Tearing :
fracture path in brittle thin sheets

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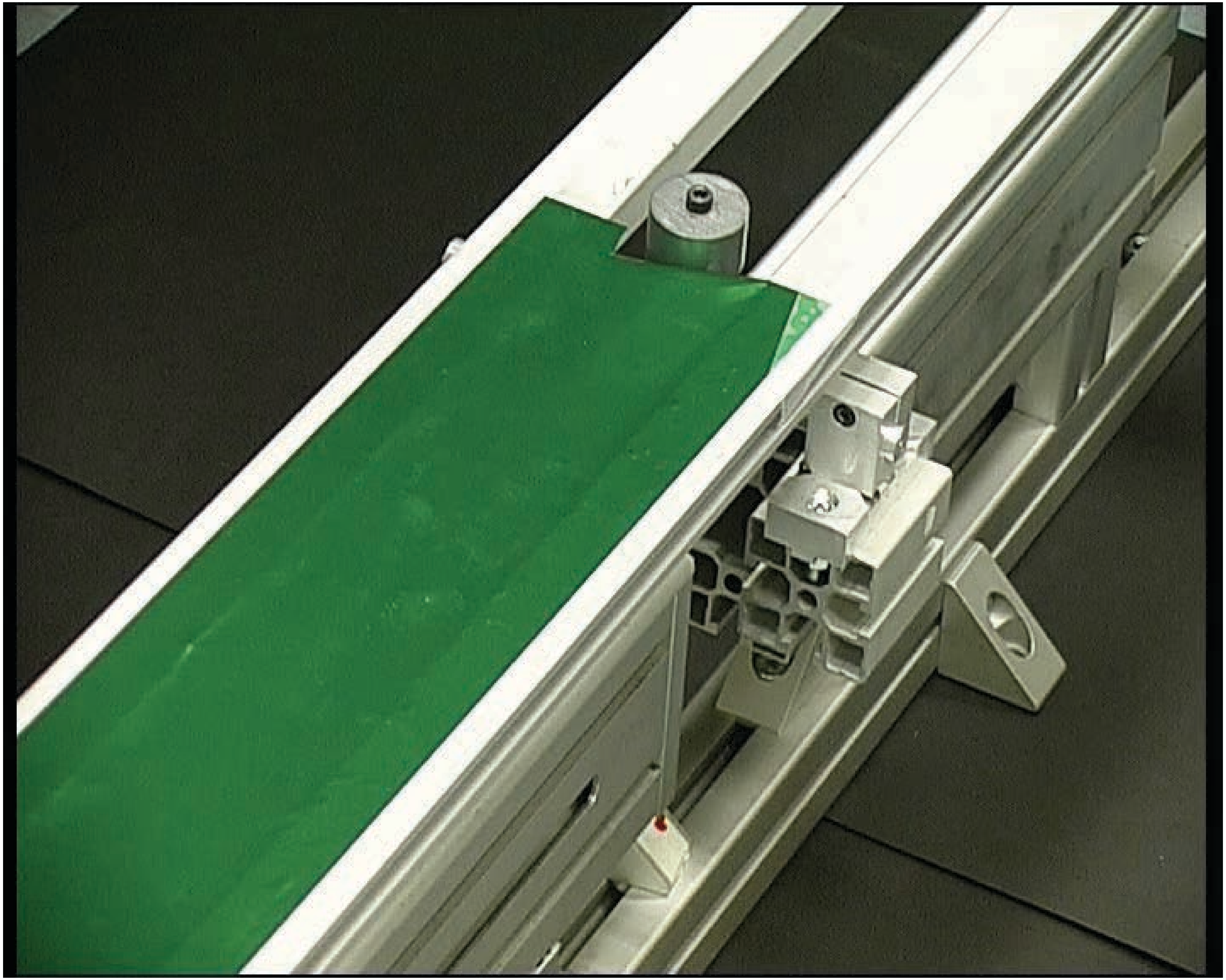
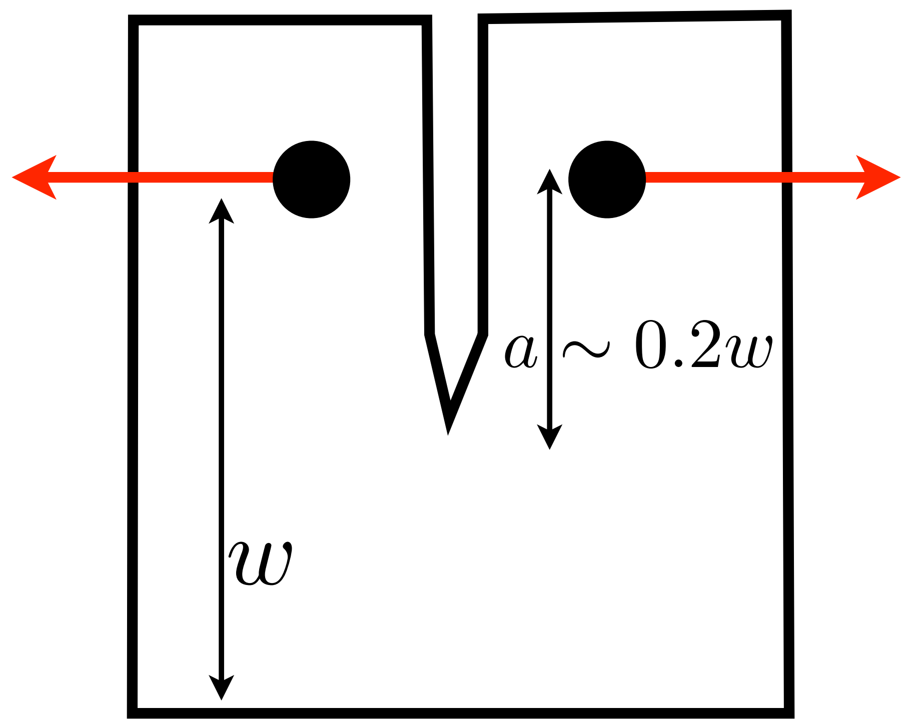


Tearing = Fracture + bending thin sheets

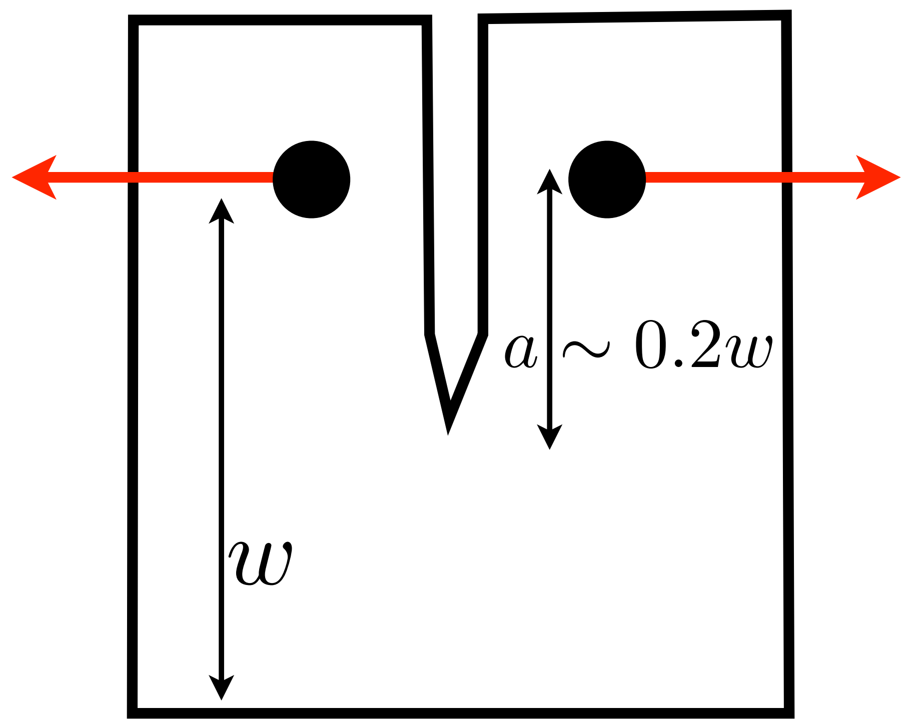
Why is it interesting ?

- a situation in real life (applications)
- very robust (intriguing) behavior
- challenging



fracture in thin sheets ::
always bending

Always out-of-plane displacements



forced in-plane

$$K_I \sim \frac{F}{t\sqrt{w}}$$

$$\boxed{F_c \sim t\sqrt{w}K_{Ic} \sim t\sqrt{G_c E w}}$$

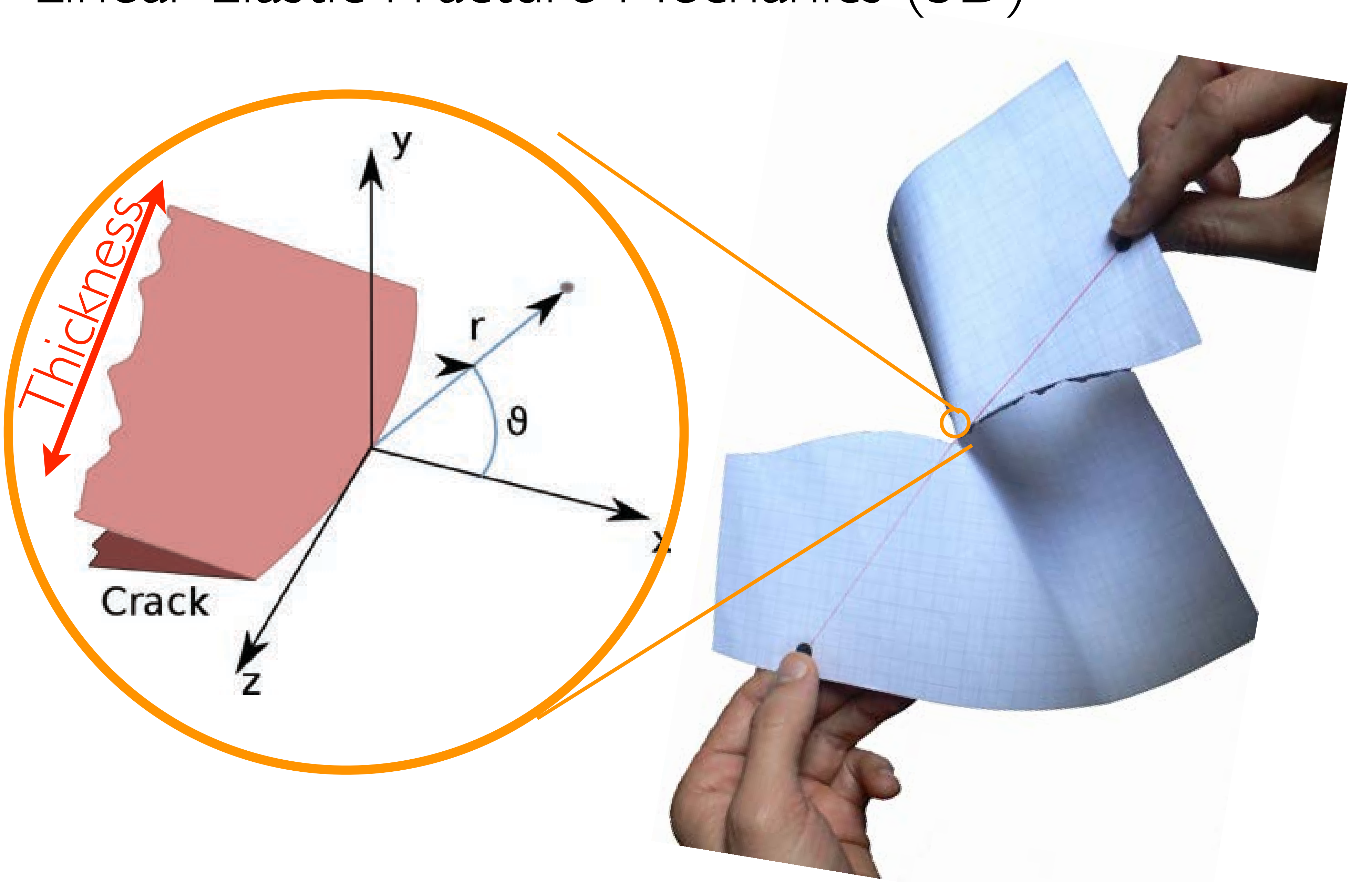
tearing

$$\boxed{F \sim G_c t}$$

$$\times\sqrt{w\frac{E}{Gc}}$$

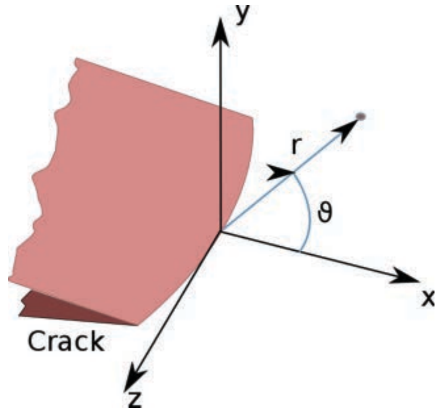
Approach :

- Linear Elastic Fracture Mechanics (3D)



Approaches :

- Linear Elastic Fracture Mechanics (3D)



- Stress intensity factors for plate mechanics

  which plate model? (Zucchini et al 2000, .Zehnder et al 2005…)
  crack path?

- Variational approach in non-linear plate mechanics
  (Li, Arroyo, Arias)

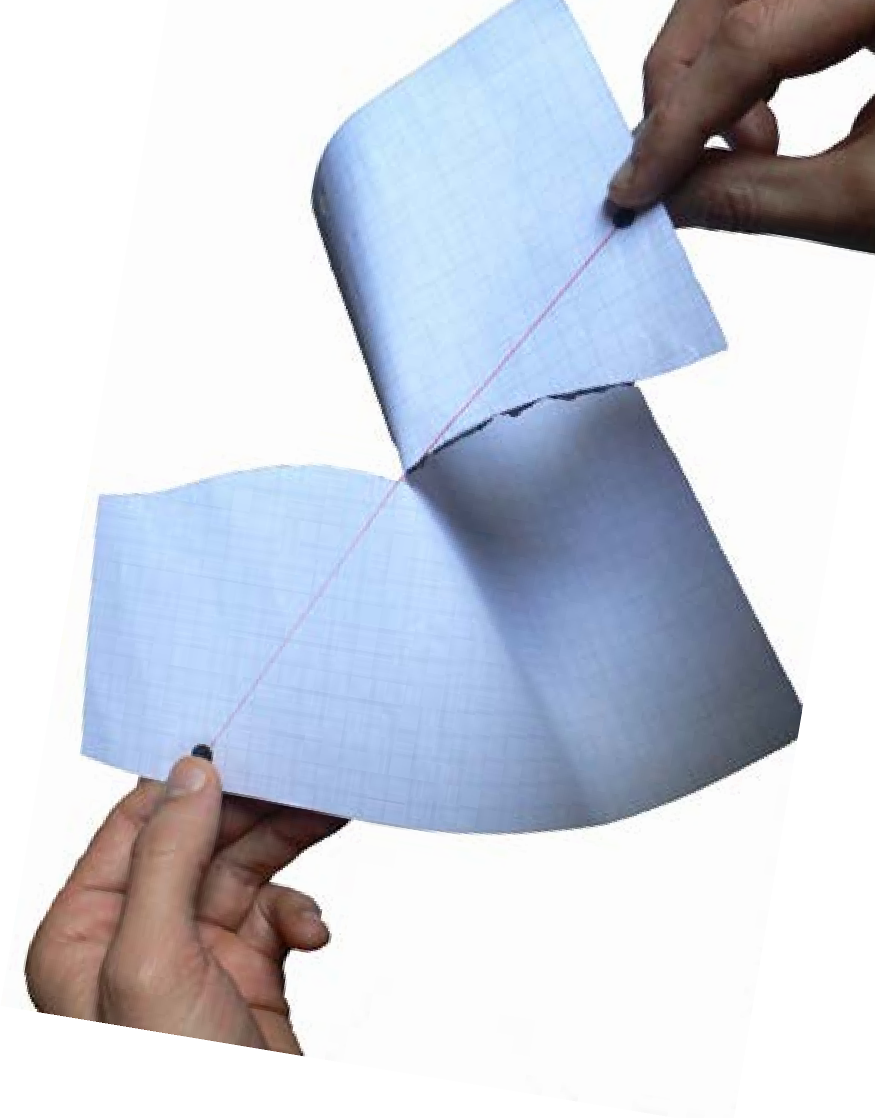
Griffith crack + crude approximations…

elastic plates :

elastic energy

Bending

Stretching



Griffith's criterion

$$\boxed{\frac{dW - dE_{el}}{t dl} = G} = \boxed{G_c}$$

Energy Release Rate

Fracture energy

«inextensible fabric» model

(0th order approximation)

The sheet is

- inextensible $\quad w \gg \frac{G_c}{E}$
- infinitely bendable $\quad w \gg \frac{Et^2}{Gc}$

NO ELASTIC ENERGY !

2D analog of a string

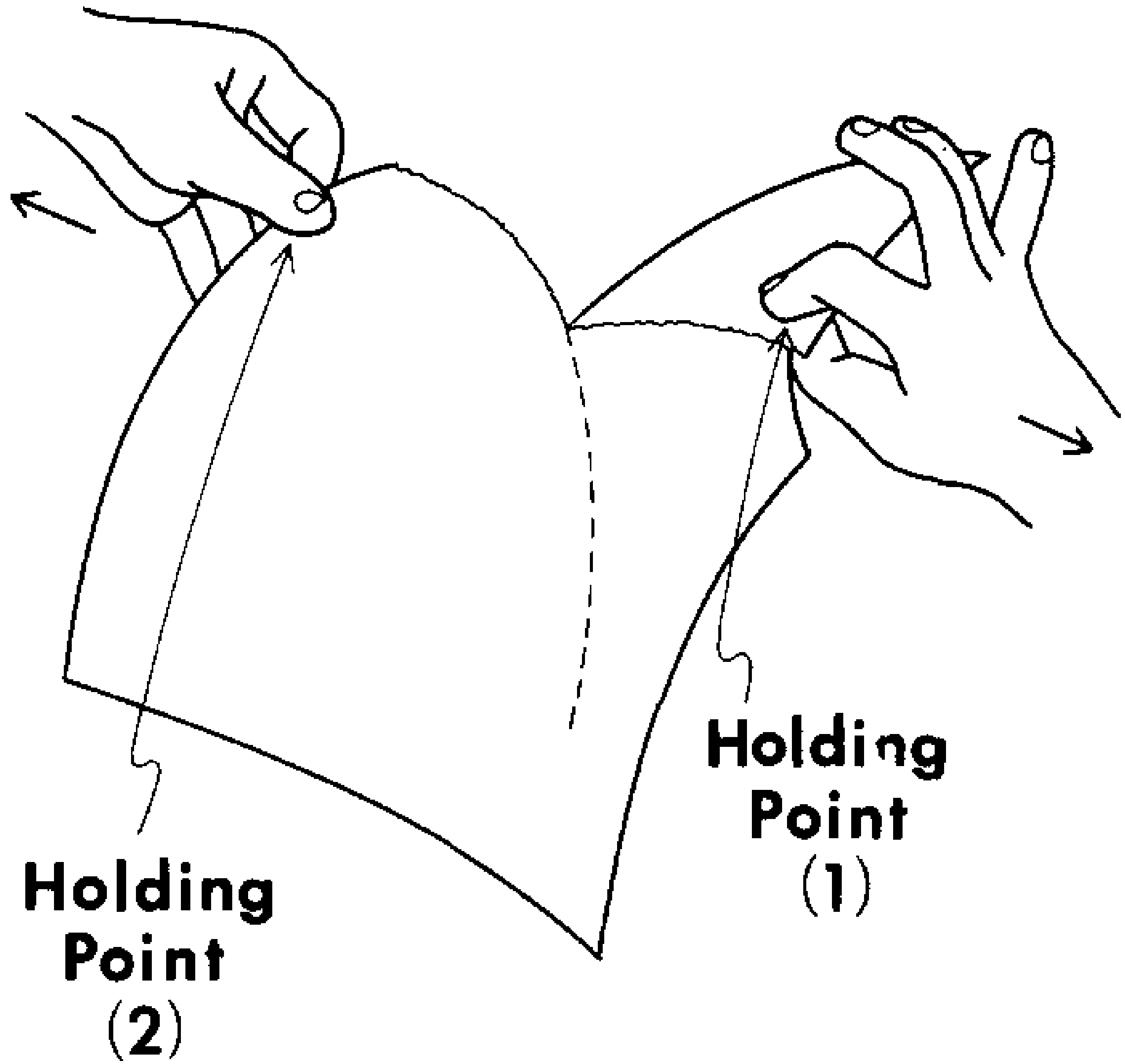
propagation

$$\boxed{\frac{dW - \cancel{dE_{el}}}{tdl} = G} = \boxed{G_c}$$

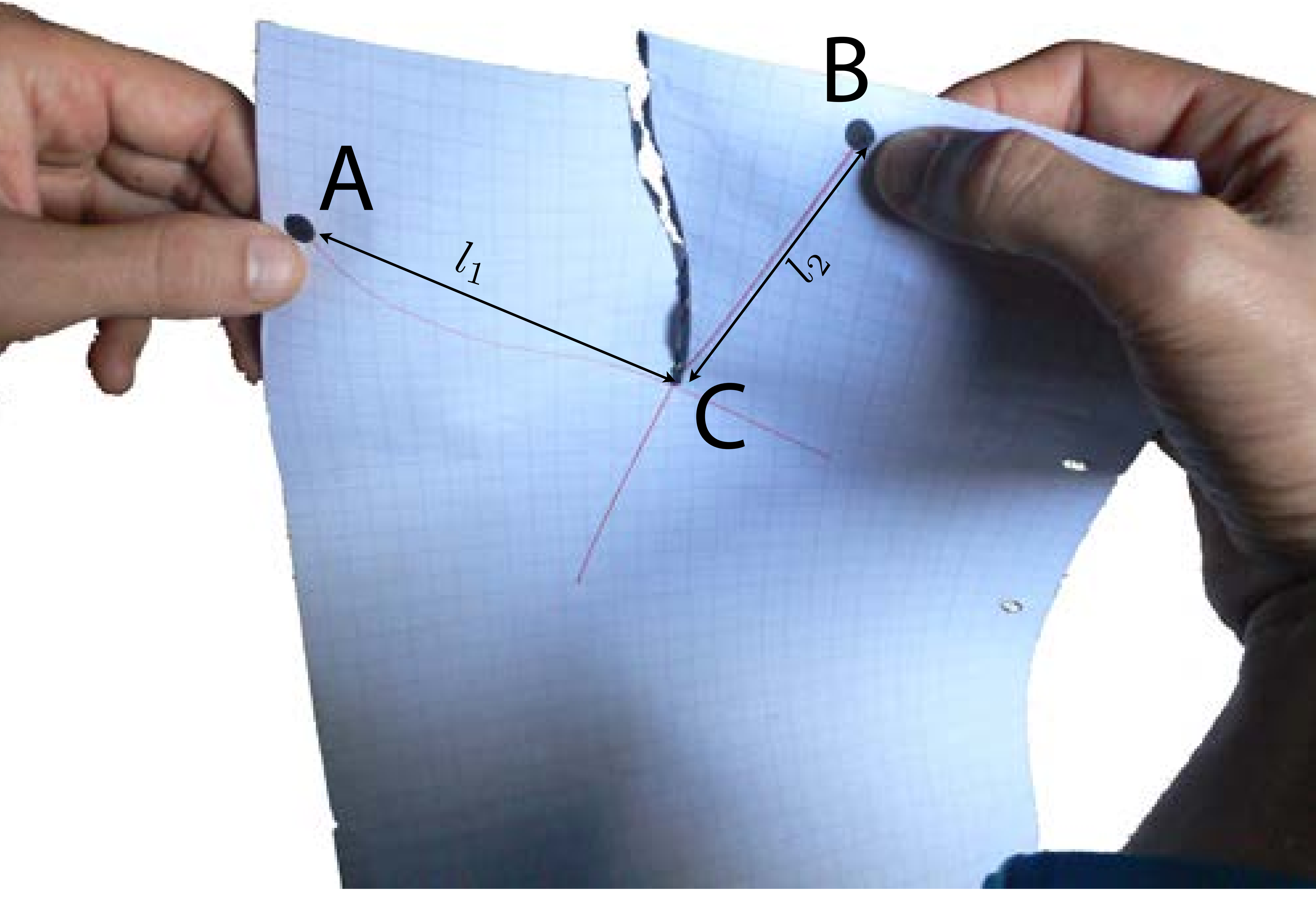
Energy Release Rate

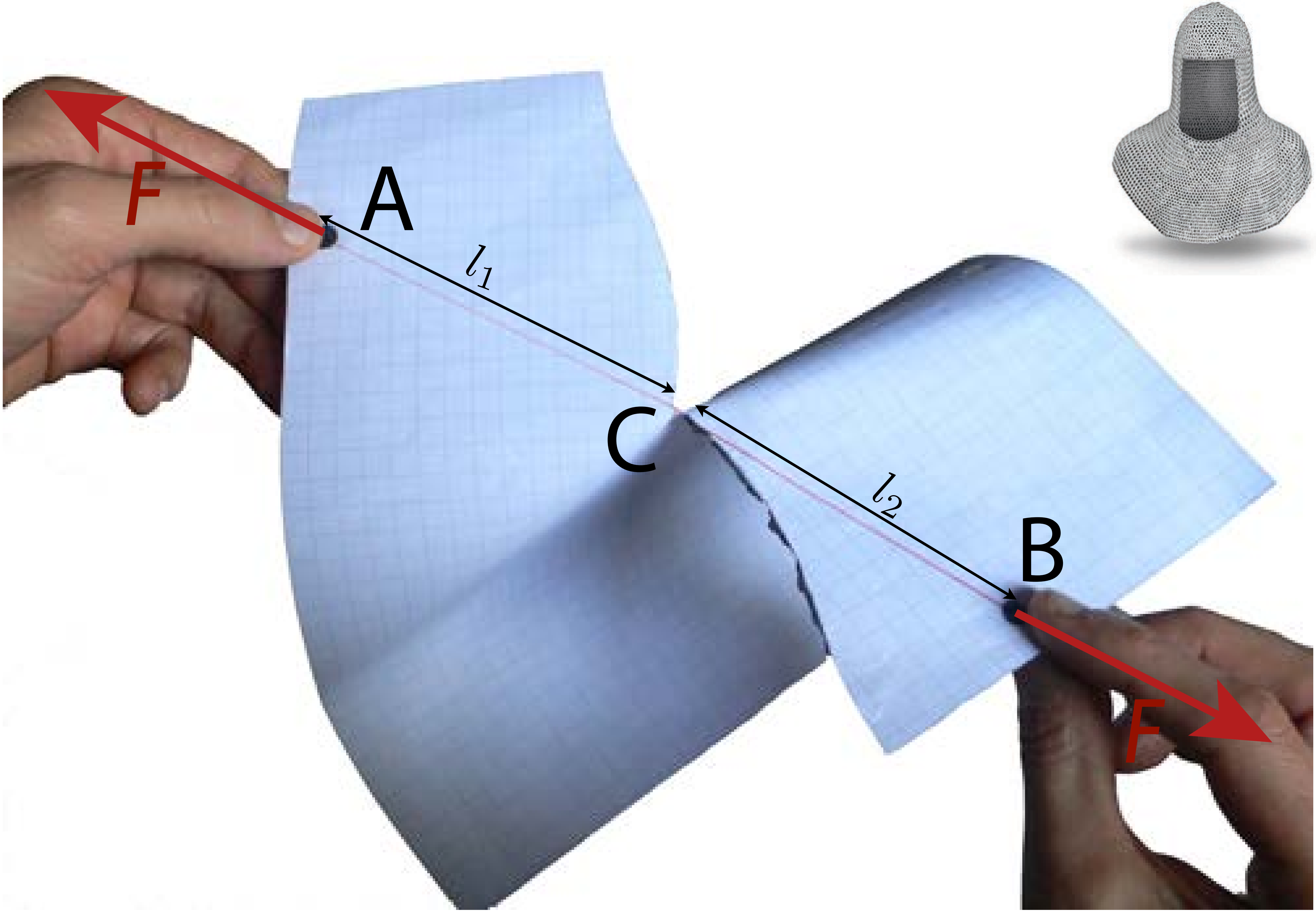
Fracture energy

Tearing problem

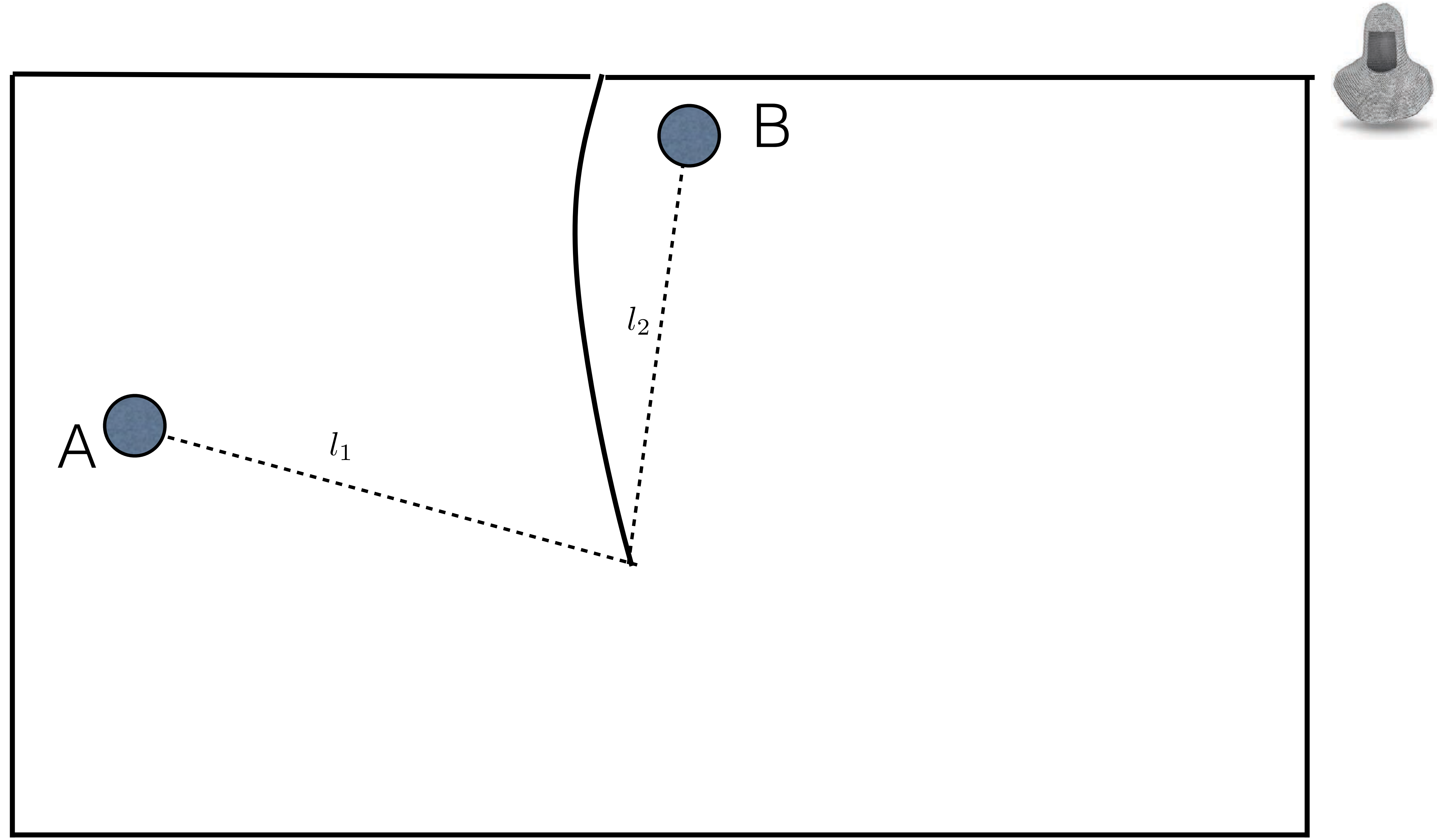


O'Keefe **Am. J. Phys. 62 (4), April 1994**

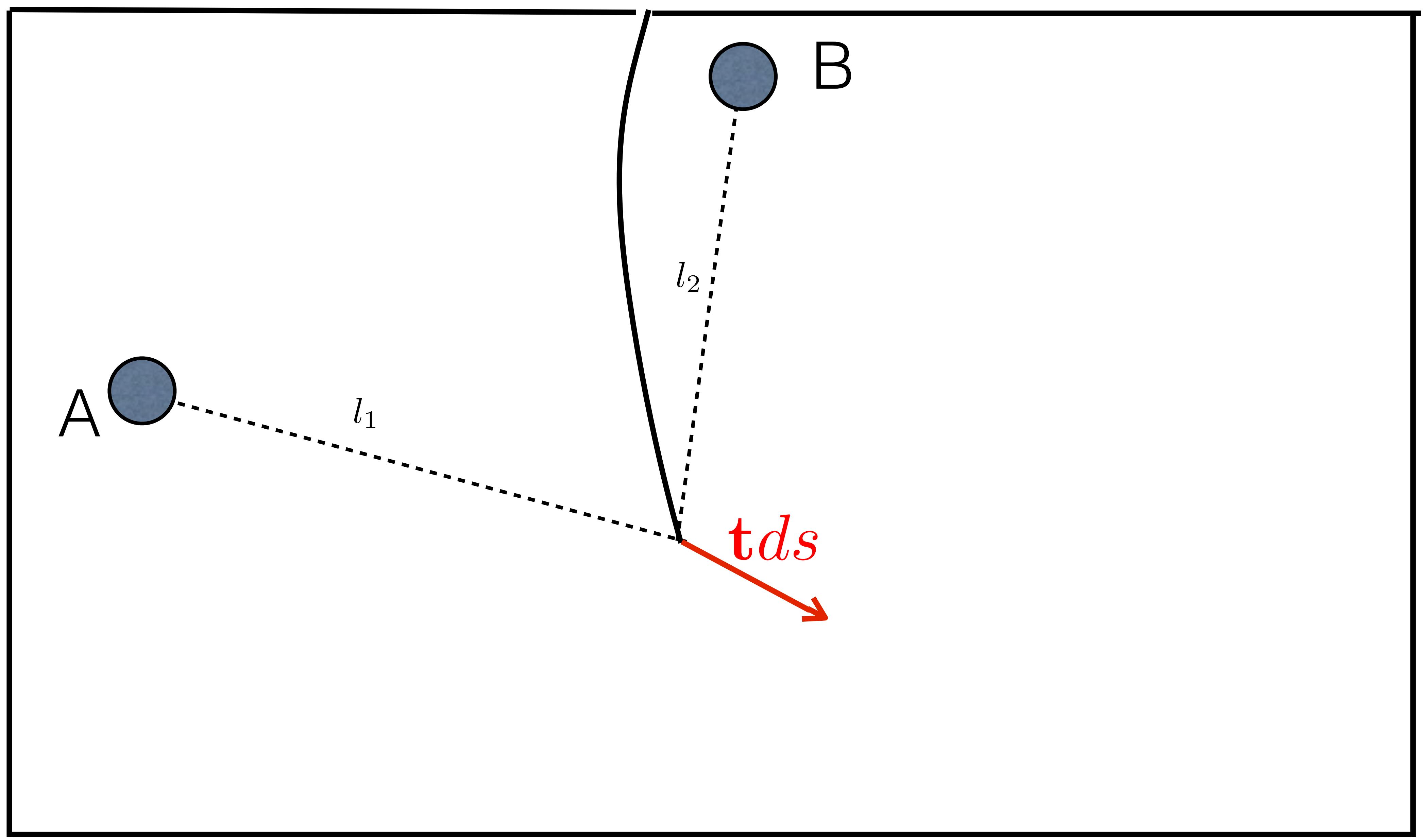

A
B
C
l_1
l_2



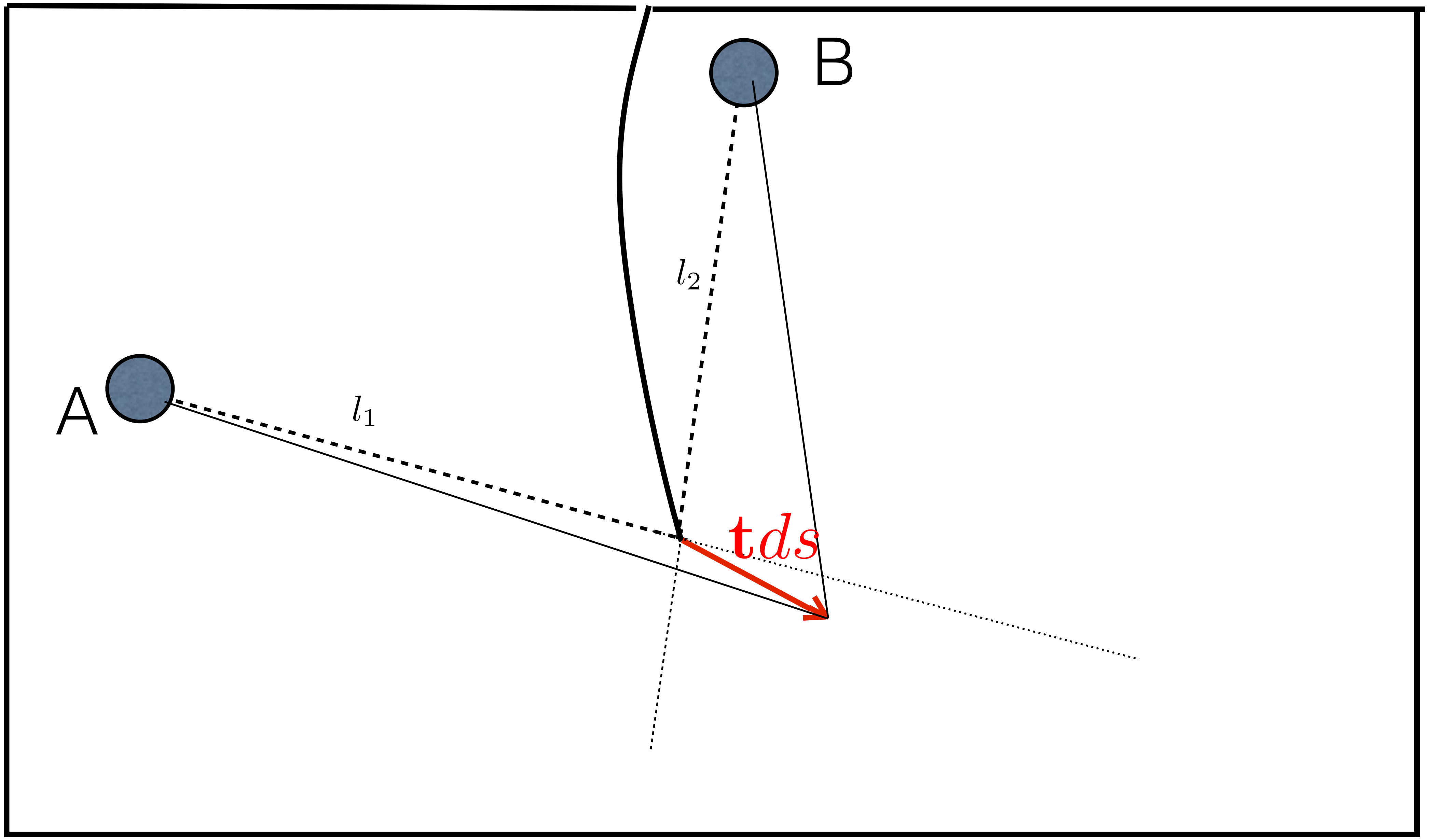
F
A
l_1
C
l_2
B
F



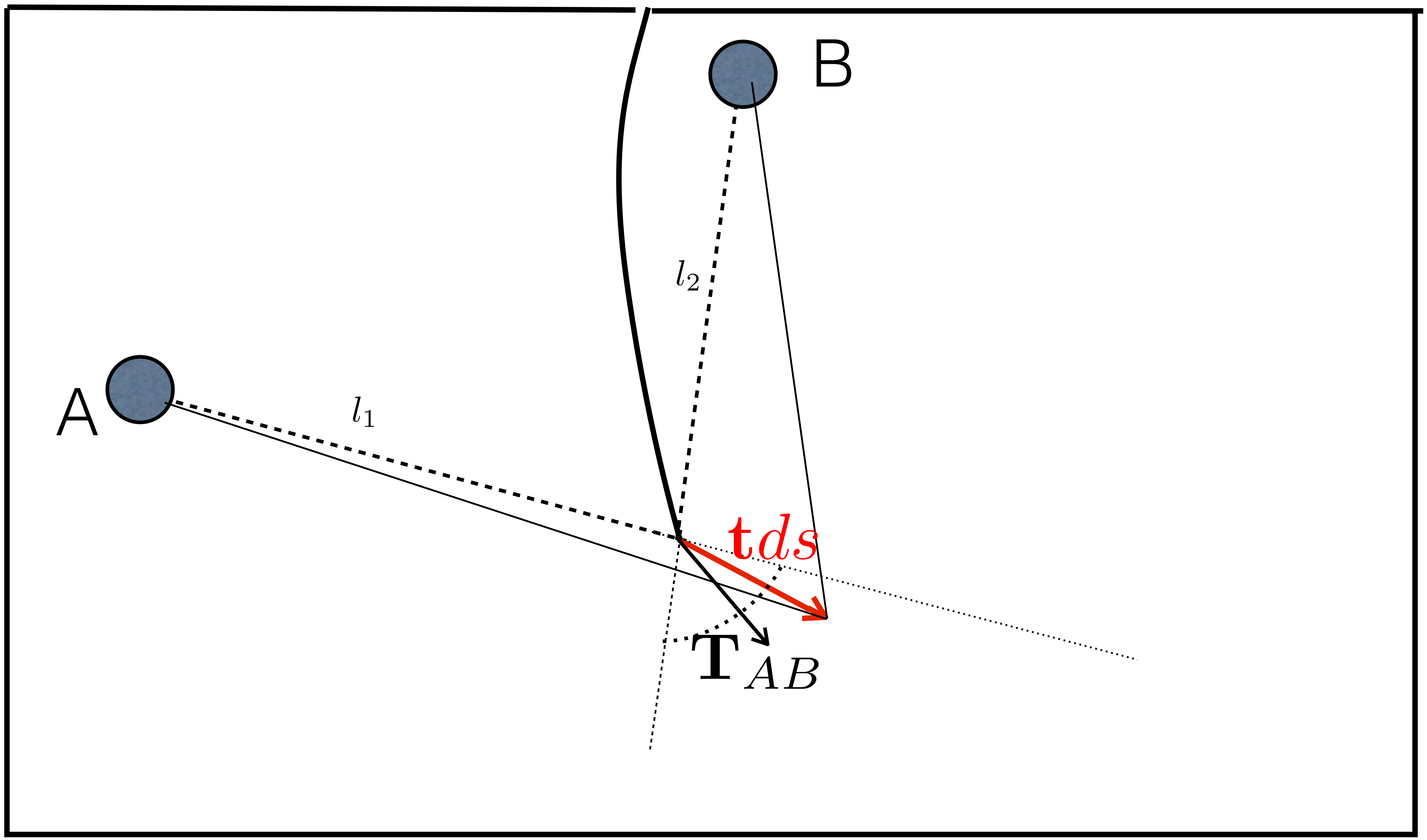
B
A
l_2
l_1



B
A
l_1
l_2
$\mathbf{t}ds$

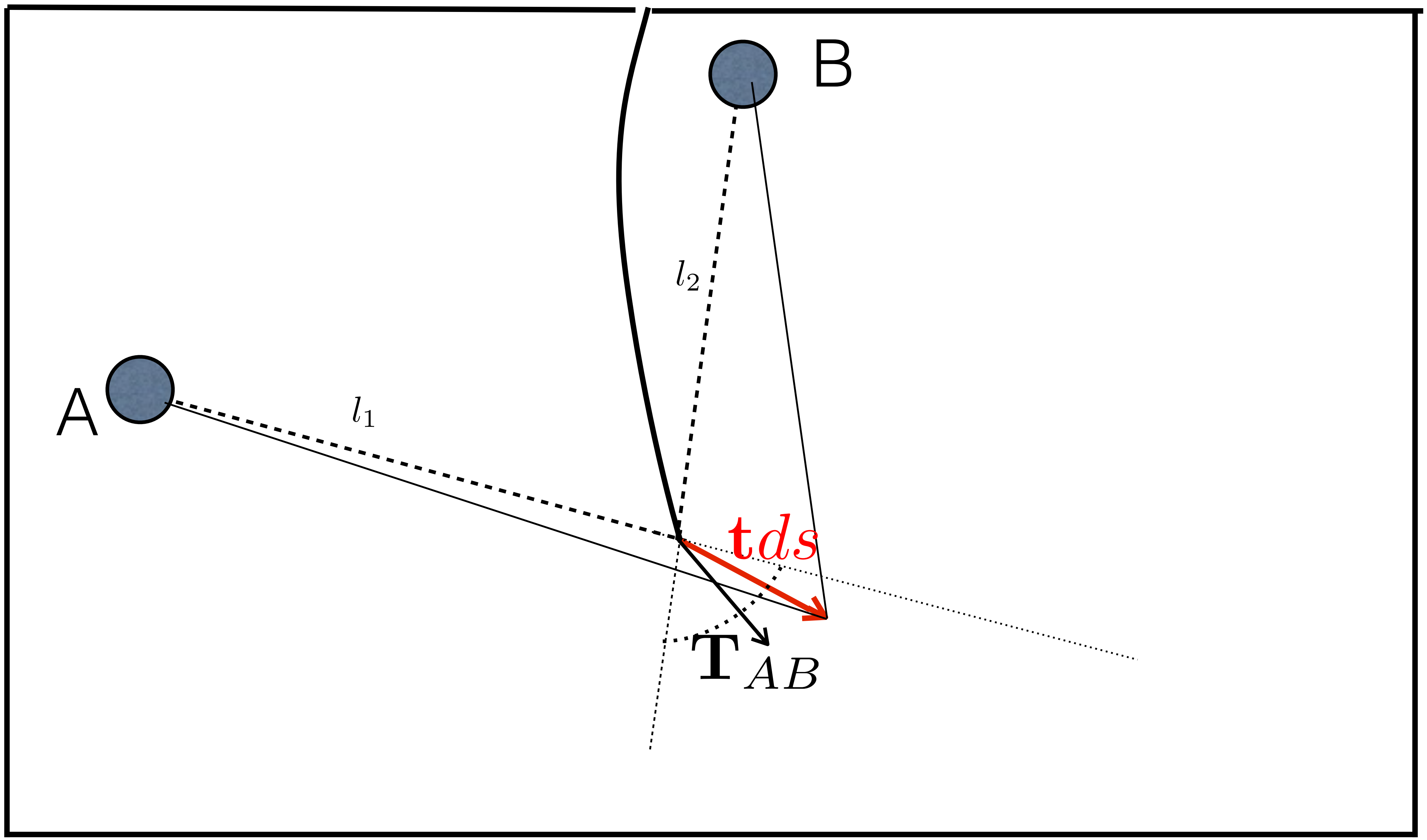


$$dW = F(dl_1 + dl_2)$$



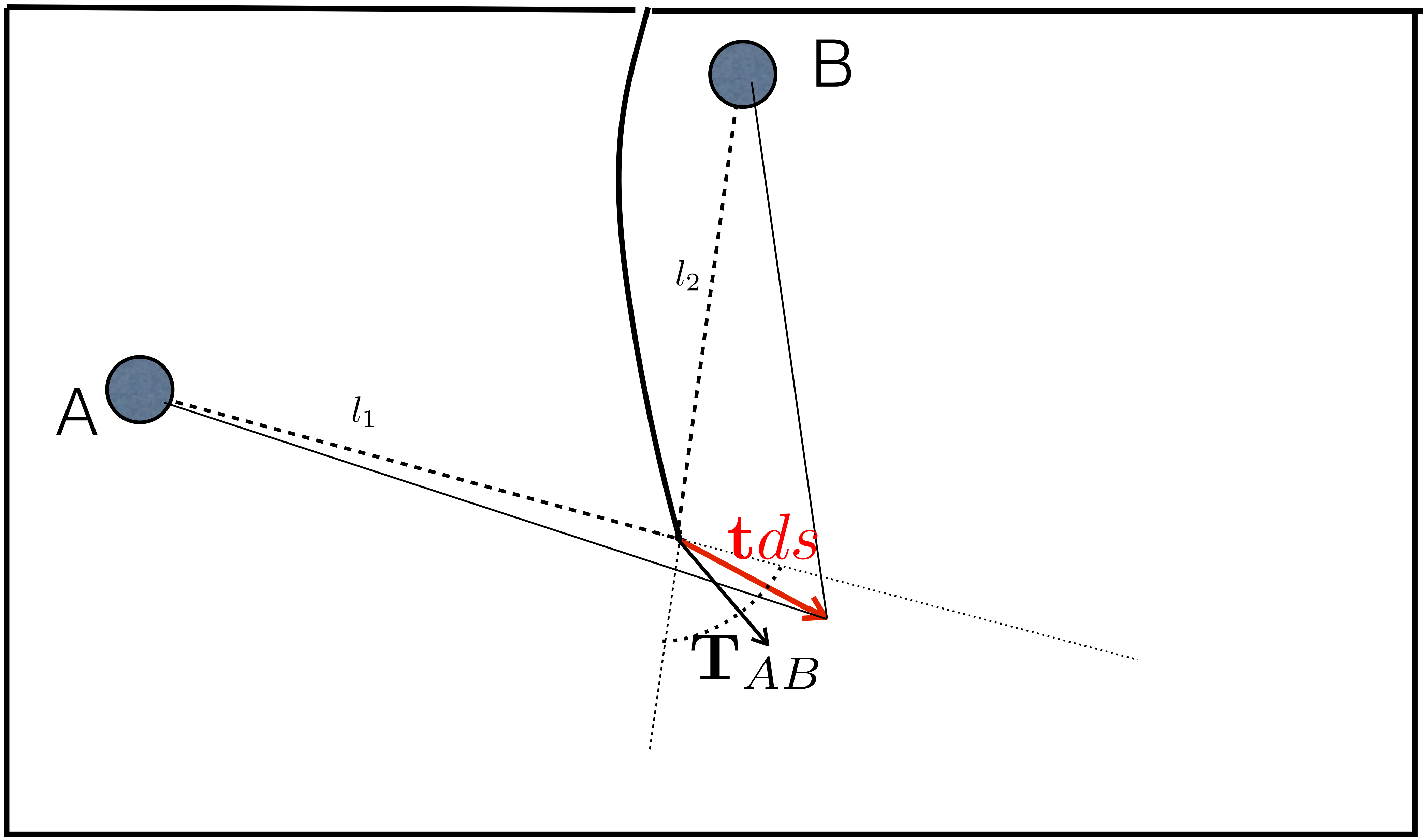
$$dW = F(dl_1 + dl_2) = F\mathbf{T}_{AB}.\mathbf{t}ds$$

bisector



$$G = \frac{F}{t}\mathbf{T}_{AB}.\mathbf{t}$$

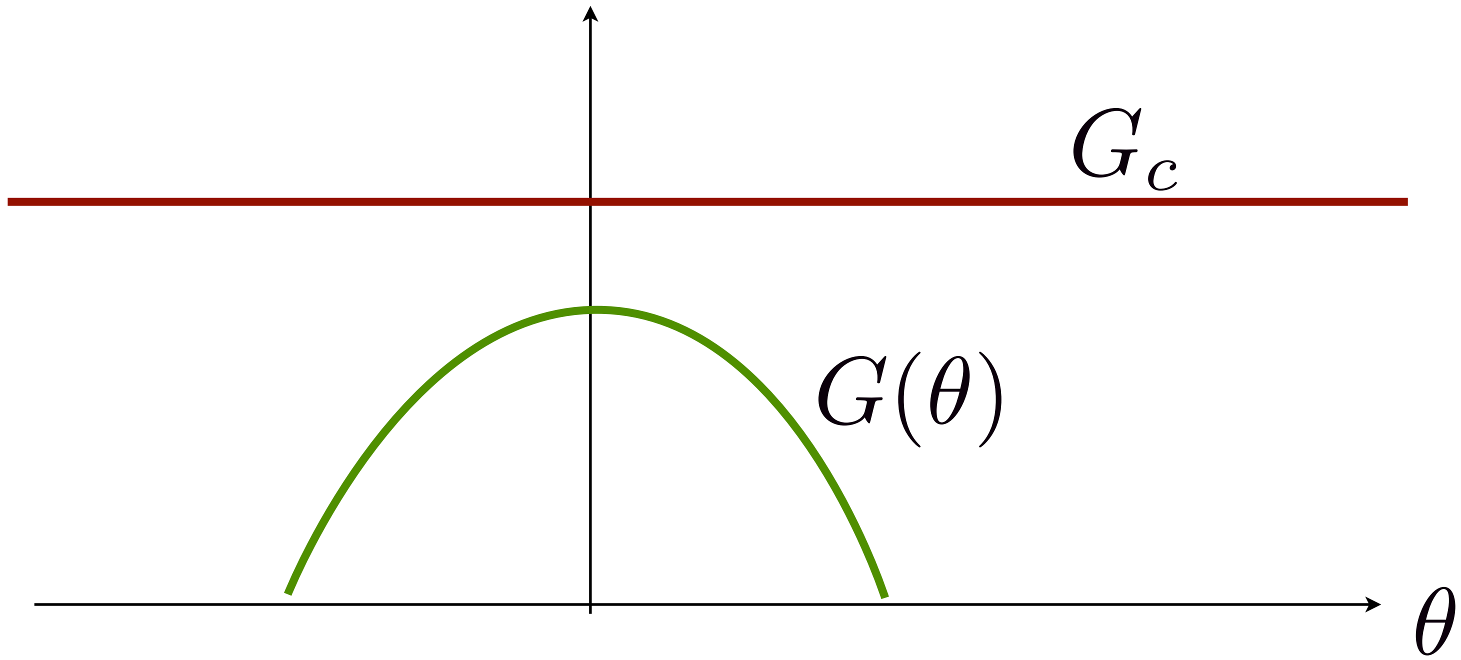
a configurational force?
(fracture tip has no memory of its path)



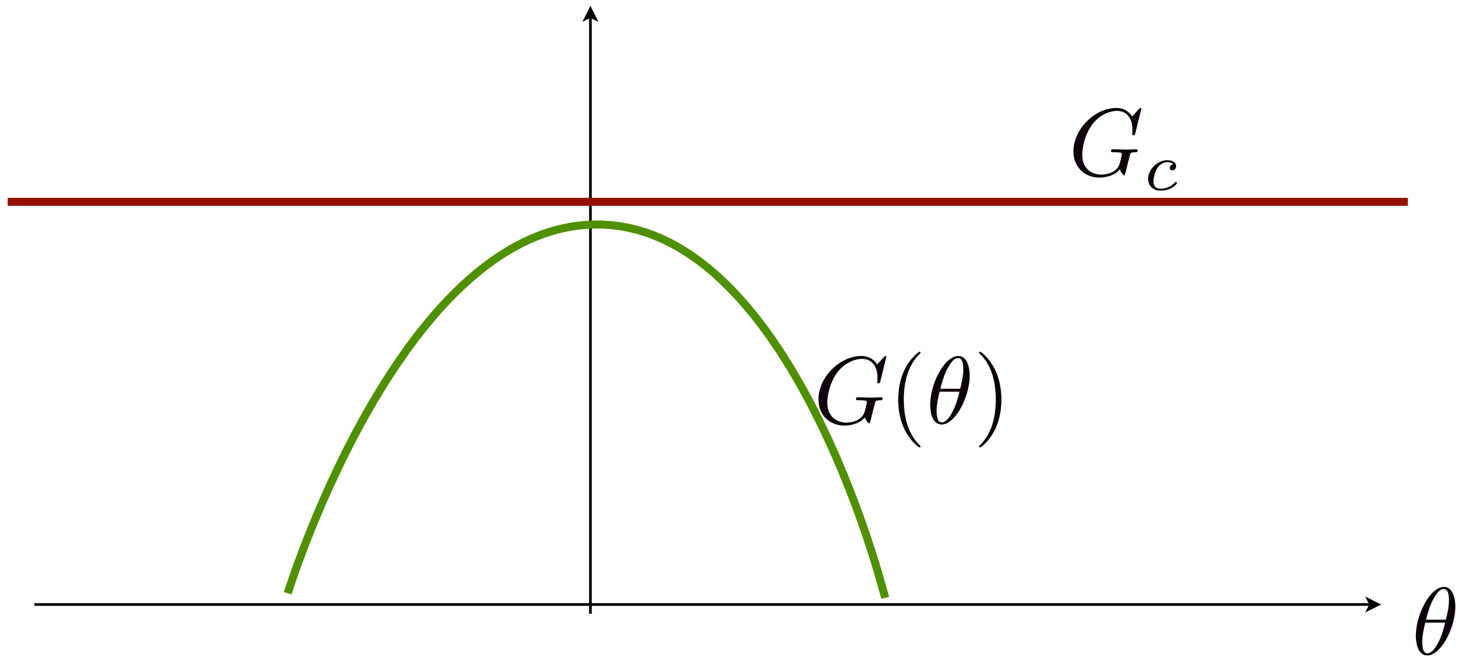
$$G = \frac{F}{t}\mathbf{T}_{AB}.\mathbf{t}$$

Which direction ?

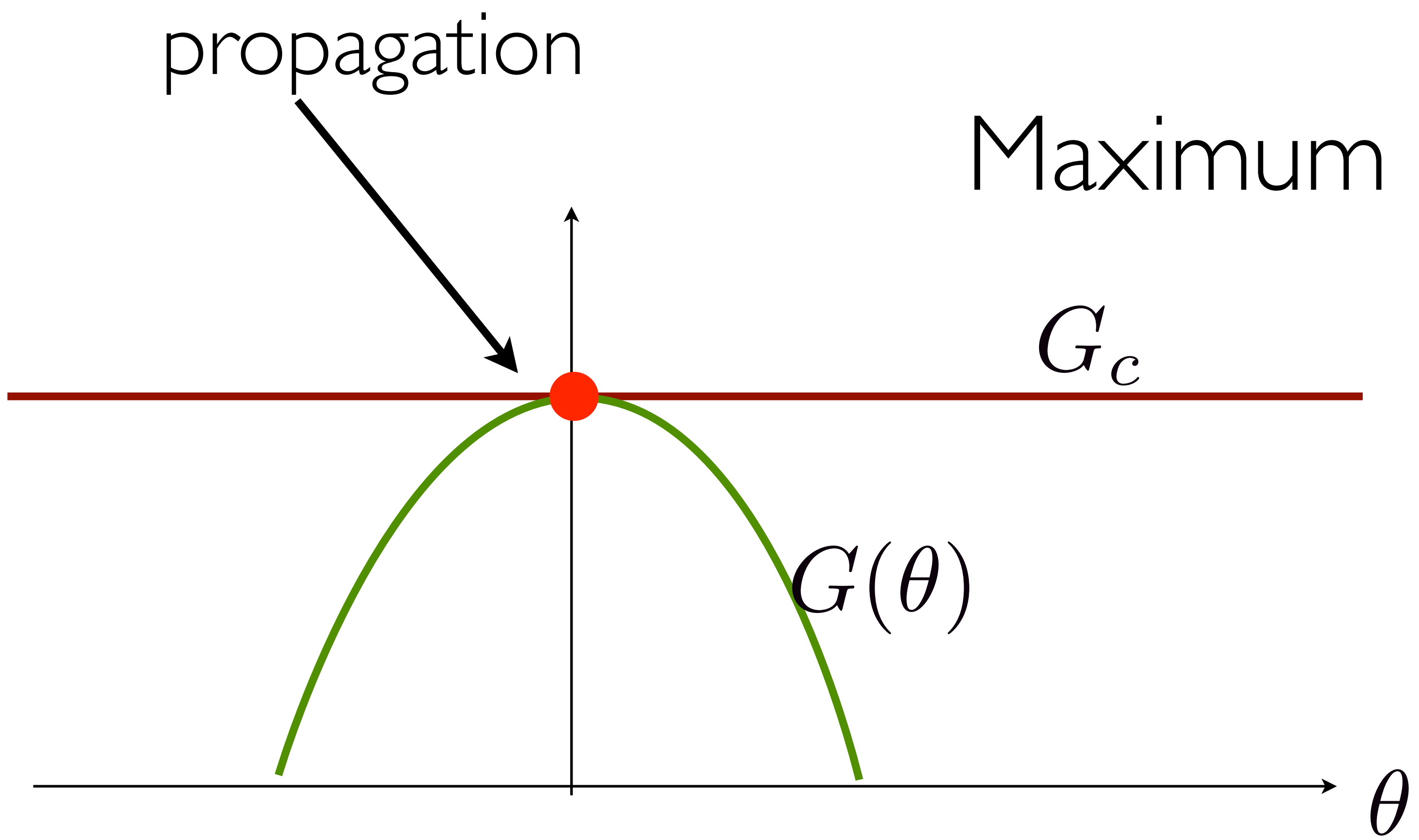
Which direction ?



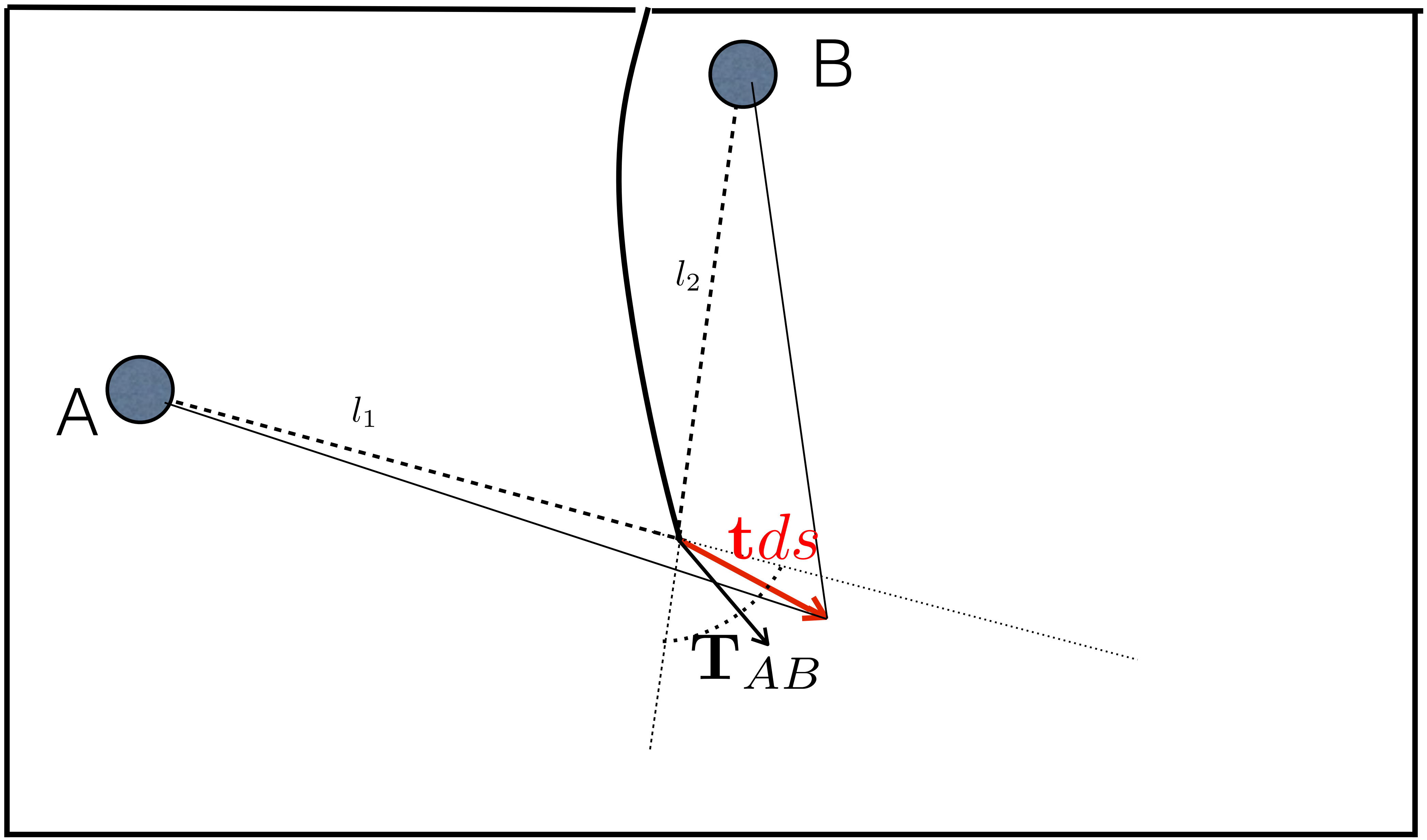
Which direction ?



Which direction ?



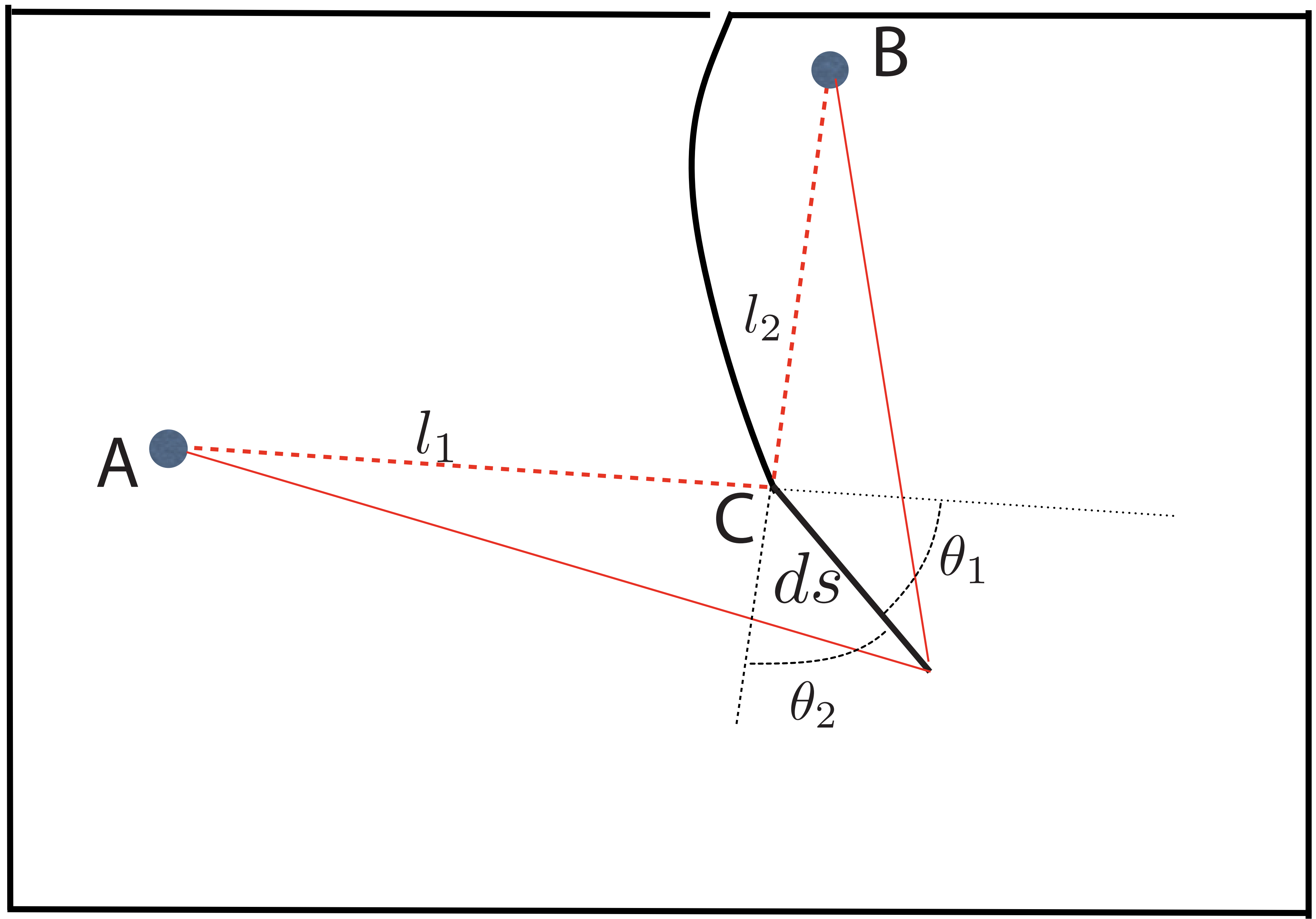
Maximum of Energy Release Rate



$$G = \frac{F}{t}\mathbf{T}_{AB}.\mathbf{t}$$

Maximum Energy Release rate : bisector

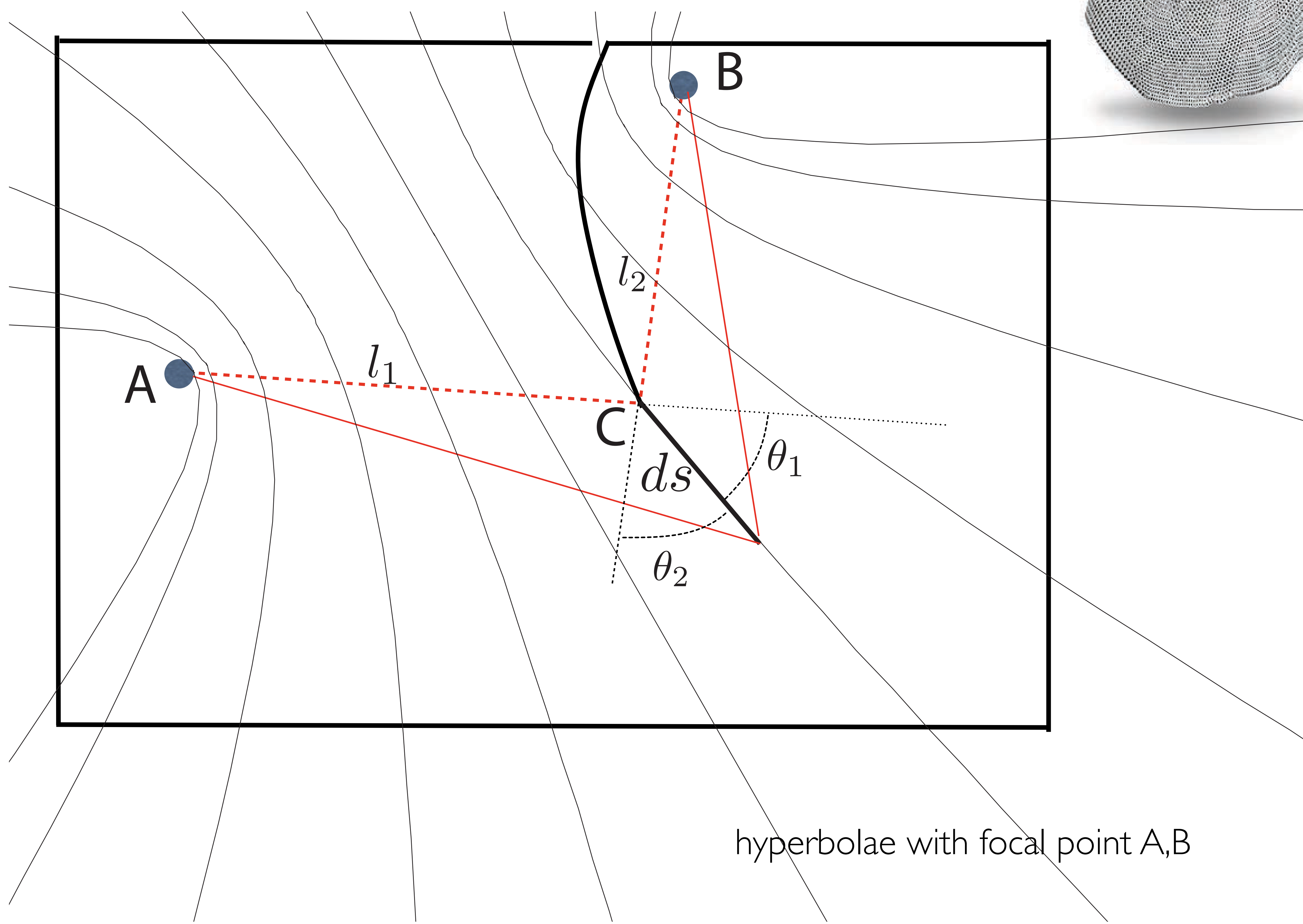
$$dW = F(dl_1 + dl_2)$$

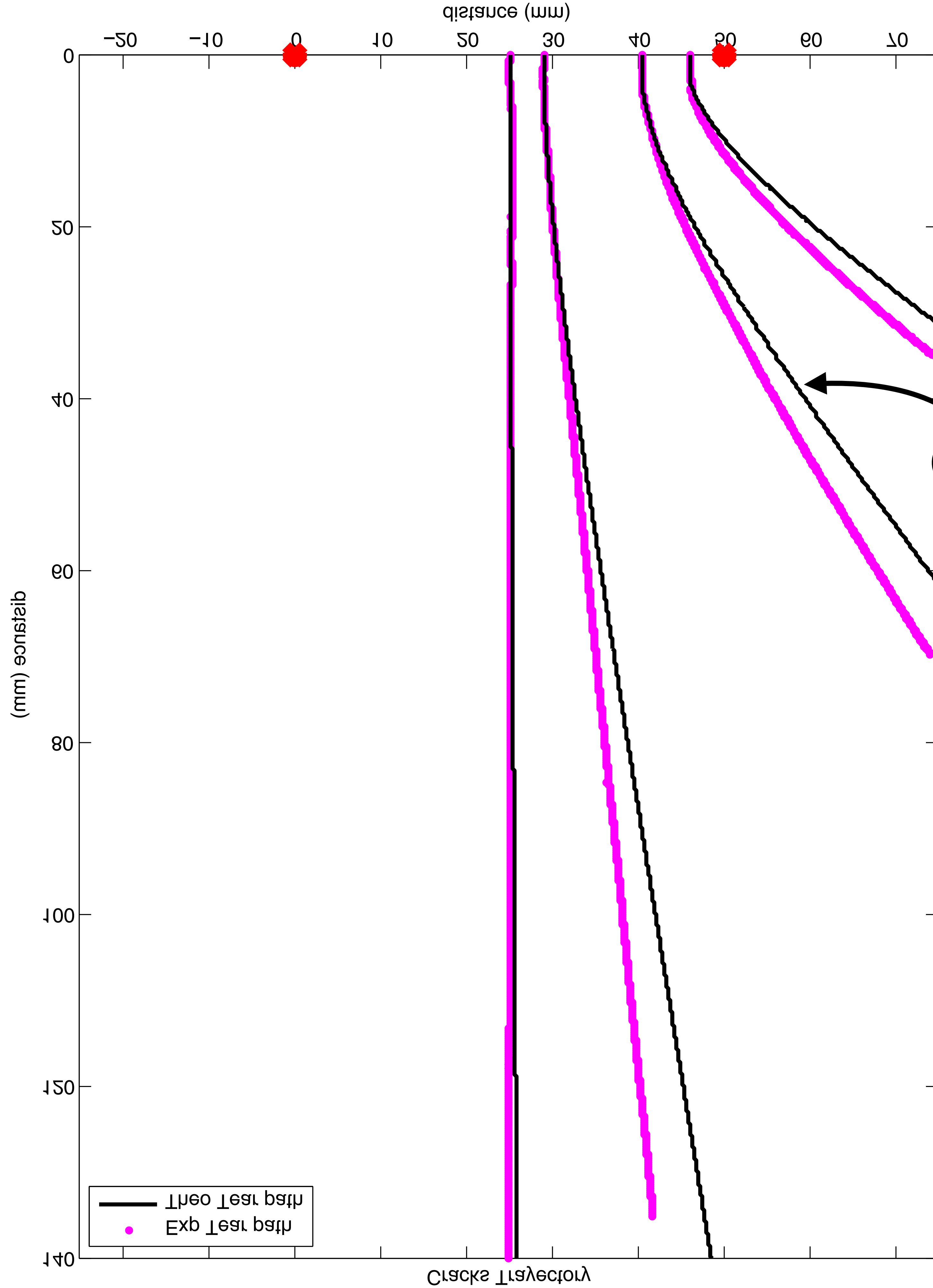


$$G = \frac{F}{t}\mathbf{T}_{AB}.\mathbf{t}$$

Maximum Energy Release rate : bisector $\theta_1 = \theta_2$

$$\theta_1 = \theta_2$$





0th order theory:
hyperbolae

J-F Fuentealba

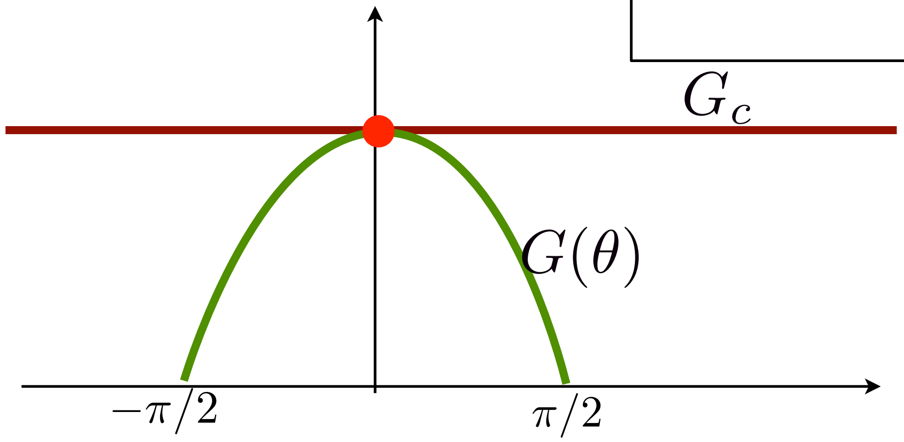
possible deviation due to **material anisotropy**

isotropic medium

- Principle of local symmetry $k_{II} = 0$
- maximum energy release rate

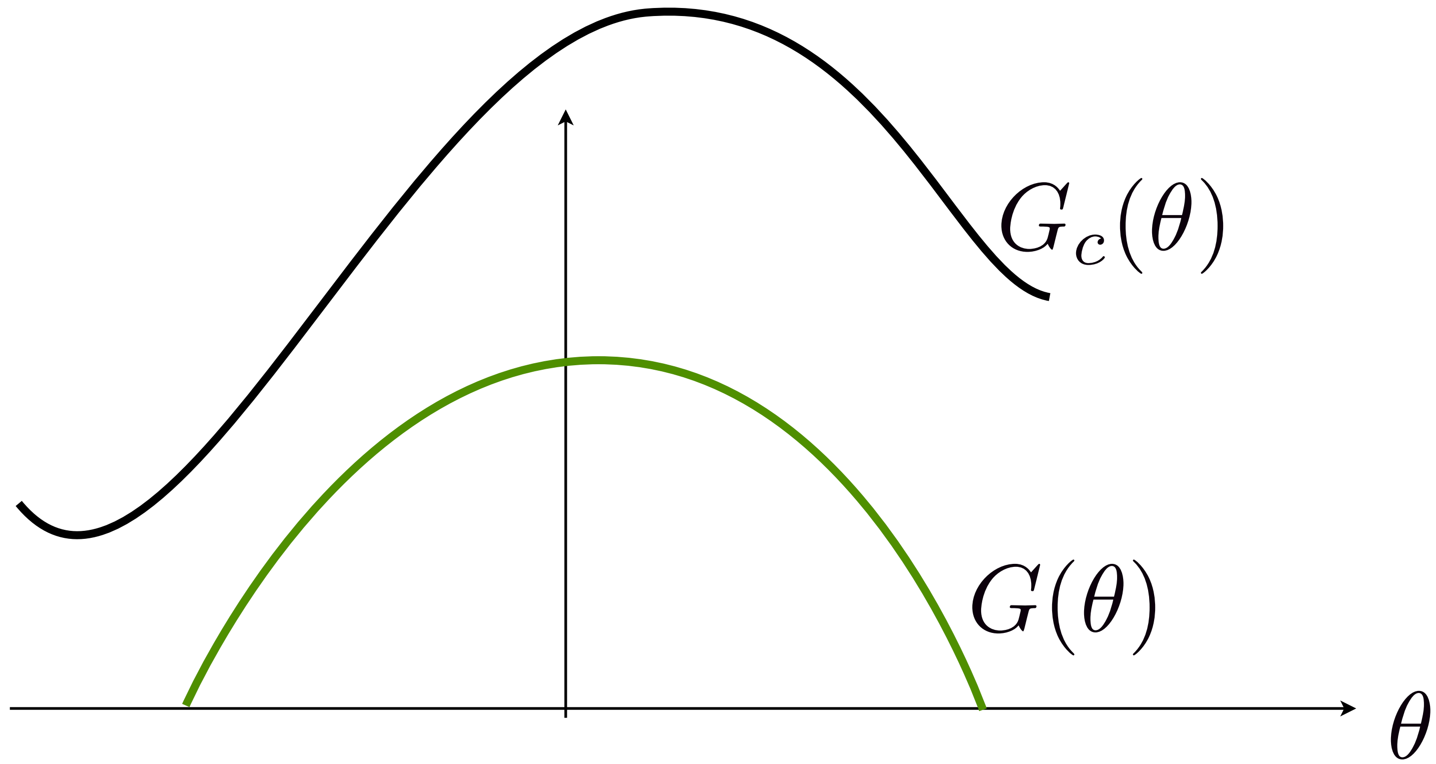
θ Maximizes $G(\theta)$
(fixed loading)

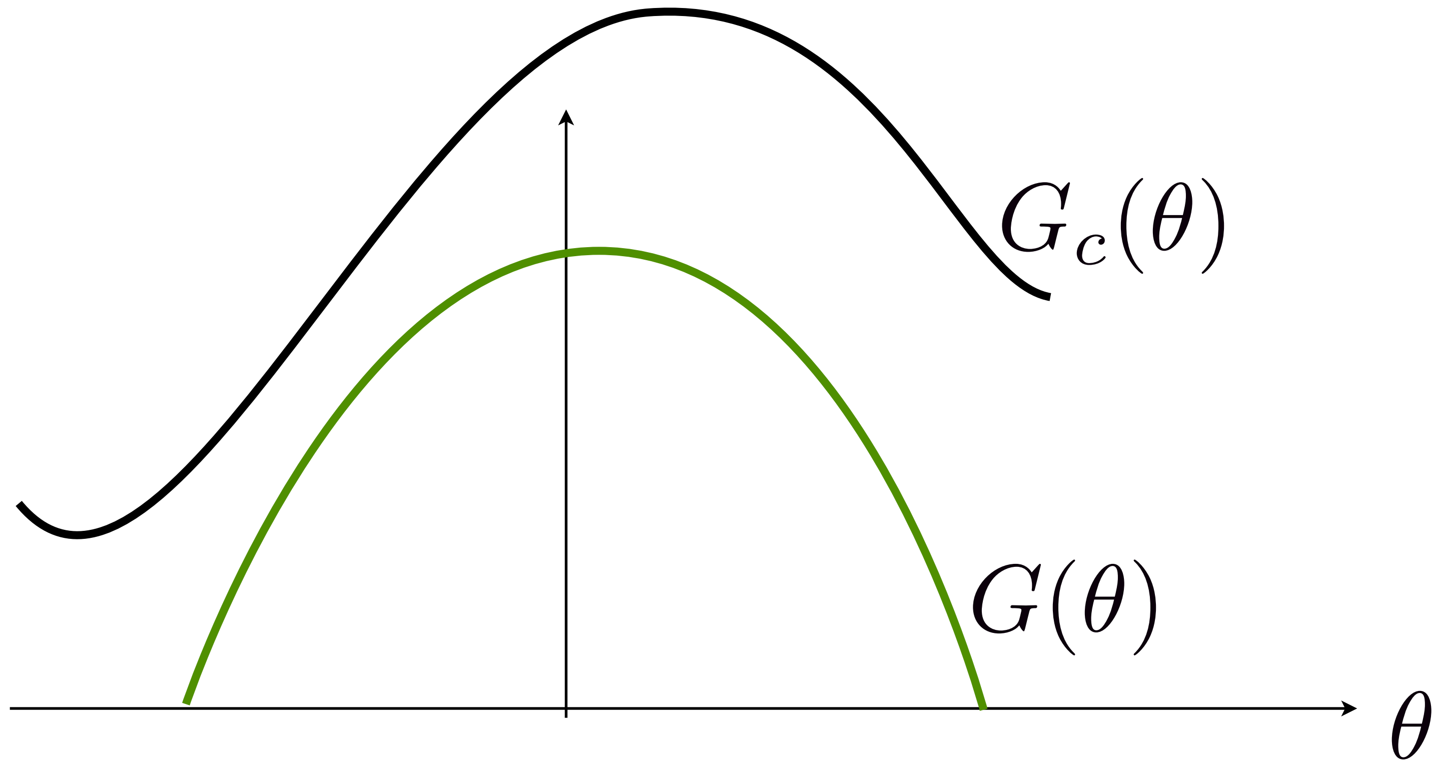
equivalent

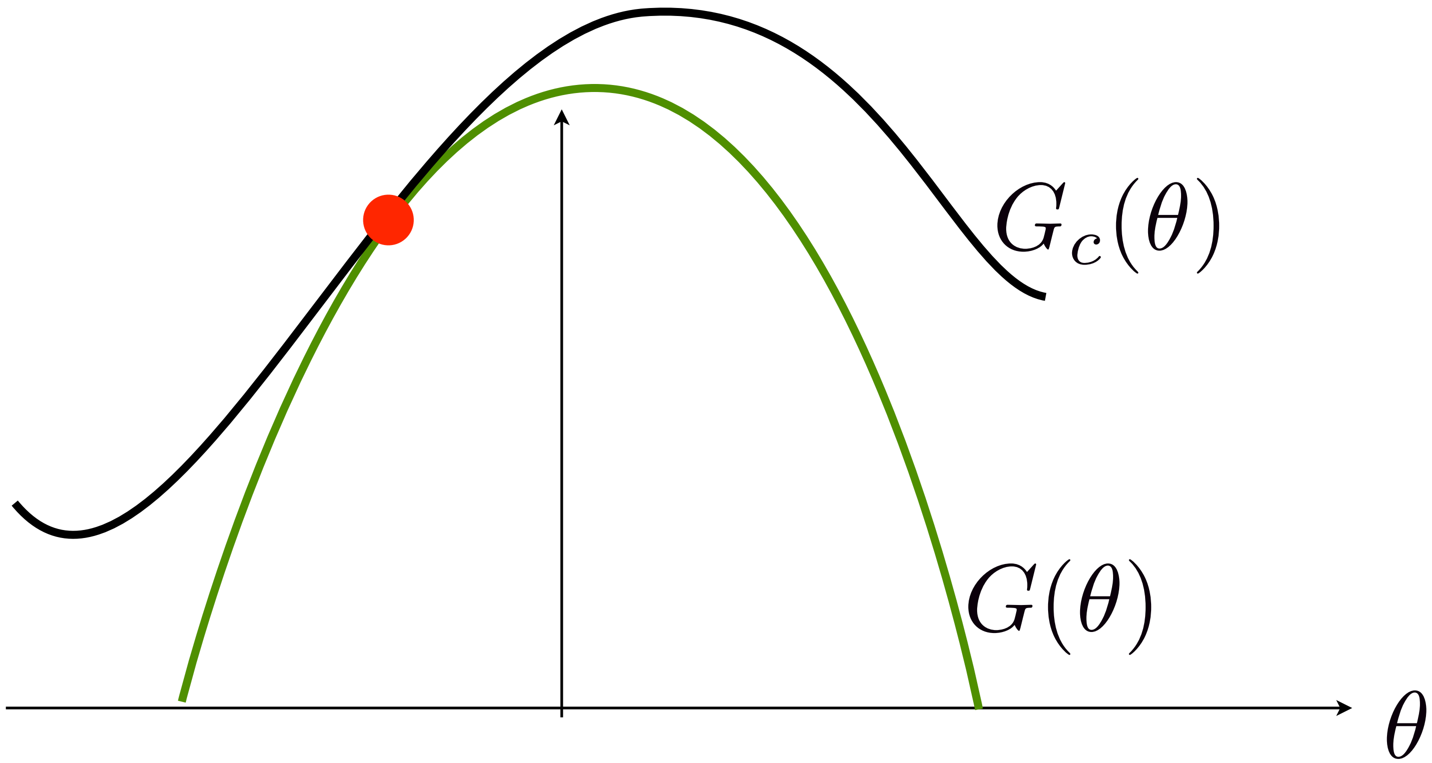


$$G(\theta) = \frac{F}{2t}\cos(\theta)$$

anisotropic medium direction of fracture? open question !

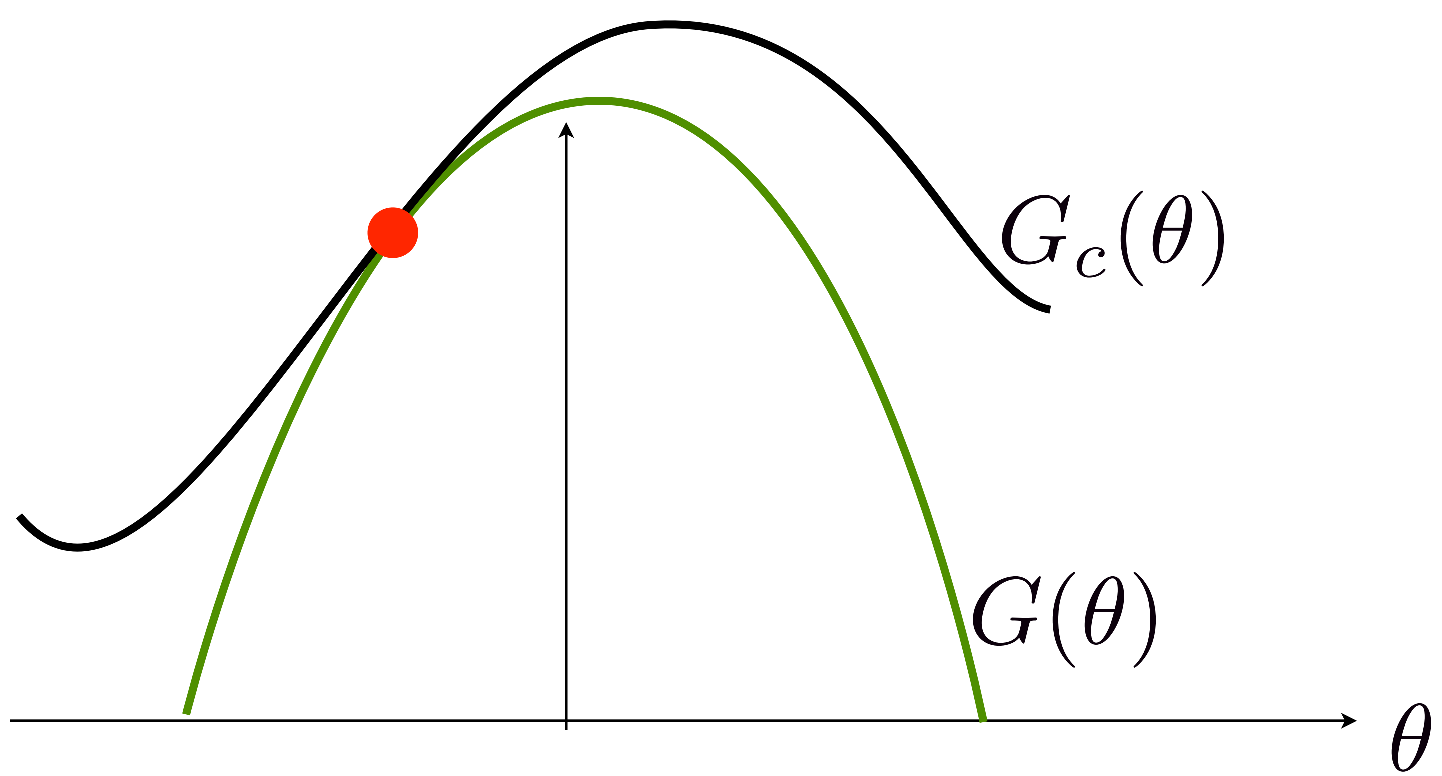

$G_c(\theta)$
$G(\theta)$
θ


$G_c(\theta)$
$G(\theta)$
θ



Propagation as soon as possible:

~~*«Maximum energy release rate»*~~



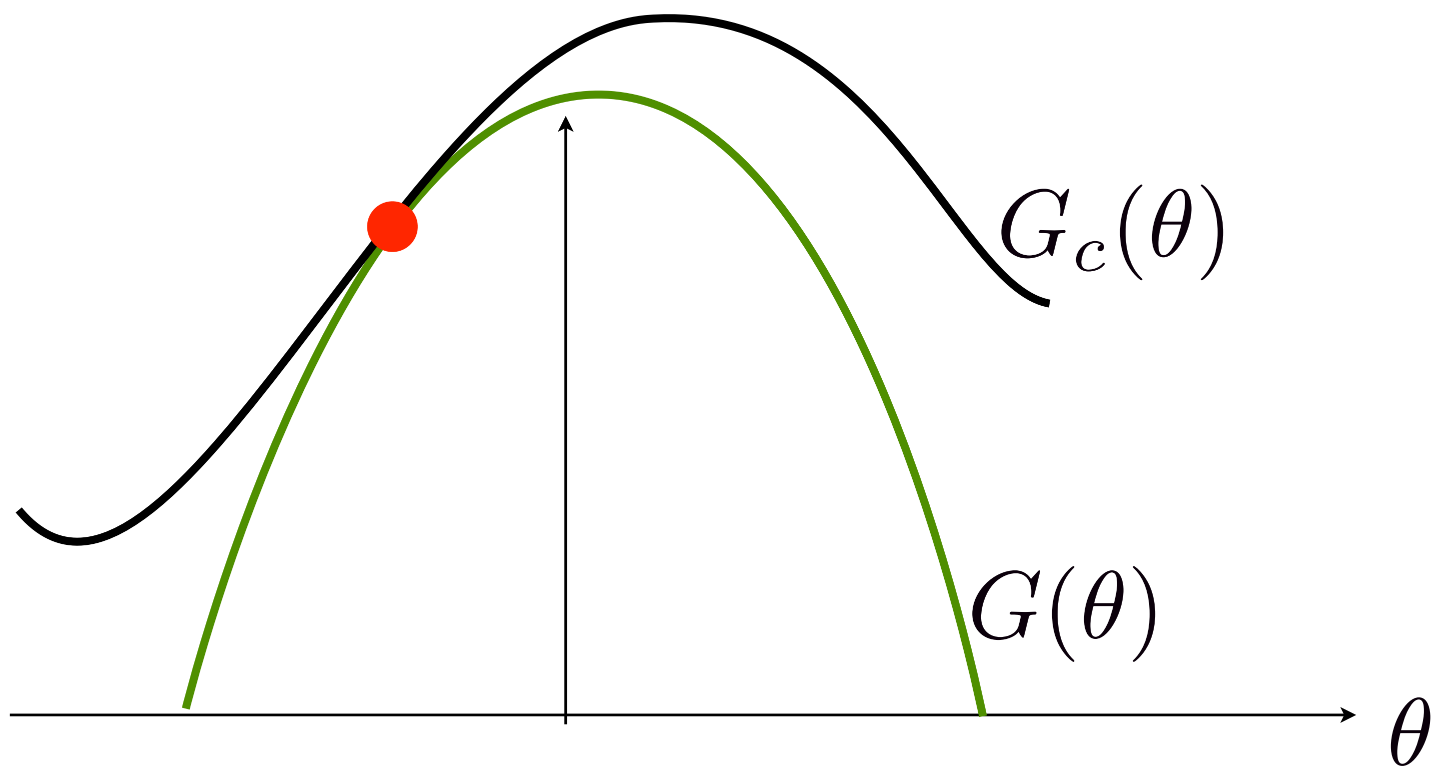
Propagation as soon as possible:

«Maximum energy release rate»

Maximizes $\dfrac{G(\theta)}{G_c(\theta)}$

(fixed loading)

i.e. Minimize loading consistent with propagation



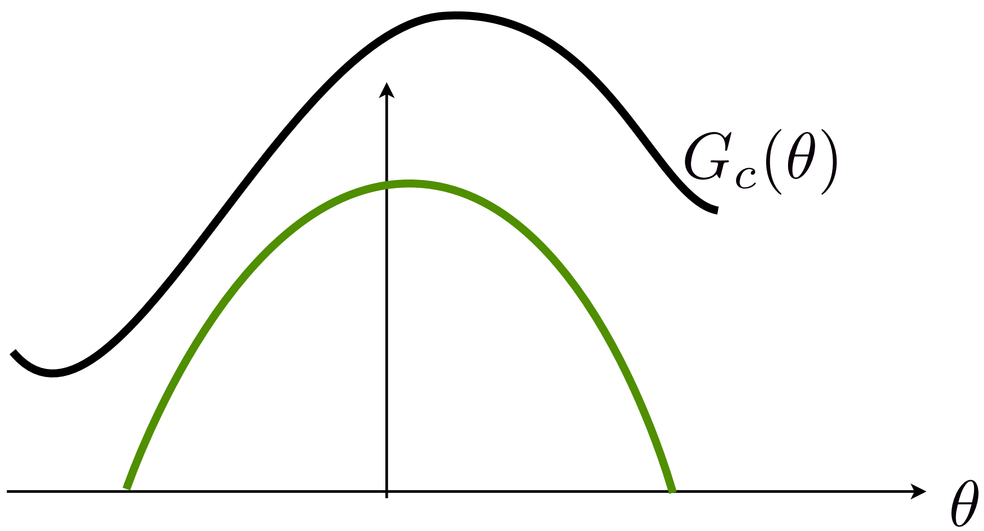
Maximizes $\dfrac{G(\theta)}{G_c(\theta)}$

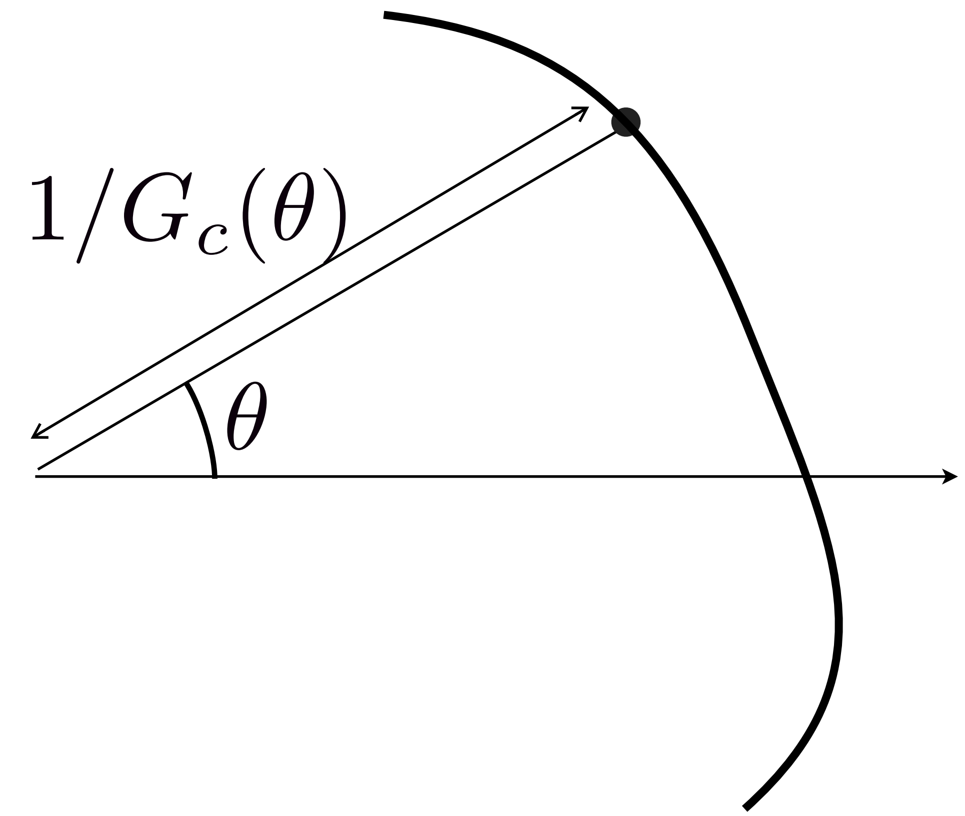
(fixed loading)

Palaniswamy *et al*	1978
Gurtin *et al*	1998
Hakim *et al*	2005,2009
Chambolle *et al*	2009

difficult to check experimentally
EASIER in thin sheets!

inverse polar plot

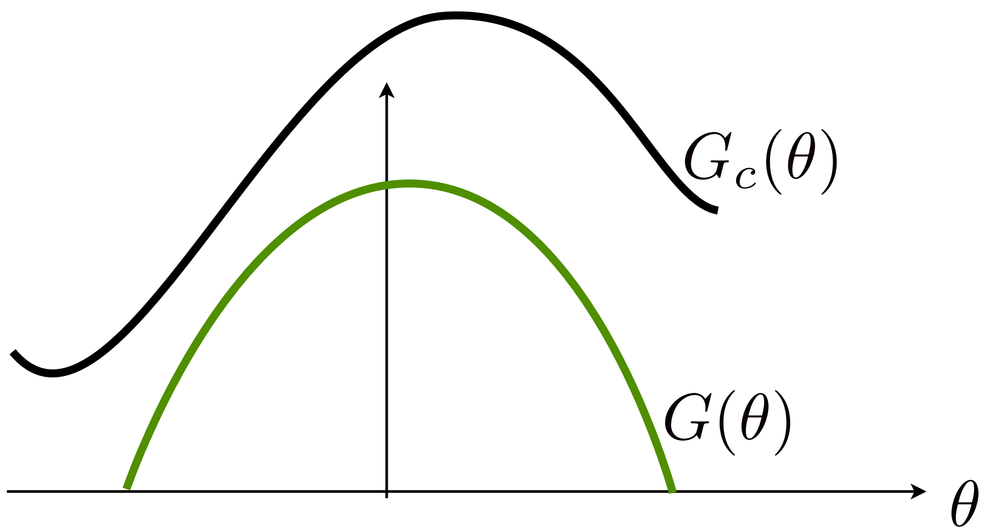




used crystal growth (inverse Wulff plot)

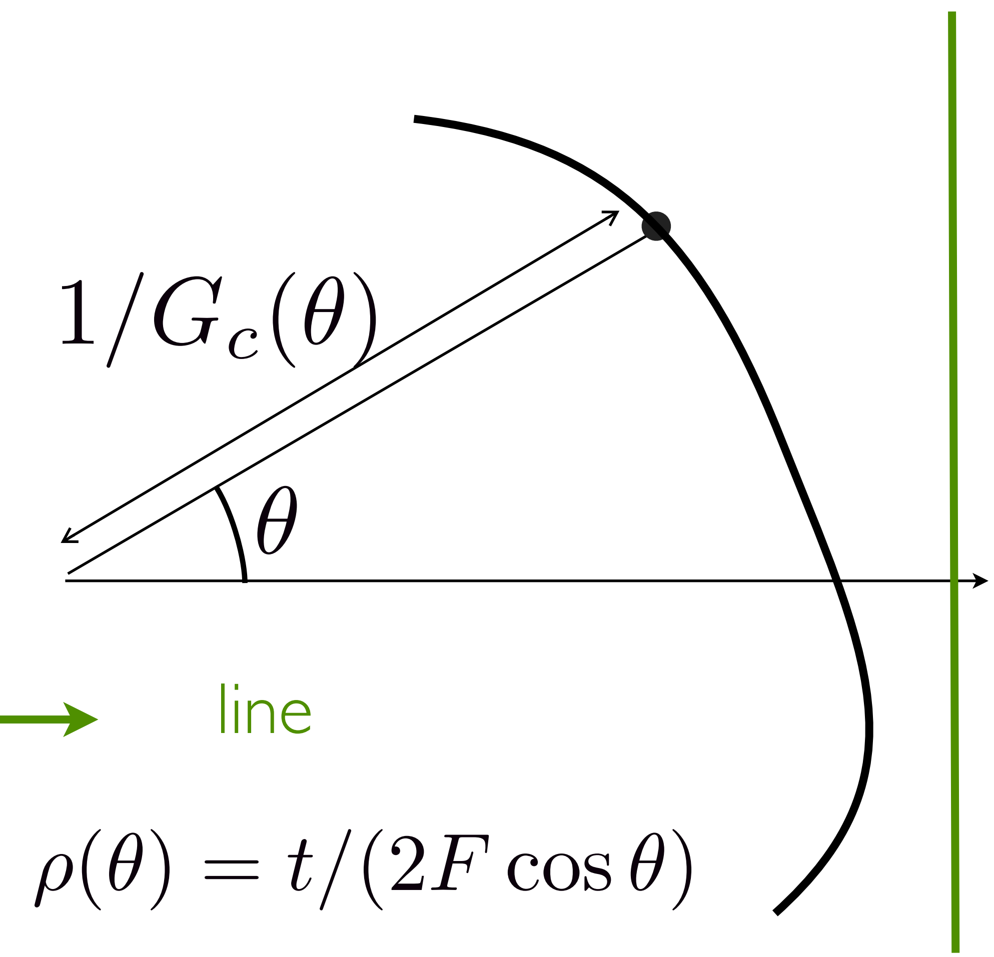
also Gurtin et al. 1998

inverse polar plot



sinusoïde

$$G(\theta) = 2\frac{F}{t}\cos\theta$$

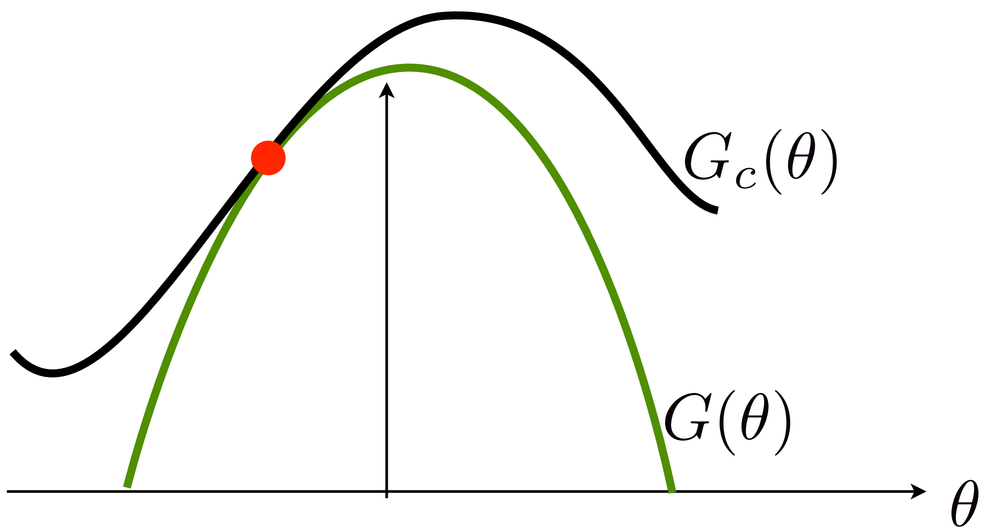


line

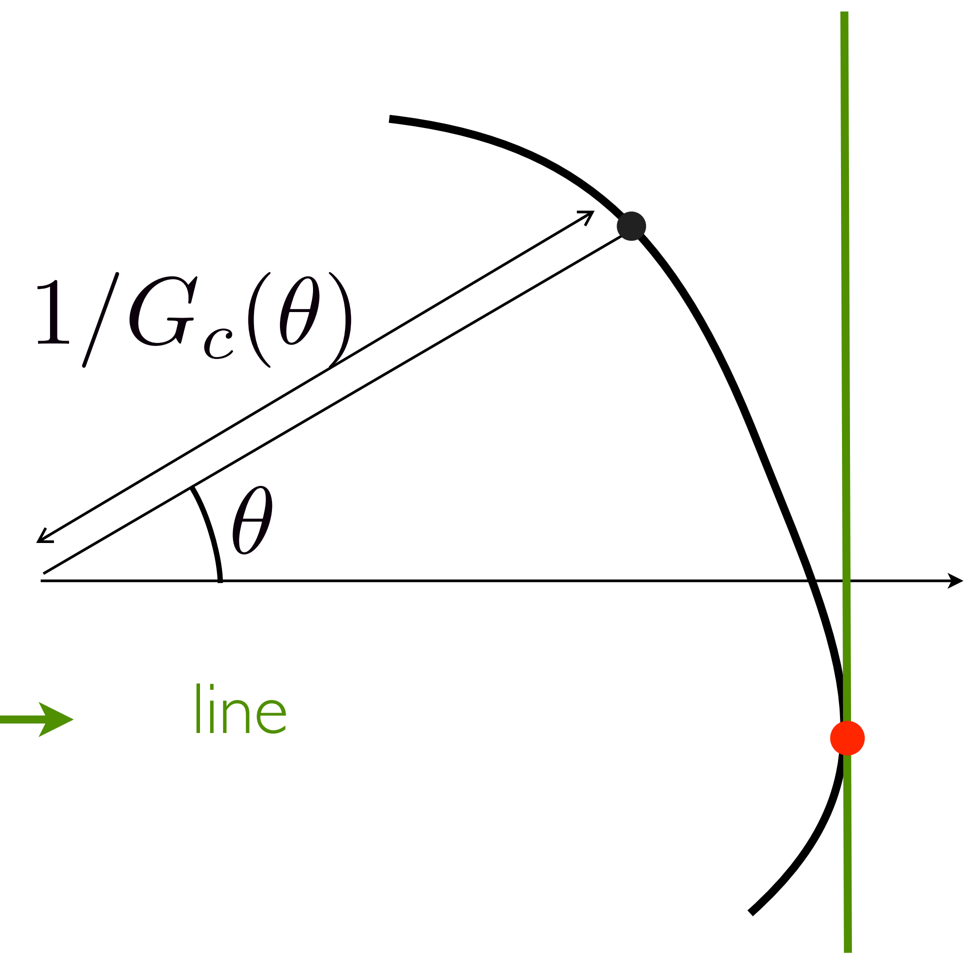
$$\rho(\theta) = t/(2F\cos\theta)$$

<u>specific to tearing</u> (not true for regular crack)

inverse polar plot



sinusoïde



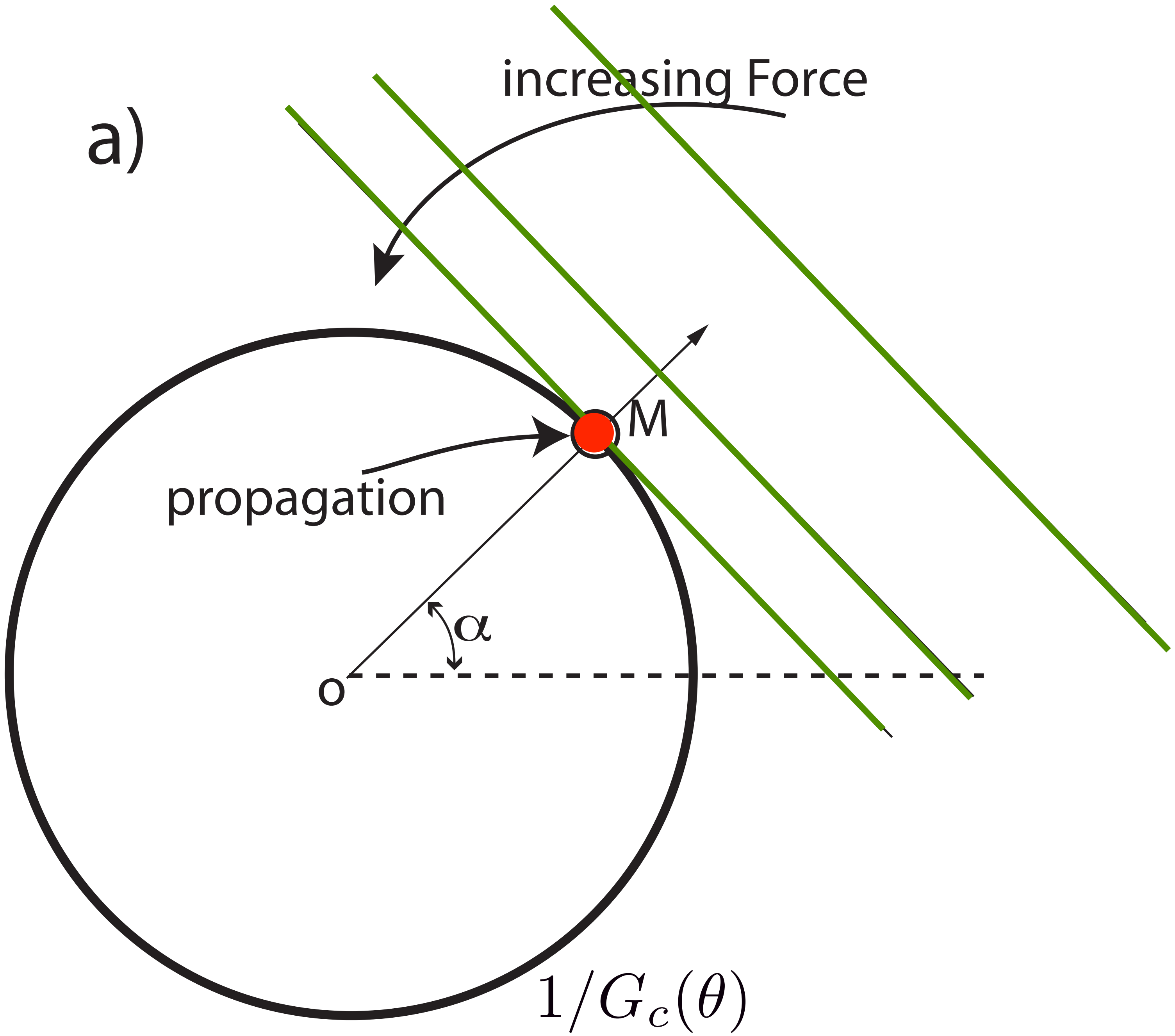
line

$$G(\theta) = 2\frac{F}{t}\cos\theta$$

exterior tangent

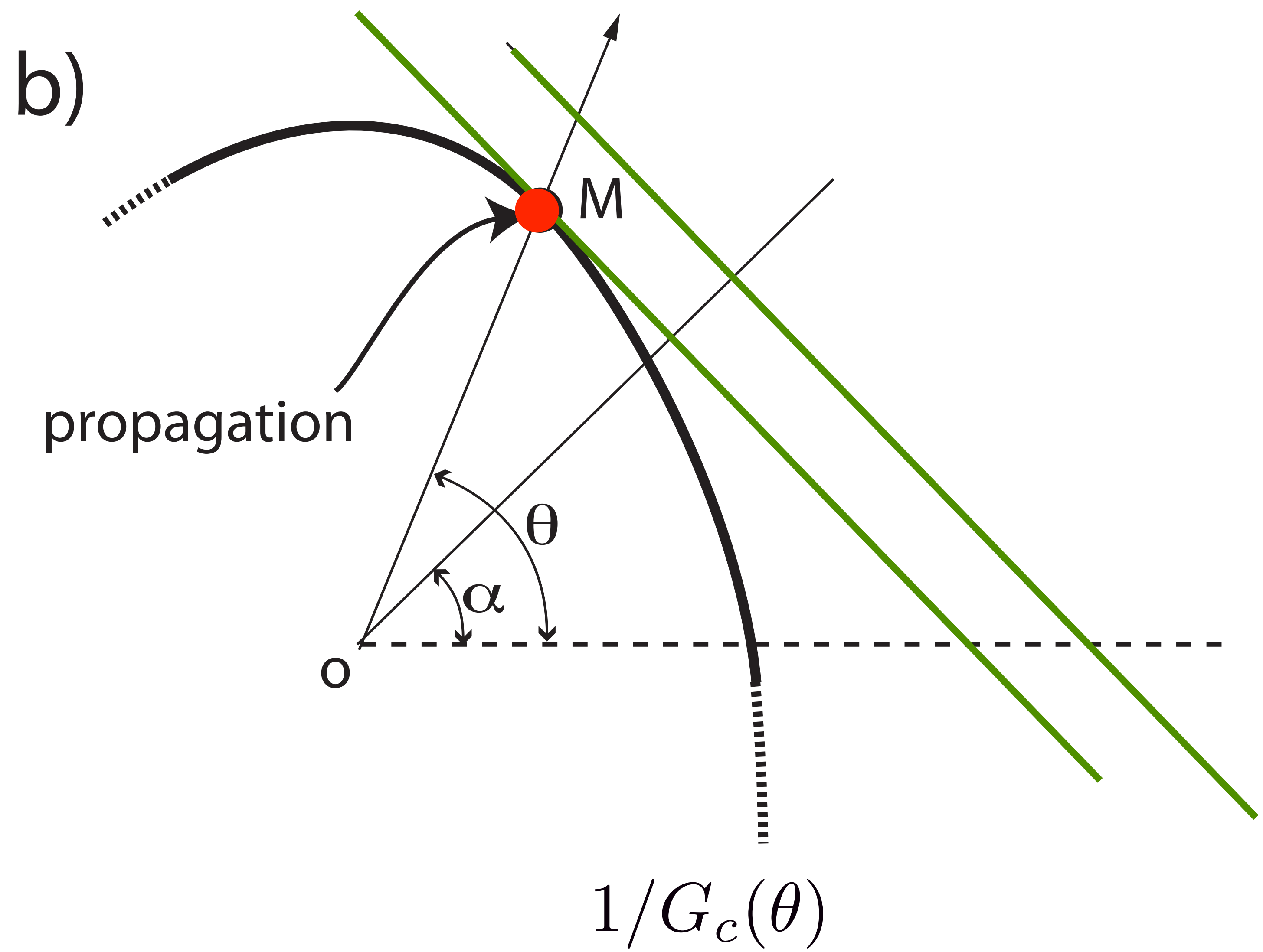
<u>specific to tearing</u> (not true for regular crack)

isotropic case



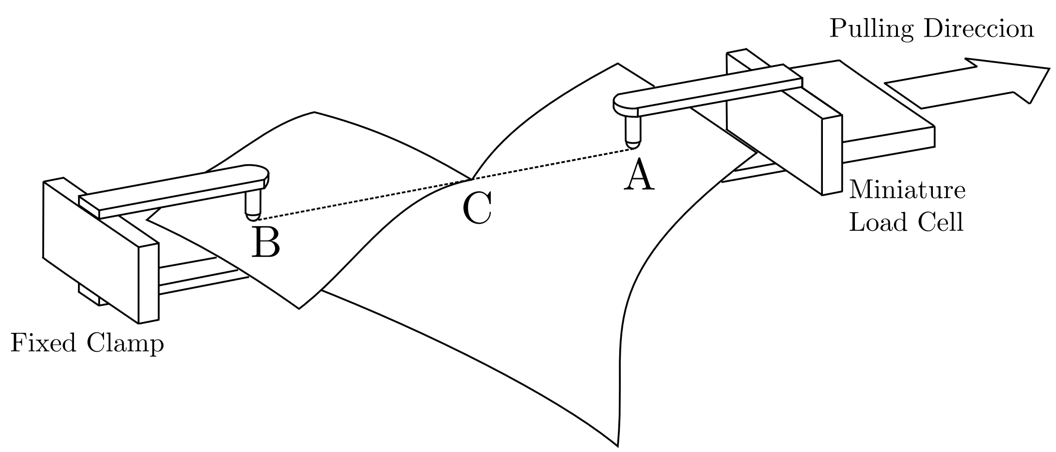
direction of energy release rate vector: α

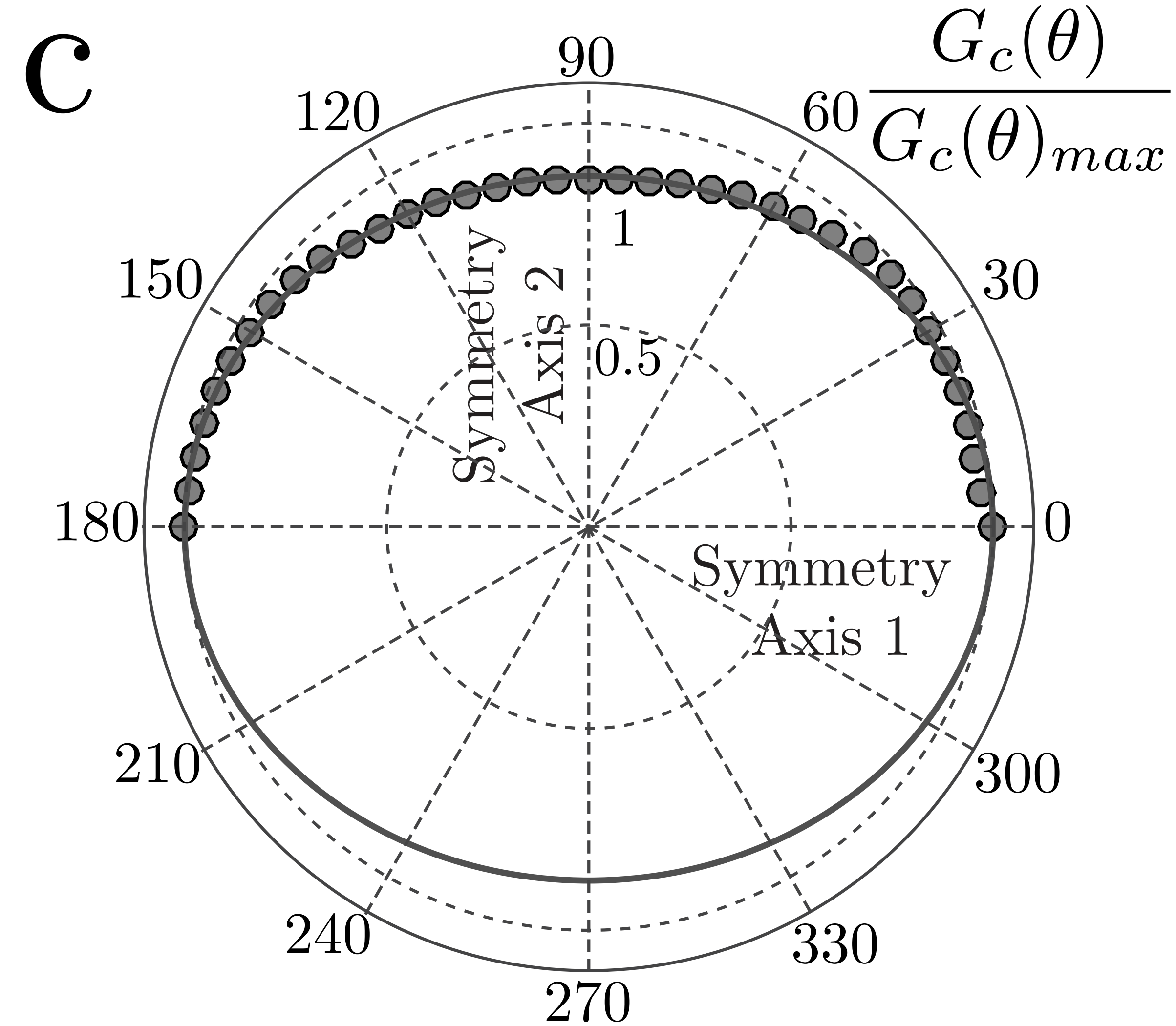
anisotropy



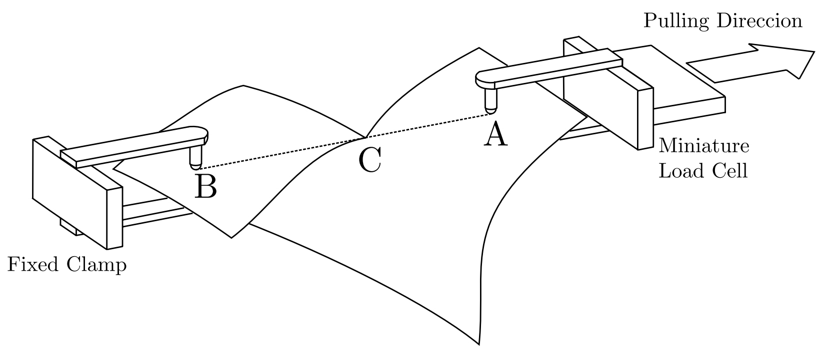
direction of energy release rate vector: α

Weakly anisotropic material

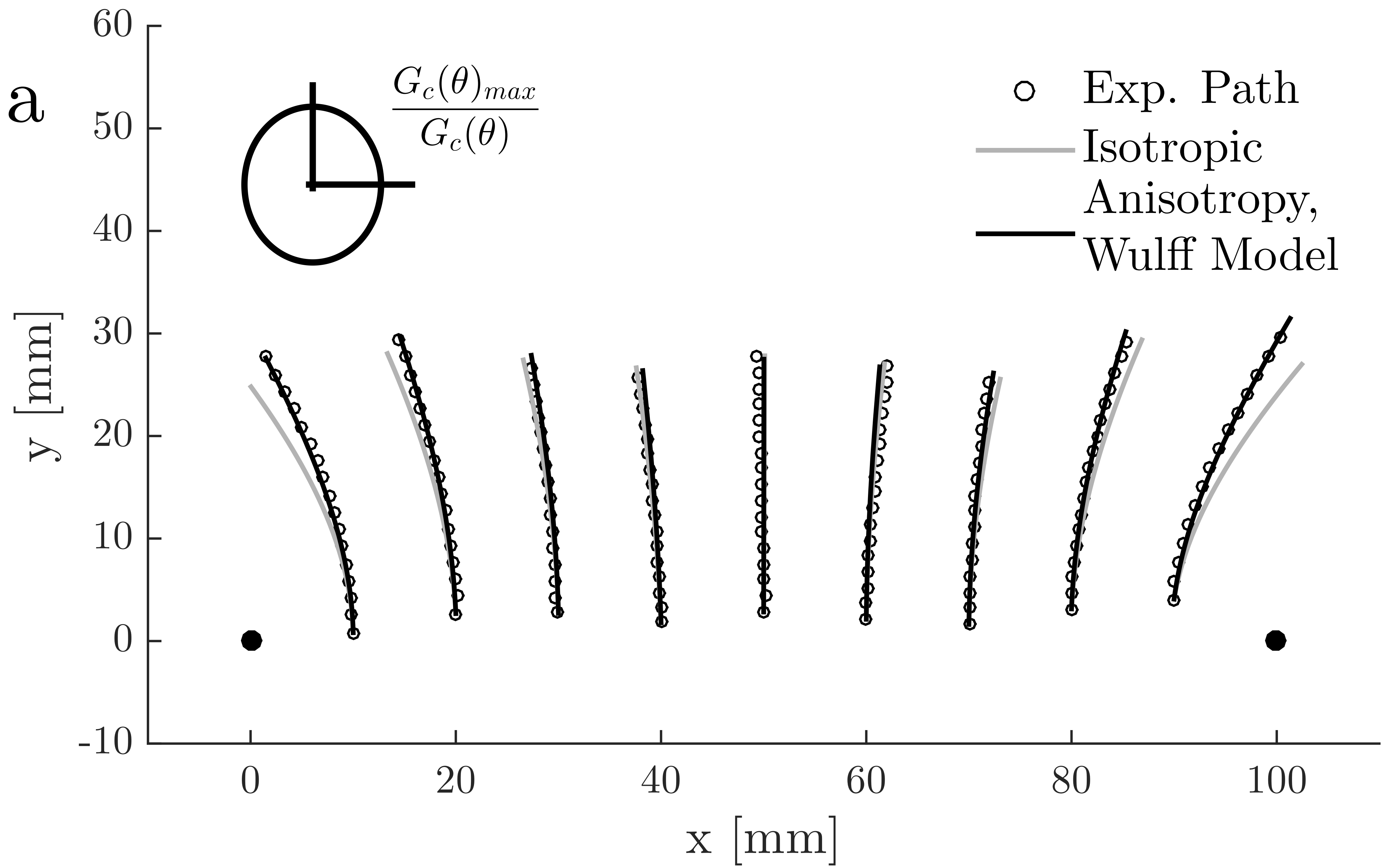




F.Melo, BR, A.Ibarra submitted



modified hyperbola



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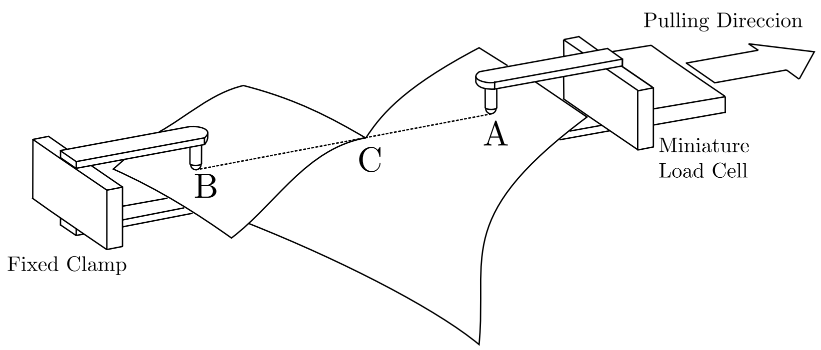
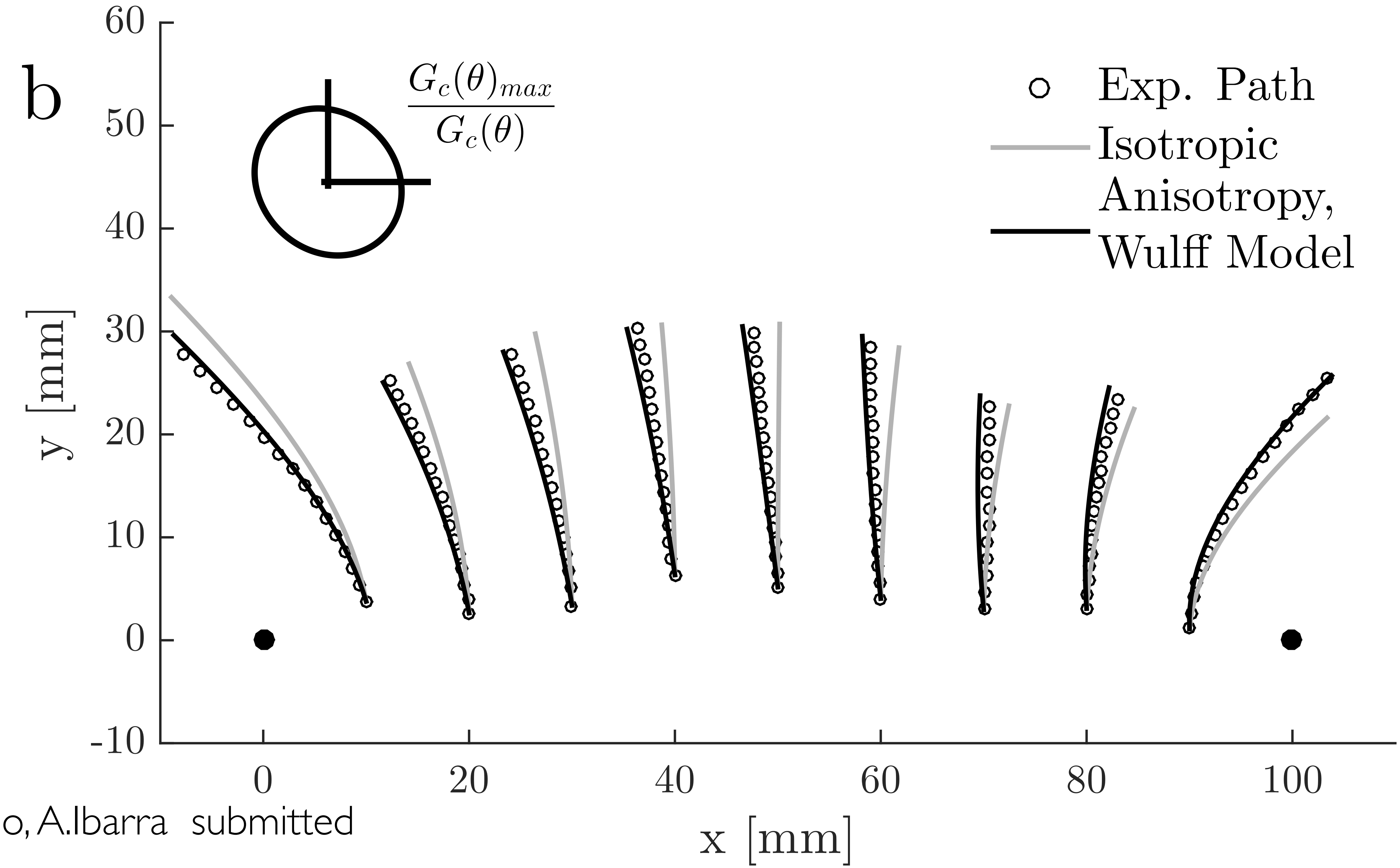
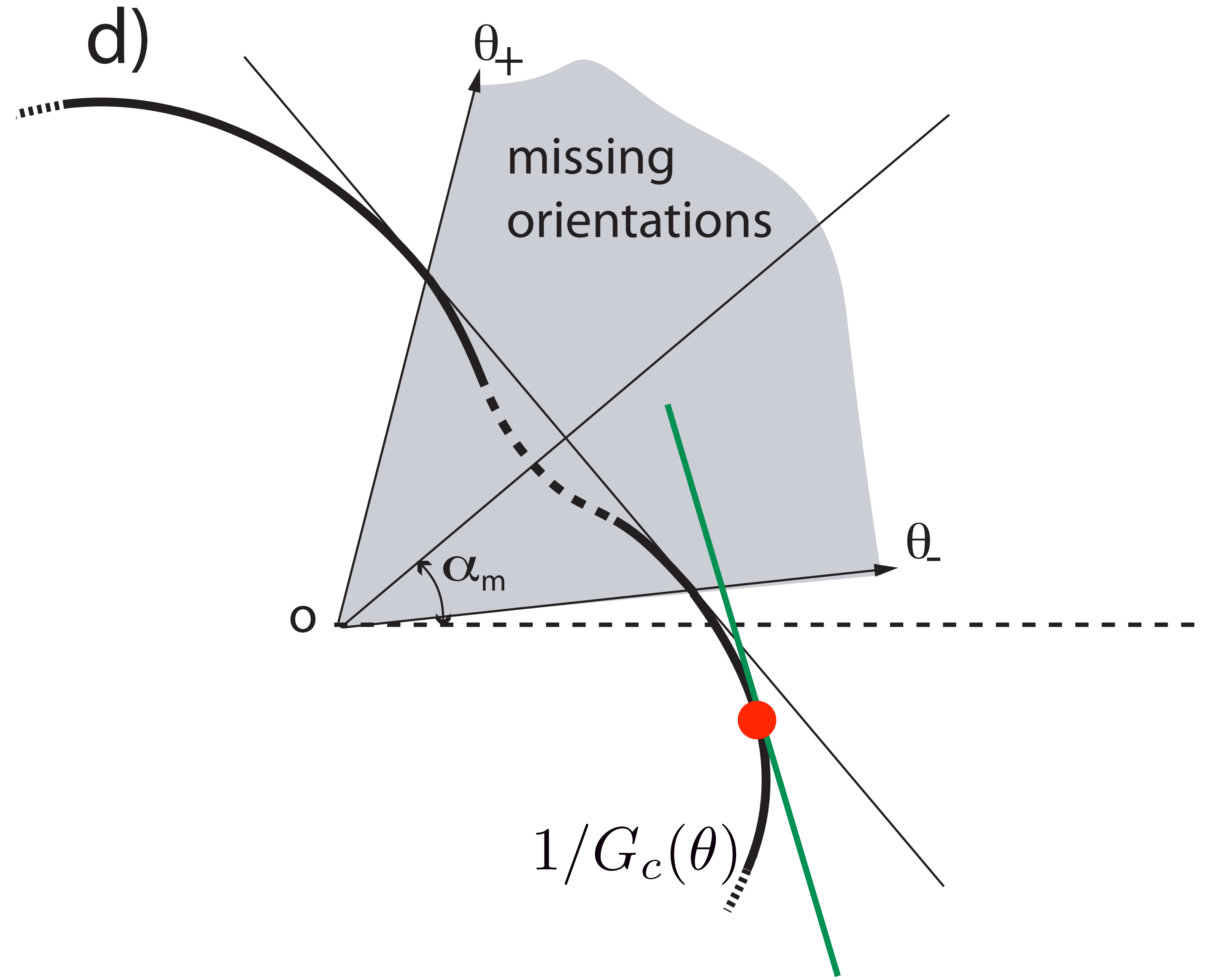


modified hyperbola

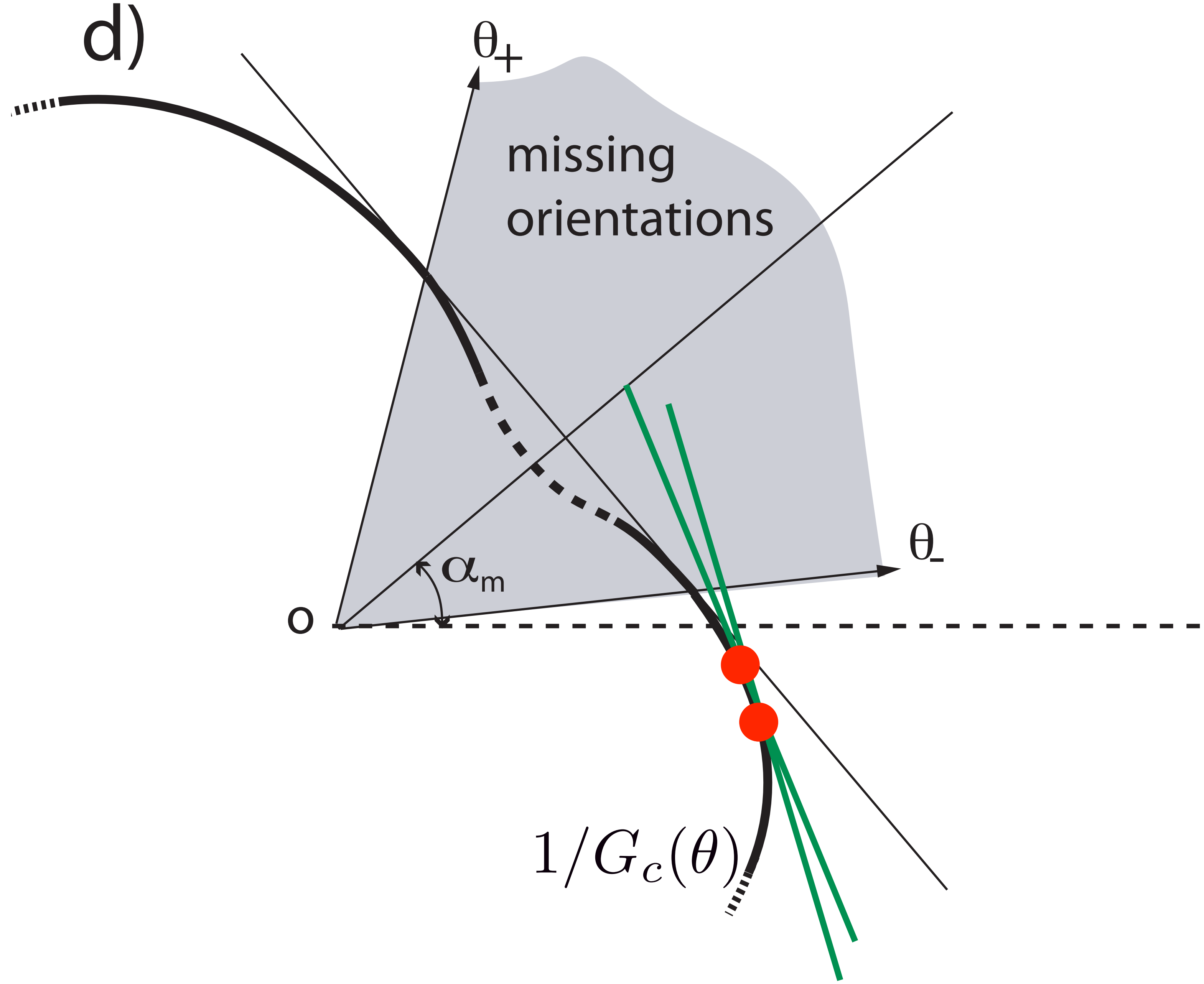


F.Melo, A.Ibarra submitted

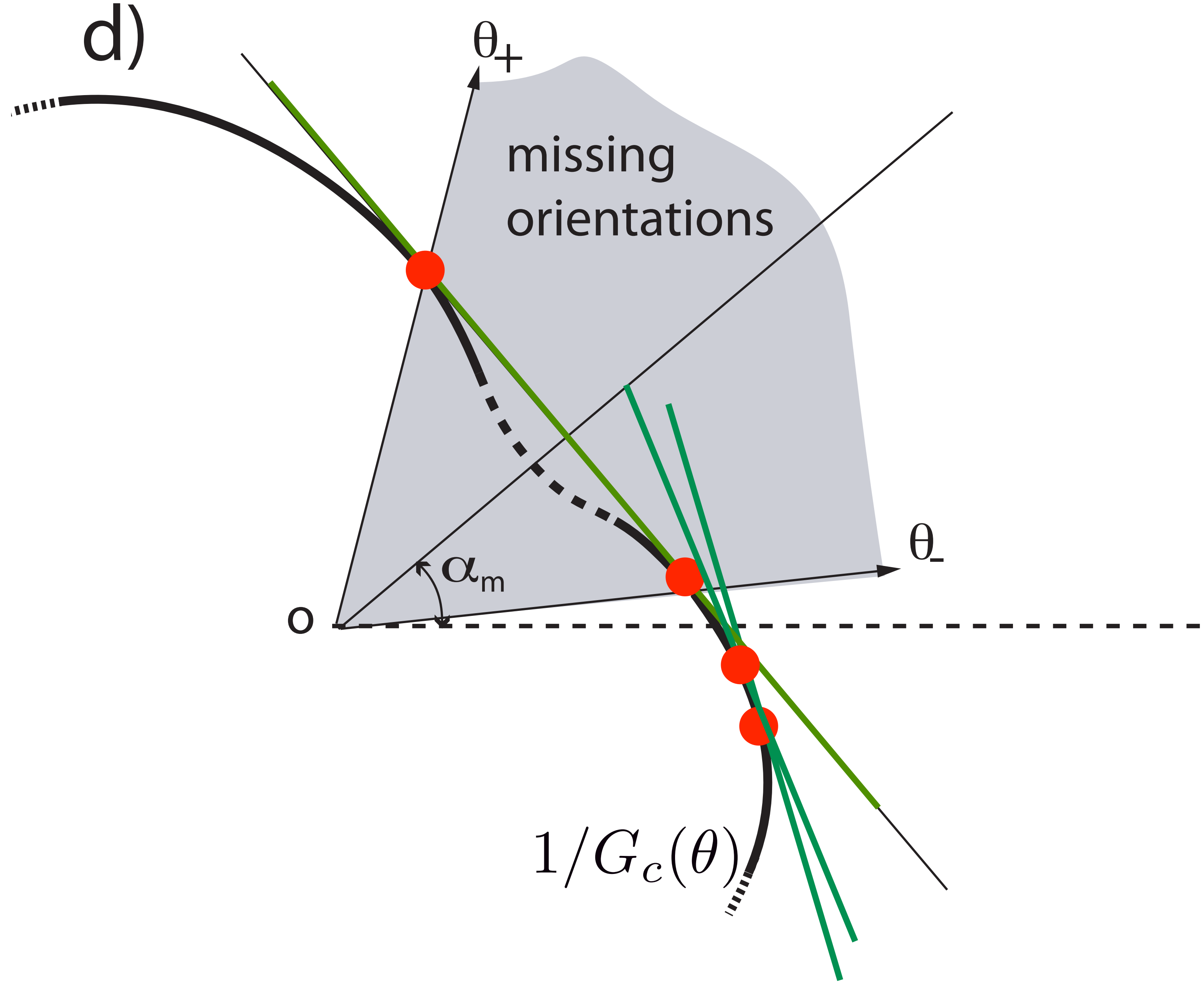
strongly anisotropic case



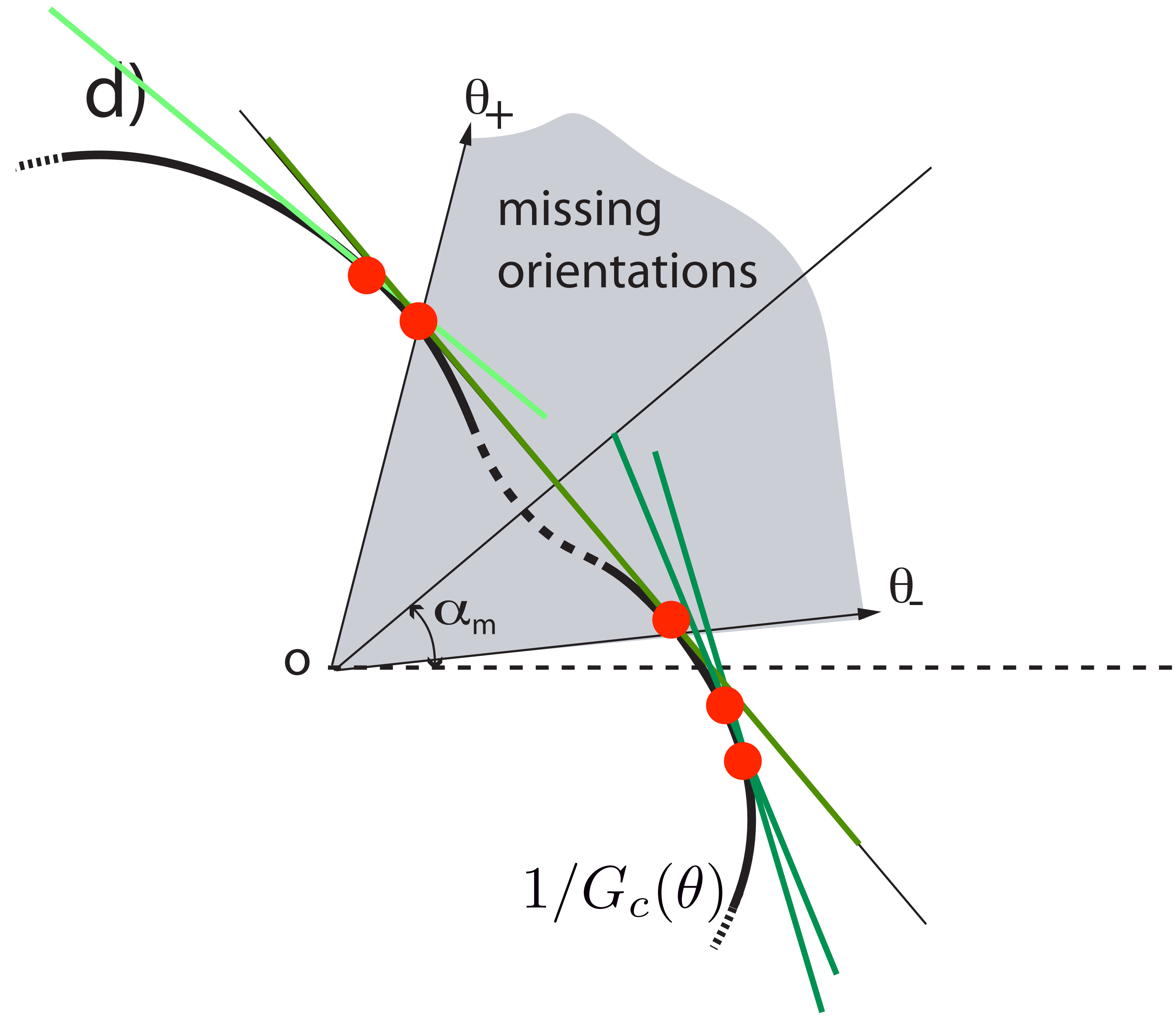
strongly anisotropic case



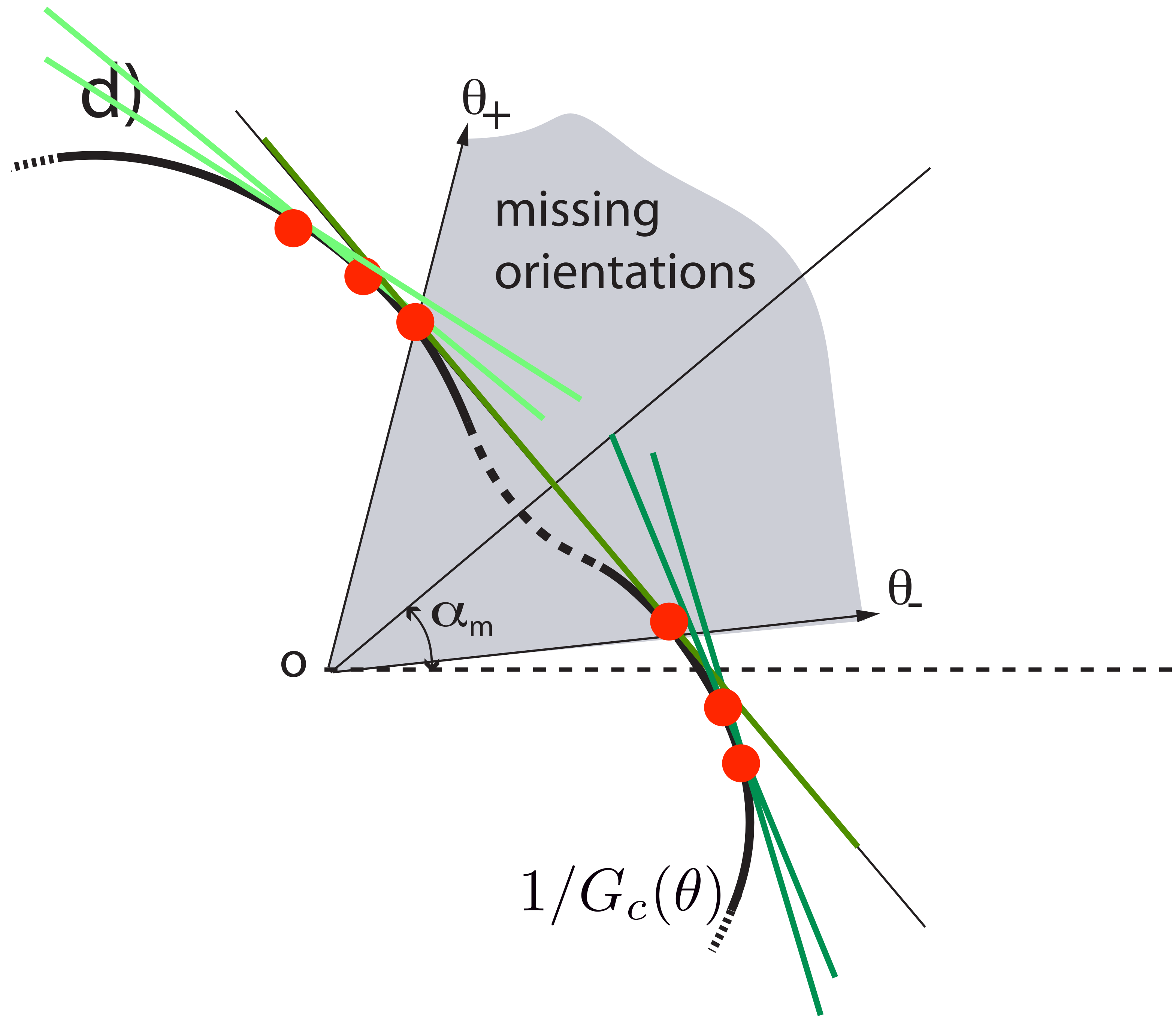
strongly anisotropic case



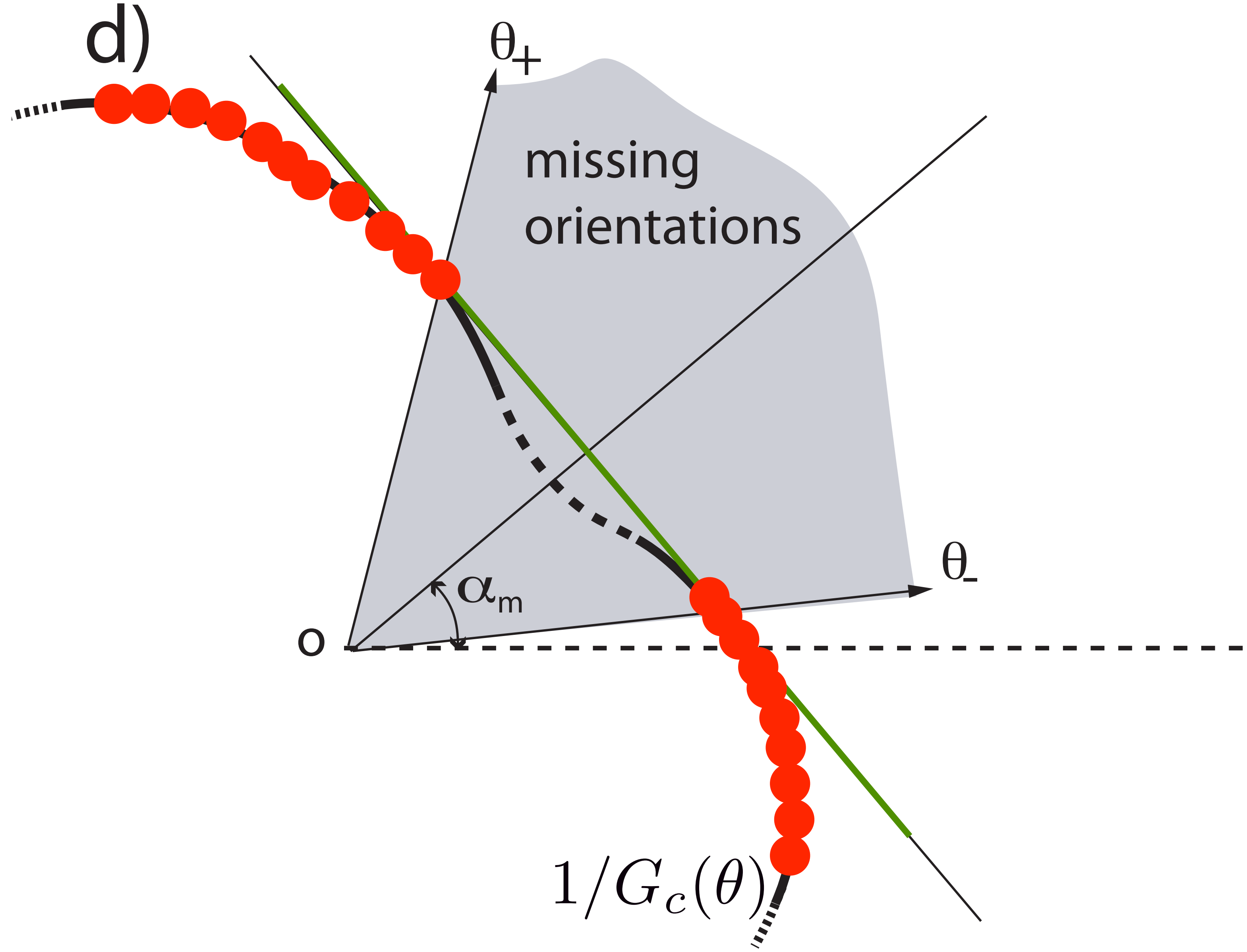
strongly anisotropic case



strongly anisotropic case



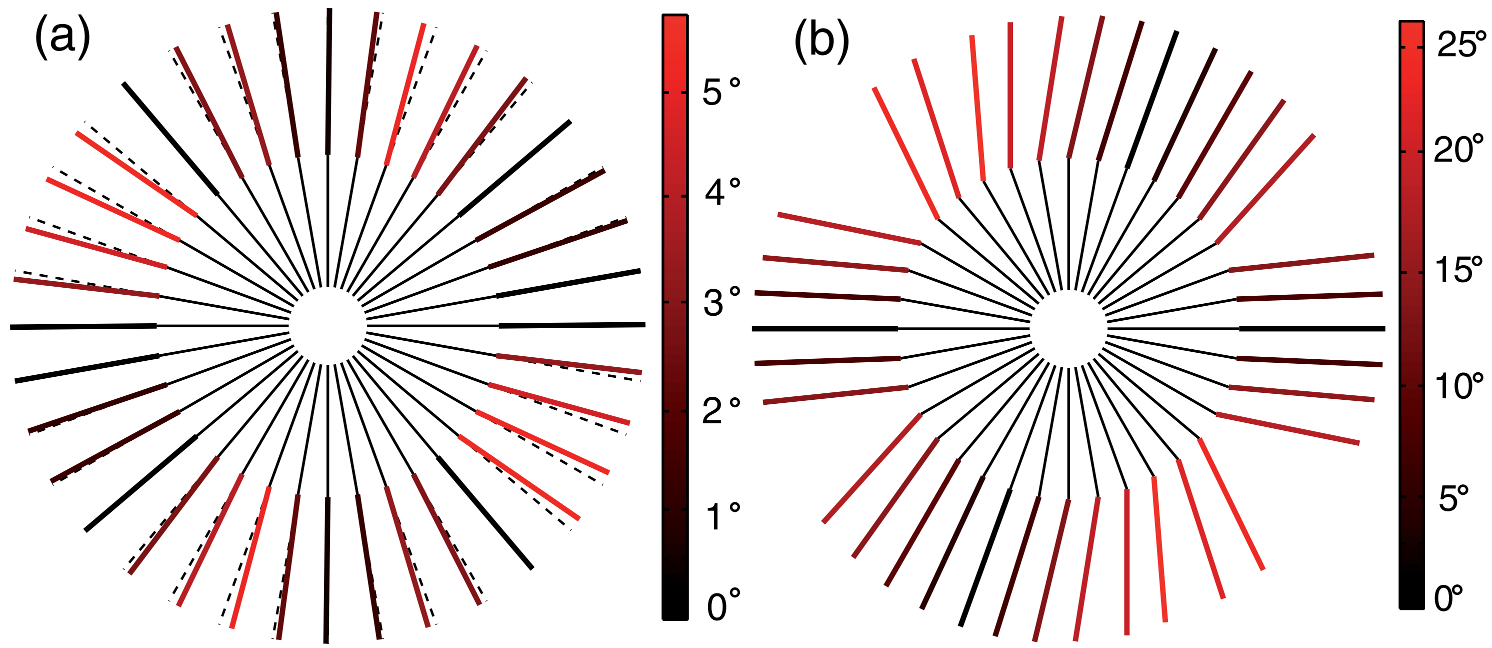
strongly anisotropic case



no propagation in sector $\theta_- < \theta < \theta_+$

non-convex curve = forbidden angles

Forbidden direction observed in tearing

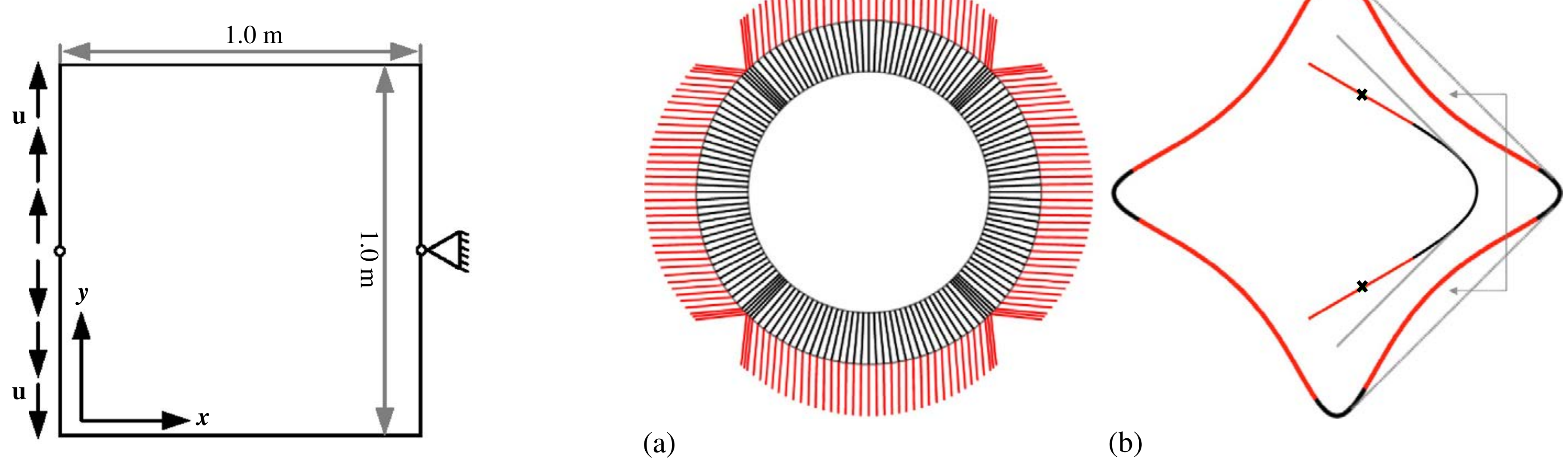


weak anisotropy *strong anisotropy*

A.Takei et al (2013)

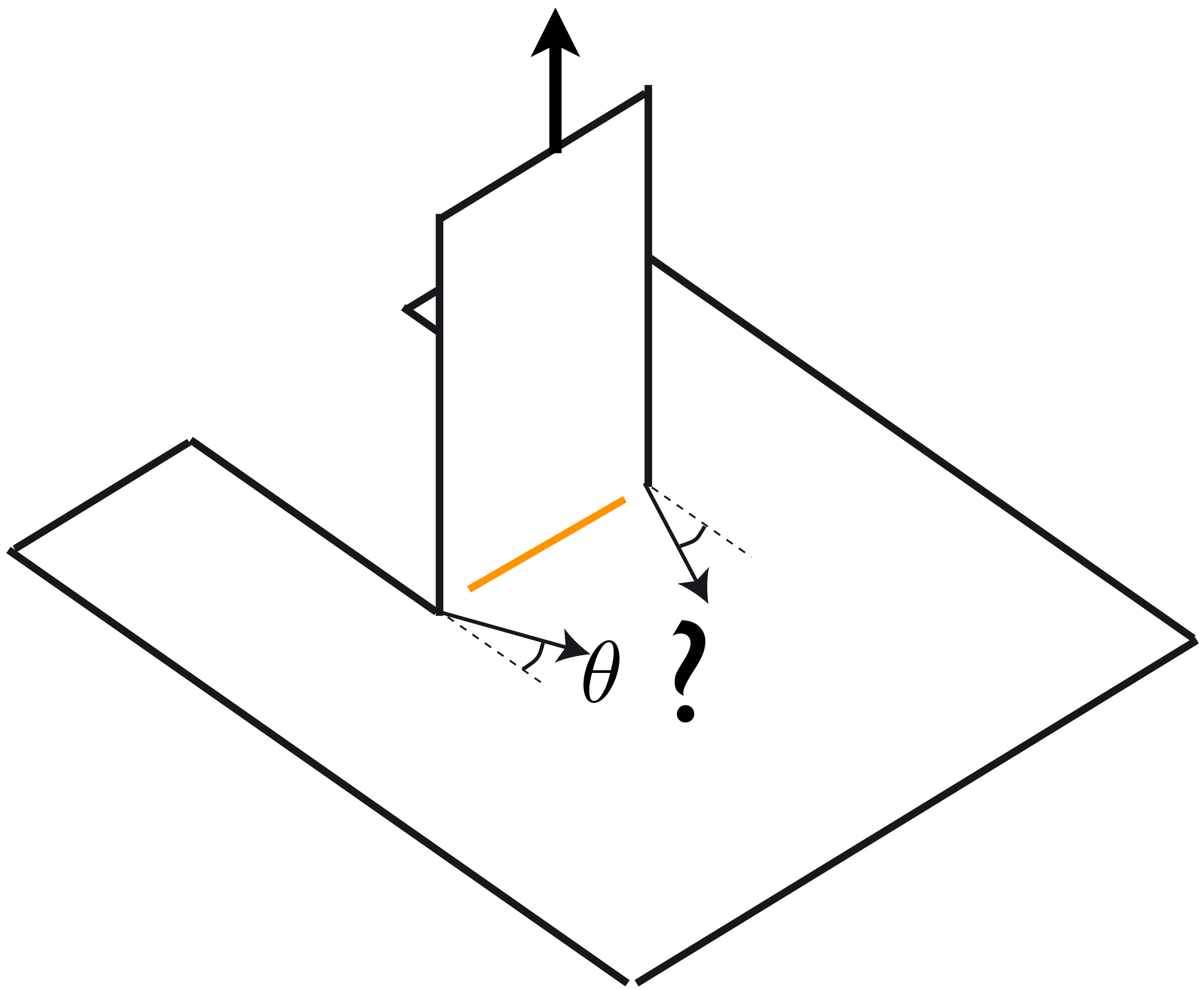
Phase-field approach (in-plane fracture)

symmetric opening mode

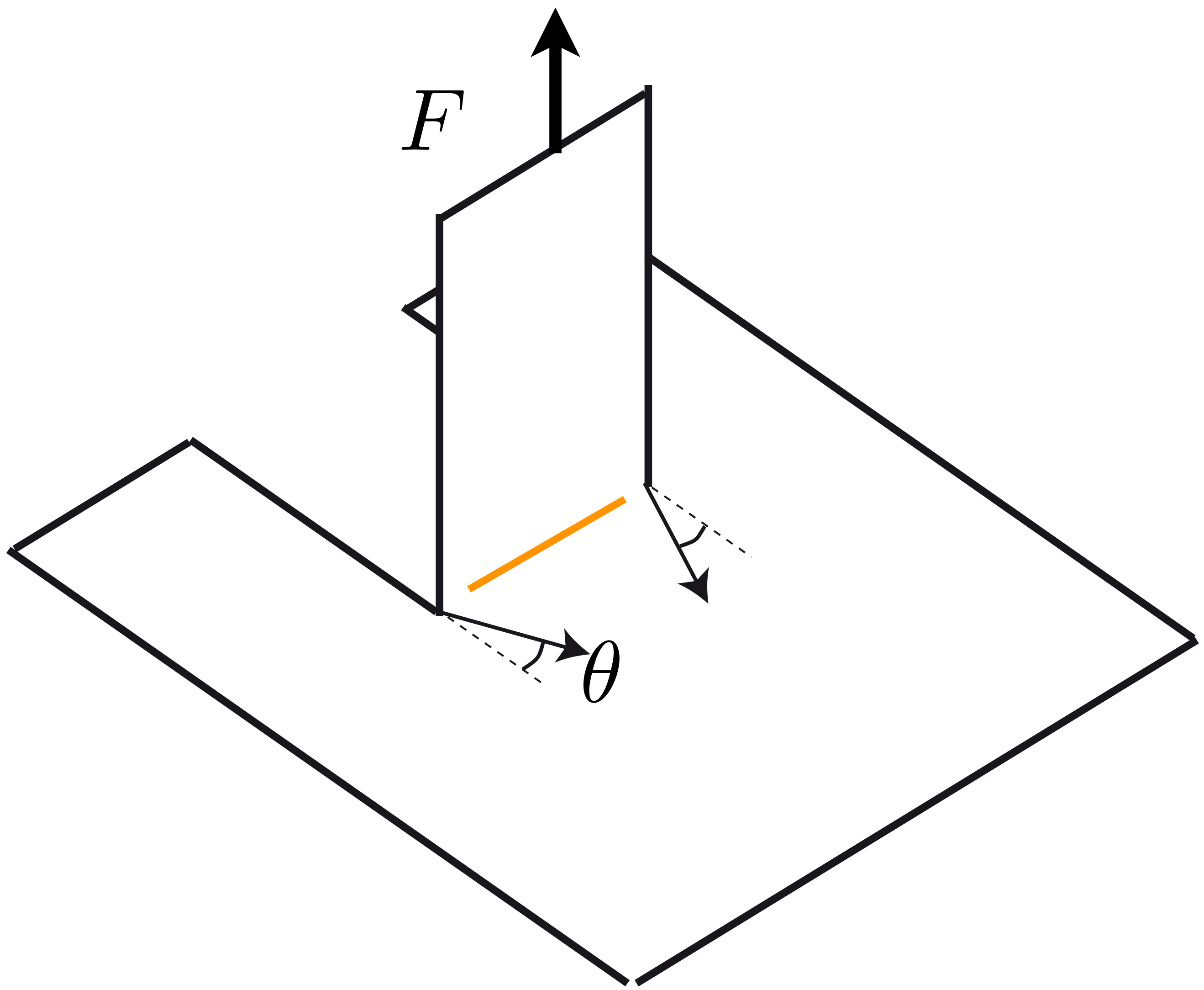


B.Li, et al(2015)

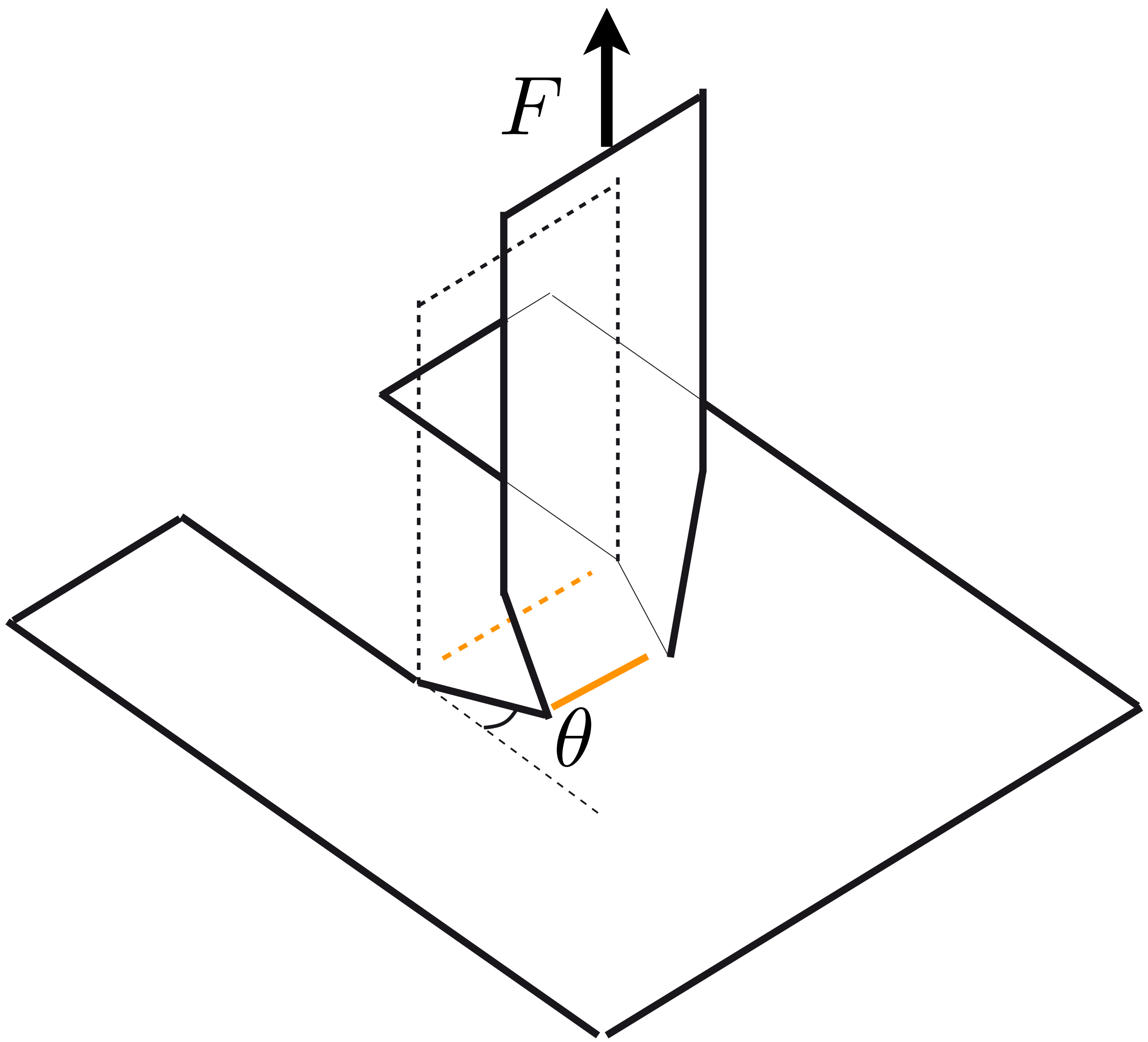
Does the construction apply in this case ?

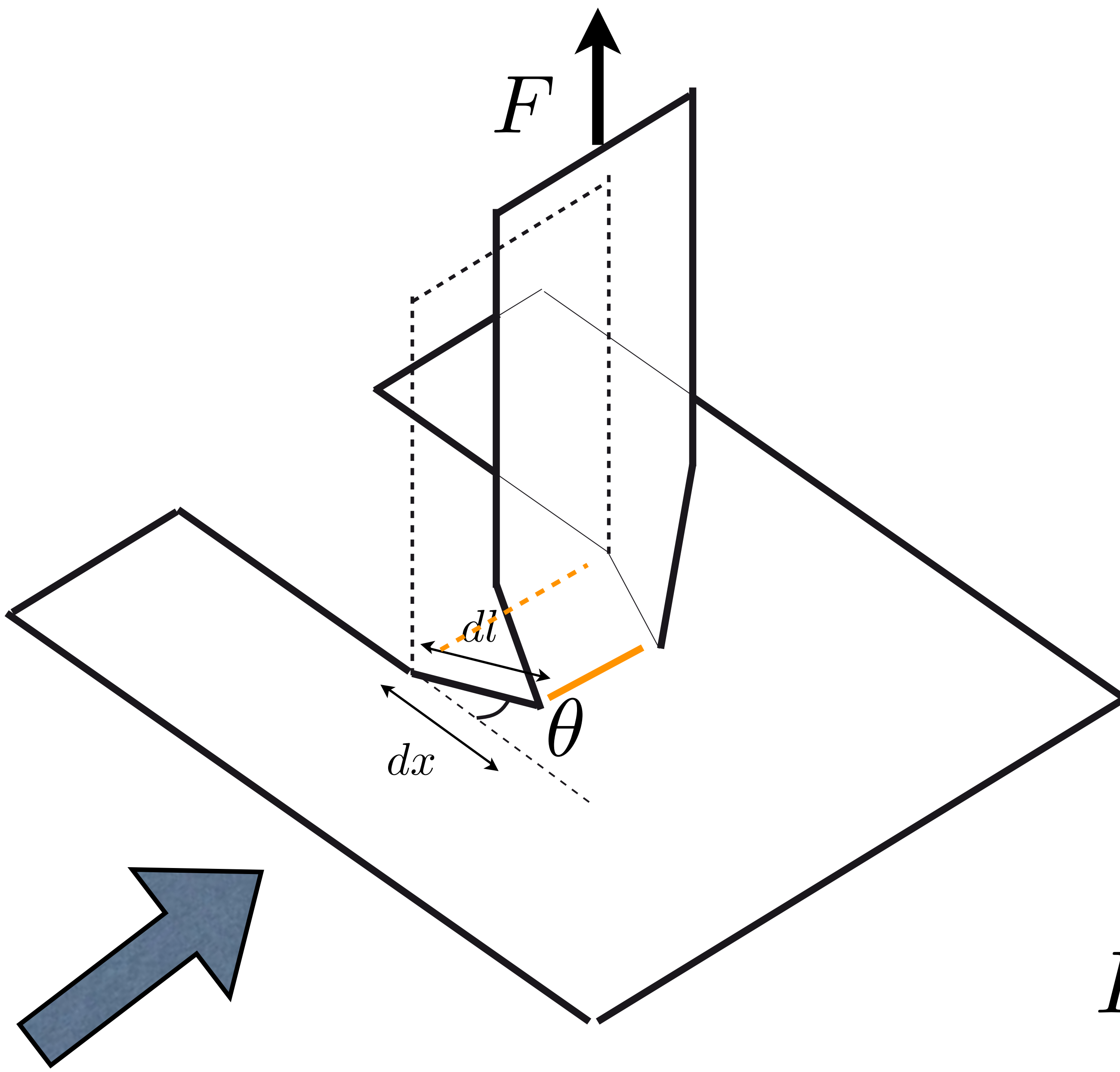


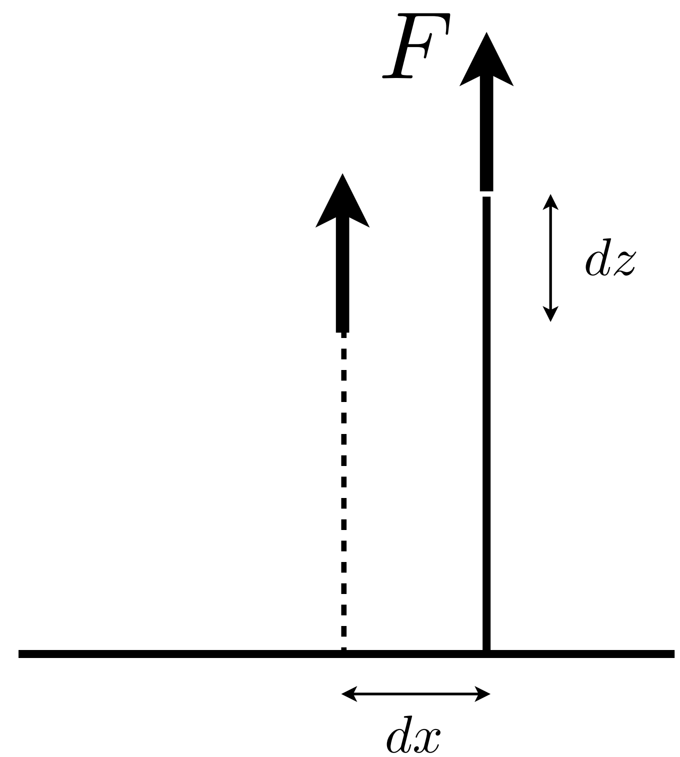
direction of propagation?



suppose propagation in direction θ


F
θ

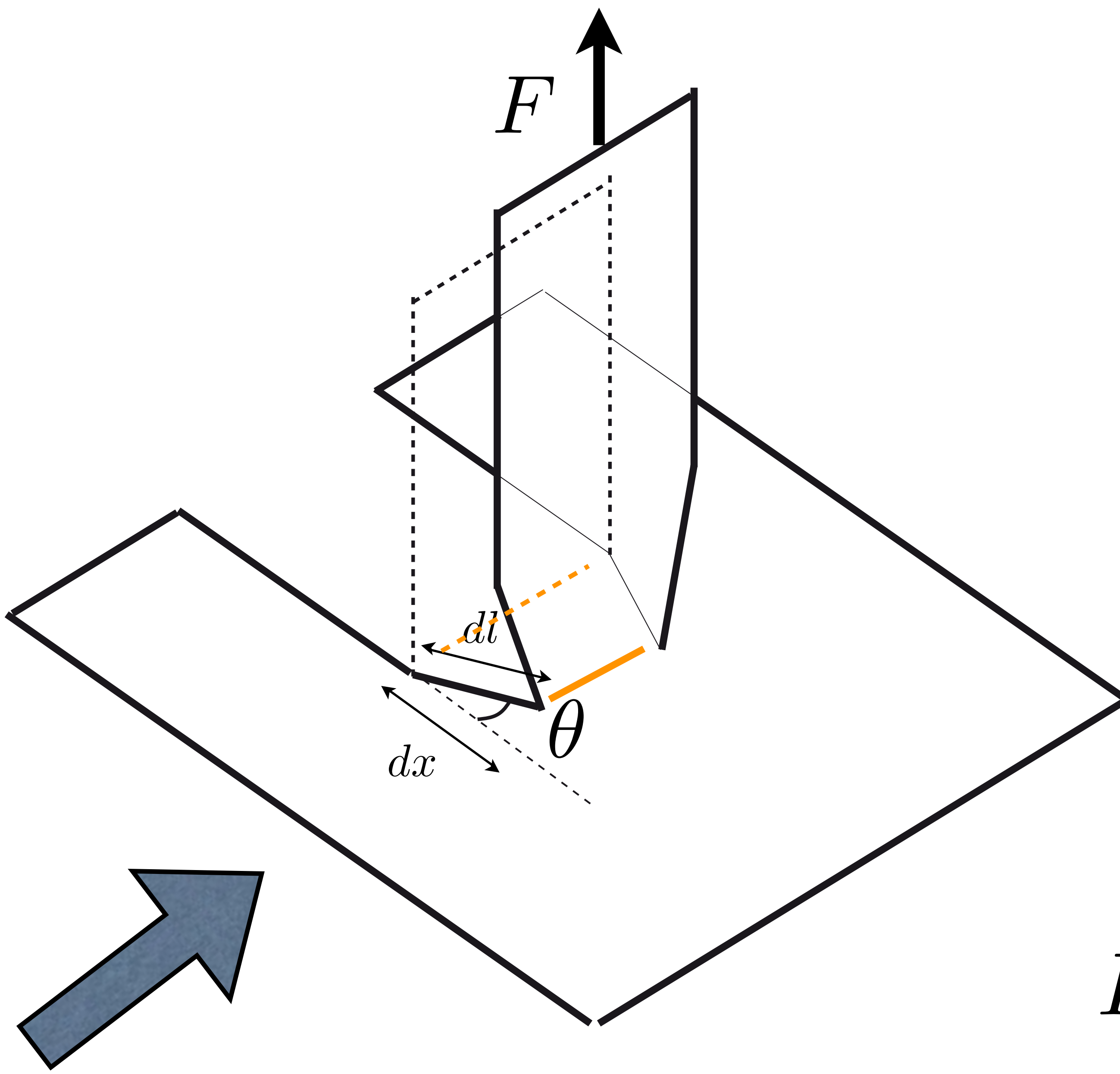


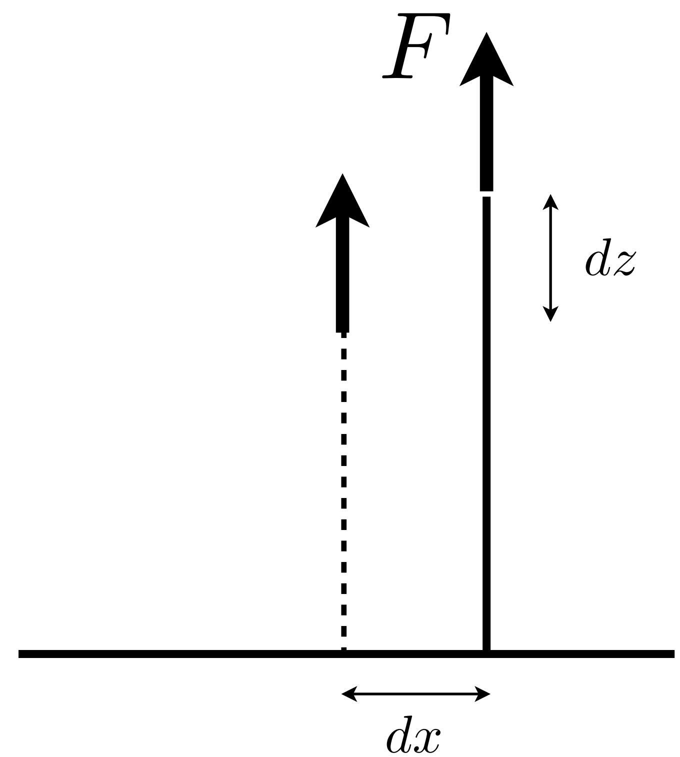


$$dx = dz = dl\cos\theta$$

propagation if:

$$F dz = dE_{el} + 2G_c t dl$$



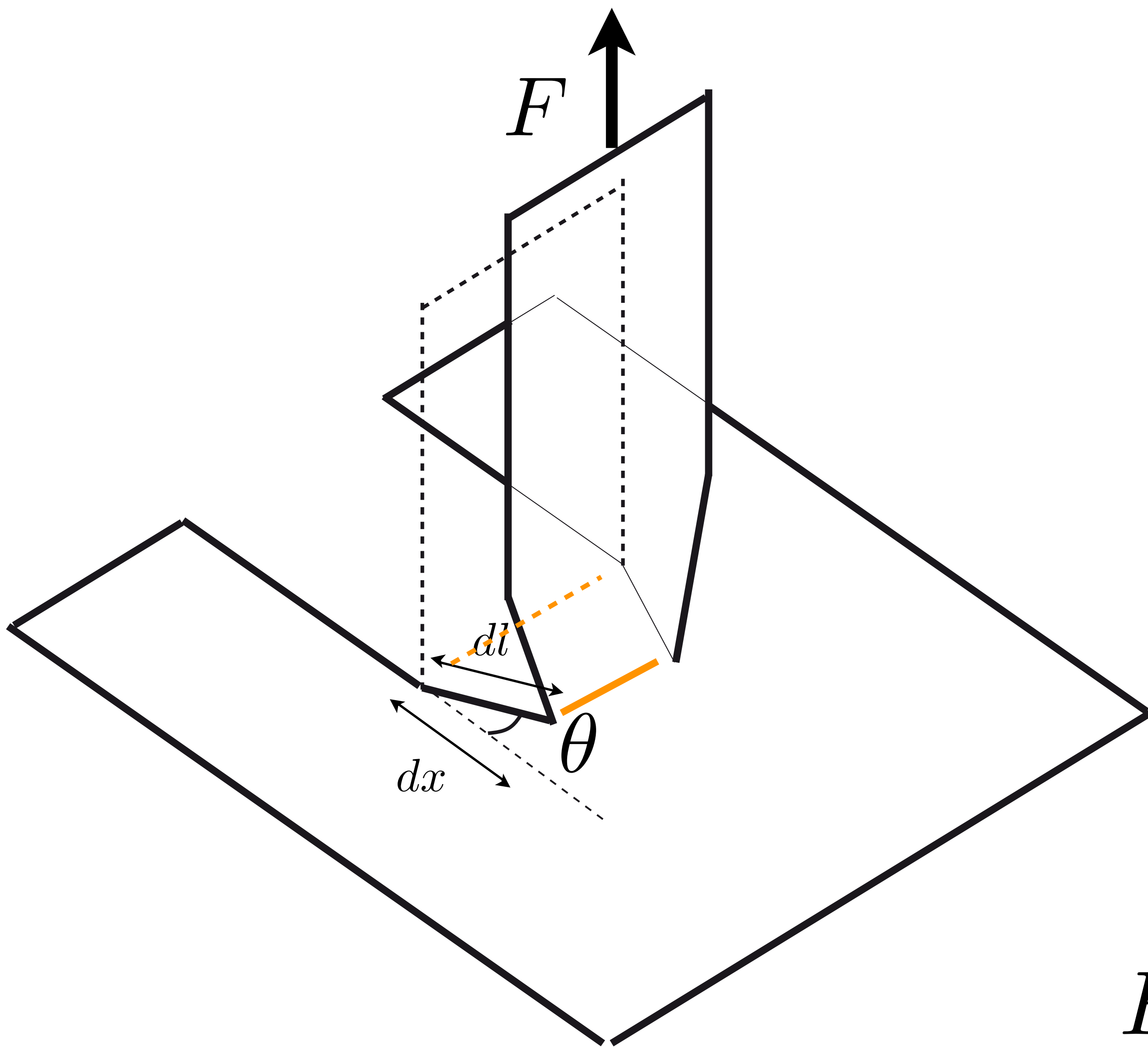


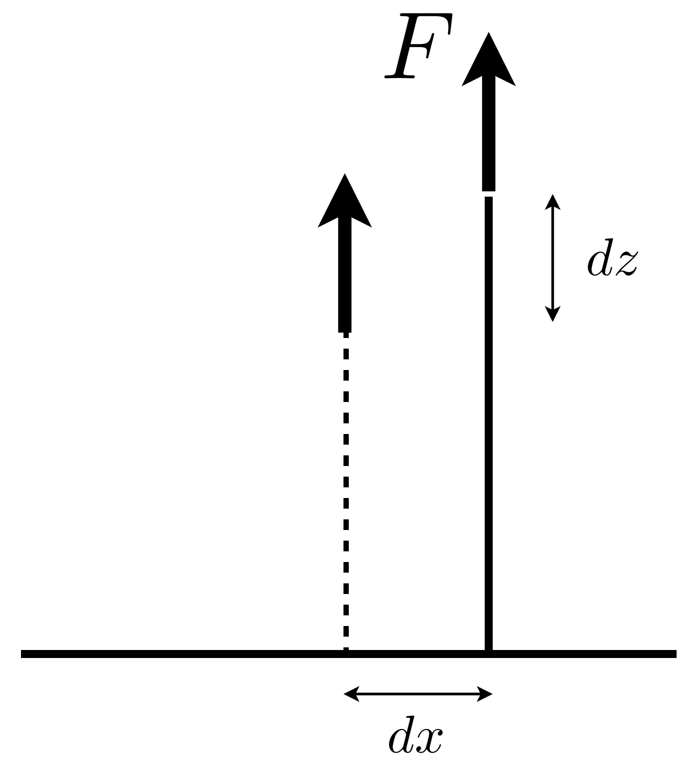
$$dx = dz = dl\cos\theta$$

propagation if:

$$Fdz = d\cancel{E}_{el} + 2G_ctdl$$

inextensible fabric : infinitely easy to bend, inextensible



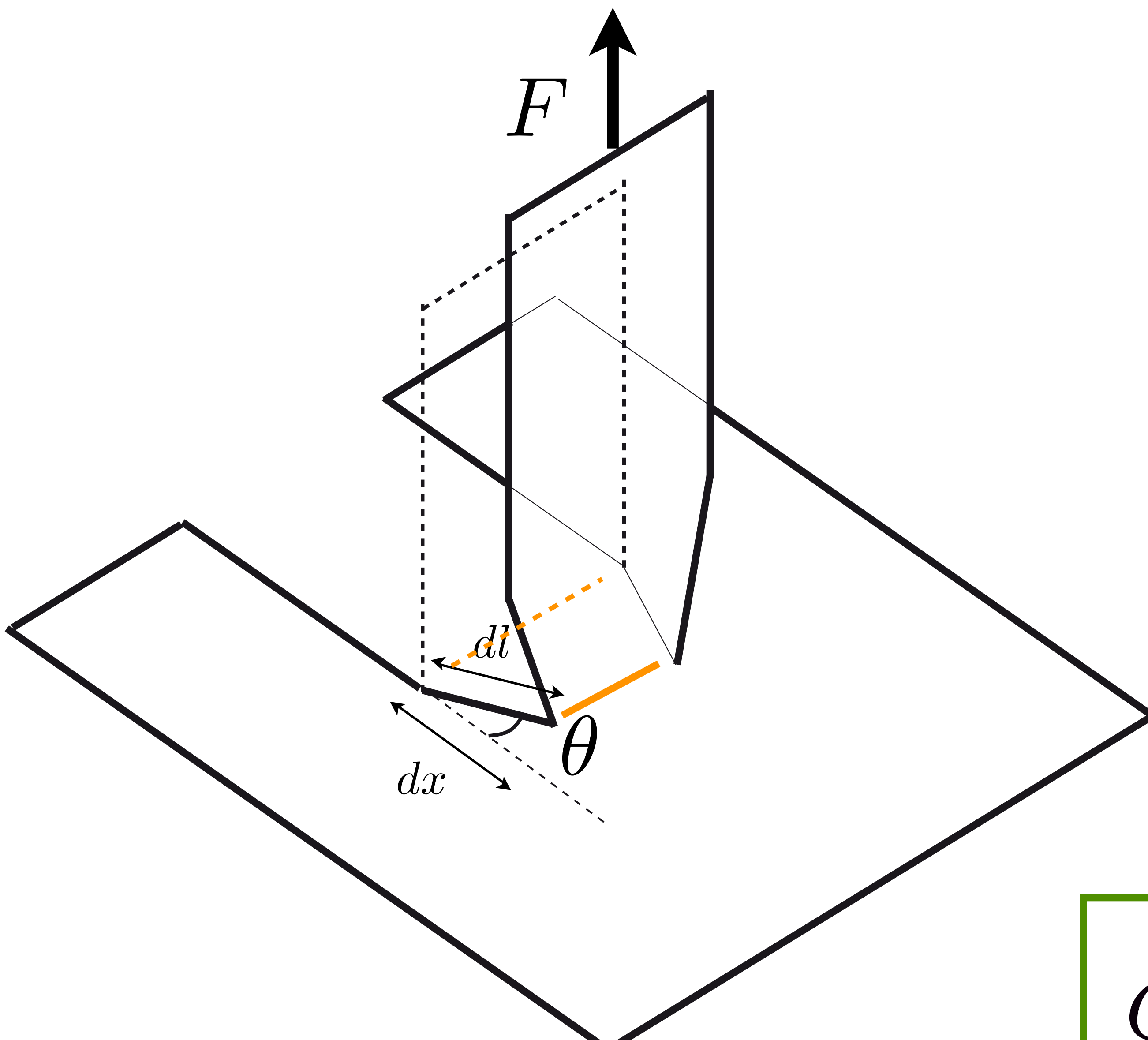


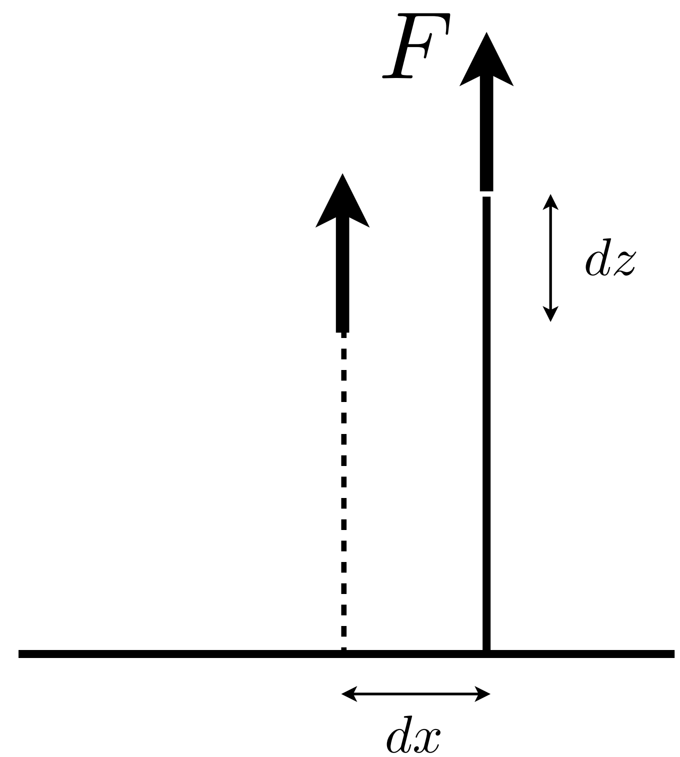
$$dx = dz = dl\cos\theta$$

propagation if:

$$\frac{F\,dz}{2tdl} =$$

$$\frac{2G_c t dl}{2tdl}$$





$$dx = dz = dl \cos\theta$$

propagation if:

$$\boxed{G(\theta) = \frac{F}{2t}\cos(\theta)} \quad = \boxed{G_c}$$

Energy Release Rate

Fracture energy

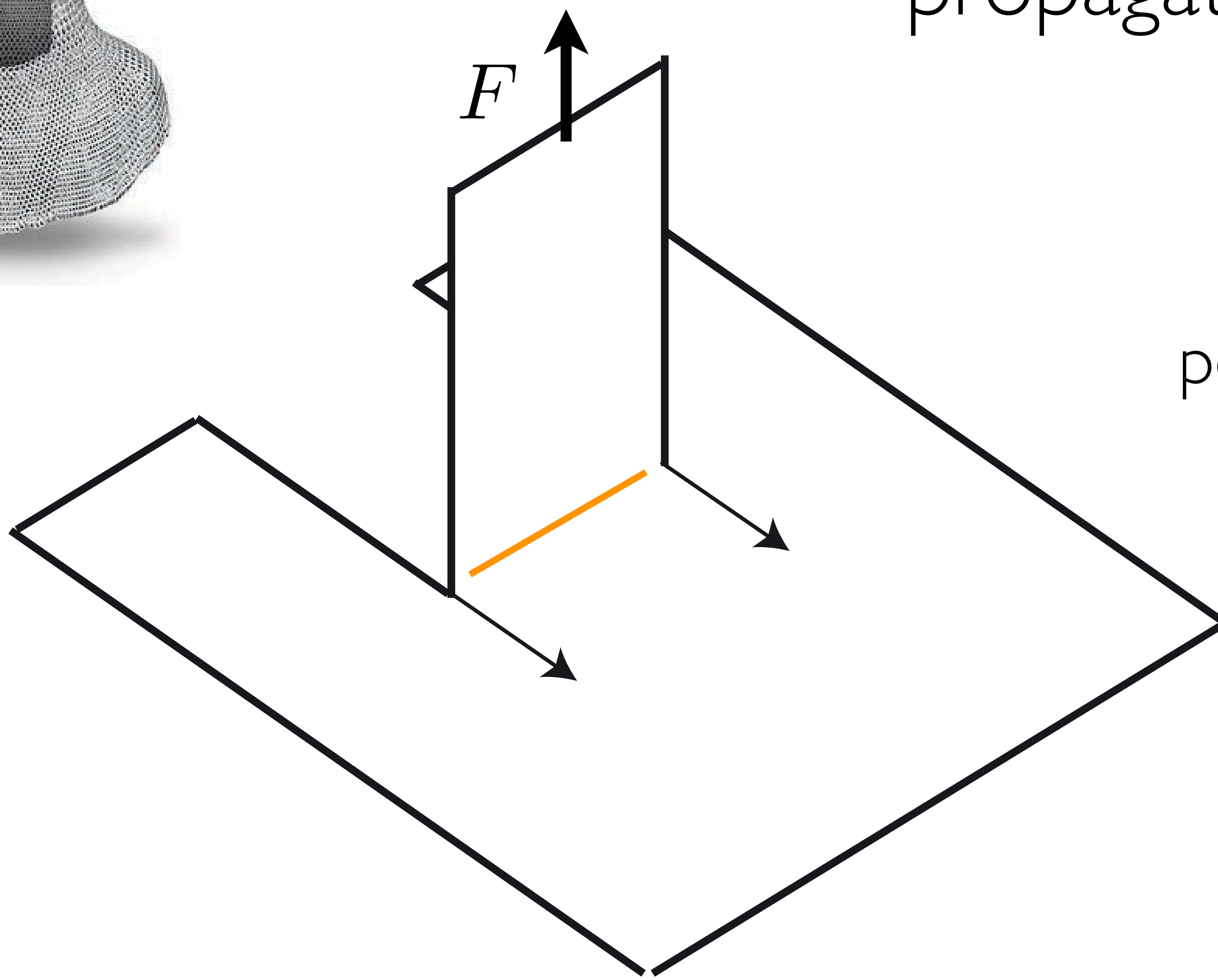
independent of previous path
independent of material property

Robust path!

propagation

$$F = 2G_c t$$

$$\theta = 0$$



perpendicular to fold

optimal direction to extract operator's work

1th order approximation:

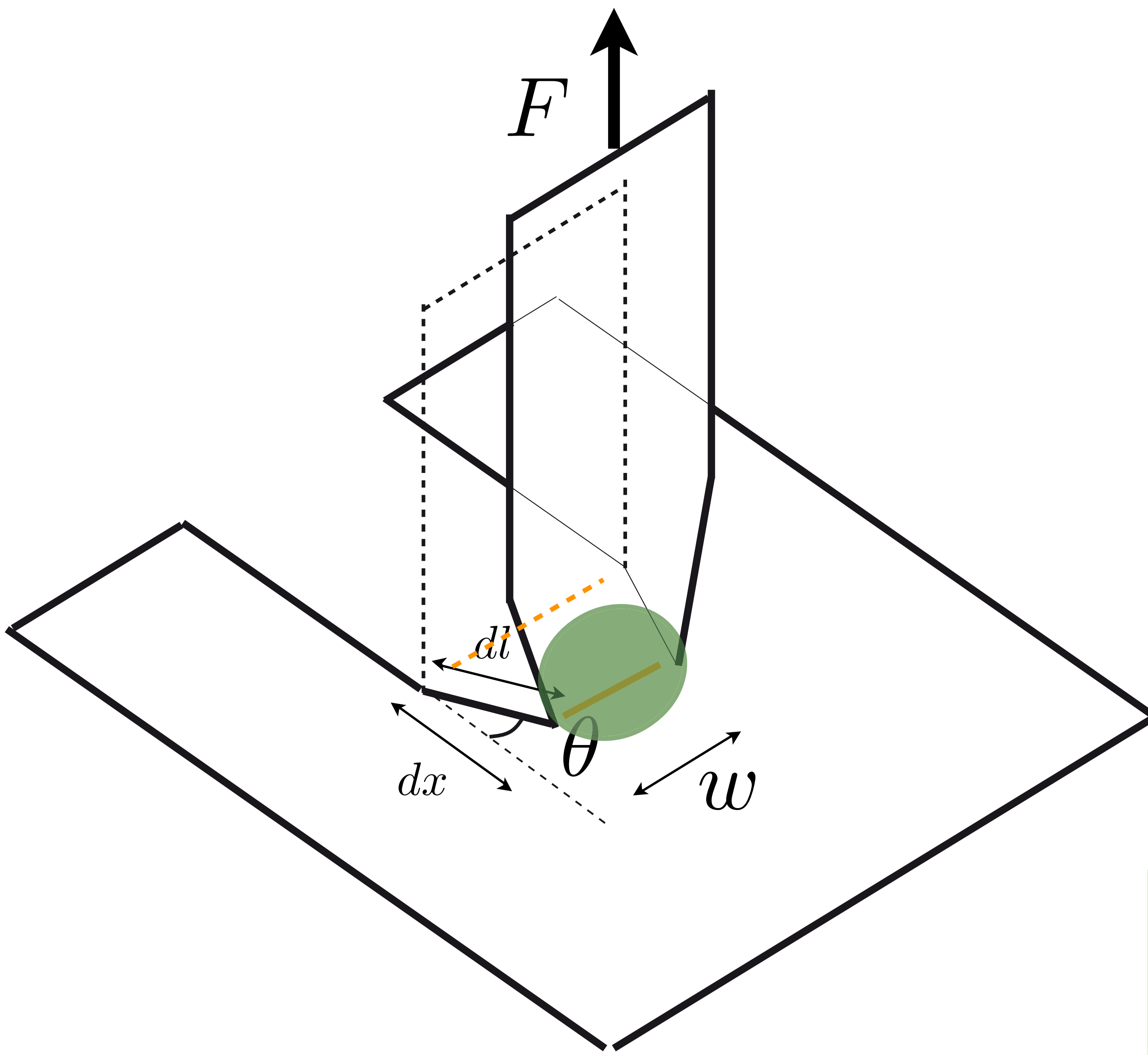
The sheet is

- Inextensible
- ~~infinitely bendable~~

include bending energy

$$Fdz = dE_{el} + 2G_c t dl$$

a new source for energy release rate

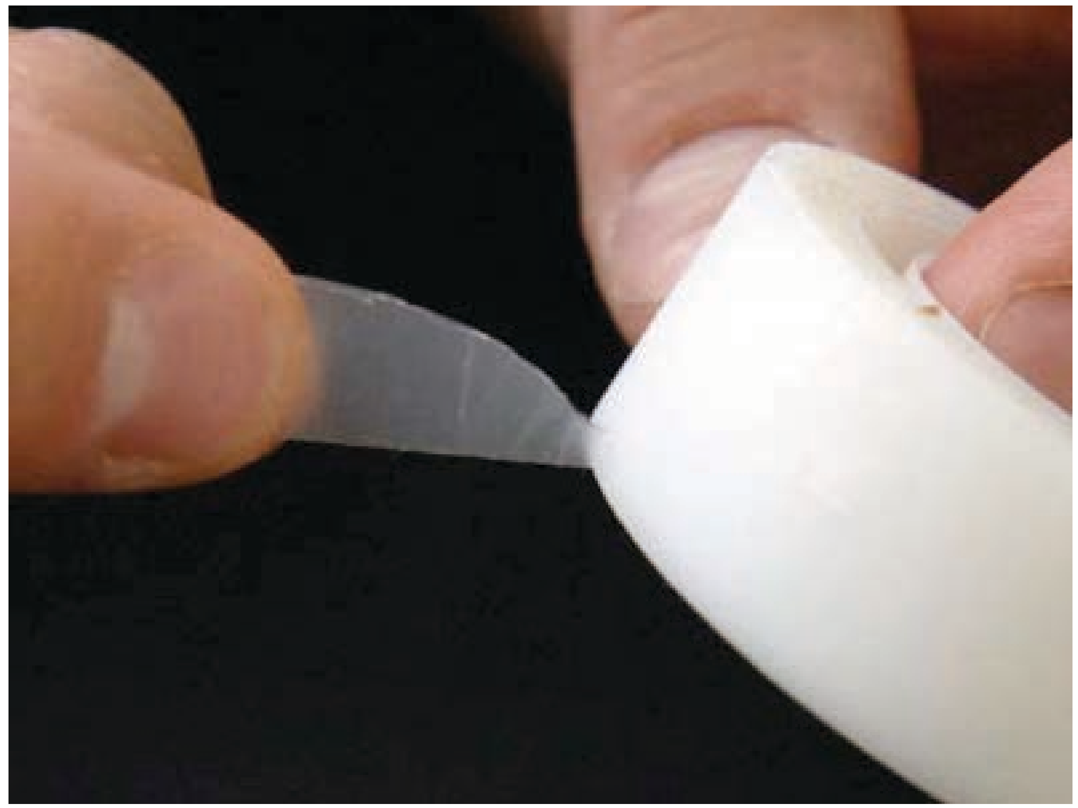


propagation if:

$$\underbrace{\boxed{F\frac{dz}{dl} - \frac{dE_{el}}{dl}}}_{\text{Energy Release}} = \underbrace{\boxed{2G_c t}}_{\text{Fracture energy}}$$

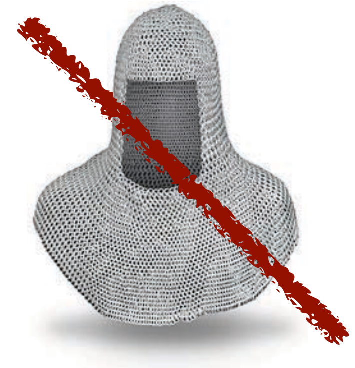
Maximum Energy Release Rate $\longrightarrow$ **always inward !**

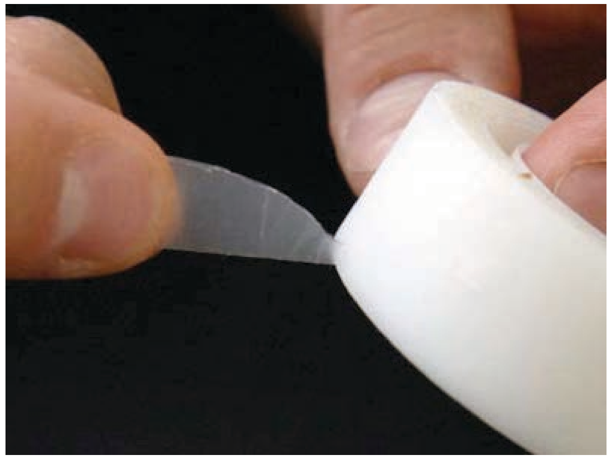
quantitative prediction : strong adhesion case



D.Coveney

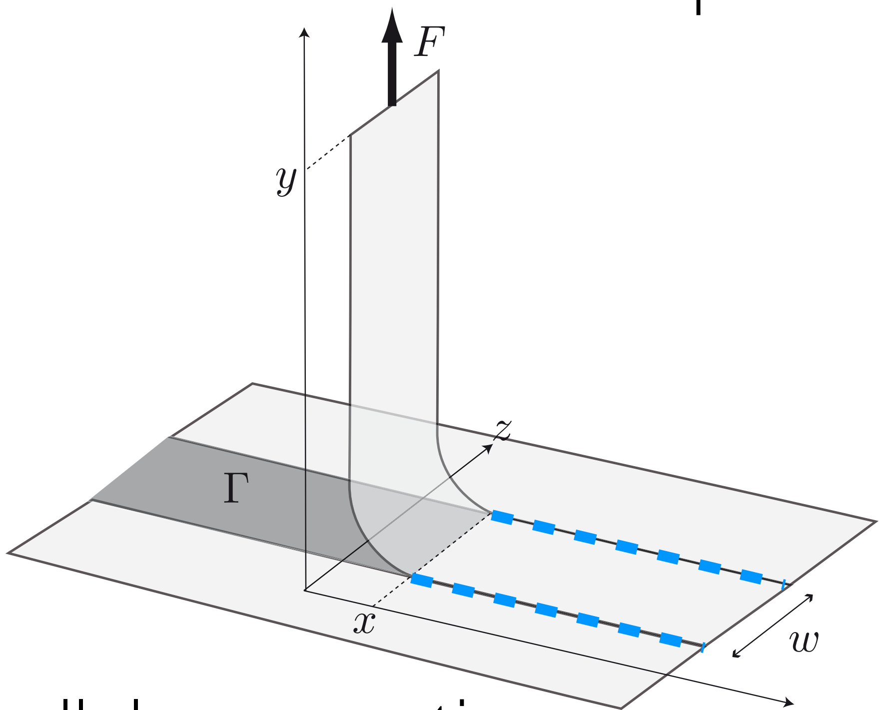
quantitative prediction : strong adhesion case



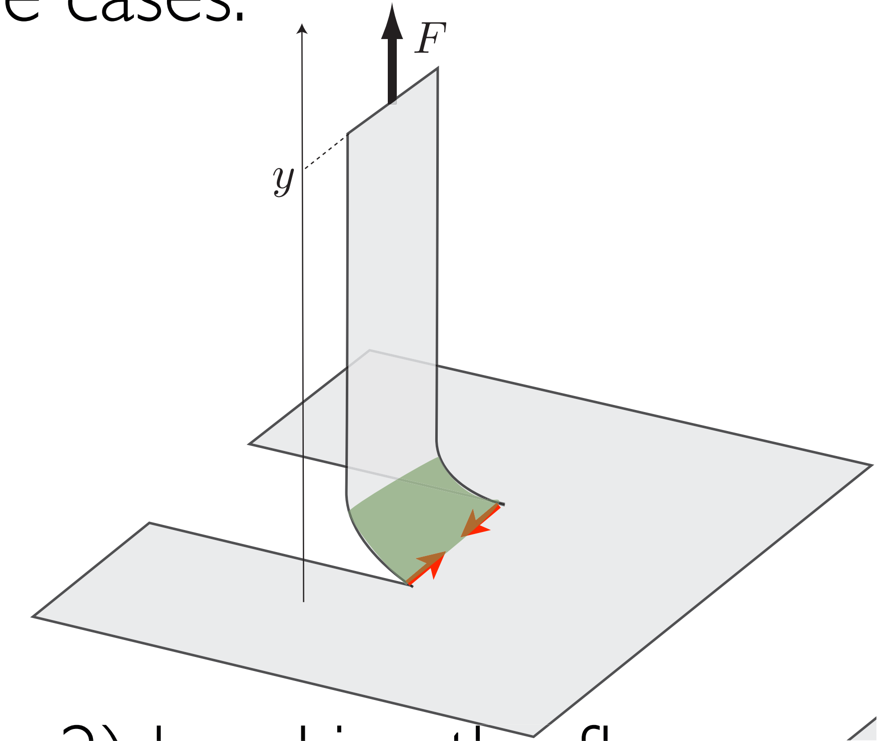


D.Coveney

Compare 2 extreme cases:

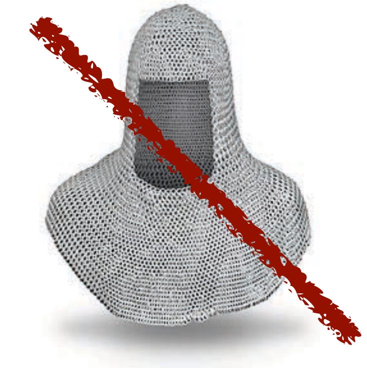


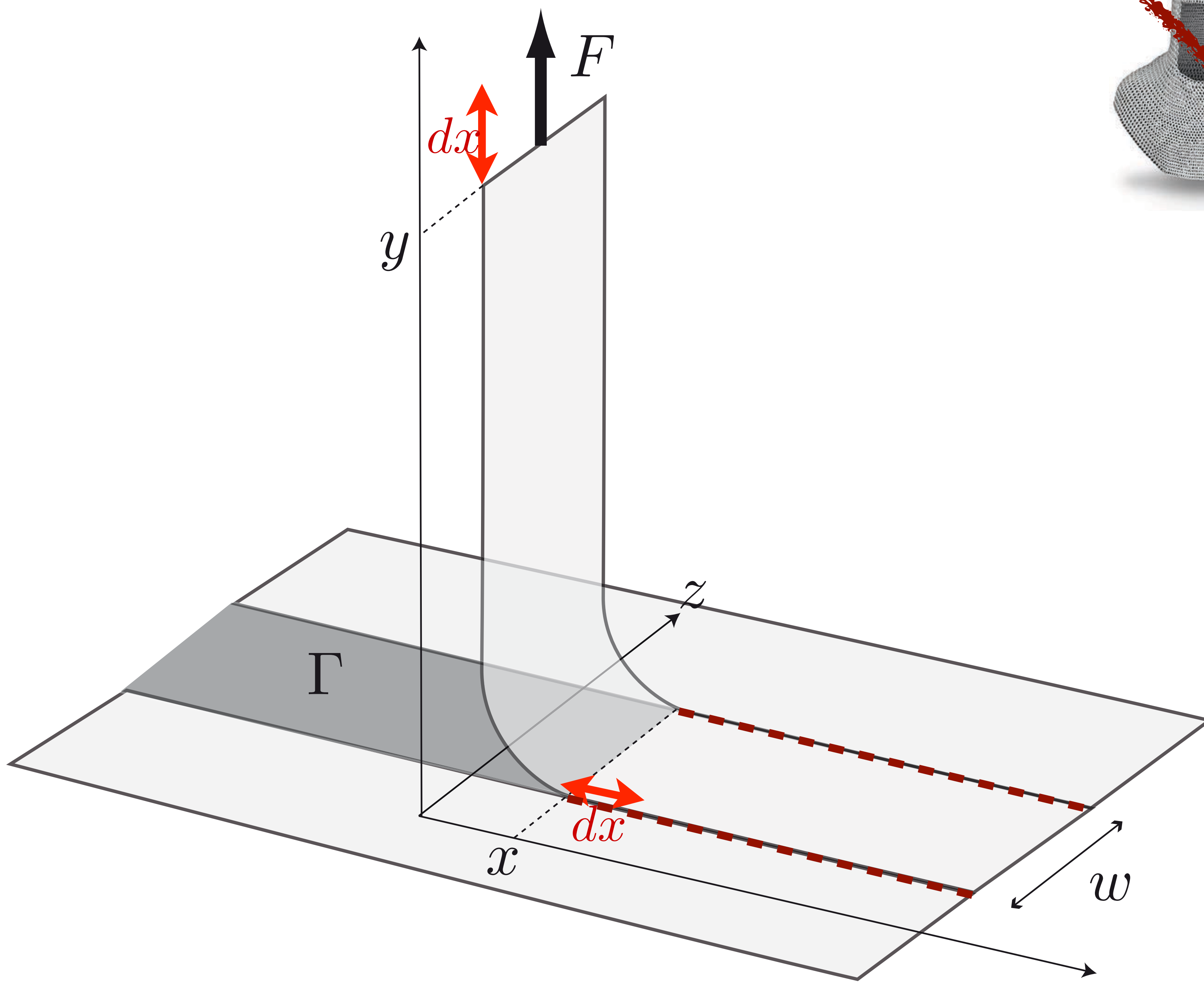
1) parallel propagation



2) breaking the flap

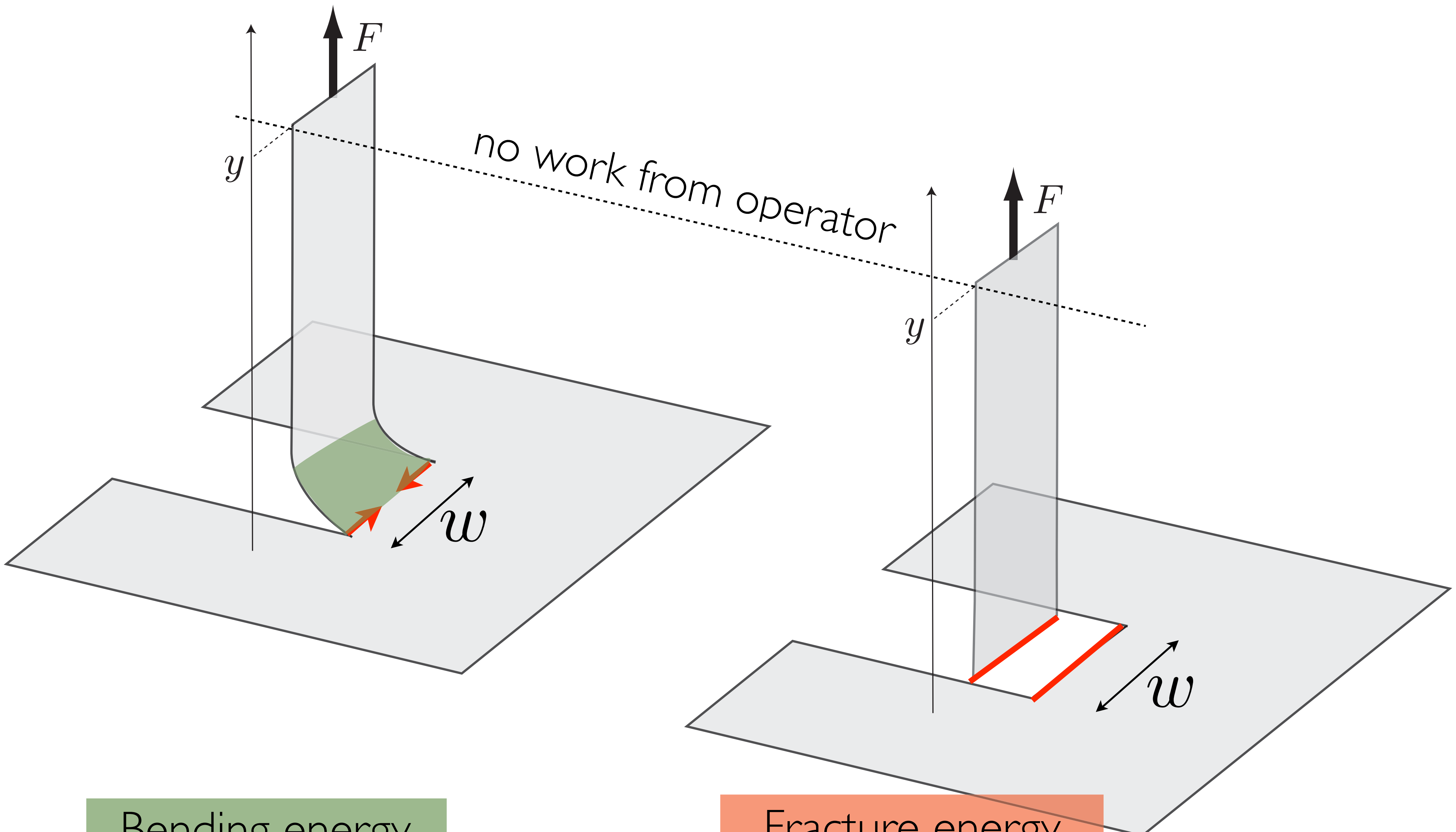
1) parallel propagation





$$F_p = \Gamma w + \cancel{2\gamma t}$$

2) breaking the flap (no work of operator)



Bending energy

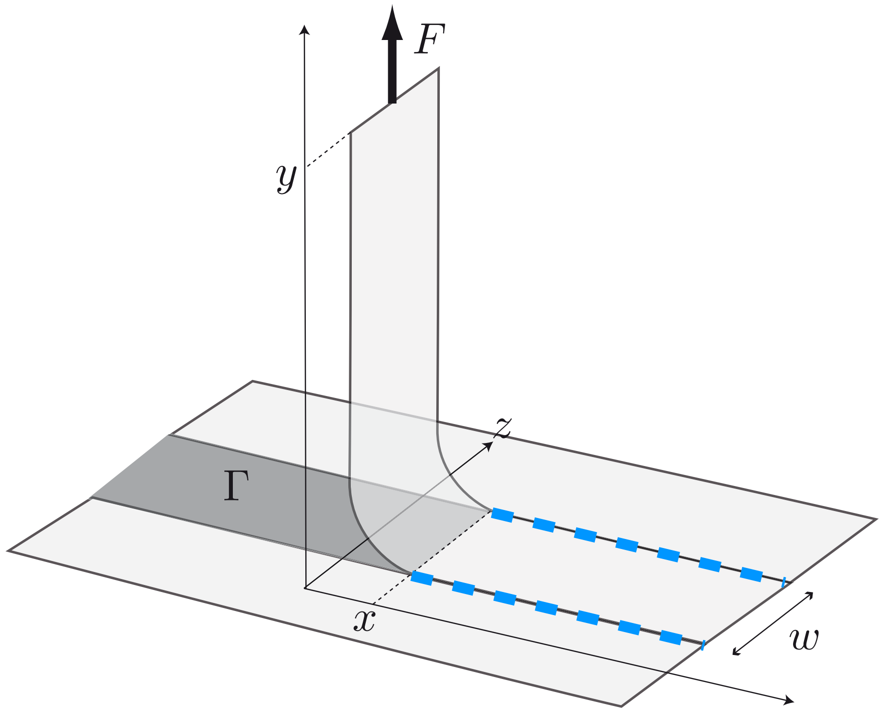
$$E_B \sim \sqrt{BwF}$$

($B \sim Et^3$ flexural rigidity)

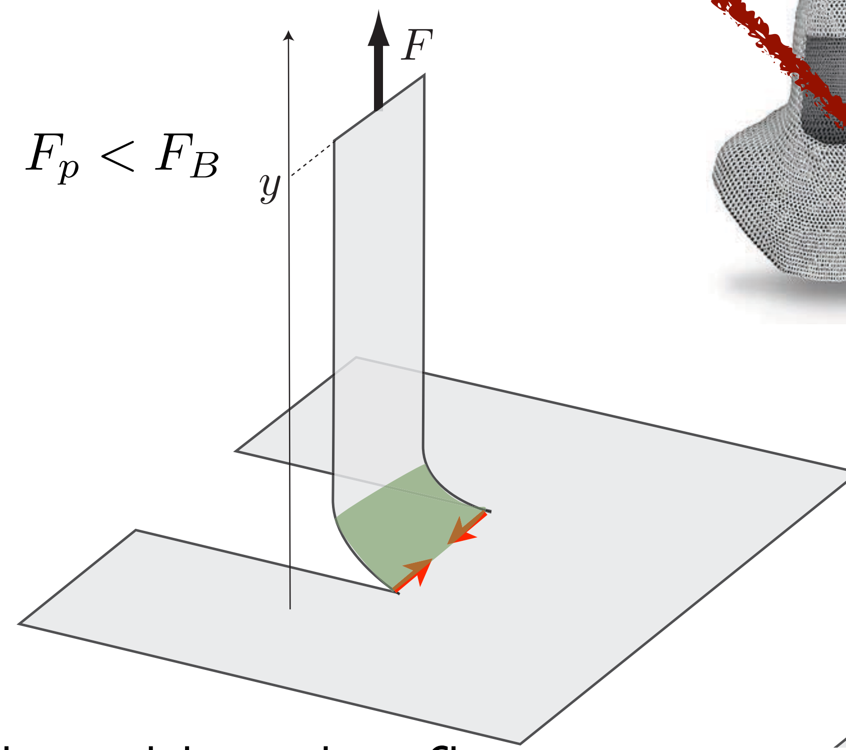
Fracture energy

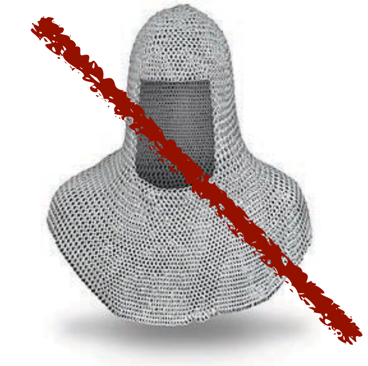
$$G_c t w$$

propagation if $\quad F_B \sim (G_c t)^2 \dfrac{w}{B}$



$F_p < F_B$





1) parallel propagation

operator's work

$$F_p = \Gamma w$$

2) breaking the flap

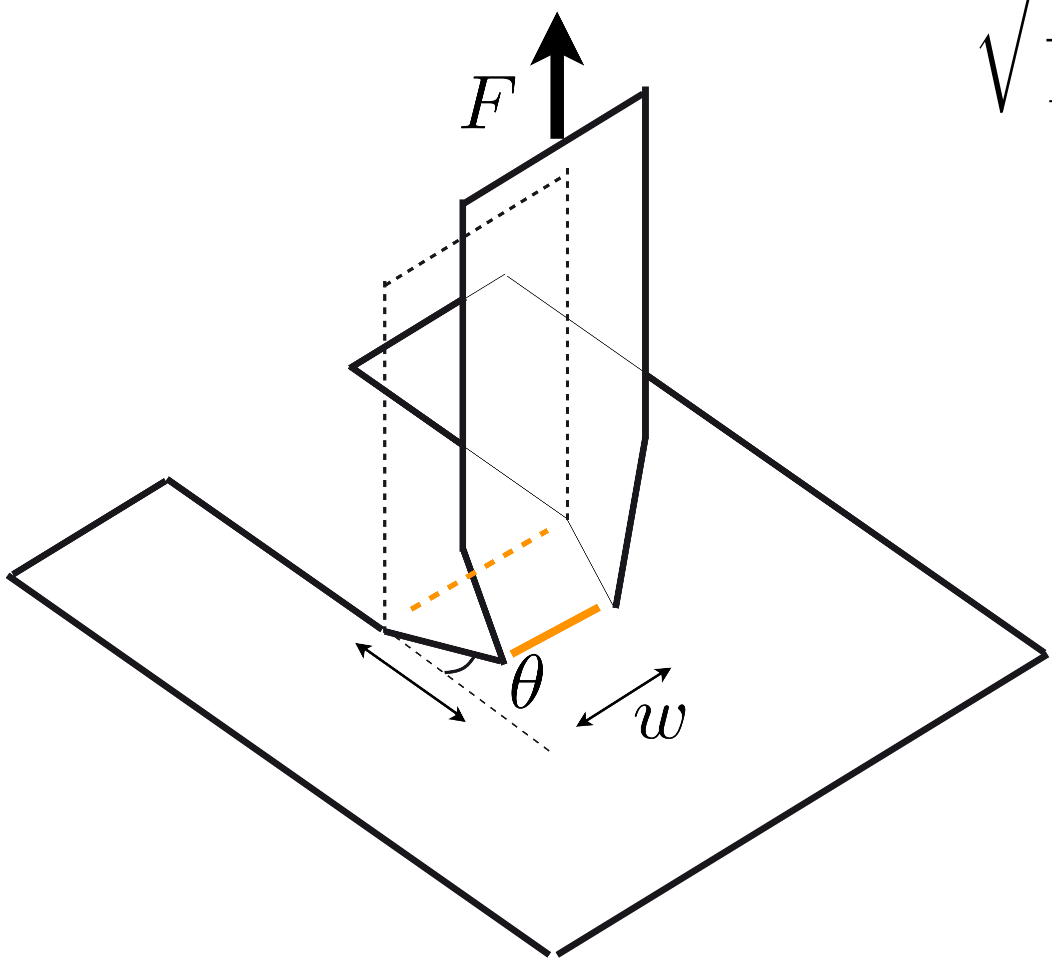
flexural energy

$$F_B \sim (G_c t)^2 \frac{w}{B}$$

$F_p < F_B$ ⟶ parallel propagation

$$\frac{F_p}{F_B} \sim \frac{B\Gamma}{(G_c t)^2}$$

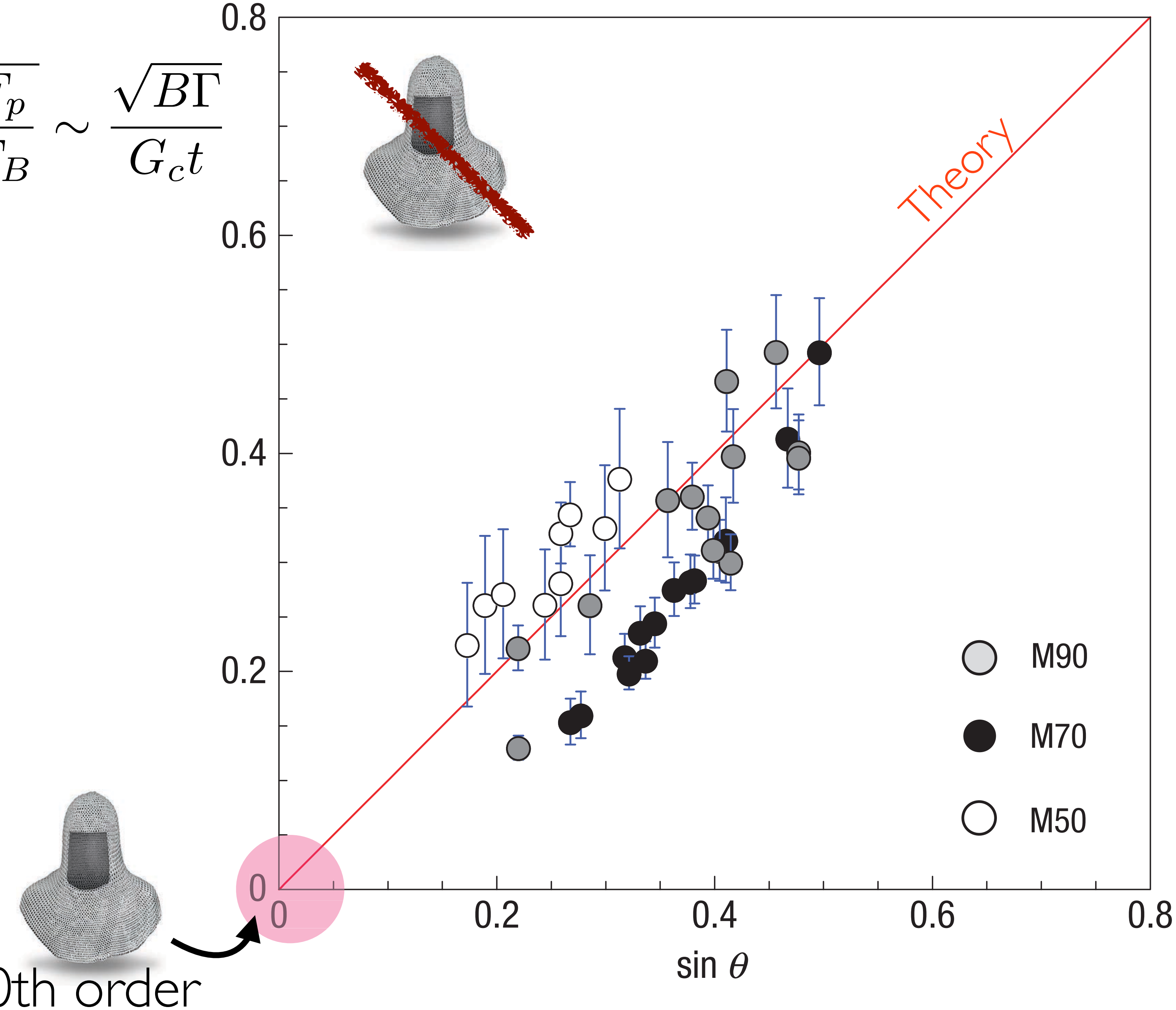
relative magnitude of converging to parallel effects



converging tears

$$\sin\theta = \frac{\sqrt{2B\Gamma}}{G_c t} \sim \sqrt{\frac{F_p}{F_B}}$$

$$\sqrt{\frac{F_p}{F_B}} \sim \frac{\sqrt{B\Gamma}}{G_c t}$$

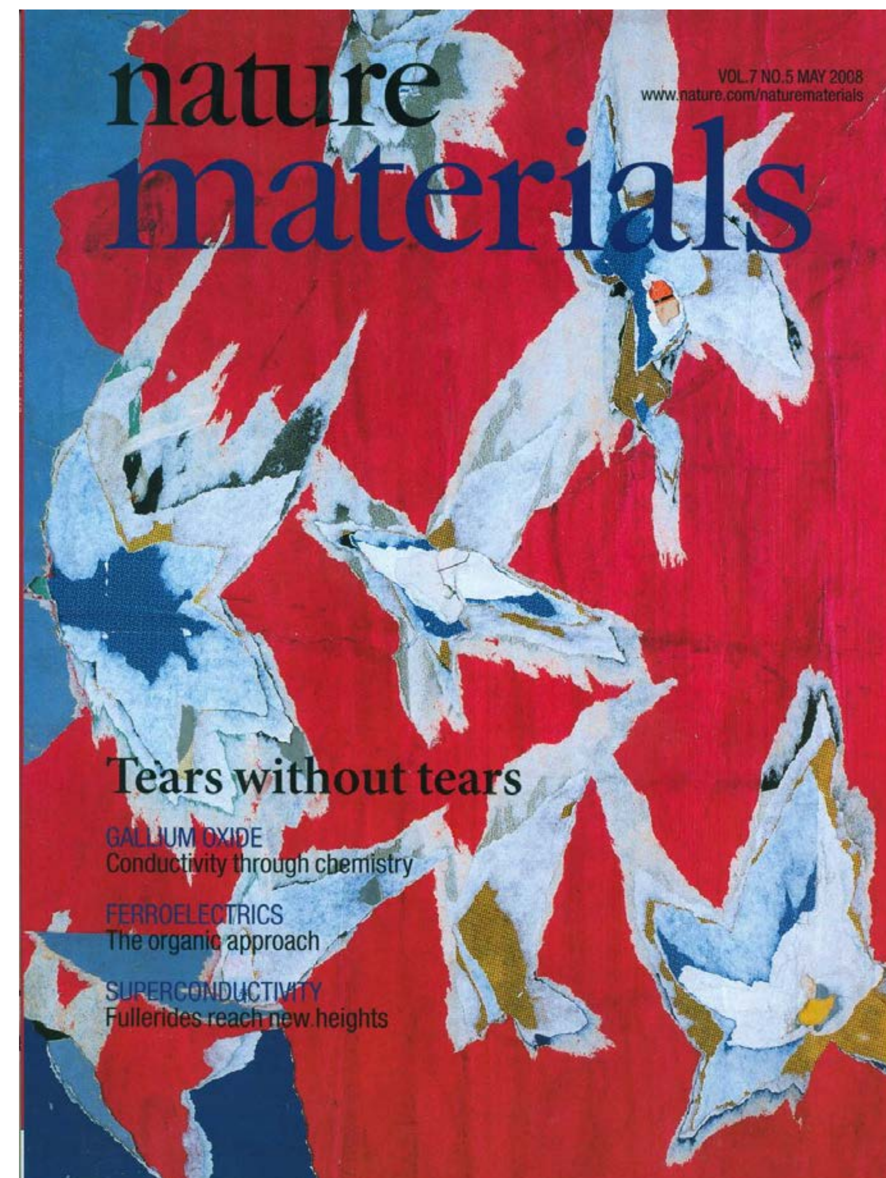


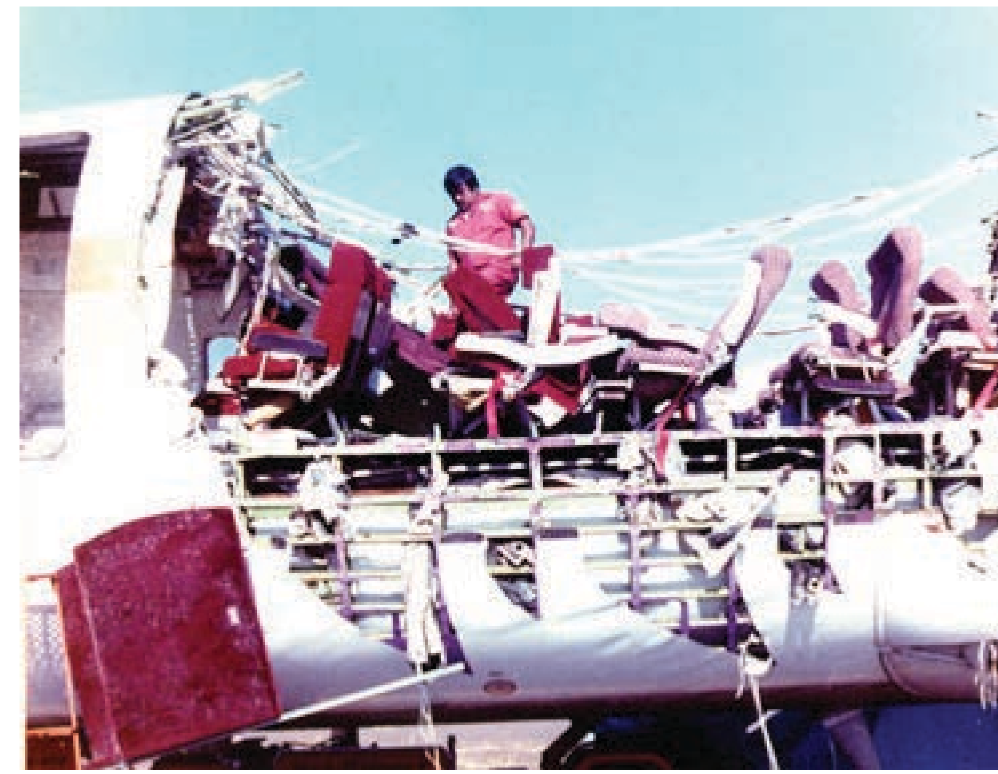
Hamm et al. , Nature Material 2008

D.Roman

Jacques Villeglé

P.Kovarik



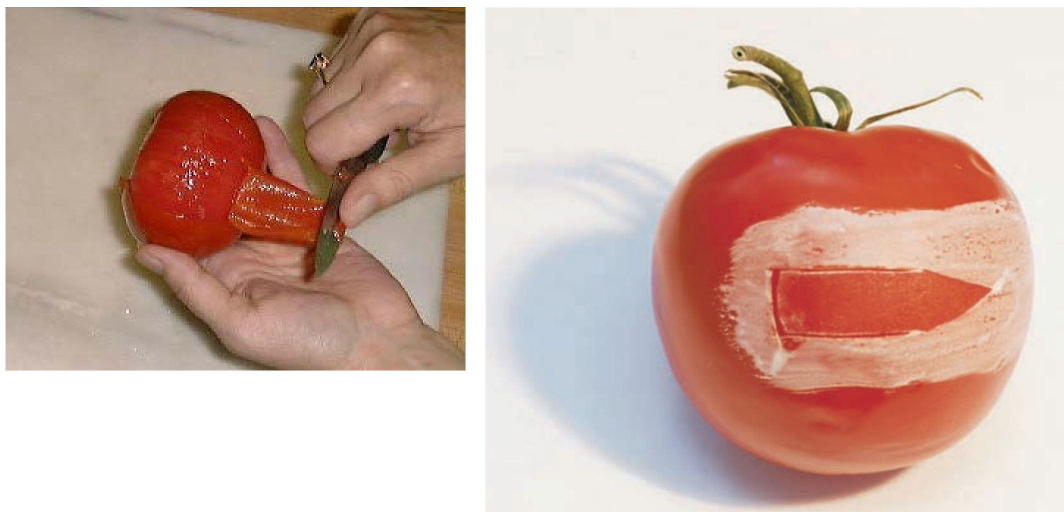


Aloha airlines 1988

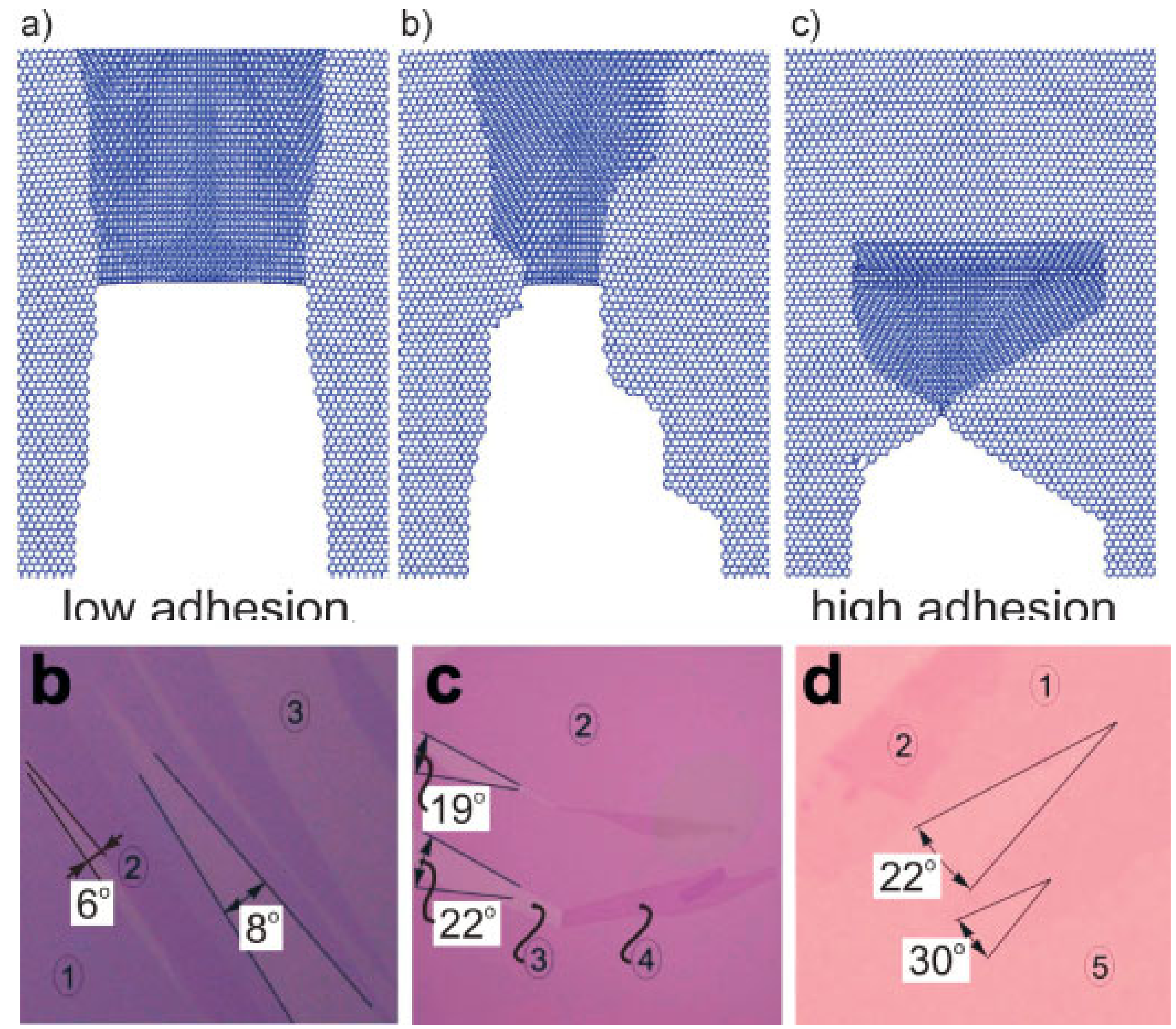
metrology ?

$$\sin\theta = \frac{\sqrt{2B\Gamma}}{G_c t}$$

peeling tomato



graphene



Adhesion 80 N/m

toughness $\gamma t = 8.10^{-2} N$

Angle $\theta = 20^\circ$

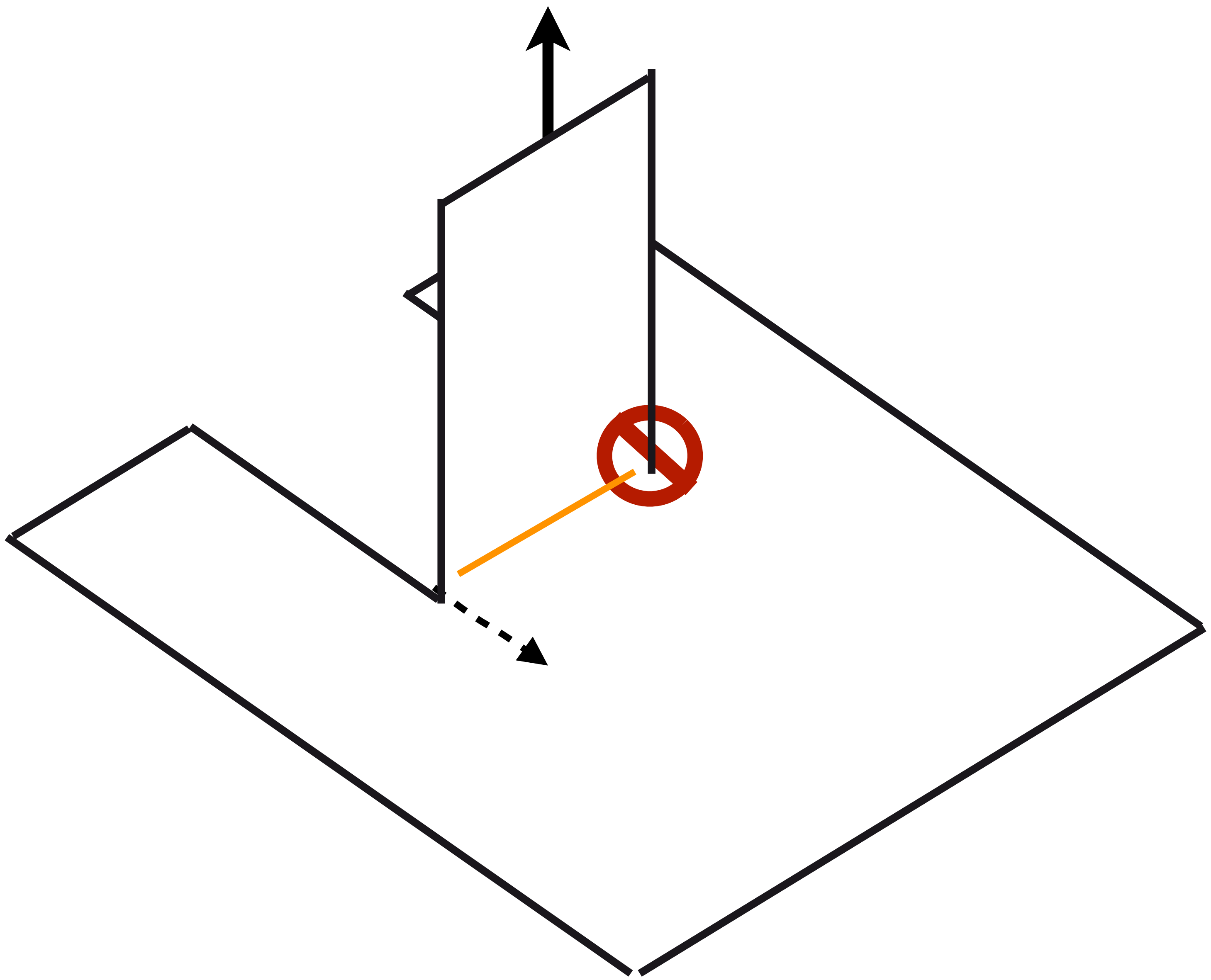
$$B = (\eta\gamma t\sin\theta)^2/2\tau \approx 4\times 10^{-6}\ \mathrm{J}$$

Young modulus: $E \approx 70$ MPa.

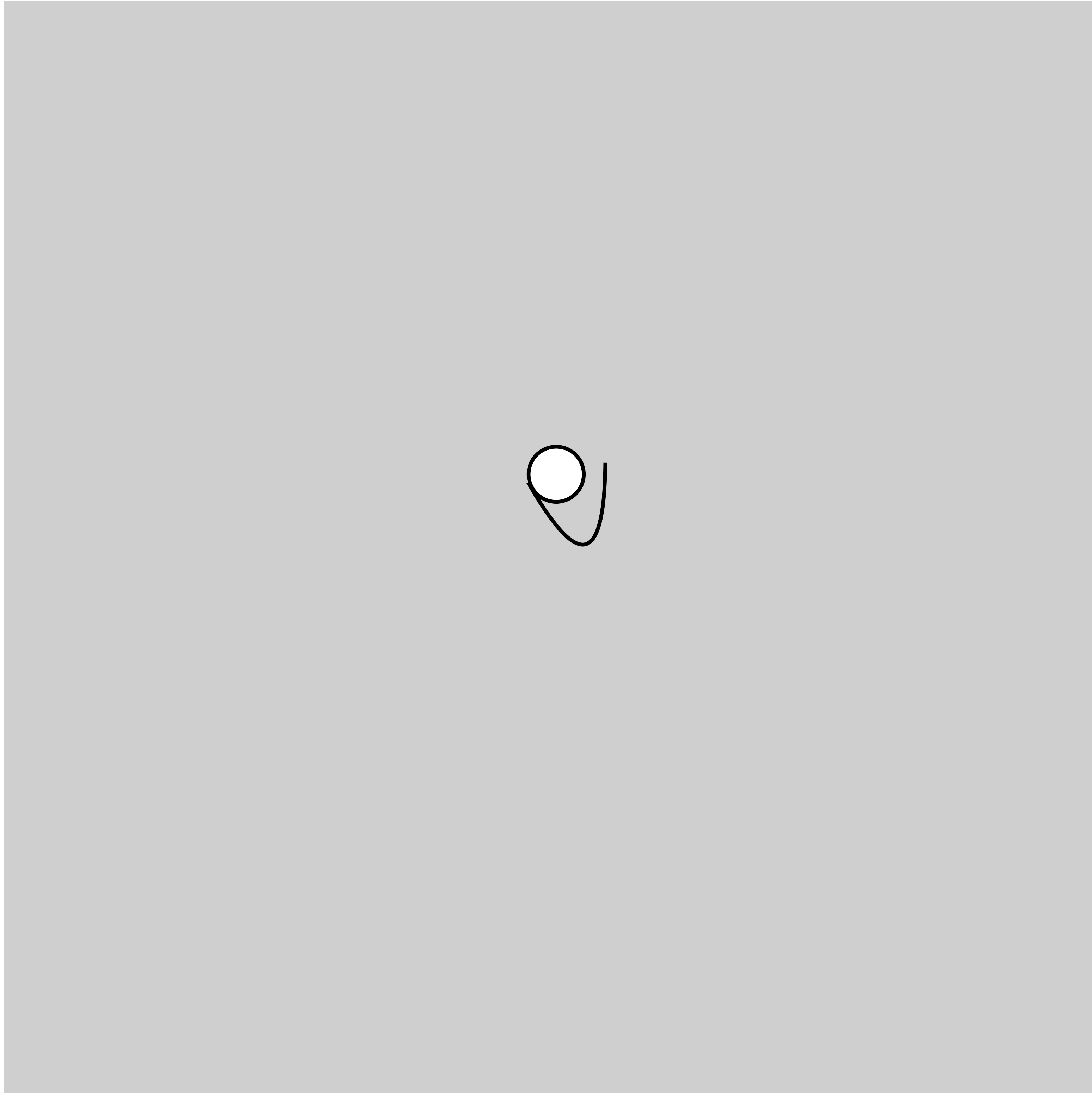
consistent with other estimations 5 MPa to 50 MPa

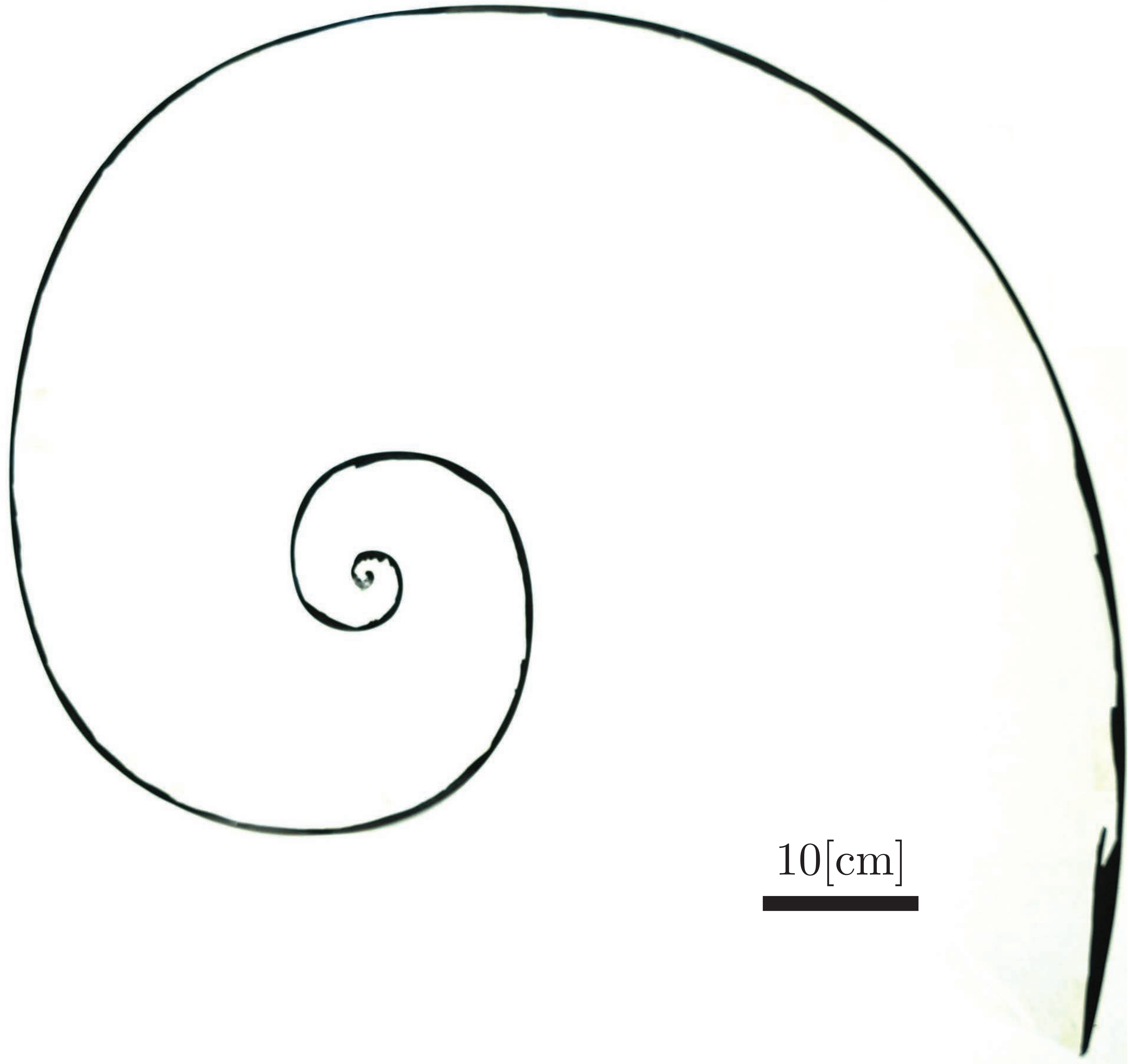
Sen et al. ,Small 2010

one crack fixed ?

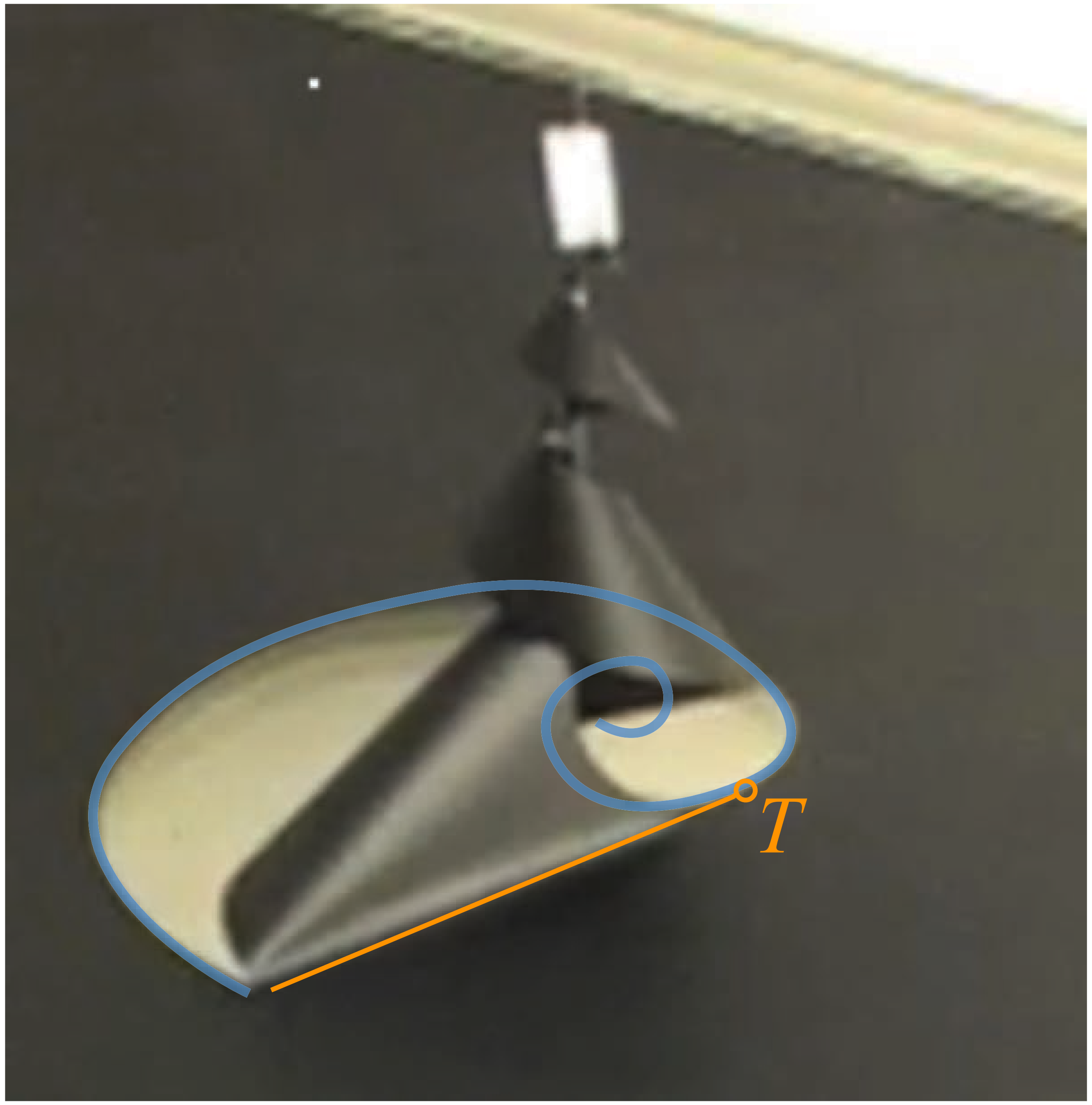


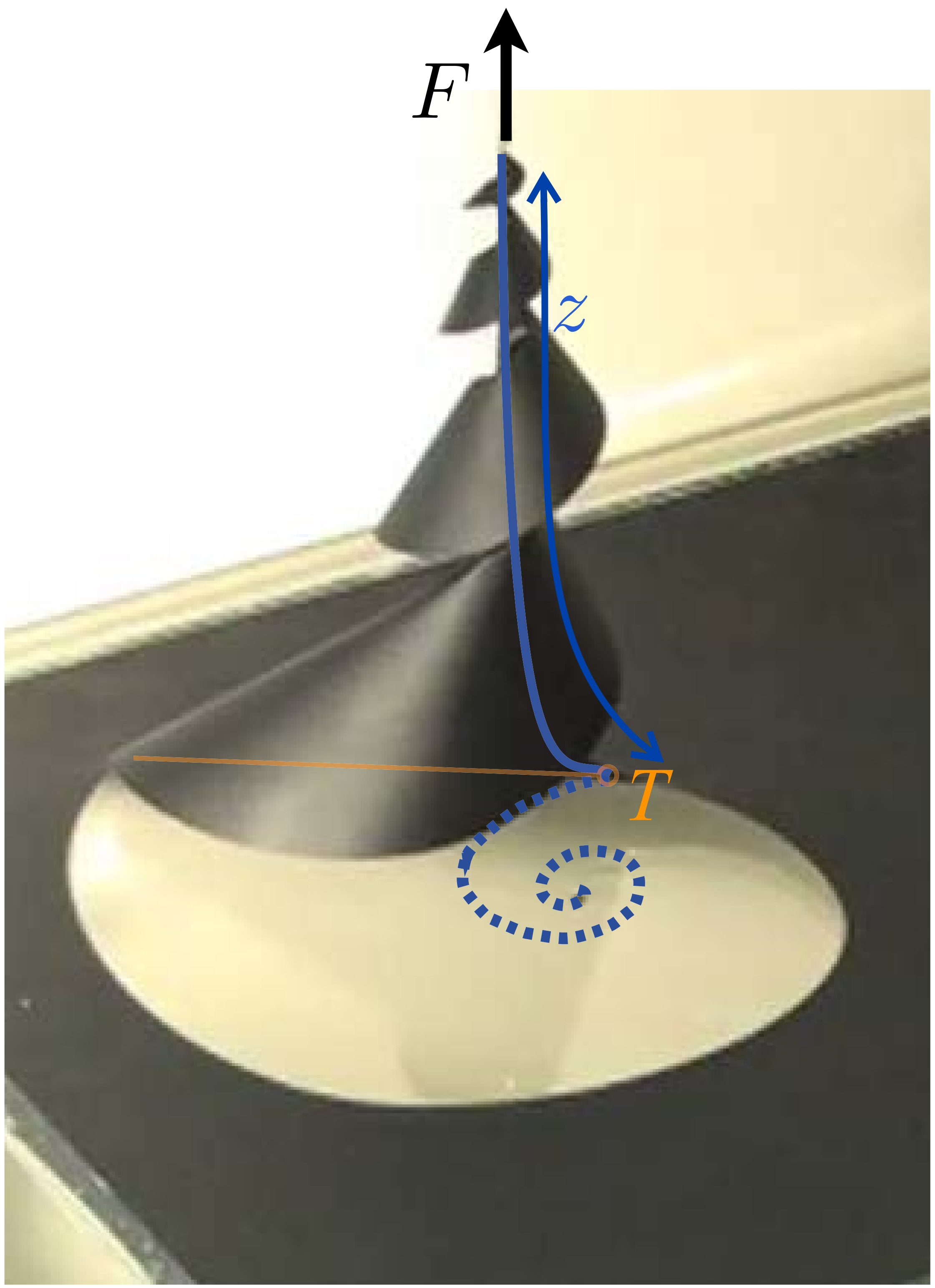
V.Romero





V.Romero


T



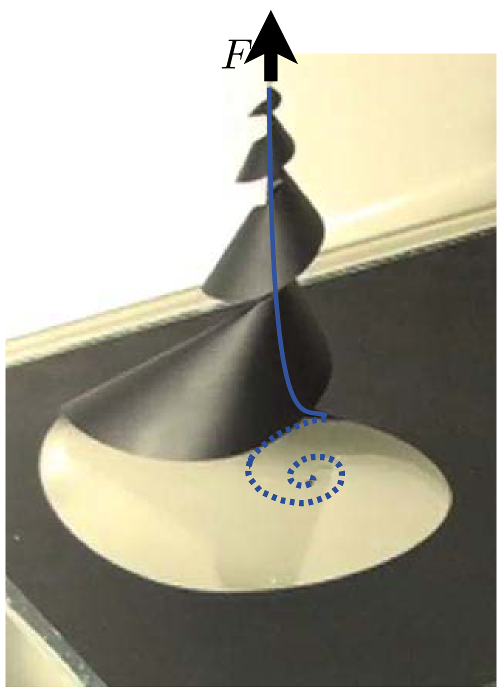
$$\delta W = F dz = G_c t dl$$

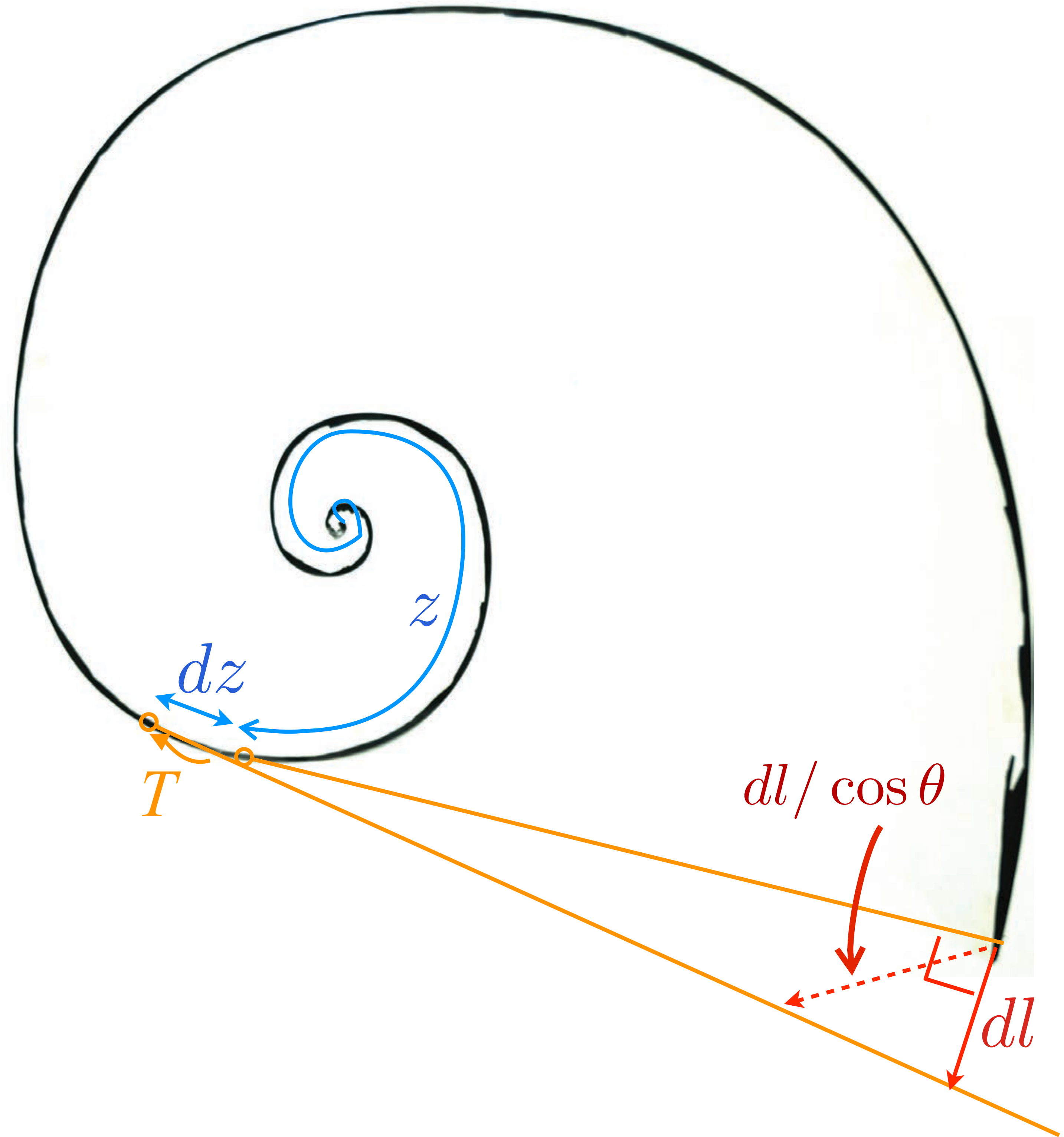
maximum E.R.Rate

$$G = \frac{F}{t}\frac{dz}{dl}$$

maximum advance dz of pushing object for a fixed crack step dl

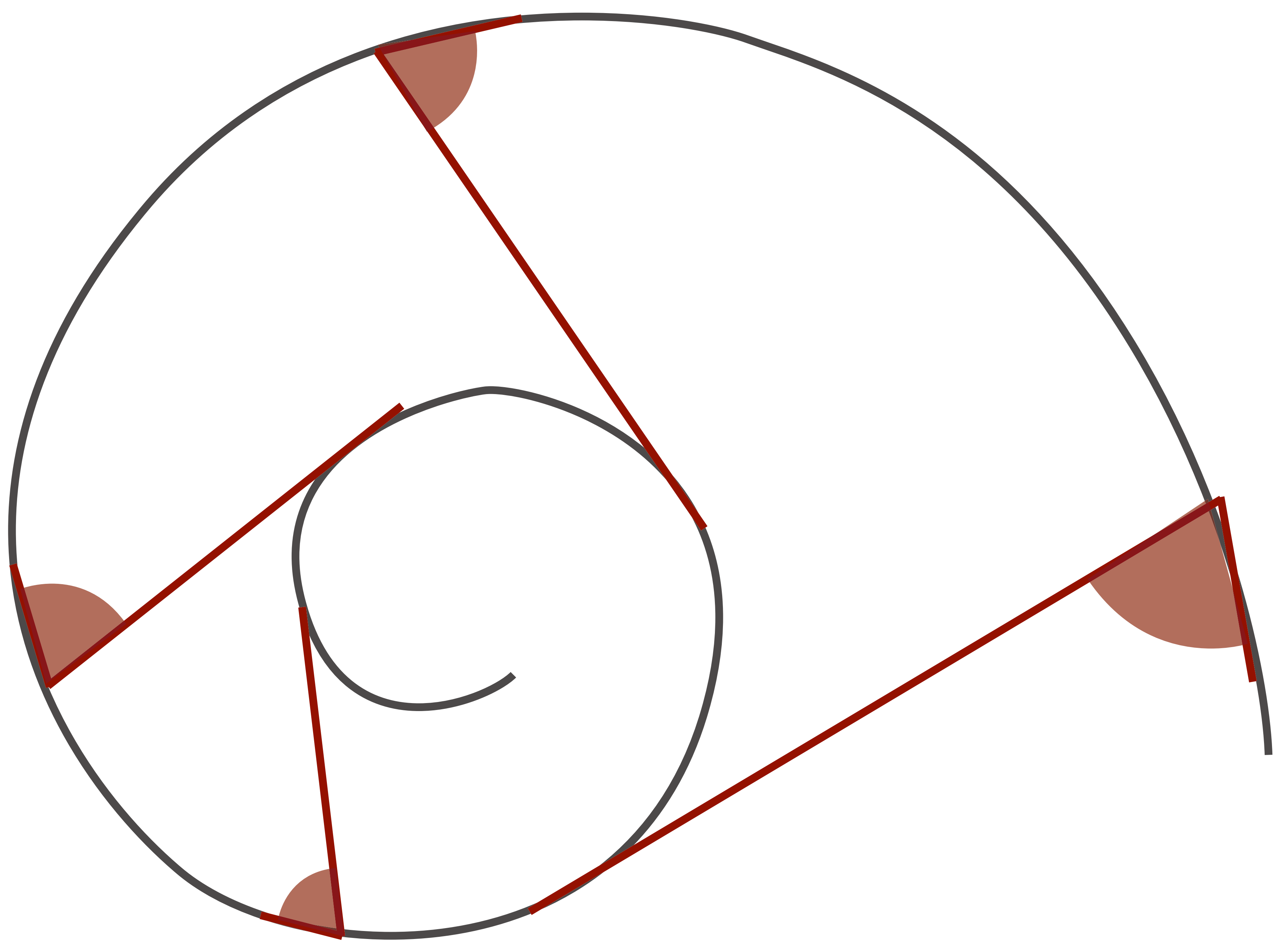
minimum advance dl of crack for fixed object step dz



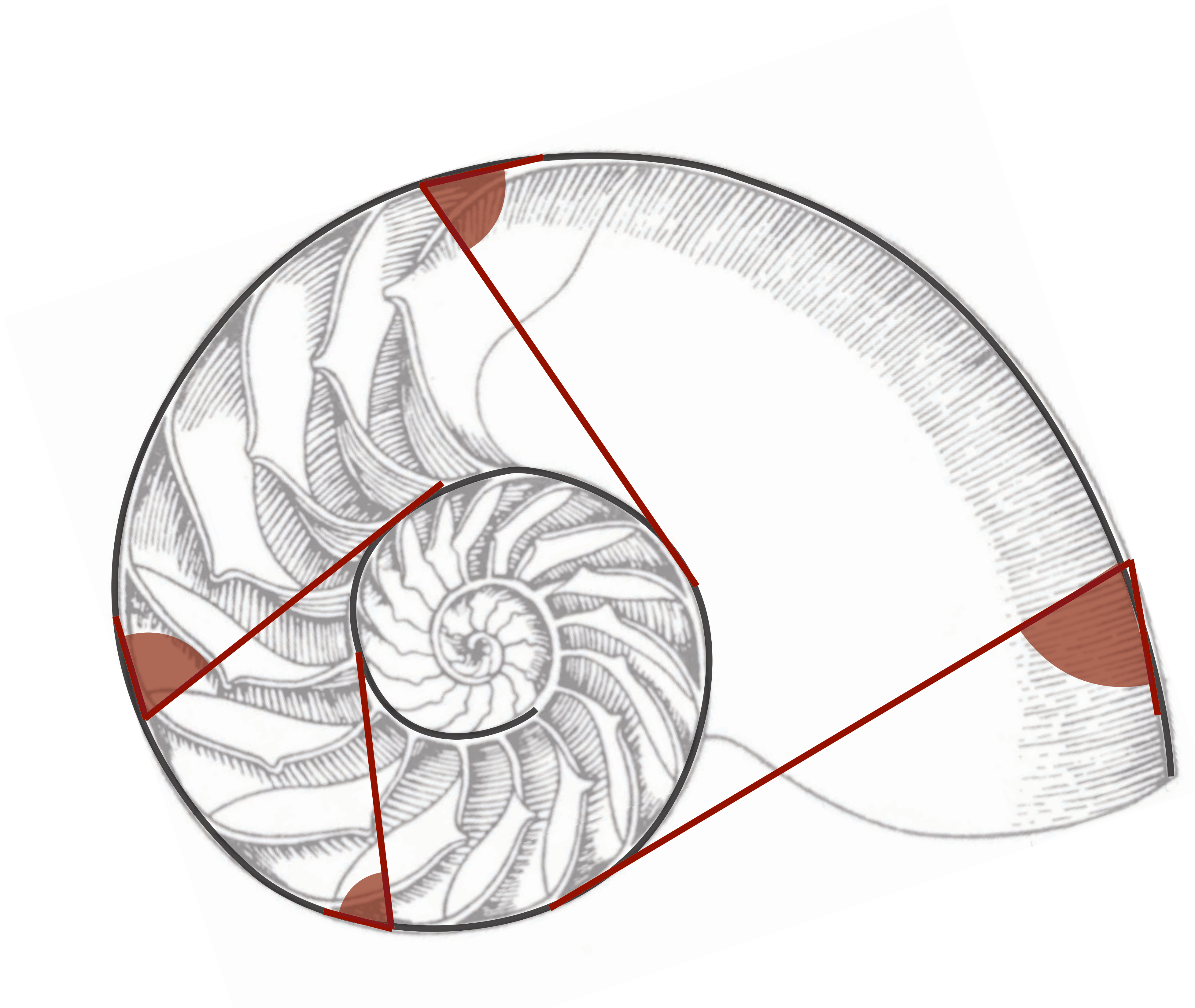


minimum advance dl of crack
for fixed object step dz

propagation perp. to fold

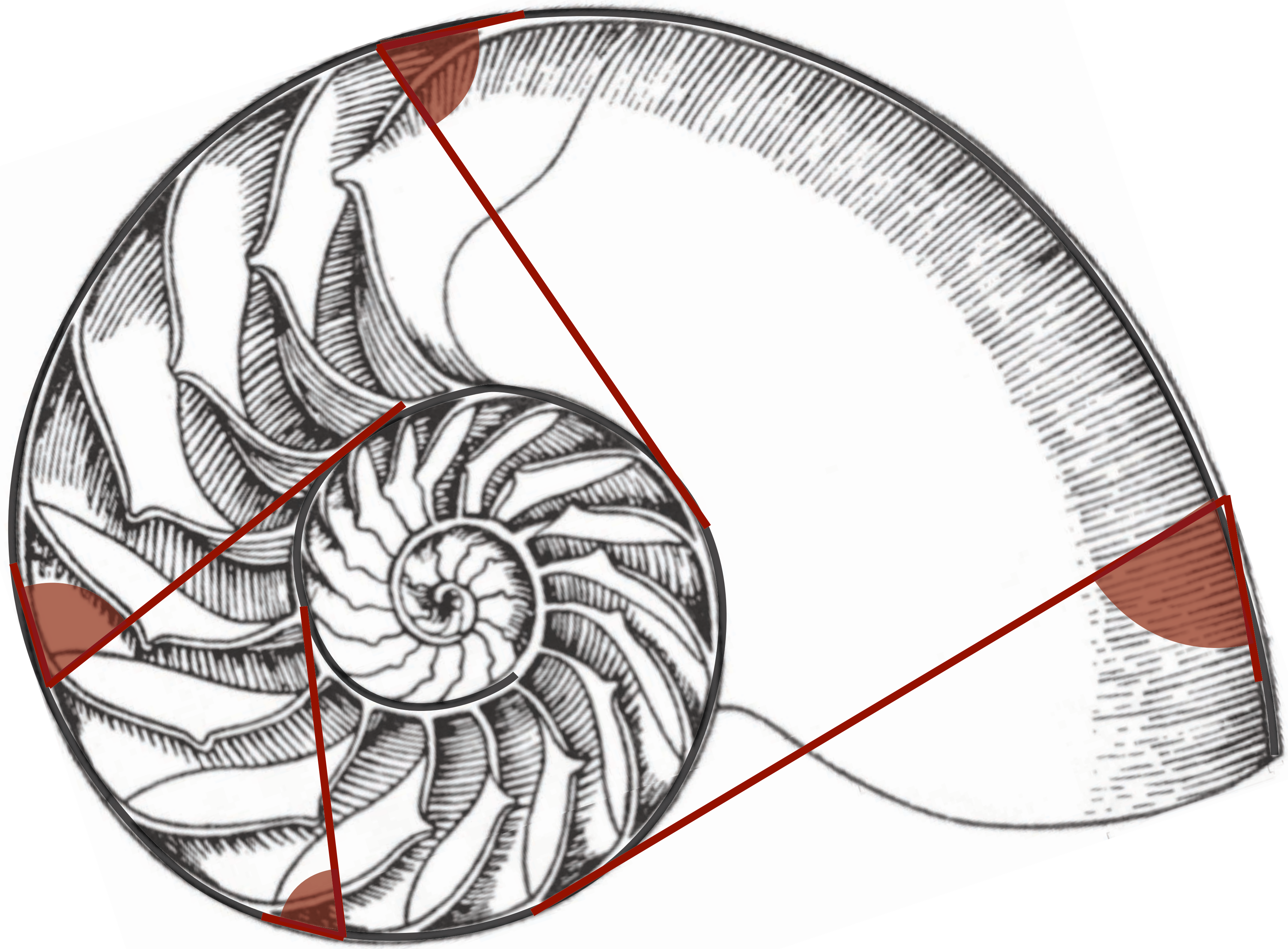


V.Romero



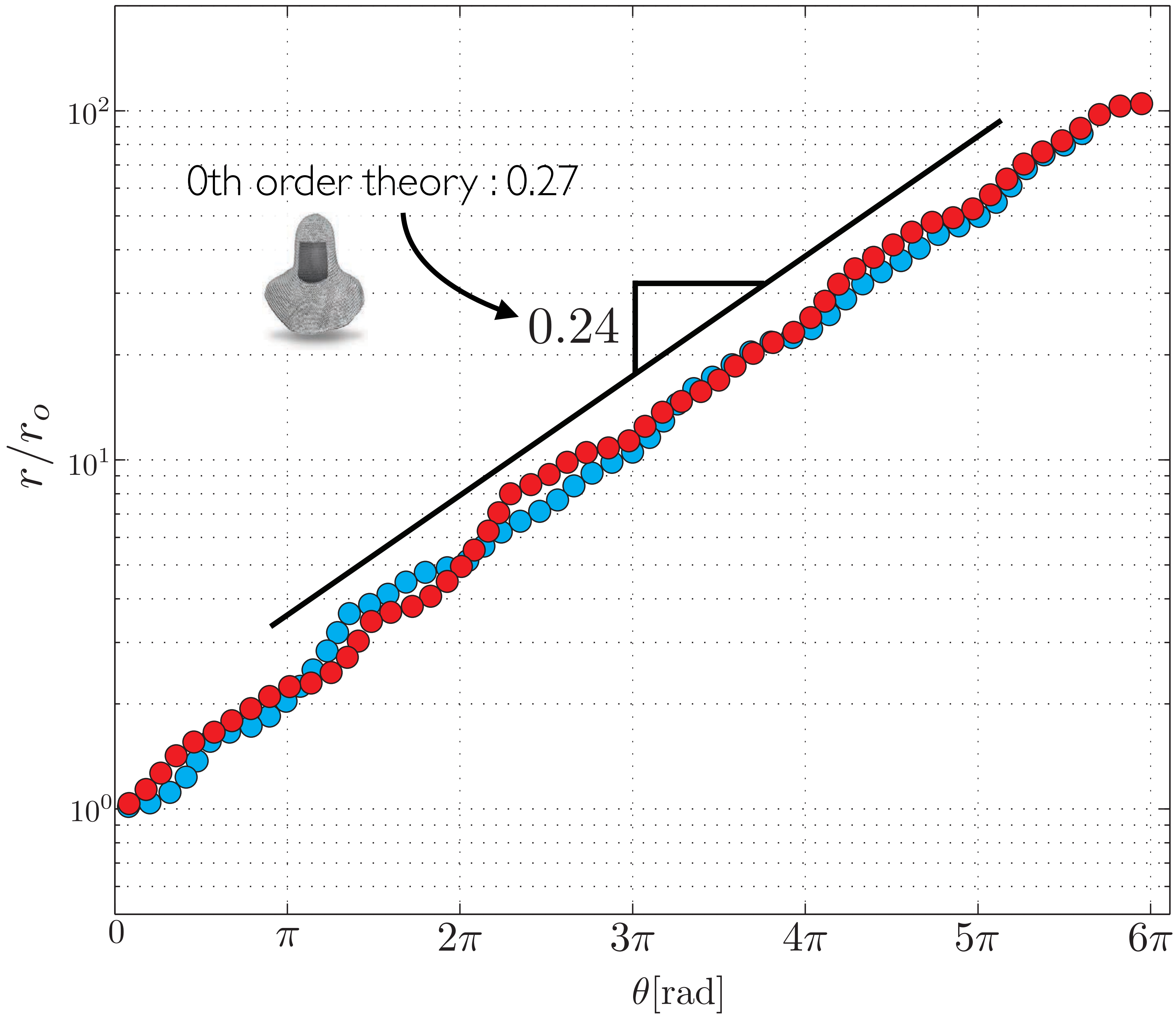
V.Romero

«Equiangular» logarithmic spiral



same process at different scale

V.Romero



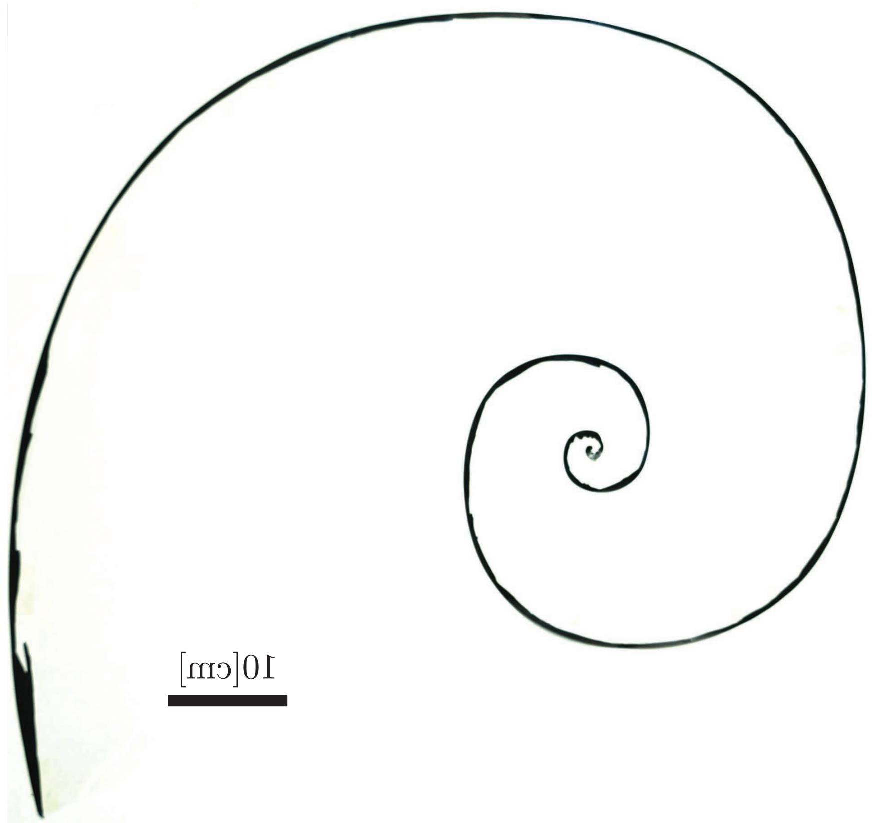
exponential growth

V.Romero

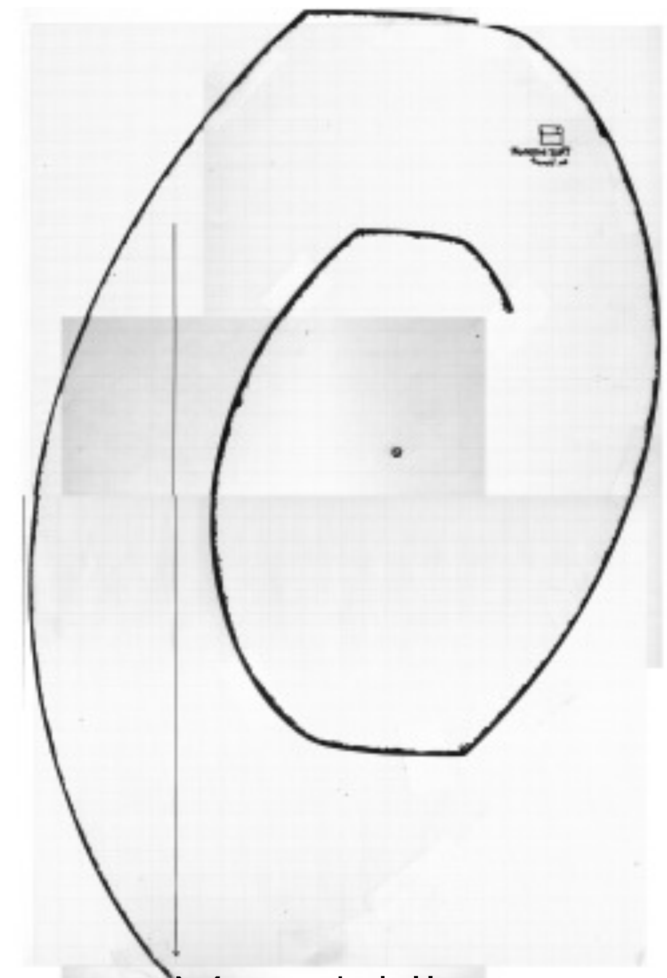
application for packaging?

independent of
- pulling angles and speed,
- type of boundaries
- thickness, elastic properties

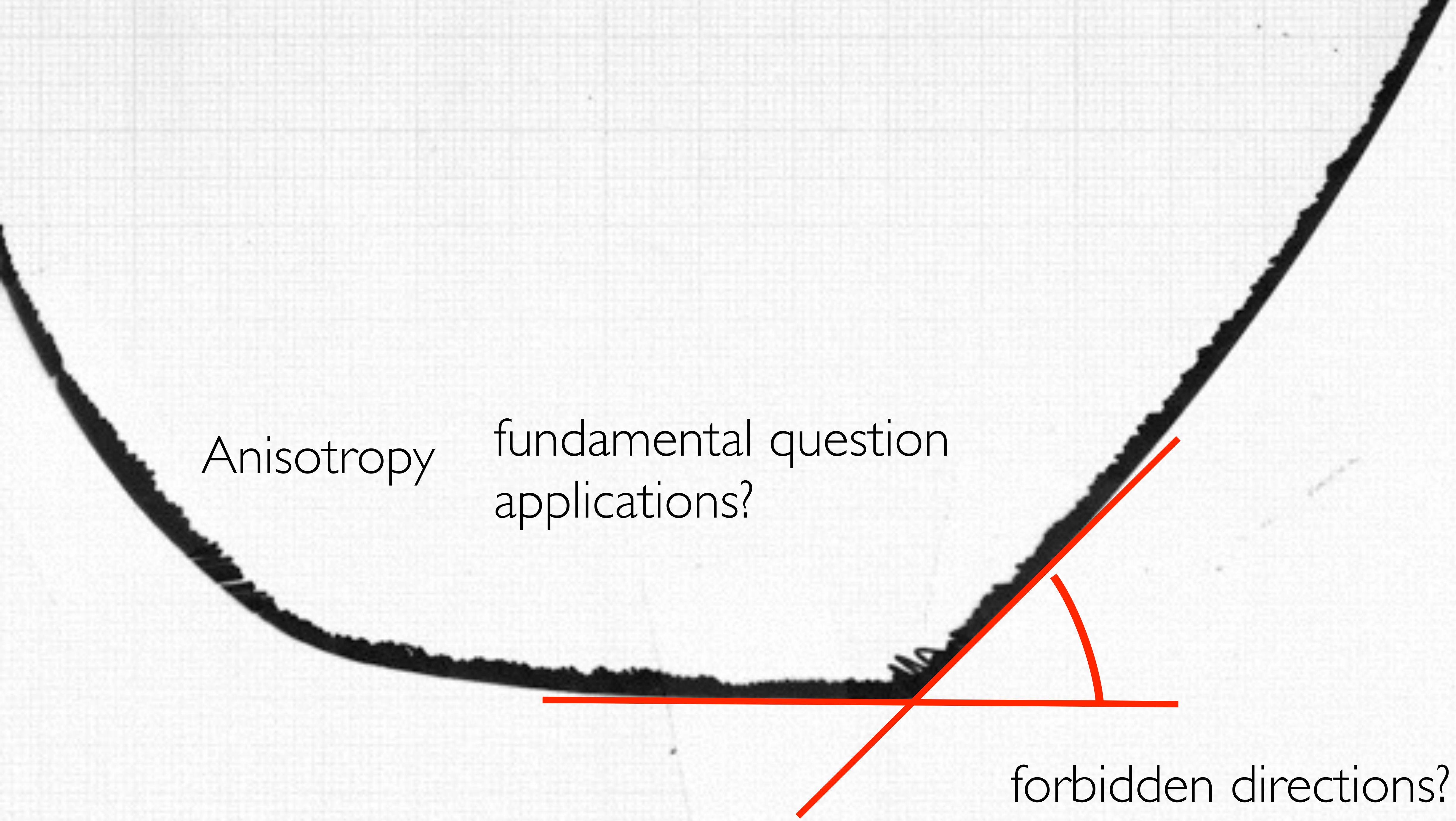
«Easy opening patent»
with E.Cerda, E.Hamm, V.Romero



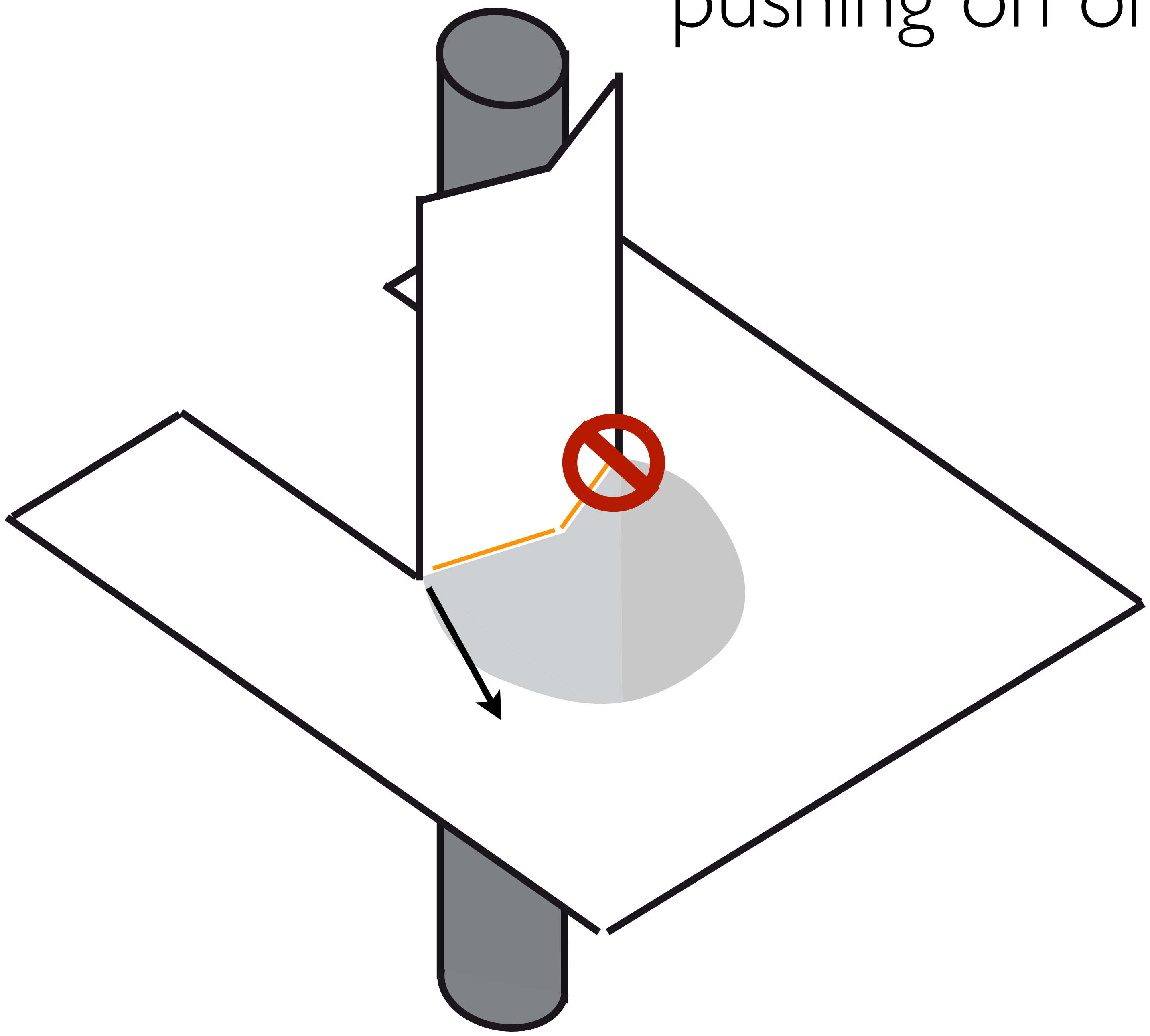
material I
Balanced Bi-oriented Polypropylene

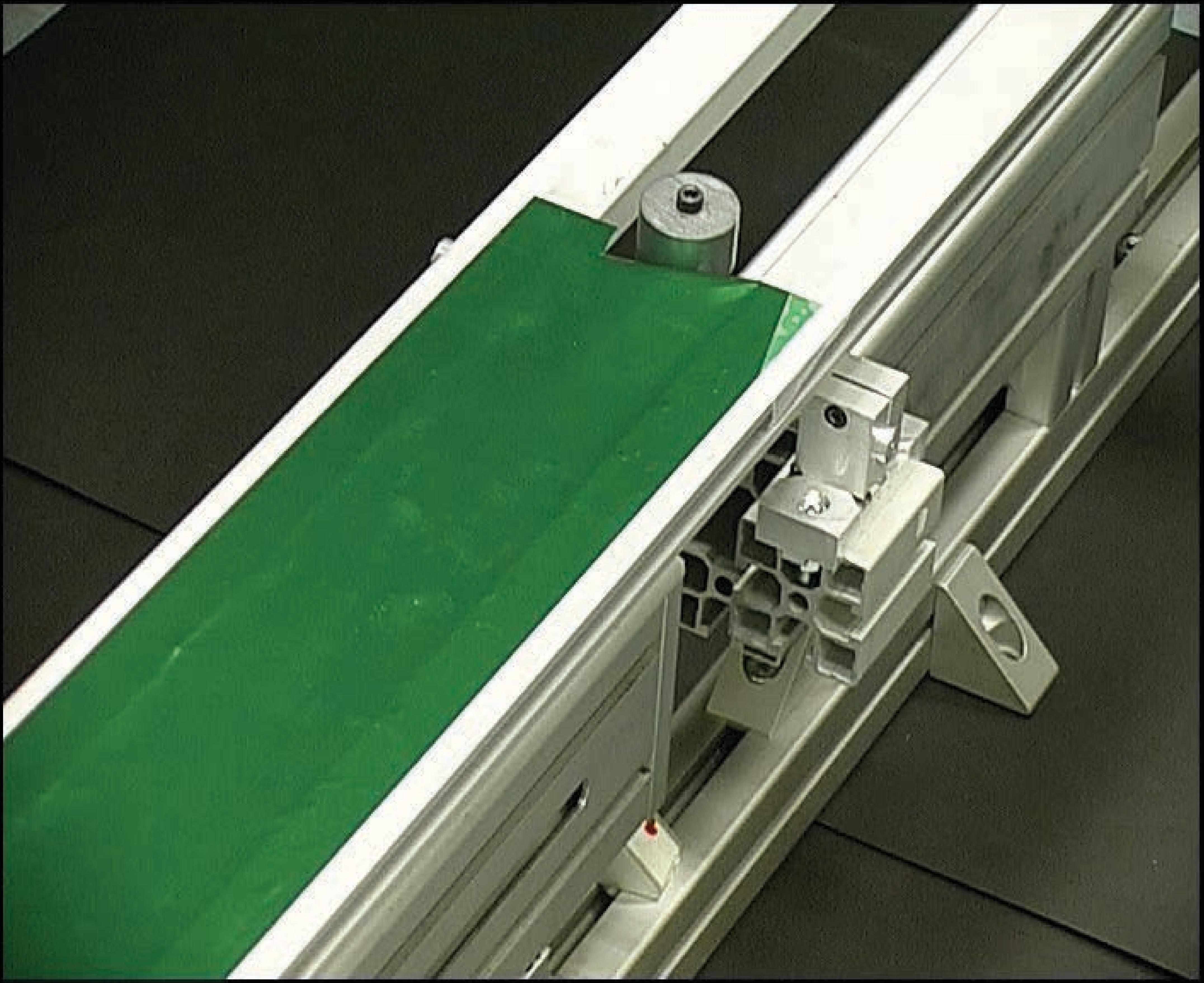


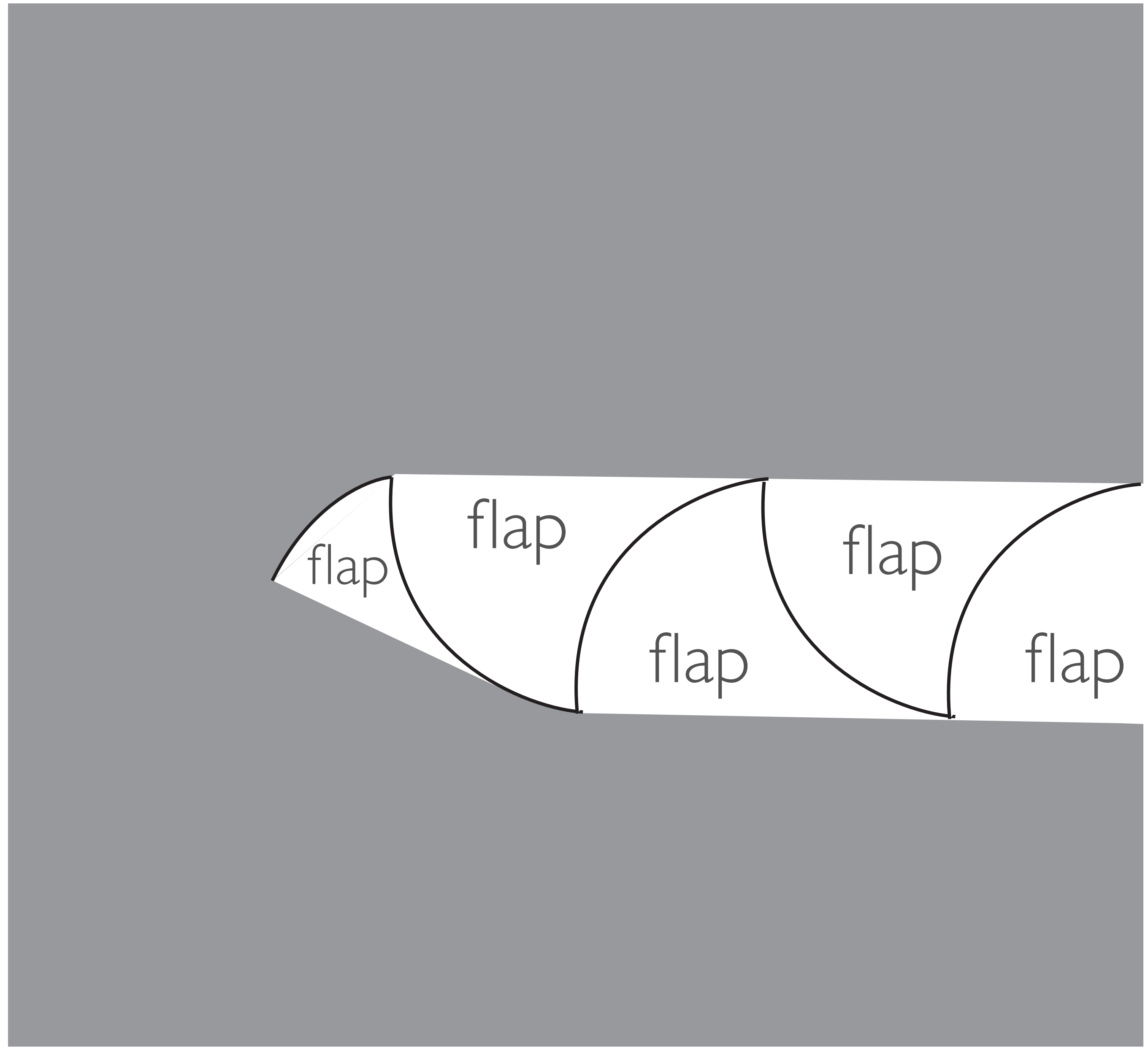
Material II
Bi-oriented Polypropylene

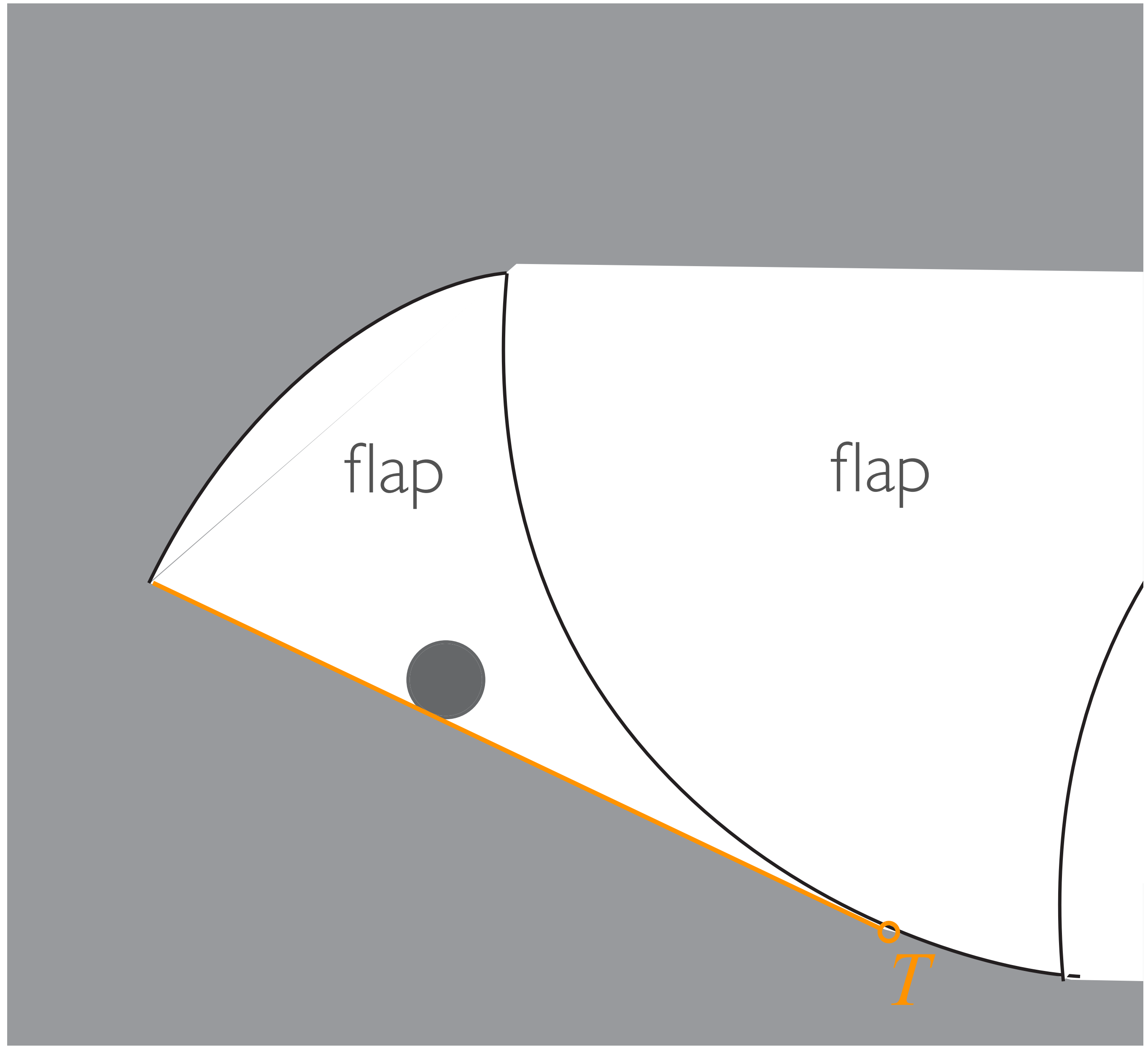

Anisotropy
fundamental question
applications?
forbidden directions?

pushing on one crack

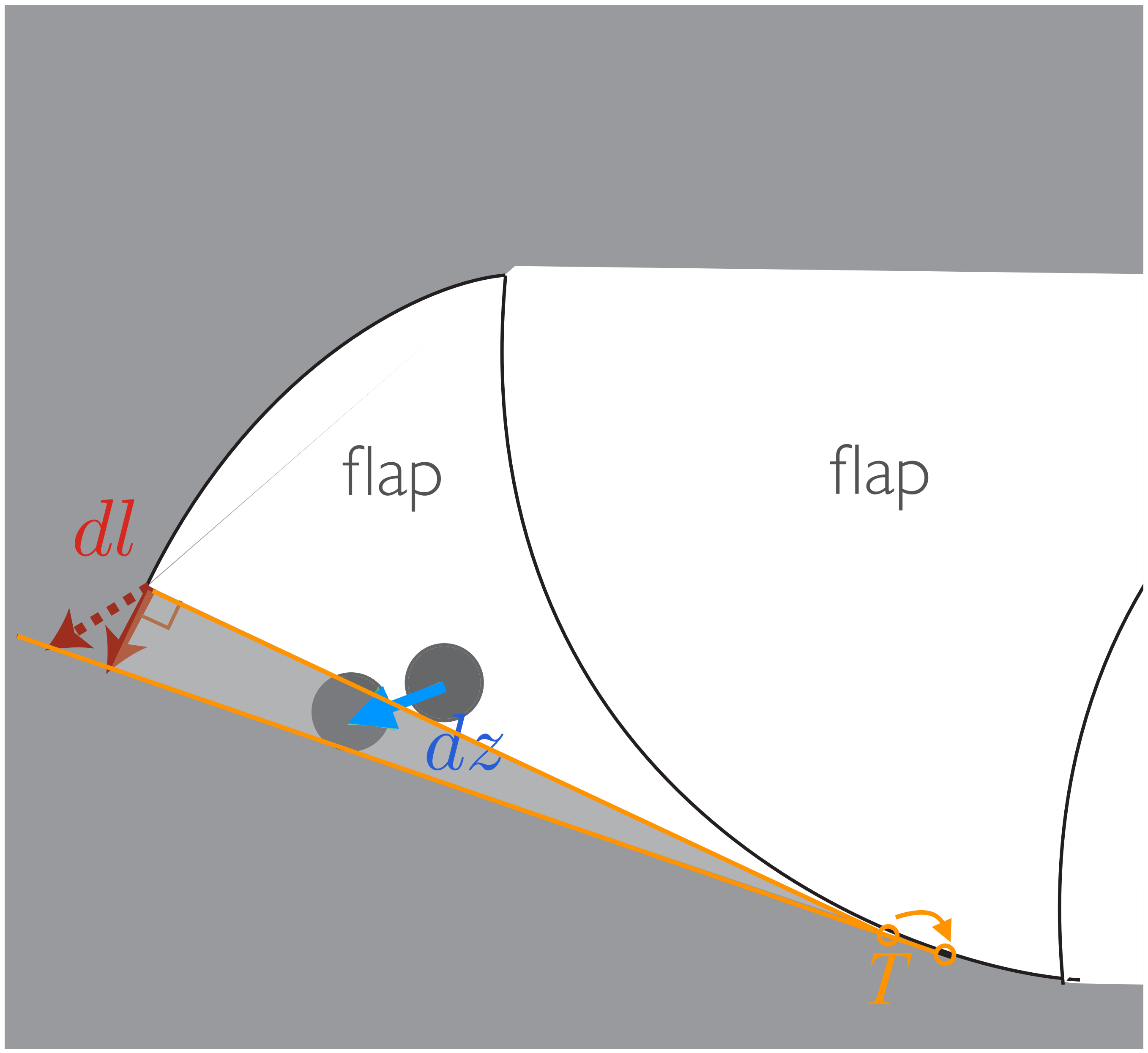





flap
flap
flap
flap
flap
flap



flap
flap
T

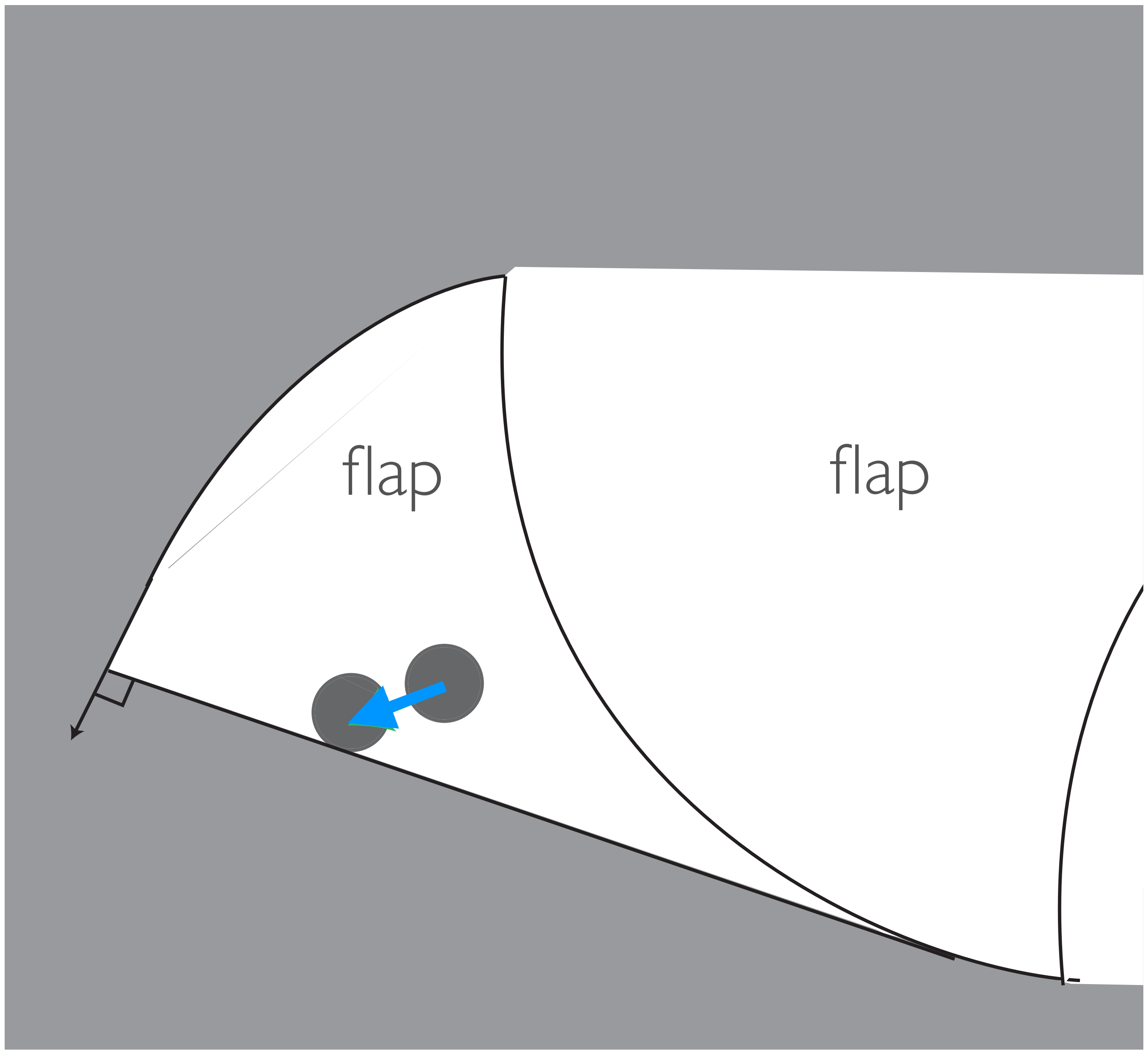


$$\delta W = F dz = G_c t dl$$

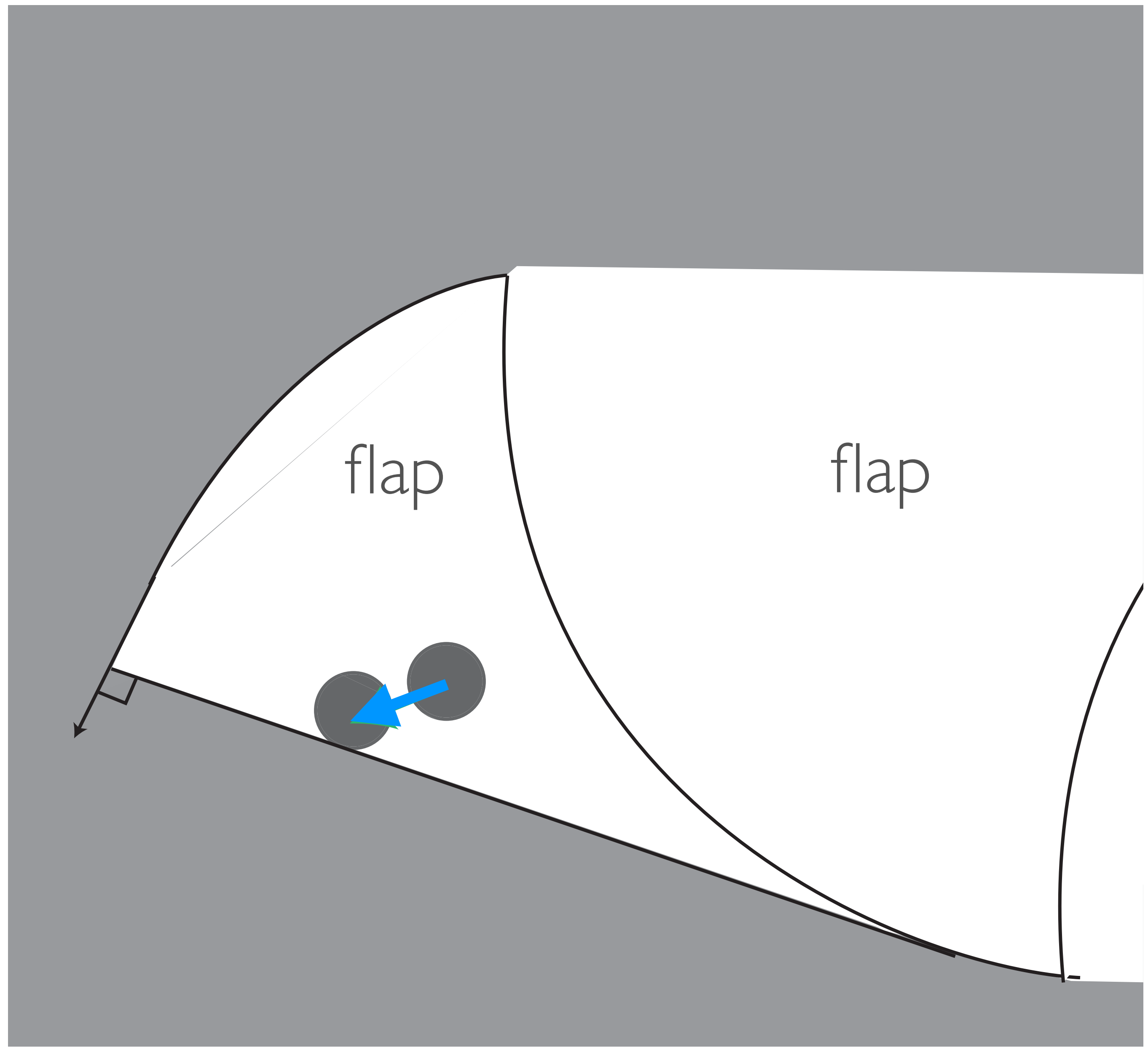
maximum E.R.Rate

$$G = \frac{F}{t}\frac{dz}{dl}$$

minimum advance dl of crack
for fixed object step dz



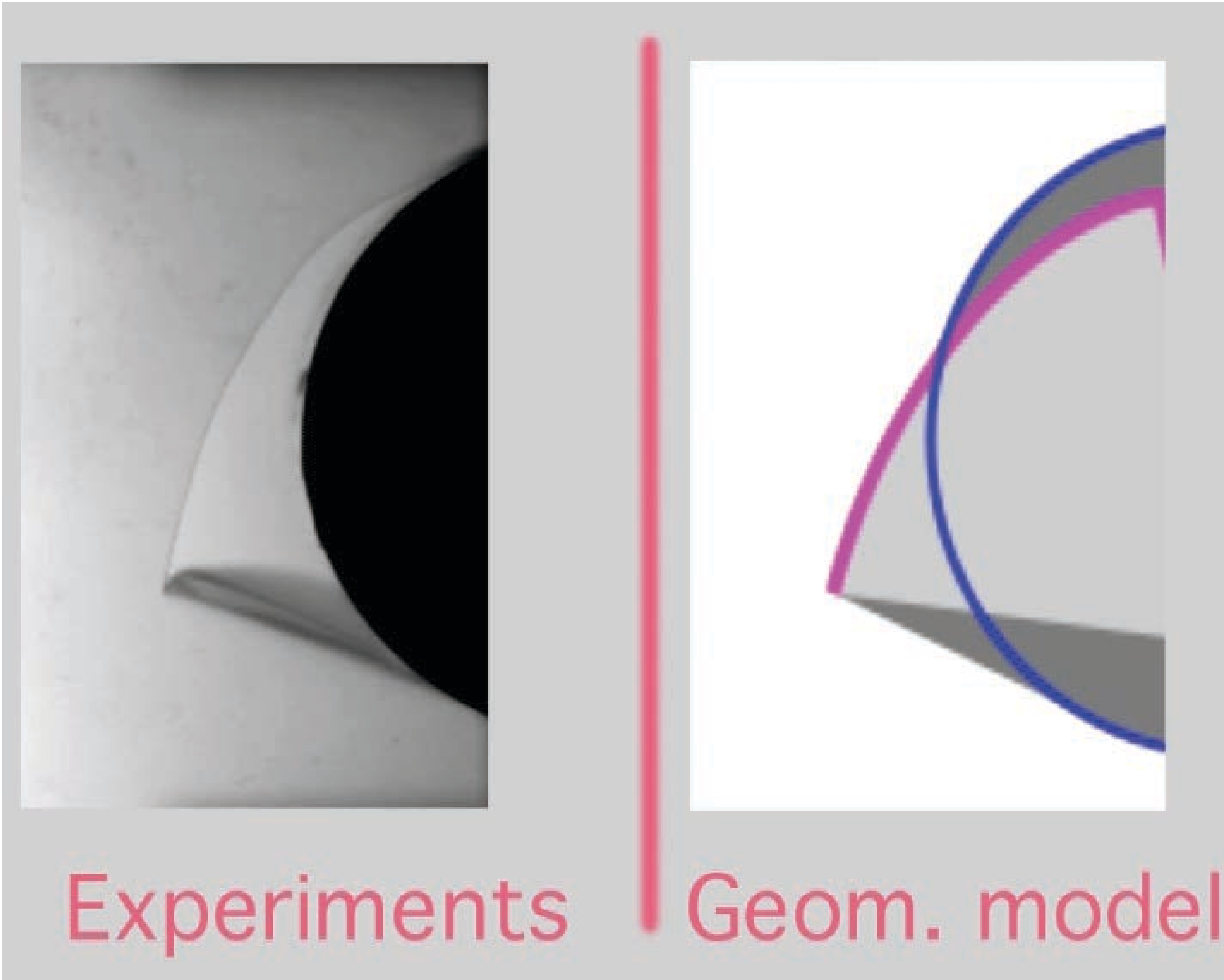
propagation perpendicular to flap limit



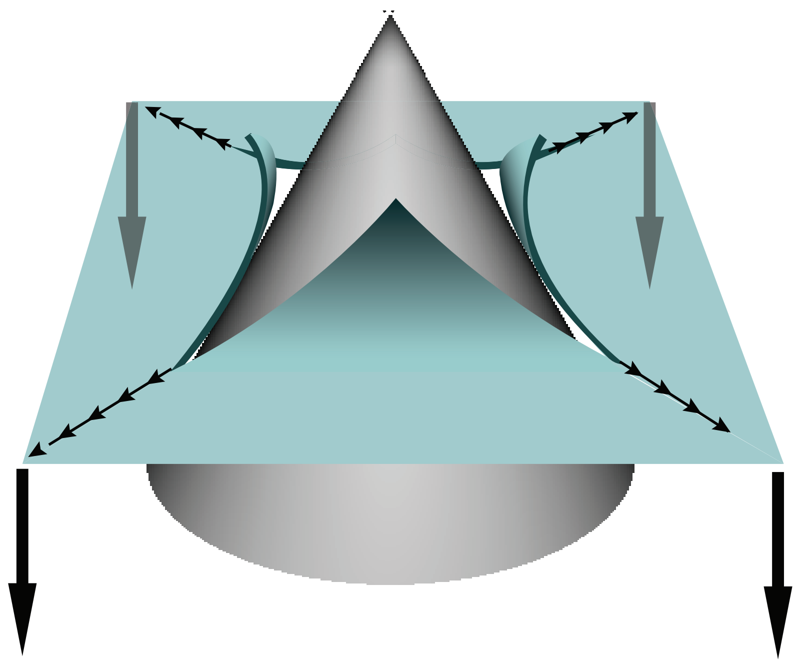
propagation perpendicular to flap limit

H

Comparing theory / experiments



(1st order theory)

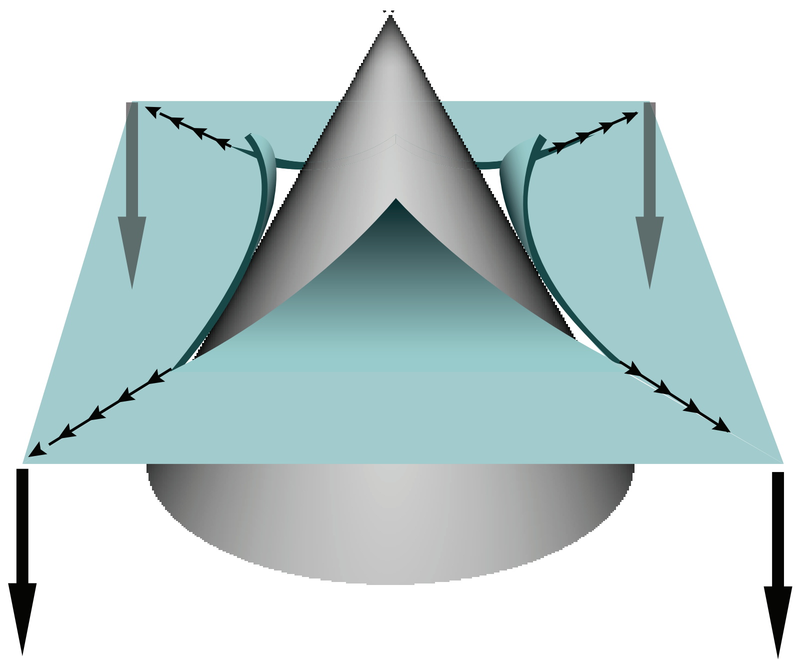


Fuentealba 2016

quasi-static perforation by a cone

4 notches

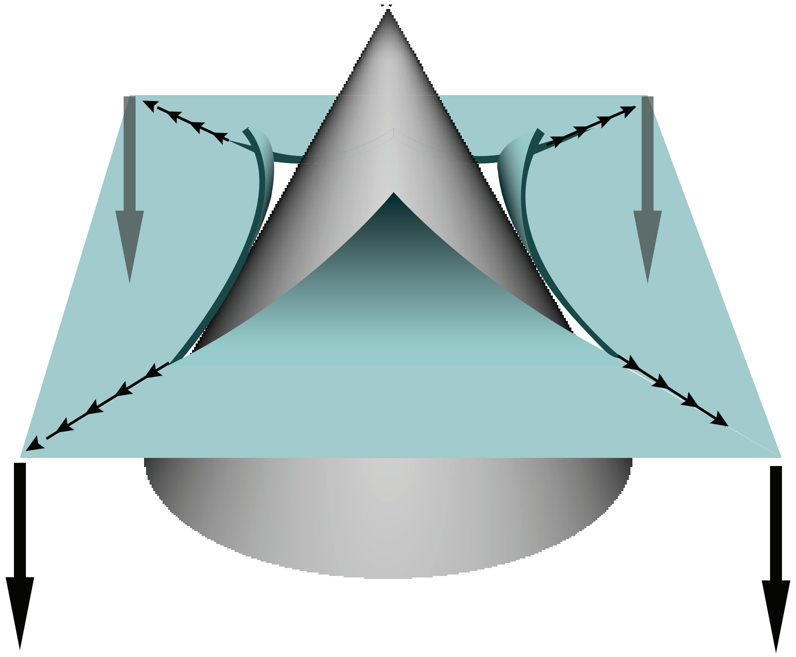
quasi-static perforation by a cone



Fuentealba 2016

3 notches

quasi-static perforation by a cone



Fuentealba 2016

2 notches

Tearing paths

Conclusion:

- applications
- rich, reproducible fracture paths — good test for theory
- a 0th-order (non-linear) framework — lab for fracture mechanics (anisotropy. dynamics?) and for teaching?

Questions for you:

- proper justification of model  (and its limits). 3D? memory?
- often not sufficient. a better set of approximations? (VA)

B.Roman **Int. J.Frac.** 2013

a lot to be done…