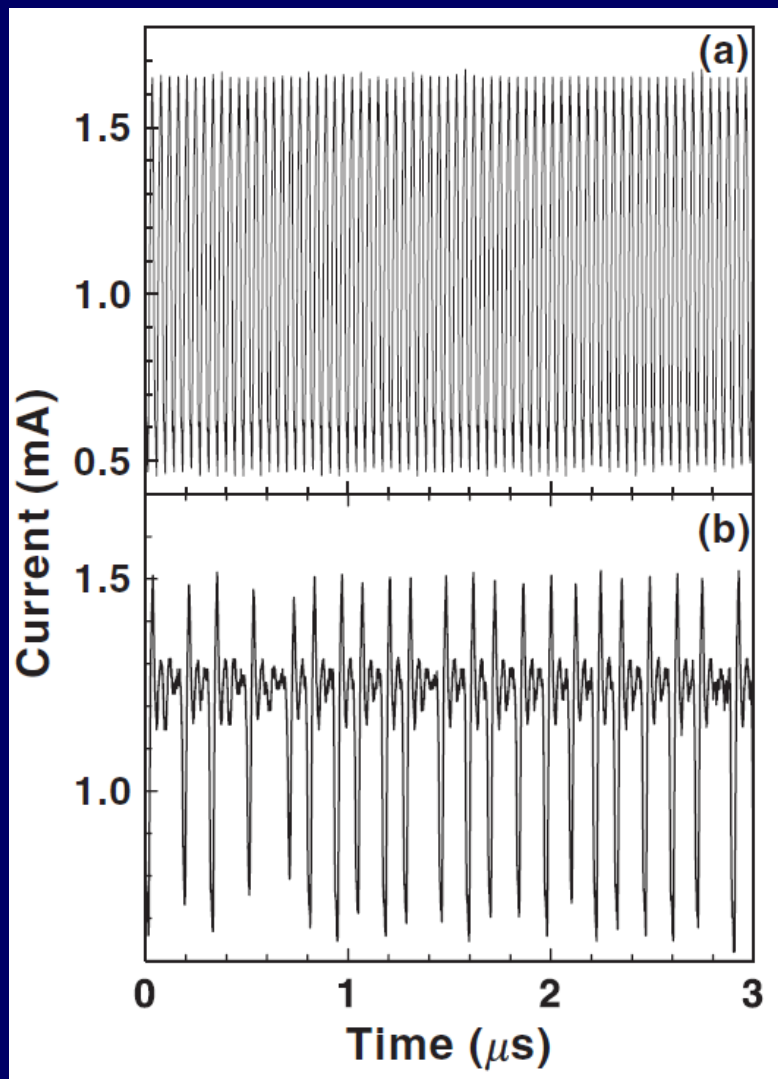
**(b) Current-voltage characteristics**

**At  $T = 50$  K, the chaotic windows become much narrower**



## Transition between synchronization and chaos in doped GaAs/AlAs superlattices

YAOHUI ZHANG, ROBERT KLANN, HOLGER T. GRAHN†, KLAUS H. PLOOG  
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**$T = 30$  K: 9.0 nm GaAs  
4.0 nm AlAs  
40 periods**

**(a) bias 7.6 V:  
periodic oscillations**

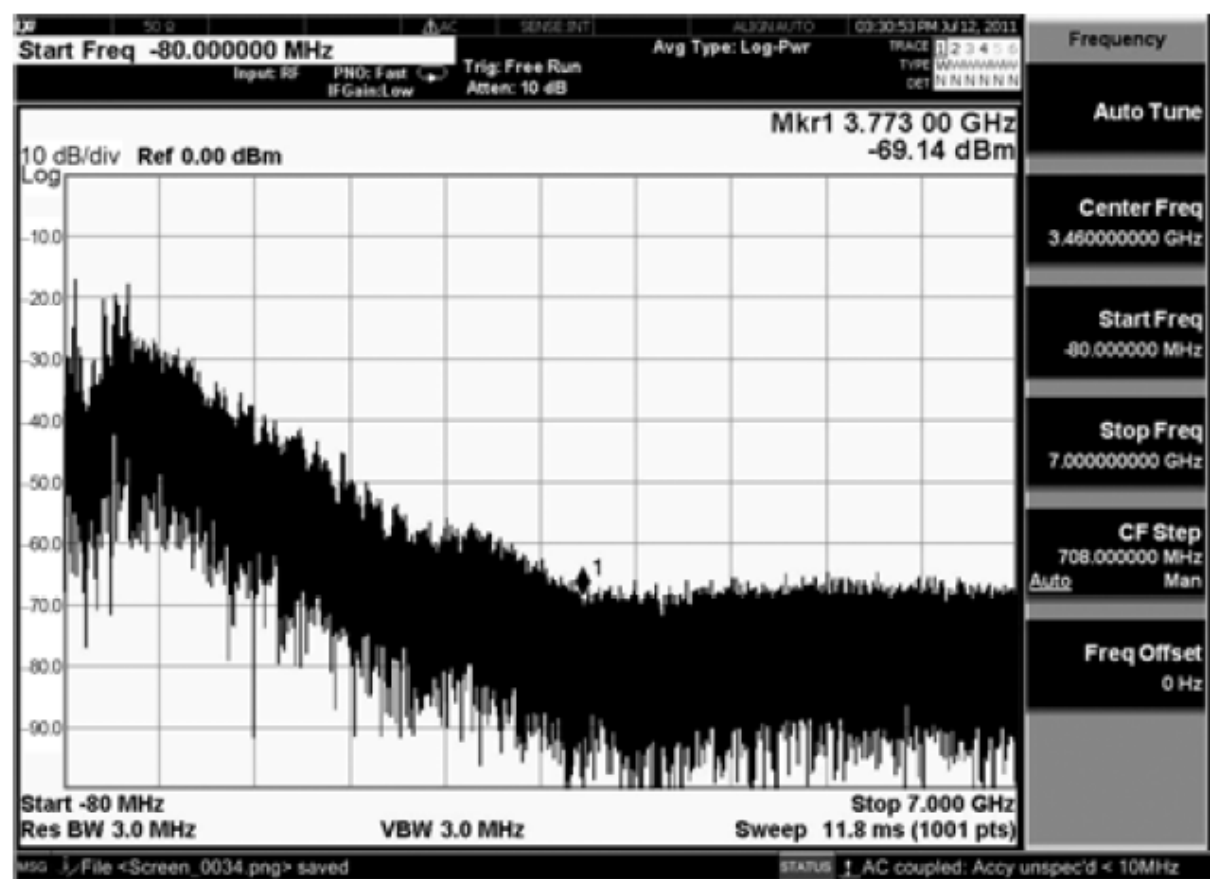
**(b) bias 7.491 V:  
chaotic oscillations**

# Experimental observation of spontaneous chaotic current oscillations in GaAs/Al<sub>0.45</sub>Ga<sub>0.55</sub>As superlattices at room temperature

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**T = 300 K:**  
**7.0 nm GaAs**  
**4.0 nm Al<sub>0.45</sub>Ga<sub>0.55</sub>As**  
**50 periods**

**Bias range:**  
**6.21–6.8 V**

**Spectrum at 6.5 V**  
**from 0 to 4 GHz**



# Fast Physical Random-Number Generation Based on Room-Temperature Chaotic Oscillations in Weakly Coupled Superlattices

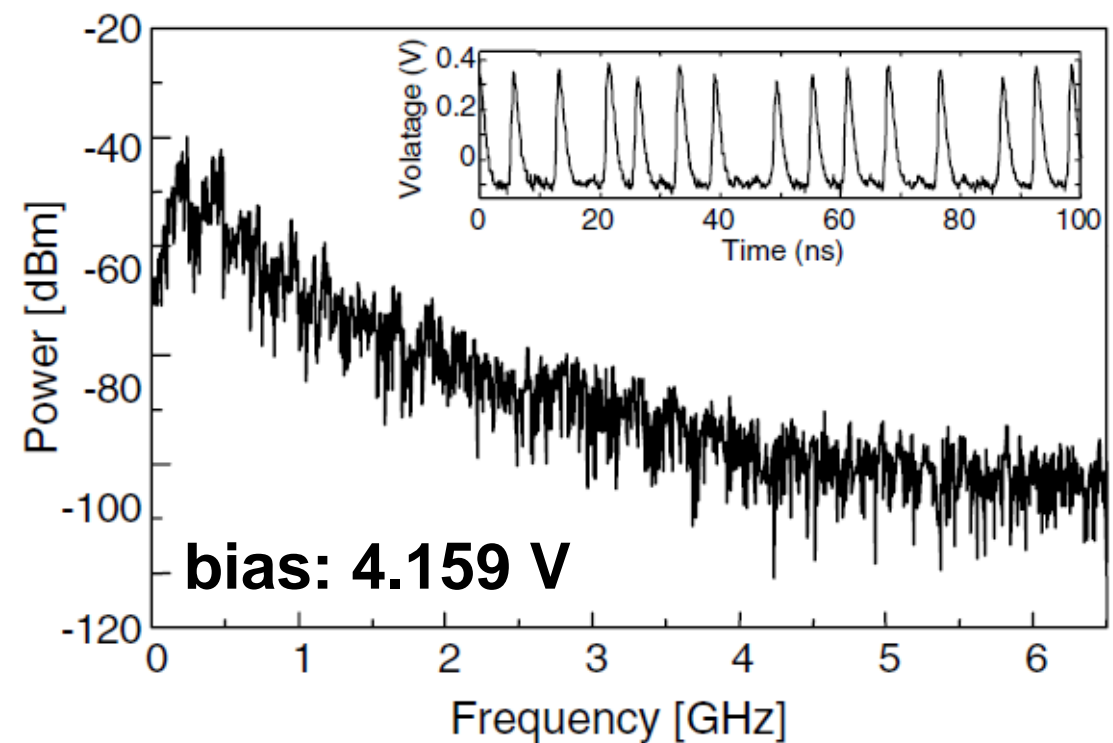
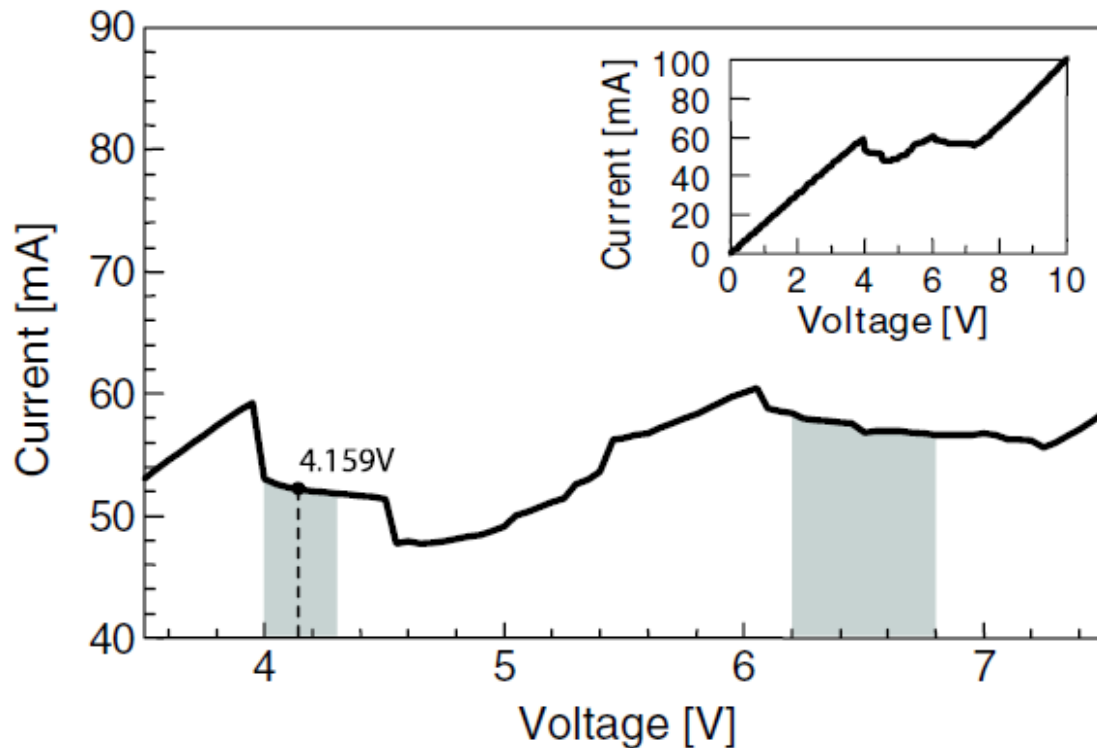
Wen Li,<sup>1</sup> Igor Reidler,<sup>2</sup> Yaara Aviad,<sup>2</sup> Yuyang Huang,<sup>1</sup> Helong Song,<sup>1</sup>  
Yaohui Zhang,<sup>1</sup> Michael Rosenbluh,<sup>2</sup> and Ido Kanter<sup>2,\*</sup>

<sup>1</sup>Suzhou Institute of Nano-tech and Nano-bionics, Chinese Academy of Sciences, Suzhou 215125, China

<sup>2</sup>Department of Physics, Bar-Ilan University, Ramat-Gan, 52900, Israel

(Received 29 March 2013; revised manuscript received 28 May 2013; published 25 July 2013)

**$T = 300$  K: 7.0 nm GaAs, 4.0 nm  $\text{Al}_{0.45}\text{Ga}_{0.55}\text{As}$ , 50 periods**

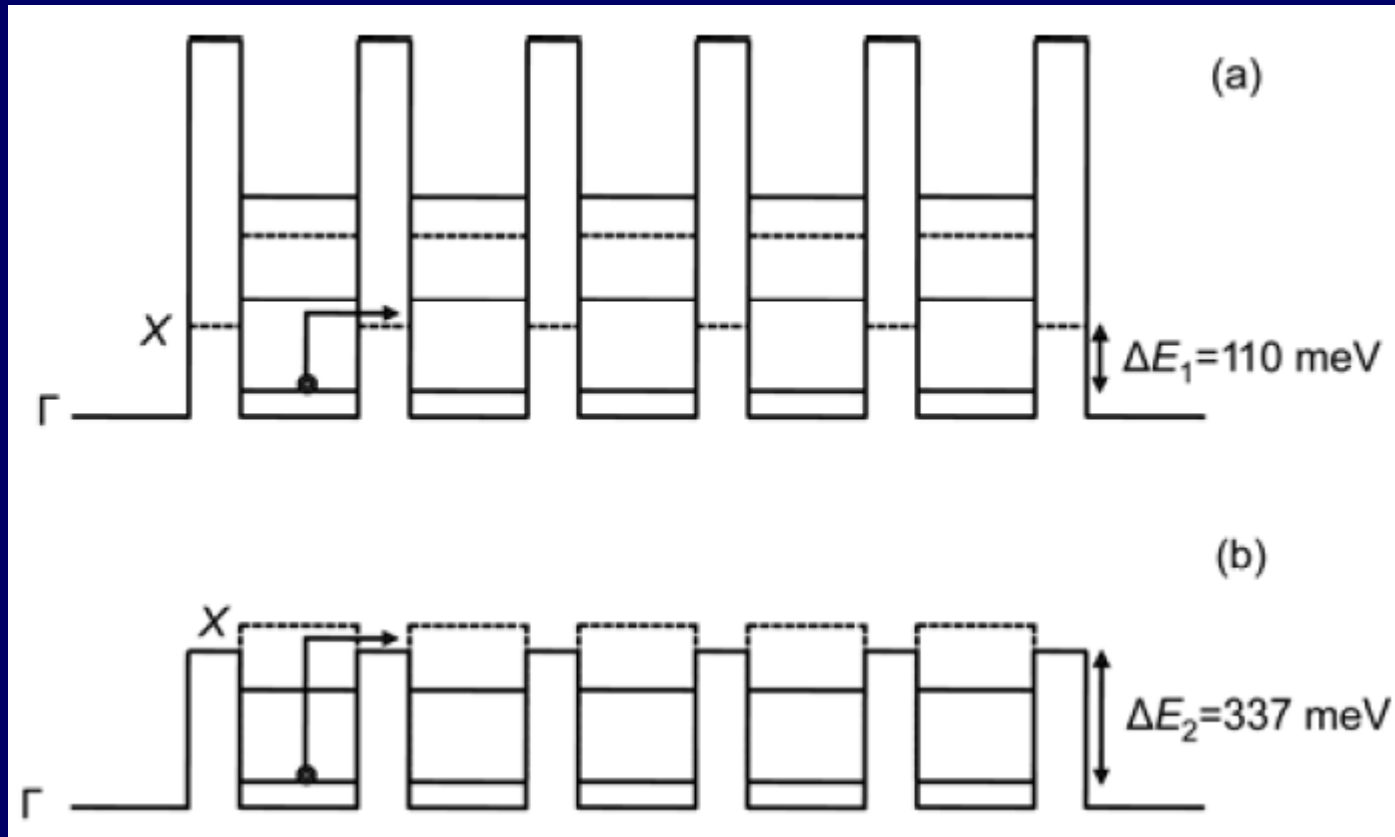


# Outline

- Motivation
- Formation of electric-field domains in semiconductor superlattices
- True random number generator based on GaAs/Al<sub>0.45</sub>Ga<sub>0.55</sub>As superlattices
- Chaos synchronization in networks of GaAs/Al<sub>0.45</sub>Ga<sub>0.55</sub>As superlattices
- Conclusions



# Conduction band offset in GaAs/(Al,Ga)As SLs



(a) GaAs/AlAs:  
 $\Gamma$ -X offset: 160 meV  
 $\Gamma$ - $\Gamma$  offset: 982 meV

(b) GaAs/ $\text{Al}_{0.45}\text{Ga}_{0.55}\text{As}$ :  
 $\Gamma$ -X offset: 390 meV  
 $\Gamma$ - $\Gamma$  offset: 390 meV

In GaAs/AlAs SL at higher temperatures:  
 $\Gamma$ -X transfer by thermionic emission

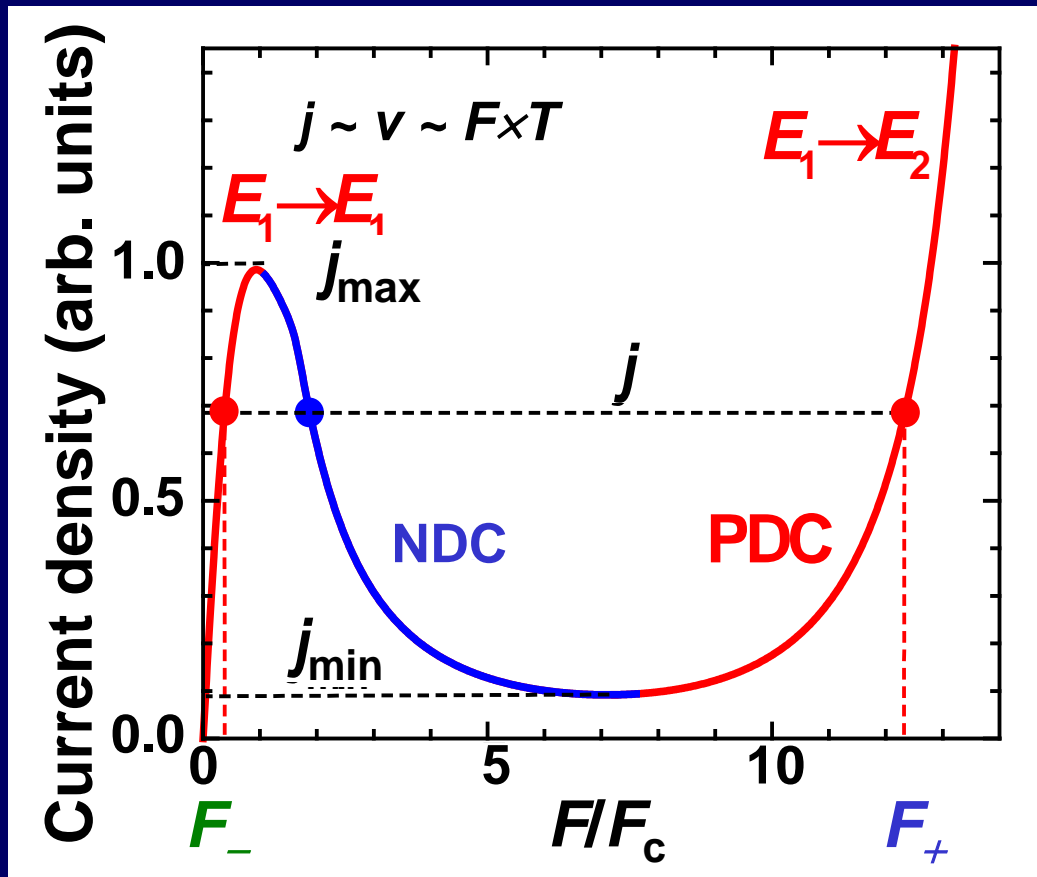
Y. Y. Huang, W. Li, W. Q. Ma, H. Qin, and Y. H. Zhang,  
Chin. Sci. Bull. 57, 2070 (2012).

# Formation of electric-field domains

## Large carrier densities

**NDC:** negative differential conductivity: unstable

**PDC:** positive differential conductivity: stable

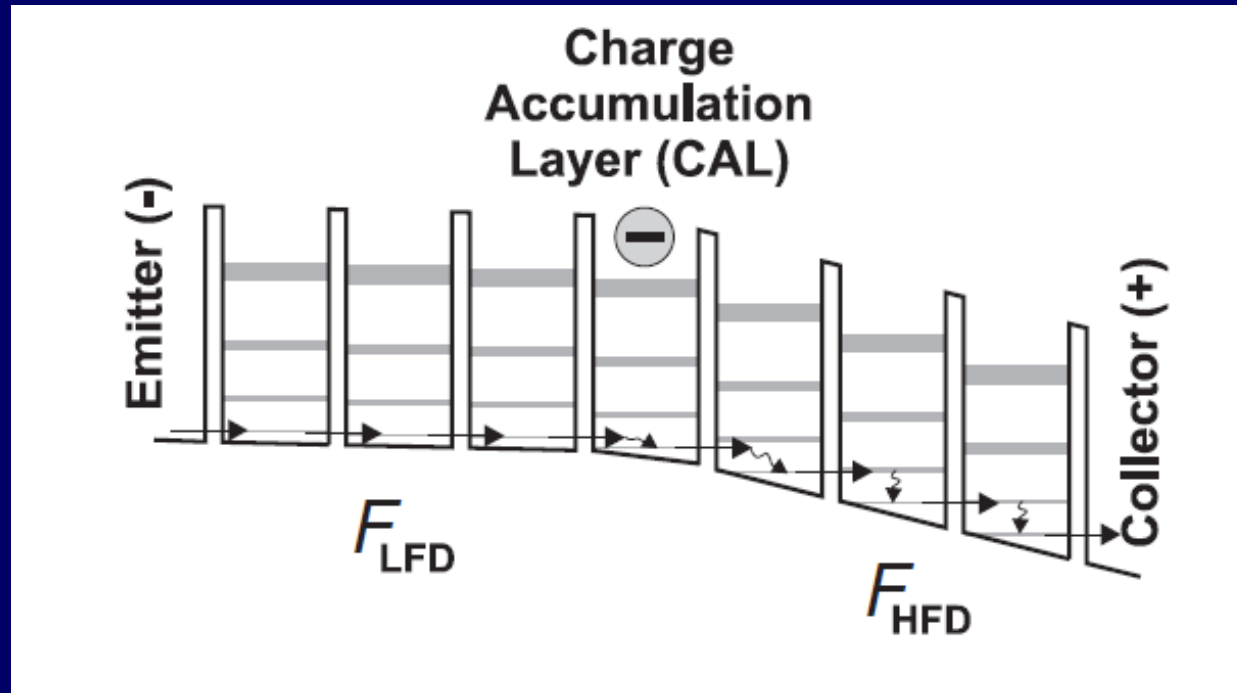


- Two domains with different field strength  $F_-$  and  $F_+$  in PDC region
- Separated by accumulation layer with  $n_{accu}$

$$n_{accu} = \epsilon_0 \epsilon \frac{F_+ - F_-}{e}$$

# Static electric-field domains

## Large carrier densities



**CAL: charge accumulation layer**

**LFD: low-field domain**

**HFD: high-field domain**

# Dynamic electric-field domains

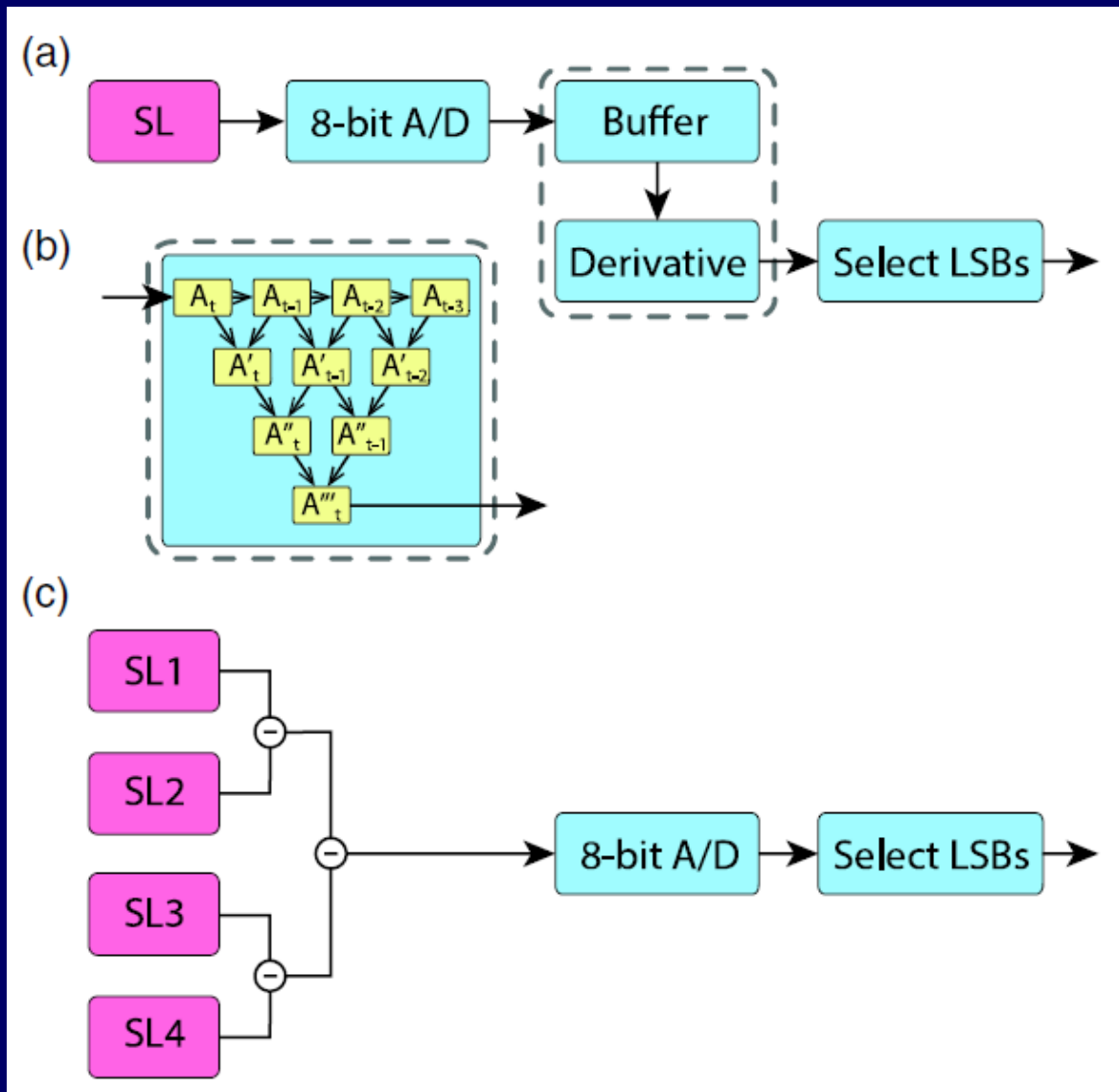
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## Intermediate carrier densities

Three types of spontaneous current oscillations:

- **Periodic: monopole and dipole oscillation modes**
- **Quasi-periodic: competition between dipole and monopole oscillation modes**
- **Chaotic: undriven (and also driven)**

# Methods for generating random sequence

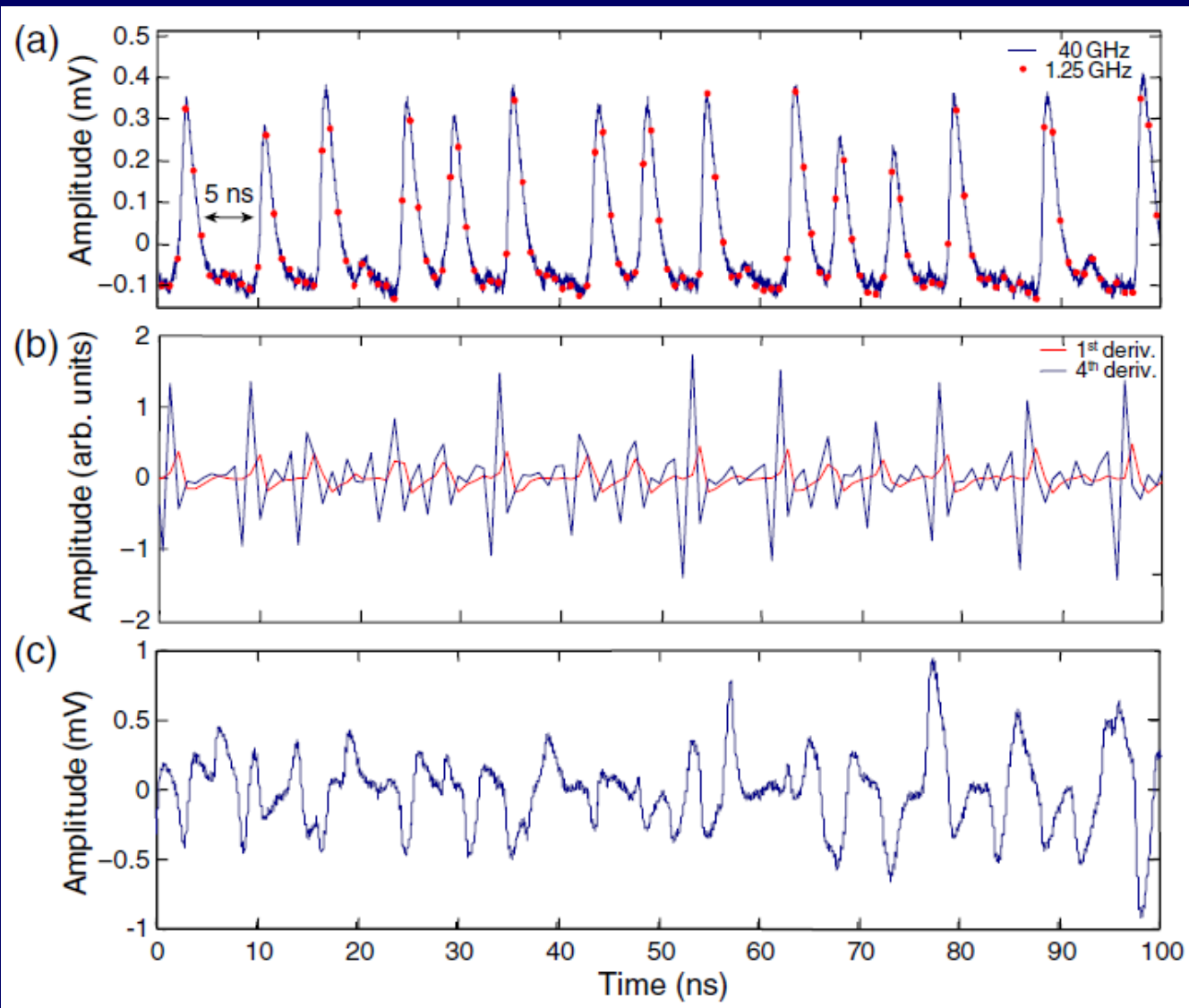


**Method 1:** a dynamical buffer of the last  $n + 1$  successively digitized current values is used to calculate the  $n^{\text{th}}$  discrete derivative as exemplified for  $n = 3$  in (b). Then, the  $m$  least significant bits (LSBs) of the resulting  $n^{\text{th}}$  derivative are appended as shown in (a).

**Method 2:** linear combination of the analog chaotic current oscillations of several uncorrelated SLs

W. Li, I. Reidler, Y. Aviad, Y. Y. Huang, H. L. Song, Y. H. Zhang, M. Rosenbluh, and I. Kanter, Phys. Rev. Lett. 111, 044102 (2013).

# True random number generator (TRNG)



**(a) Sampling frequency:**  
blue line: 40 GHz  
red dots: 1.25 GHz

**(b) Method 1:**  
Discrete derivative:  
red line: 1<sup>st</sup> order  
blue line: 4<sup>th</sup> order  
for 1.25 GHz sampling

**(c) Method 2:**  
Linear combination  
of 4 recorded SL  
oscillation traces  
digitized at 40 GHz

W. Li, I. Reidler, Y. Aviad, Y. Y. Huang, H. L. Song, Y. H. Zhang, M. Rosenbluh, and I. Kanter, Phys. Rev. Lett. 111, 044102 (2013).

# True random number generator (TRNG)

LSB: least significant bit, RBG: random bit generator,  
Results based on NIST and TestU01 statistical test suites

| Number of combined SL devices | 1    | 2  | 4   | 6   |
|-------------------------------|------|----|-----|-----|
| Derivative                    | 4    | 3  | ... | ... |
| Max sampling rate (GHz)       | 1.25 | 5  | 10  | 20  |
| Retained LSBs                 | 5    | 4  | 4   | 4   |
| RBG rate (Gbit/s)             | 6.25 | 20 | 40  | 80  |

**Typical rates of bit transfer:**

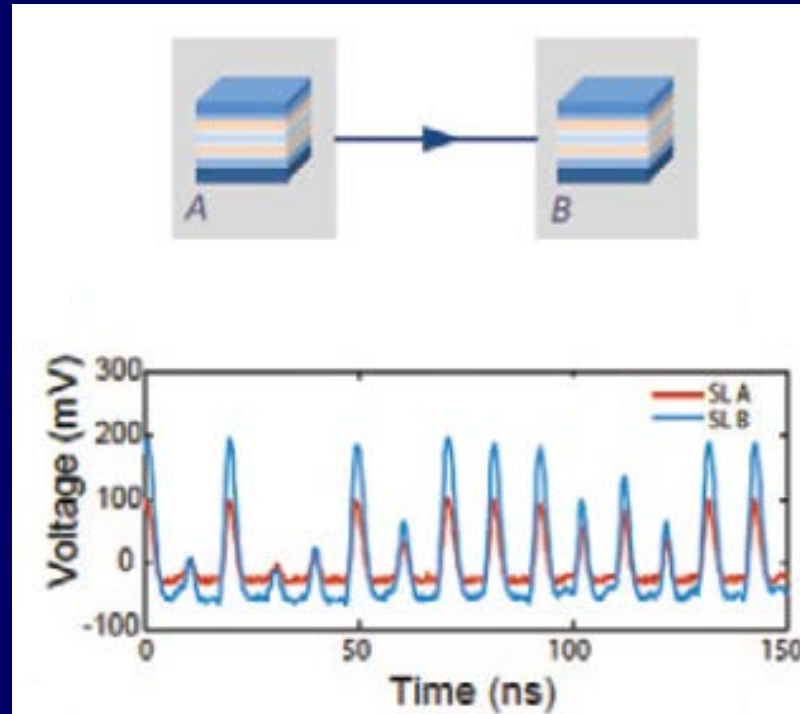
**Currently available TRNGs: up to 1–2 Gbit/s**

**Superlattice TRNG: up to 80 Gbit/s**

W. Li, I. Reidler, Y. Aviad, Y. Y. Huang, H. L. Song, Y. H. Zhang,  
M. Rosenbluh, and I. Kanter, Phys. Rev. Lett. 111, 044102 (2013).

# Synchronization: leader-laggard configuration

Unidirectional coupling with 16 m cable  
by using an amplifier from SL A to SL B.  
Chaotic SL A drives nonchaotic SL B.



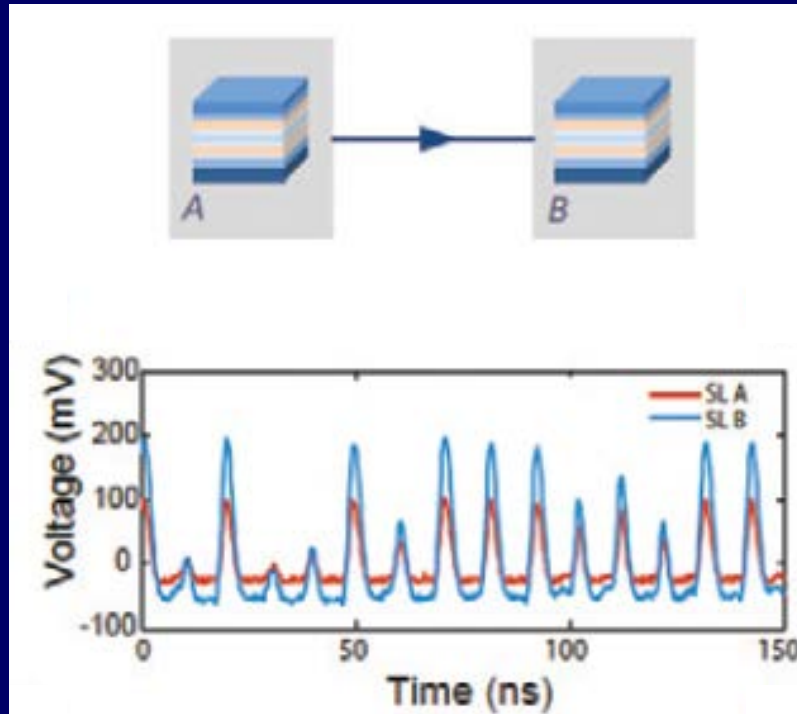
**Highly synchronized waveform with delay  
of 65 ns originating from the 16 m cable**

W. Li, Y. Aviad, I. Reidler, H. L. Song, Y. Y. Huang, K. Biermann, M. Rosenbluh,  
Y. H. Zhang, HTG, and I. Kanter, *Europhys. Lett.* 112, 30007 (2015).

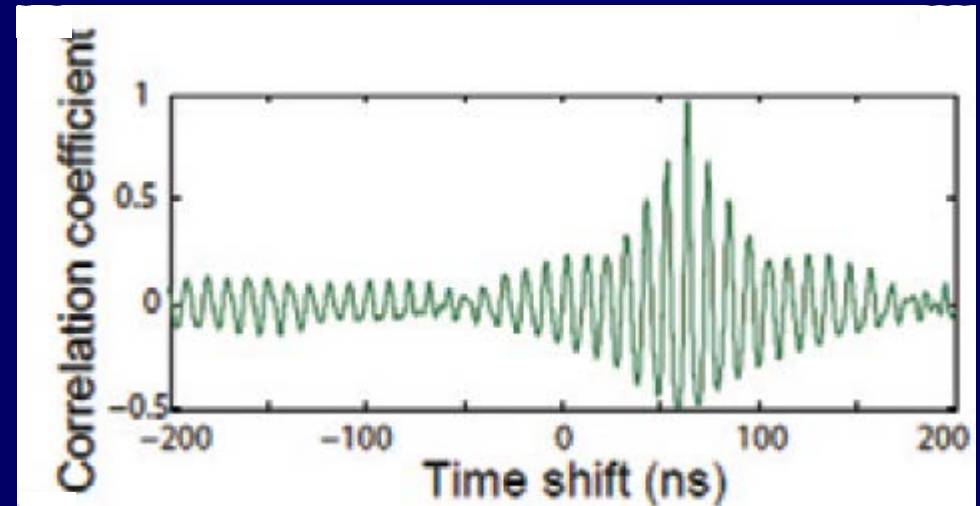


# Synchronization: leader-laggard configuration

Unidirectional coupling with 16 m cable  
by using an amplifier from SL A to SL B.  
Chaotic SL A drives nonchaotic SL B.



Cross correlation of SLs A and  
B with a value of one at 65 ns



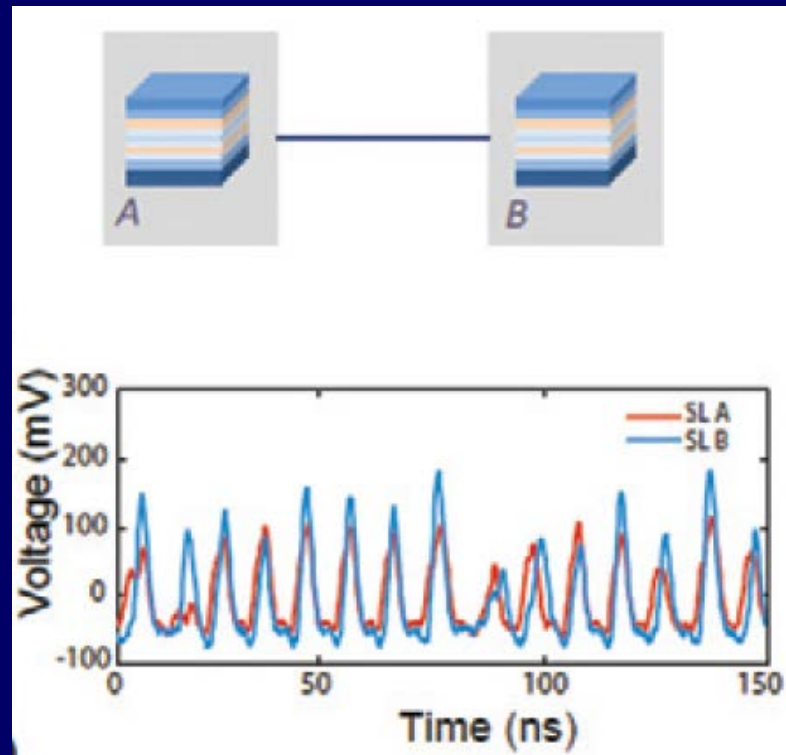
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# Synchronization: face-to-face configuration

Bidirectional coupling with 16 m cable  
without any amplifier.

Chaotic SL A coupled to chaotic SL B.



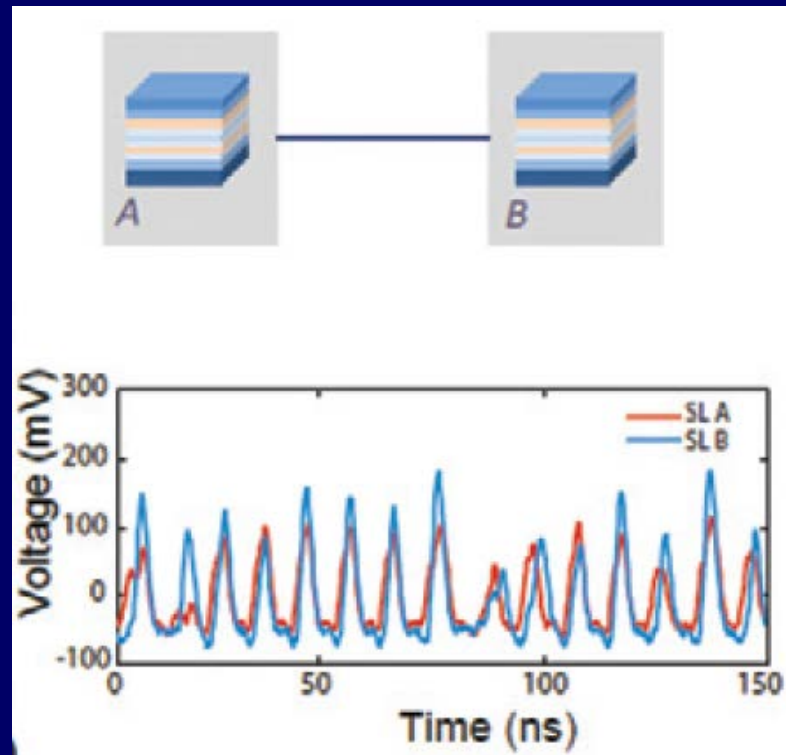
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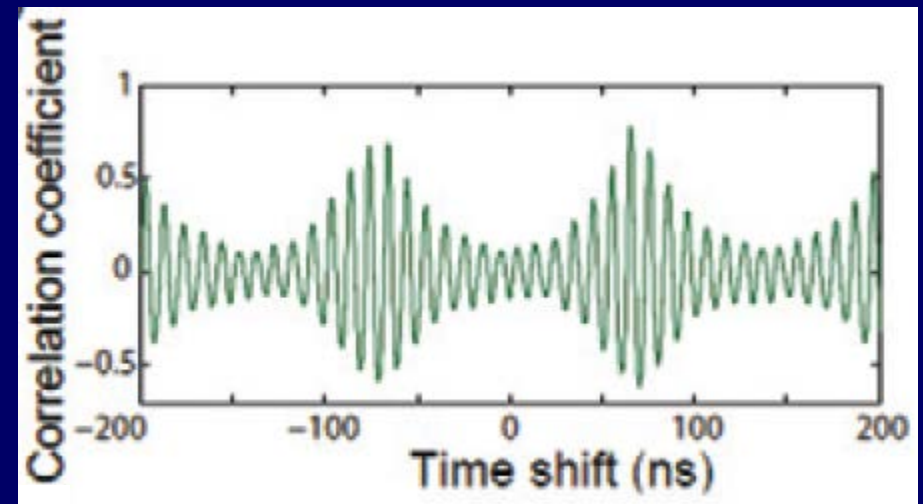
# Synchronization: face-to-face configuration

Bidirectional coupling with 16 m cable  
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Chaotic SL A coupled to chaotic SL B.



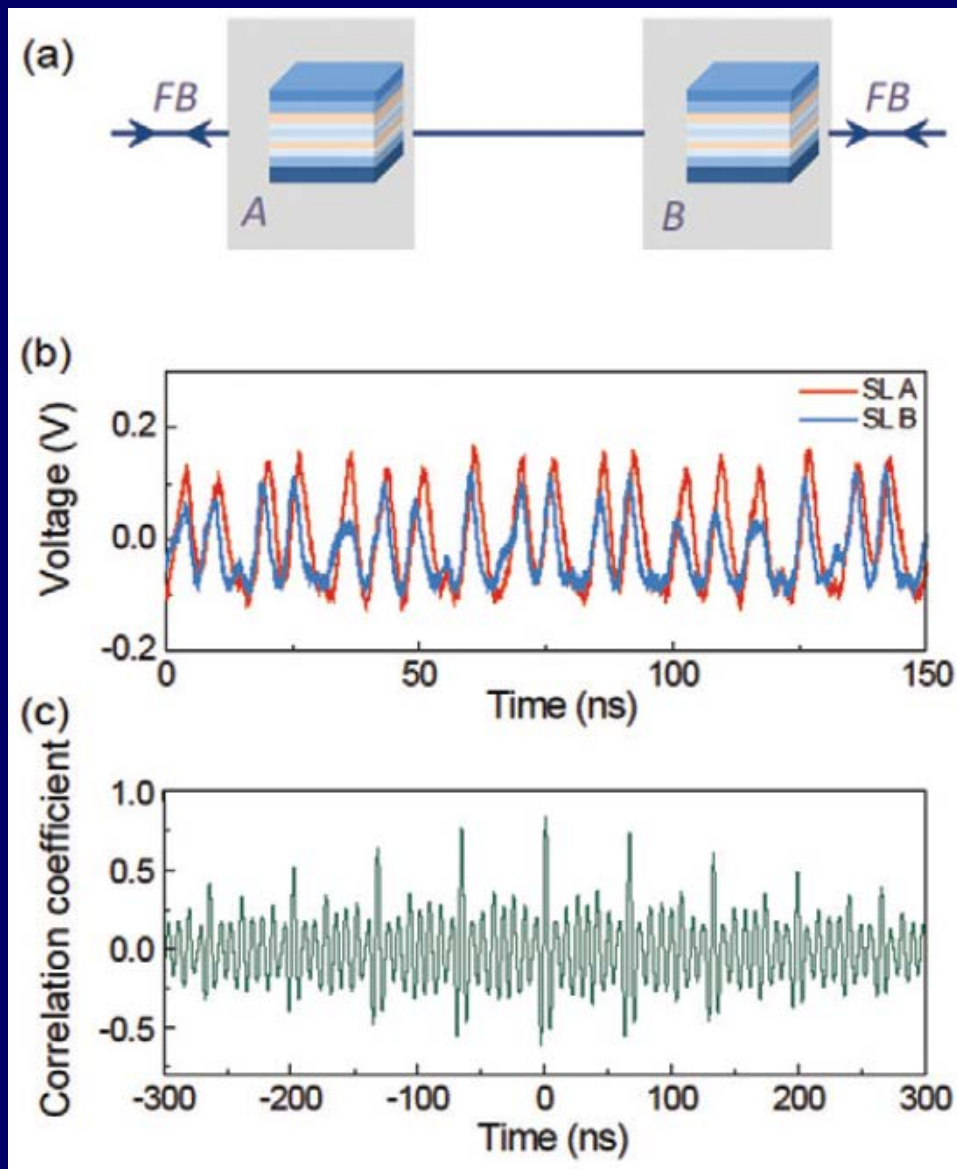
Symmetric cross correlation of SLs  
A and B with fading revivals at  
 $\pm(2n+1)\times 65$  ns, but maximum  $< 1$



Synchronized waveform with delay of  
65 ns originating from the 16 m cable

W. Li, Y. Aviad, I. Reidler, H. L. Song, Y. Y. Huang, K. Biermann, M. Rosenbluh,  
Y. H. Zhang, HTG, and I. Kanter, *Europhys. Lett.* 112, 30007 (2015).

# Zero-lag synchronization (ZLS)



(a) Two mutually coupled SLs with a 16 m cable with an additional self-feedback (FB) coupling of 8 m for each SL.

(b) Synchronized waveform with zero delay

(c) dominant peak of cross correlation with a value of 0.84 at zero time shift and fading revivals at  $\pm n \times 65$  ns

W. Li, Y. Aviad, I. Reidler, H. L. Song, Y. Y. Huang, K. Biermann, M. Rosenbluh, Y. H. Zhang, HTG, and I. Kanter, *Europhys. Lett.* 112, 30007 (2015).

# Conclusions

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- Spontaneous chaotic current oscillations at room temperature have been observed in weakly coupled GaAs/Al<sub>0.45</sub>Ga<sub>0.55</sub>As superlattices
- All-electronic true random number generator with bit rates up to 80 Gbit/s has been demonstrated using chaotic oscillations in GaAs/Al<sub>0.45</sub>Ga<sub>0.55</sub>As superlattices at room temperature
- Leader-laggard, face-to-face, and zero-lag synchronization have been demonstrated at room temperature using mutually coupled chaotic GaAs/Al<sub>0.45</sub>Ga<sub>0.55</sub>As superlattices

# Outlook

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- Improvement of the performance by using weakly coupled GaAs/ $\text{Al}_x\text{Ga}_{1-x}\text{As}$  superlattices with larger values of  $x$ , e.g. 0.5 to 0.7
- **Origin of the spontaneous chaotic current oscillations**
- **Synchronization of much larger scale networks and development of reliable as well as advanced secure communication protocols**

**Thank you very much  
for your attention**

**THE END**