# *BAYESIAN HIERARCHICAL MODELS FOR EXTREME EVENT ATTRIBUTION*

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**University of North Carolina and SAMSI**

(Joint with Michael Wehner, Lawrence Berkeley Lab)

Uncertainty Modeling in the Analysis of Weather, Climate and Hydrological Extremes

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# *SAMSI Program on Mathematical and Statistical Methods for Climate and the Earth System 2017-18*

- **Program Leaders**
  - Amy Braverman, Jet Propulsion Laboratory
  - Peter Challenor, Exeter
  - Doug Nychka, National Center for Atmospheric Research
  - Michael Wehner, Lawrence Berkeley National Lab
  - Mary Lou Zeeman, Bowdoin College, Mathematics,
- **Directorate Liaison**
  - Richard Smith, SAMSI
- **Local Scientific Coordinators**
  - Montse Fuentes, North Carolina State University
  - Christopher Jones, University of North Carolina at Chapel Hill
- **National Advisory Committee Liaisons**
  - Habib Najm, Sandia National Laboratory
  - Michael Stein, University of Chicago

# *Activities*

- **Opening Workshop**
  - August 2017, at SAMSI (Research Triangle Park, North Carolina)
- **Closing (Transition) Workshop**
  - May 2018, at SAMSI
- **Visitors in residence for all or part of program**
- **Postdoc opportunities**
- **Additional workshops**
  - Possible venues LBNL, NCAR, CICS (at NCDC), possible joint workshop with Newton Institute
- **Current proposed topics for working groups:**
  - Reconstructing climate databases using remote sensing data;
  - Global carbon cycle;
  - Parameter estimation in climate models;
  - Data assimilation;
  - Applications of data analytics to climate science;
  - Climate prediction;
  - Climate extremes;
  - Stochastic parameterizations;
  - Climate and health;
  - Applications of dynamical systems and agent-based models to food systems.

# *How To Get Involved*

- **Workshops by invitation or open registration**
  - For most participants, SAMSI will pay travel and hotel costs
  - Registration via www.samsi.info
- **Visitor positions open for application**
  - Travel and accommodation expenses
  - For some participants, partial salary support
  - Email rls@samsi.info
- **Postdoc applications: closing date in December 2016**
  - Please encourage qualified applicants to apply!
  - Application details on www.samsi.info
- **Working group participation is open to all**
  - Webex

# *Objectives of This Talk*

- Two problems
  - *How to "attribute" extreme events to anthropogenic or natural causes (Fraction of Attributable Risk; Stott, Stone, Allen 2004 and much since))*
  - *Projection of future extreme events (Christidis, Jones, Stott 2014)*
- The specific idea: develop a method that can be applied to archived climate model runs, without the need for intensive model runs tailored to a specific event
  - *A caveat: the results are only as good as the models that generate them – absence of evidence (of an anthropogenic effect) is not evidence of absence*

# *The Method of Pall et al. (Nature, 2011)*

- Pall et al. proposed a simpler method based on counting of extreme events in a large ensemble of "several thousand model runs" (climateprediction.net)
- The method seems effective if you have a large ensemble and the probabilities are not too small
- However, power calculations show that the method could become extremely data intensive if the estimated probabilities are truly small

## Power Calculation:

Sample size required to distinguish two event probabilities in a test of size 0.05 at power 0.8.

| Null Probability | Ratio of Probabilities 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|
| 0.05 | 422 | 71 | 31 | 18 | 11 |
| 0.025 | 880 | 144 | 67 | 41 | 28 |
| 0.01 | 2,239 | 384 | 170 | 104 | 73 |
| 0.001 | about 23,000 | 3,863 | 1,728 | 1,057 | 743 |

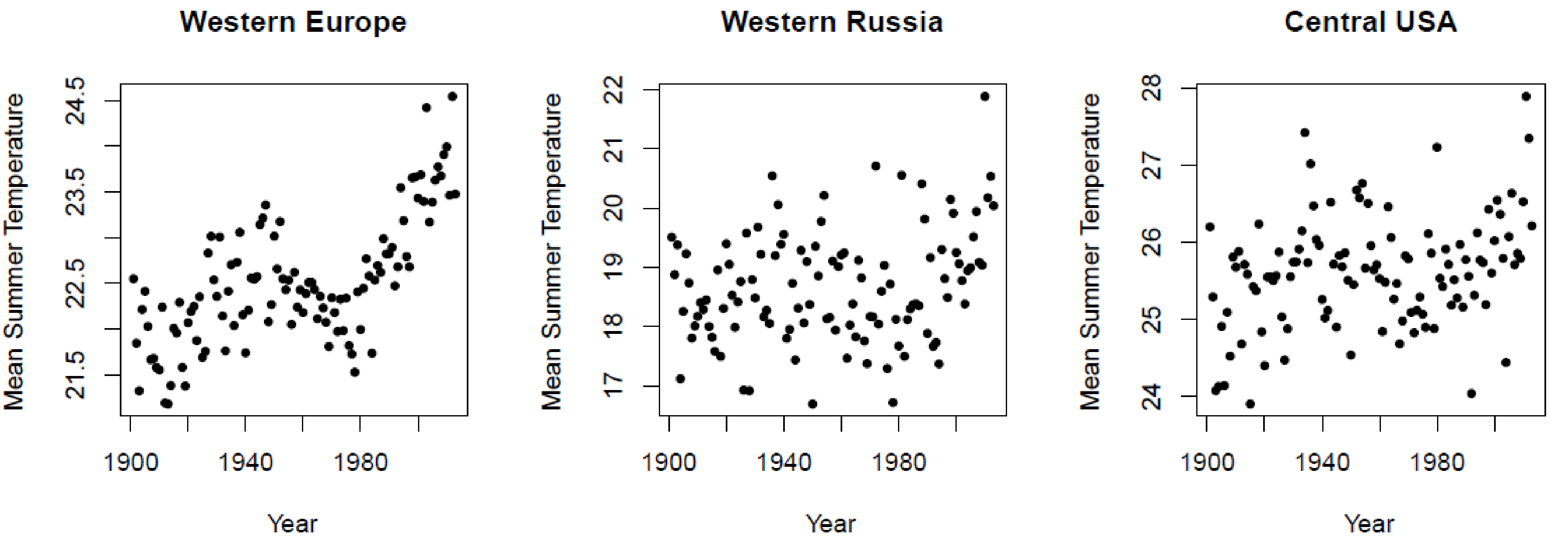
Highlighted cases correspond to two versions of the analysis by Pall *et al.*, and the probability values given in Stott, Stone and Allen (2004)

Conclusion: the method could become *extremely* data intensive

# *Data*

- Observational data from CRU TS 3.22 – land surface monthly average temperatures ($^oC$) on $0.5^o x 0.5^o$ grid boxes, aggregated to JJA over
  - Europe: spatial averages over $10^oW$-$40^oE$, $30^oN$-$50^oN$
  - Russia: spatial averages over $30^oE$-$60^oE$, $45^oN$-$65^oN$
  - Central USA: spatial averages over $90^oW$-$105^oW$, $25^oN$-$45^oN$
- Climate model data from CMIP5
  - 40 climate models
  - 142 total runs "historical"
  - 60 total runs "historical natural"
  - 74 total runs rcp8.5
  - Same spatial regions as observational data

# Observational Data



# Approach for a Single Series

*Generalized Extreme Value Distribution (GEV) with covariates to represent trend*

$$
\begin{aligned}
\Pr\{Y_t \le y\} &= \exp\left[-\left\{1+\xi_t\left(\frac{y-\mu_t}{\sigma_t}\right)\right\}_+^{-1/\xi_t}\right], \\
\mu_t &= \beta_0 + \sum_{k=1}^{K} \beta_k x_{kt}, \\
\sigma_t &= \sigma_0, \\
\xi_t &= \xi_0,
\end{aligned}
$$

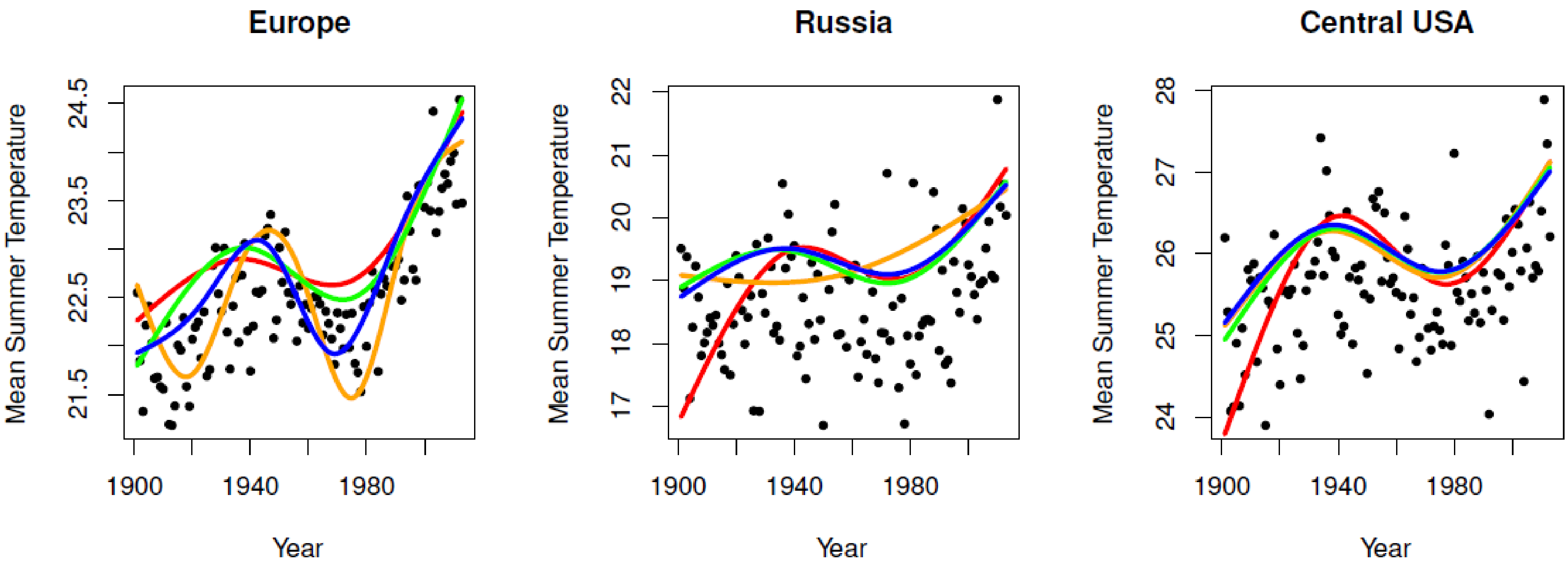
- *Peaks over threshold* approach: fit GEV to exceedances over threshold $u$, treat $Y_t < u$ as censored
- $u$ chosen as one of 75th, 80th or 85th percentile
- Covariates $\{x_{kt},\ 1 \le k \le K\}$ chosen to represent spline basis functions
- $K$ chosen by AIC, no formal selection of threshold but run different thresholds for comparison
- *Bayesian predictive analysis: use MCMC to calculate a posterior density for the probability of crossing a given high level in a given year*

# Model Selection

| Region | Threshold Percentile | | | |
|---|---|---|---|---|
| | 70% | 75% | 80% | 85% |
| Europe | 5 | 3 | 6 | 3 |
| Russia | 3 | 3 | 2 | 3 |
| CentUSA | 3 | 3 | 3 | 3 |

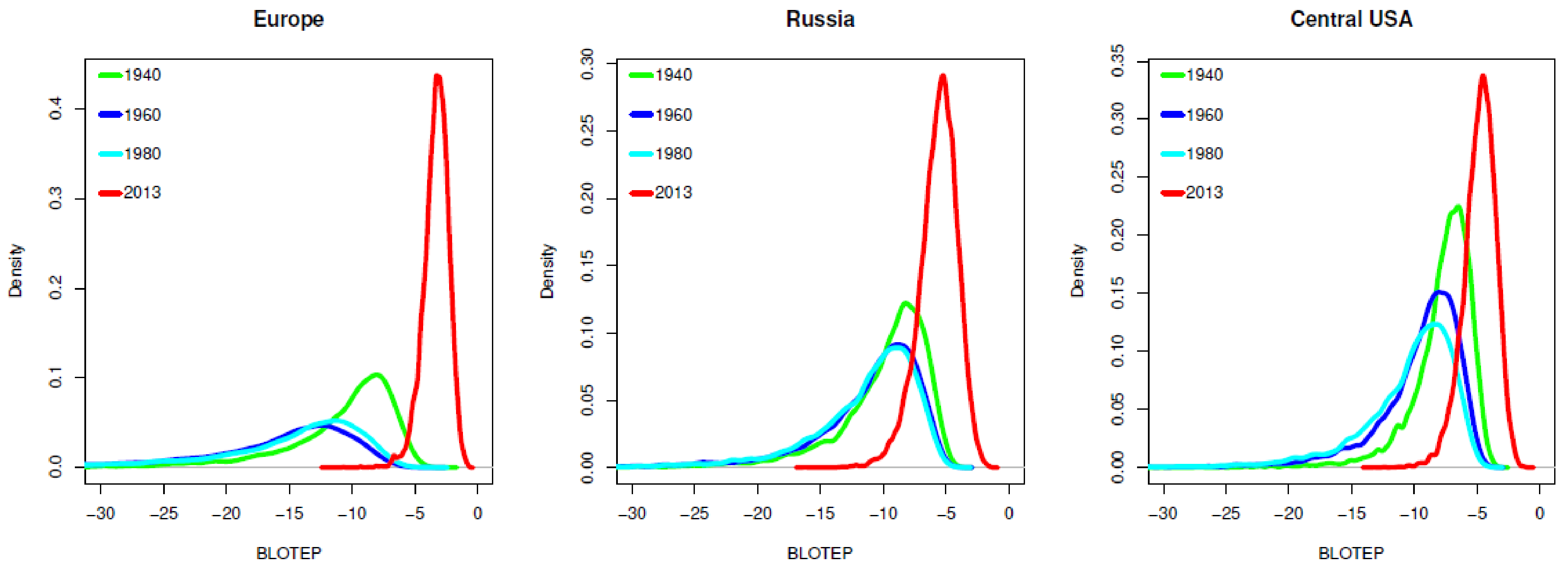
Table 2: Best value of $K$ by BIC for each region and threshold

# Observational Data with Fitted 80th Percentile for Each Year (four curves represent 70%, 75%, 80%, 85% thresholds)



# Posterior Densities for Exceedance Probabilities

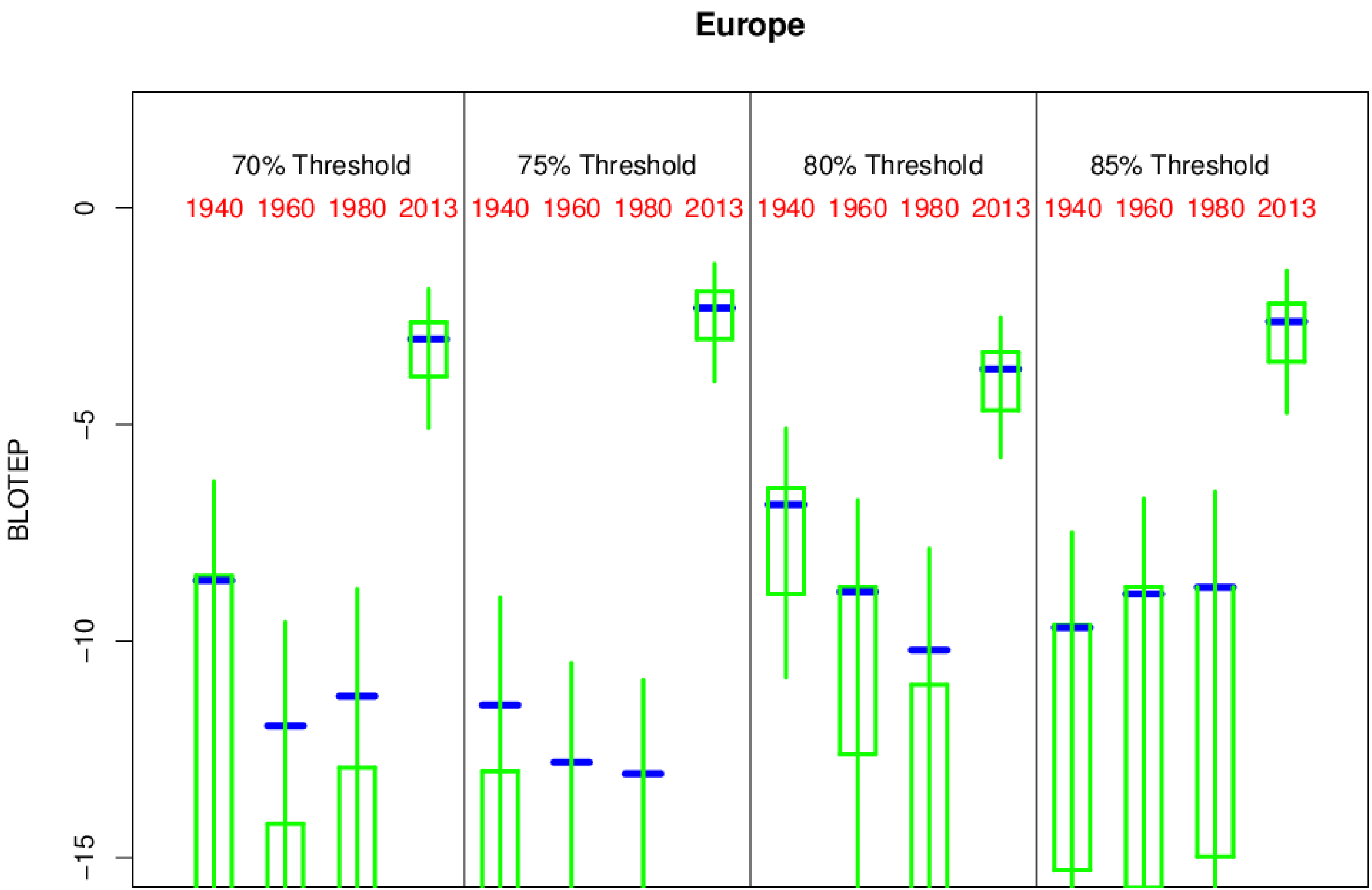
Compute posterior density for the ***B***inary ***LO***g ***T***hreshold ***E***xceedance ***P***robability of the extreme value of interest in each of 4 years (80% threshold)



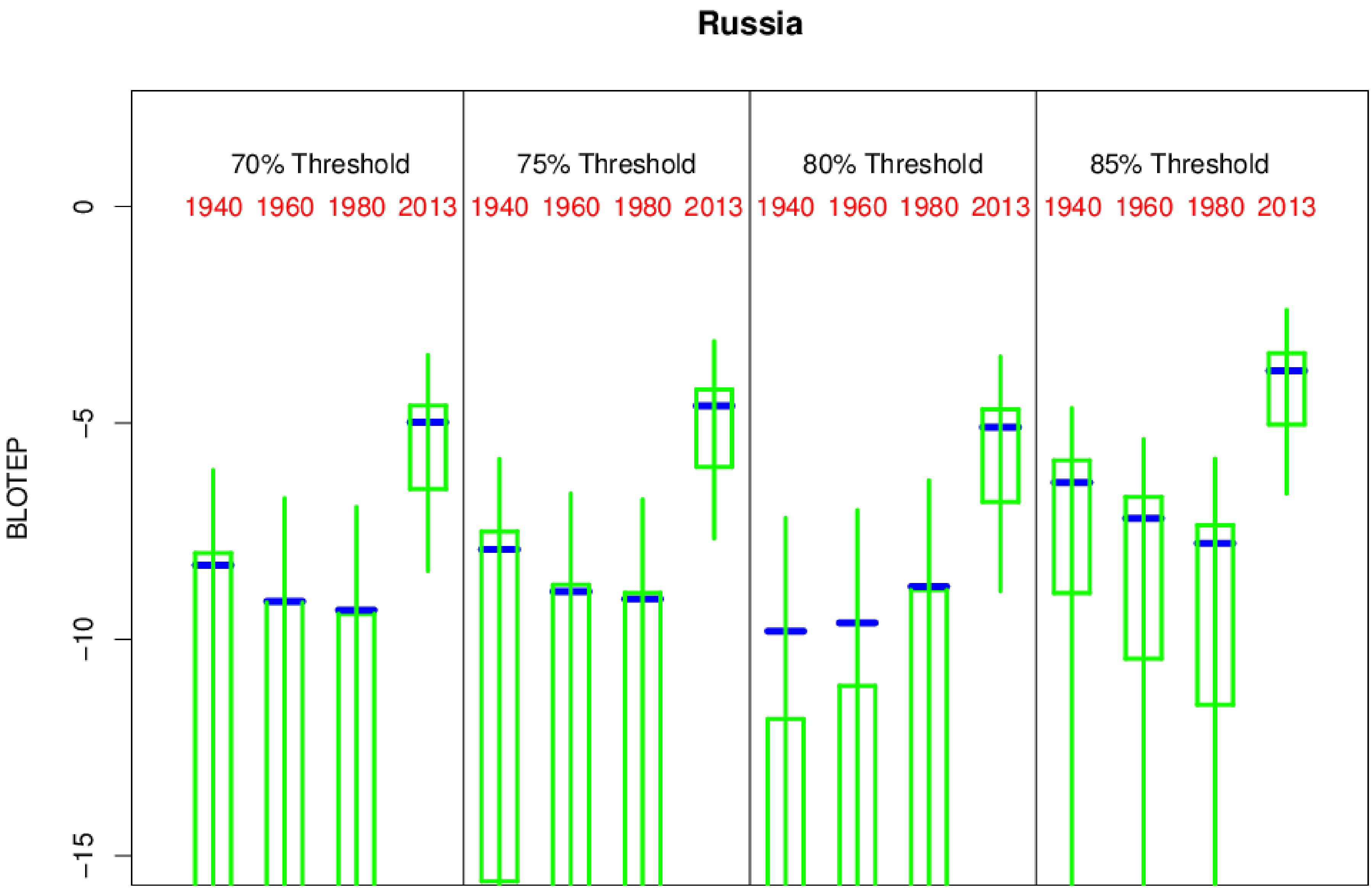
# *Question for Discussion*

Even if we have a posterior distribution for the parameter of interest, how do we display the results?

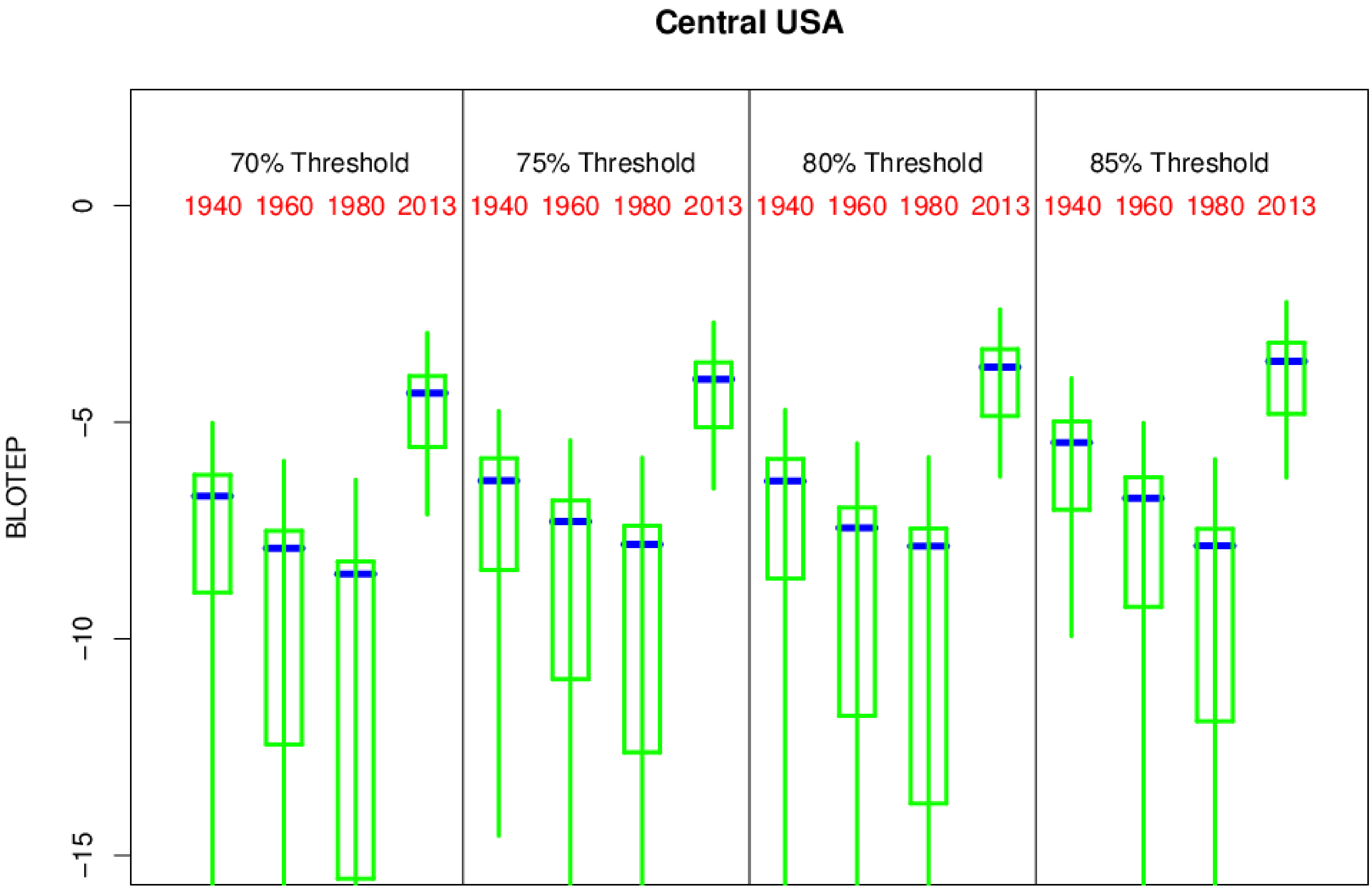
# Sensitivity to the Threshold I



# *Sensitivity to the Threshold II*



# Sensitivity to the Threshold III

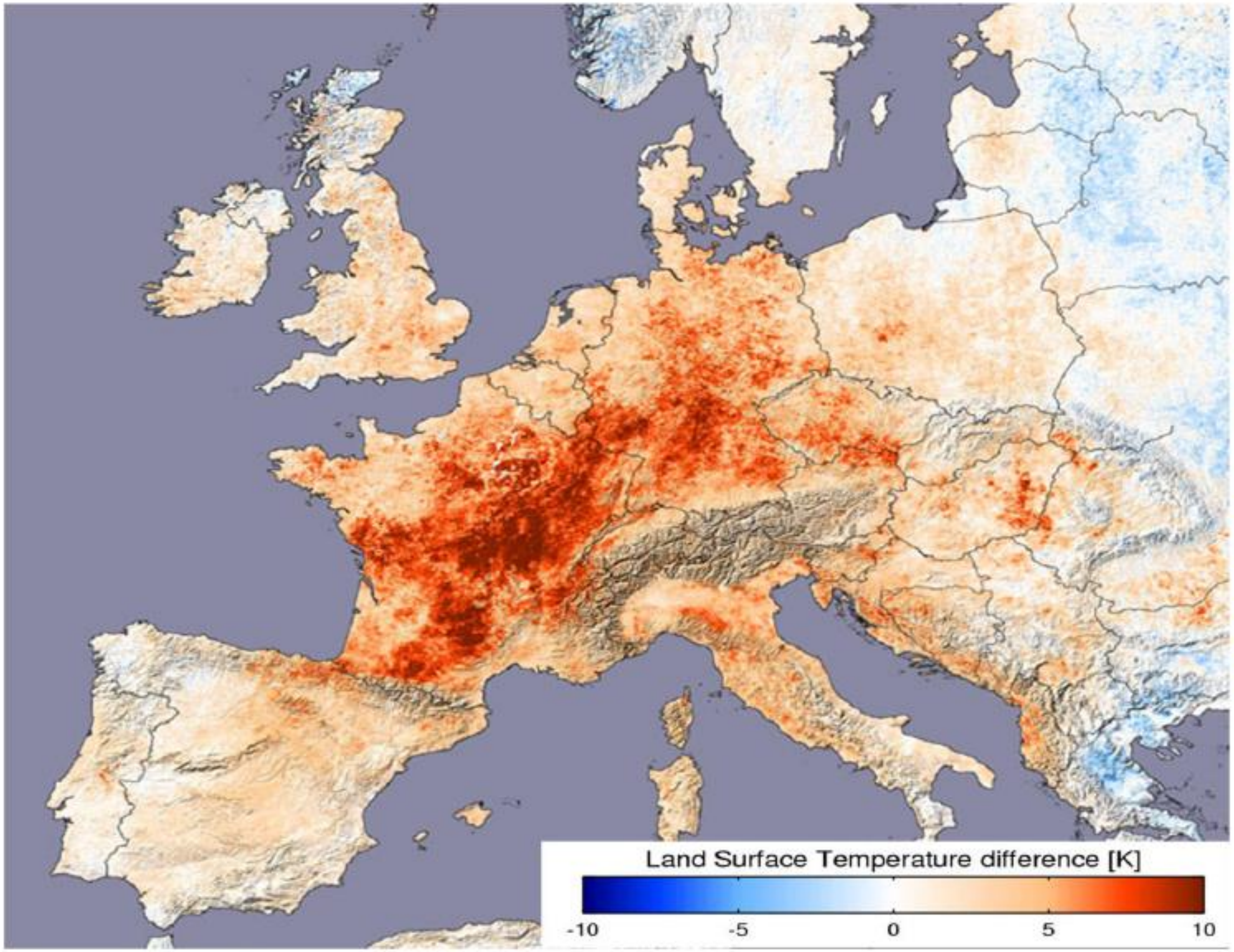


# *Question for Discussion*

Is it more meaningful to compute these probabilities using all data including the extreme event of interest, or should we “stop the clock” before that observed event?

(Results presented here used the first method)

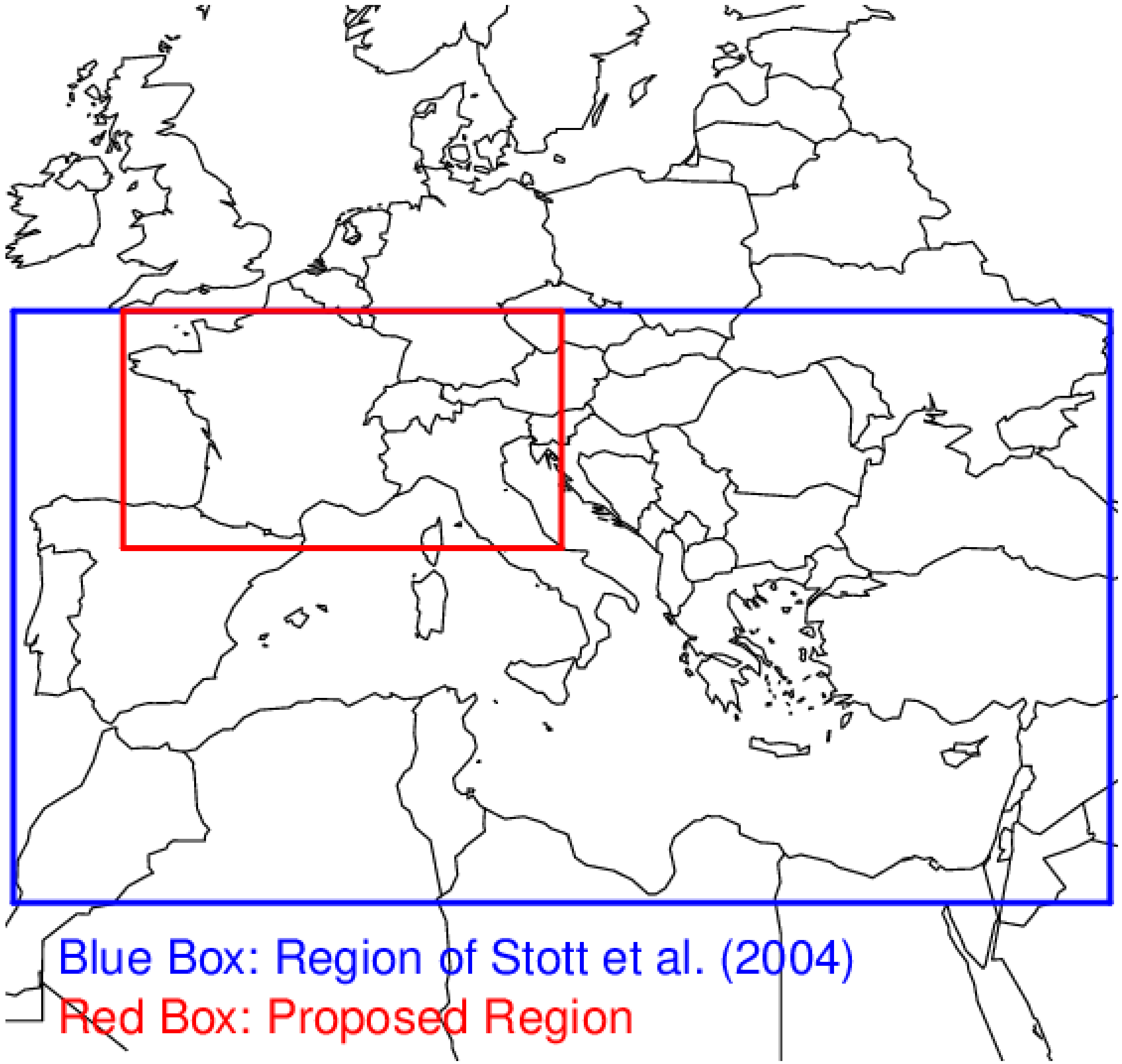
# *Other Datasets*



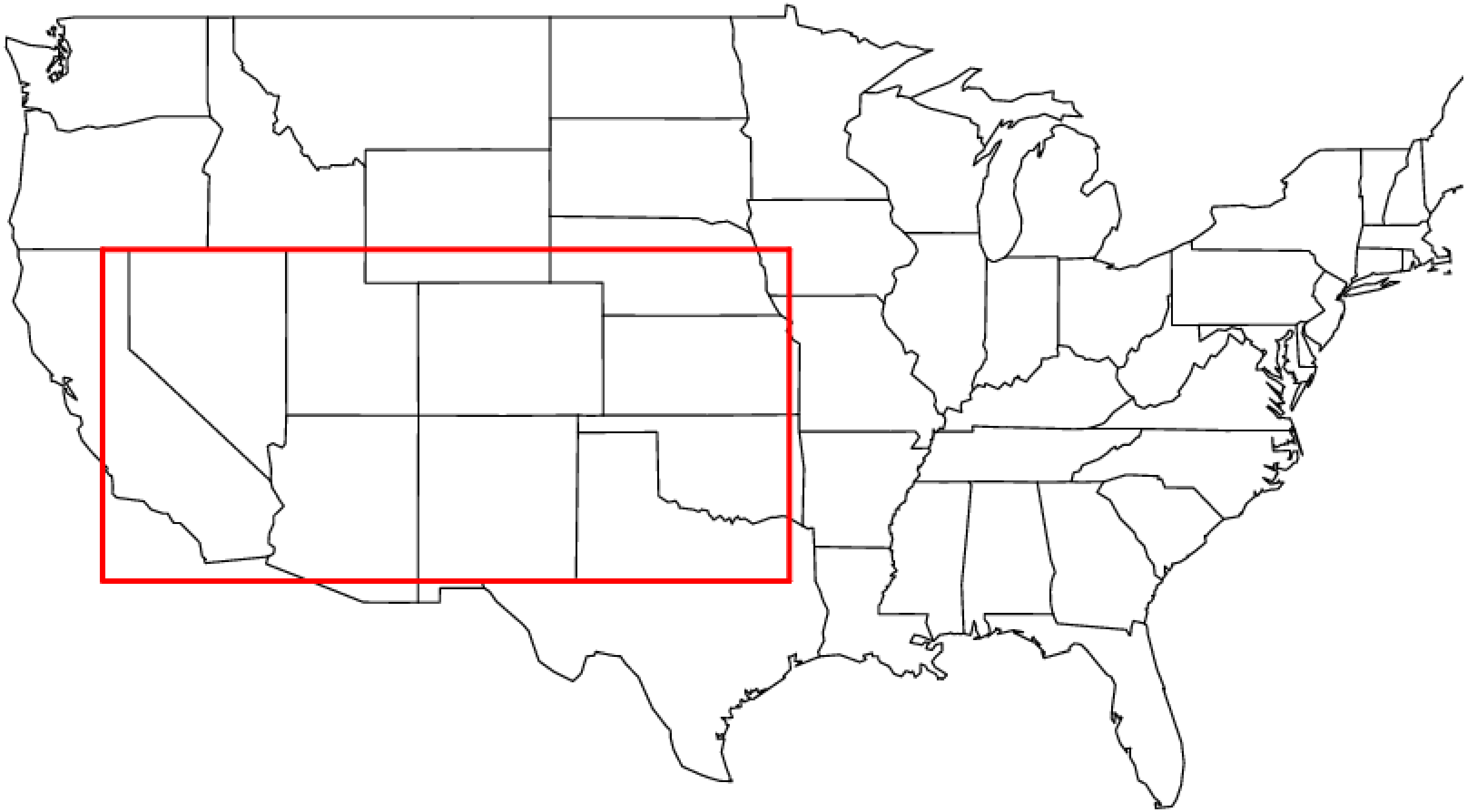

**European temperatures in early August 2003, relative to 2001-2004 average**

**From NASA's MODIS - Moderate Resolution Imaging Spectrometer, courtesy of Reto Stöckli, ETHZ**

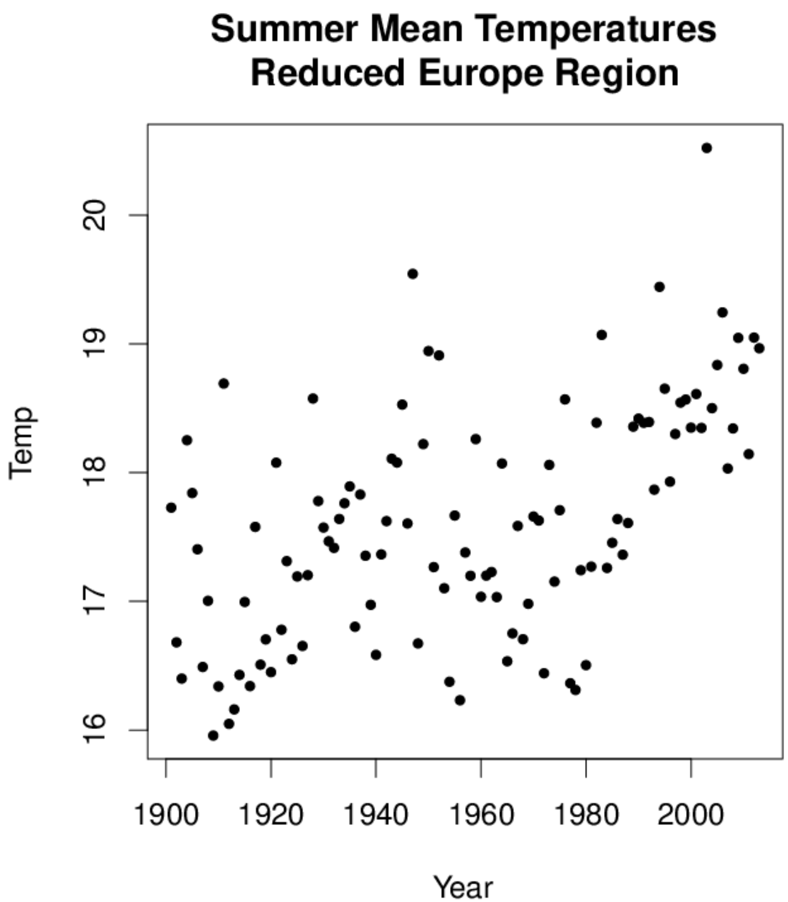
# Other Datasets – Europe Temperatures

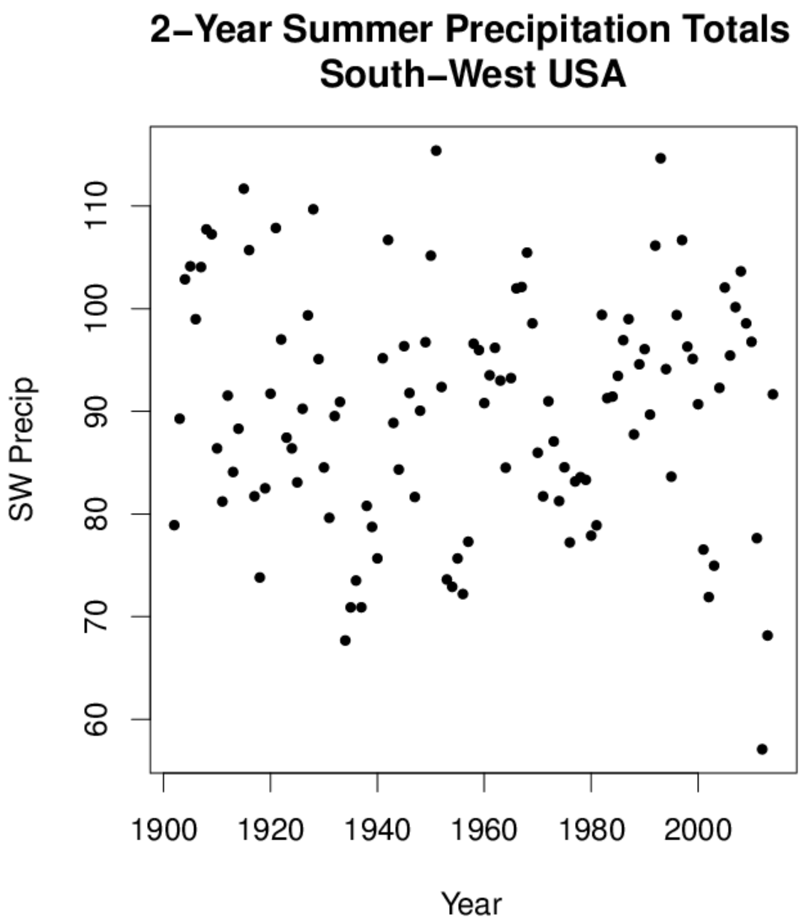


# Other Datasets – US Summer Drought

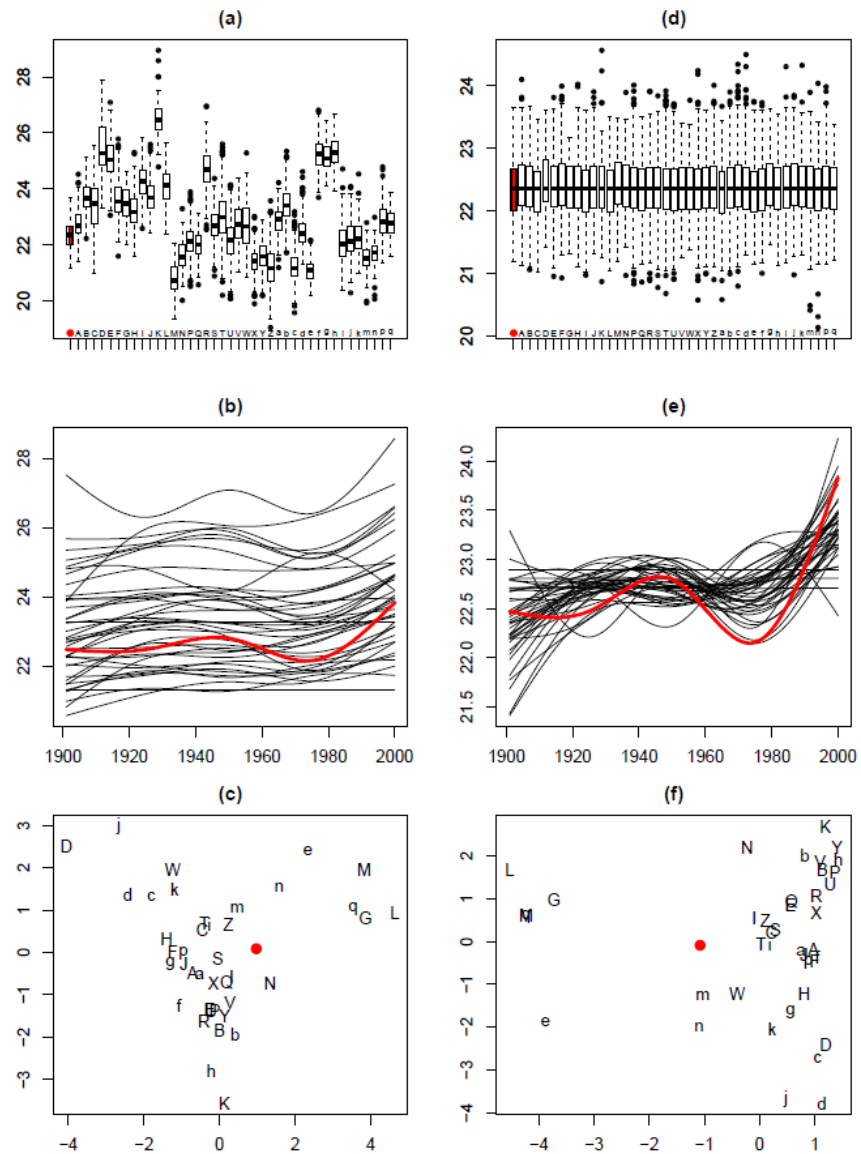


# Other Datasets – New Time Series

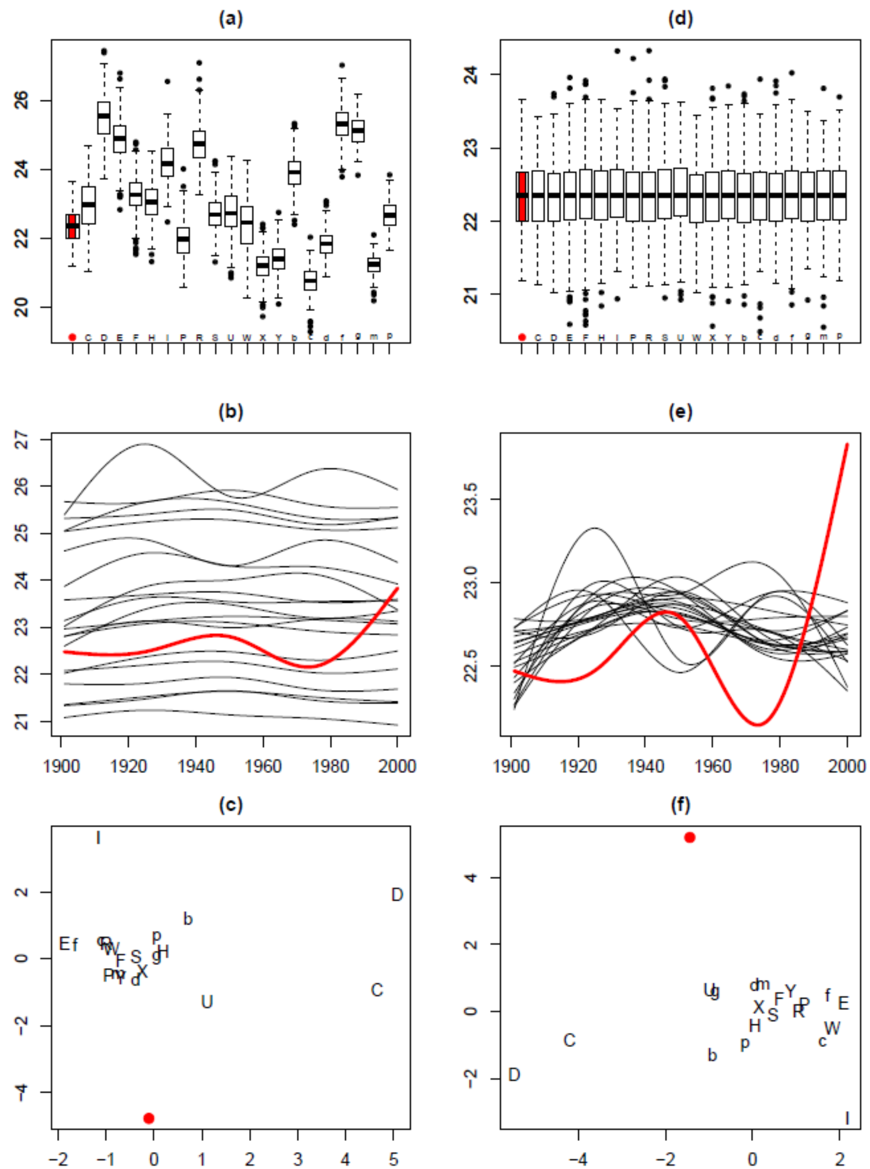


# Comparing Observations and Models I: All-forcings models (Europe)



# Comparing Observations and Models II: Natural-forcings models (Europe)



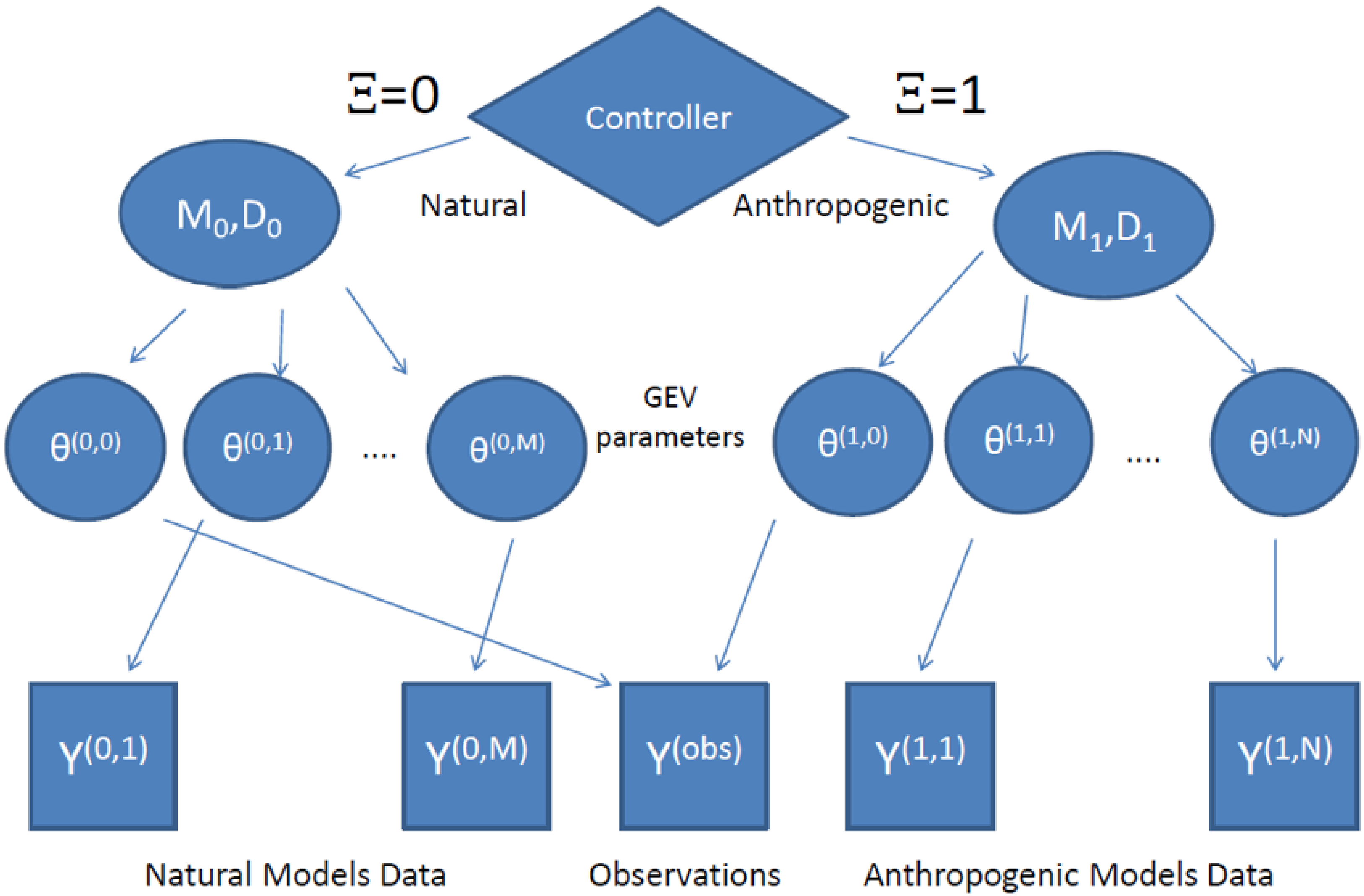
# *Summary So Far*

- Bayesian approach to extreme value modeling allows us to construct posterior distributions for a high-threshold exceedance based on observational data
- Clear differences in 2013 against any of 1940, 1960, 1980 but cannot *attribute* this to human influence
- Comparison of observational and model data suggests a *population of models* approach – naturally leads into **hierarchical modeling**

# *Outline of Hierarchical Model*

- $\Theta_j$: parameter vector for model j, j=0,…,$N_M$ where $N_M$ is the number of models (j=0: observational data)
- Prior distribution: the $\Theta_j$'s are IID from a multivariate normal distribution with mean M and precision matrix D (extension: $\psi D$ for j=0, possibly $\psi \neq 1$)
- Fit by Markov Chain Monte Carlo (MCMC)
- Do this once for anthropogenic forcings, again for natural forcings
- Extension: extrapolate to future decades under your favorite rcp scenario – basis for future projections of extreme event probabilities

# Proposed Hierarchical Model



# Bayesian Statistics Details

## Model Specification

- $(M_1, D_1) \sim WN_q(A, m, M^*, F)$, Wishart-Normal prior with density
  $\propto |D_1|^{(m-q)/2} \exp\left[-\frac{1}{2}\mathrm{tr}\left\{D_1\left(A + F(M_1 - M^*)(M_1 - M^*)^T\right)\right\}\right].$
- Given $M_1, D_1$, $\theta^{(1,0)}, ..., \theta^{(1,N)}$ are IID $\sim N_q(M_1, D_1^{-1})$.
- Given $\theta^{(1,j)}$, $\mathbf{Y}^{(1,j)}$ generated by GEV with parameters $\theta^{(1,j)}$ ($\mathbf{Y}^{(\mathrm{obs})}$ for $j = 0$, if $\Xi = 1$)
- Similar structure for $M_0, D_0$ etc.
- We can expand this model by defining $\theta^{(1,0)} \sim N_q(M_1, (\psi D_1)^{-1})$ where $\psi$ represents departure from exchangeability ($\psi = 1$ is exchangeable). However, $\psi$ is not identifiable — we can only try different values as a sensitivity check.

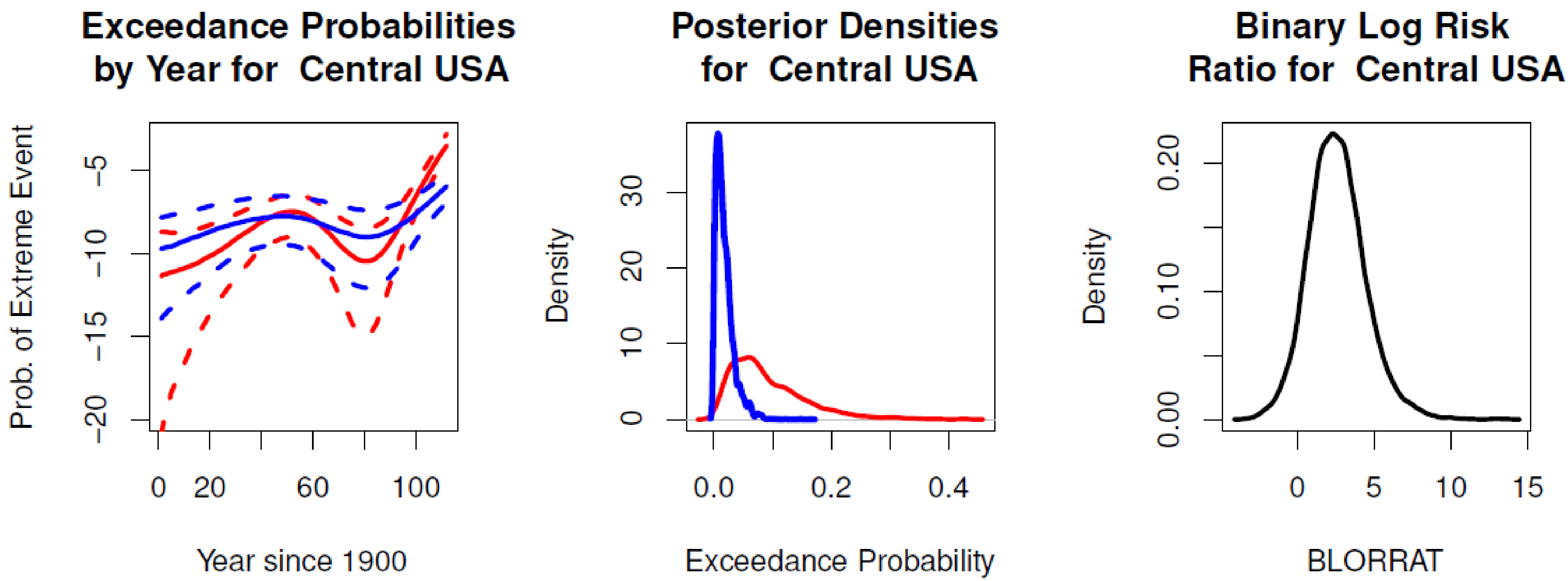
## Computation

- $(M_1, D_1) \mid \theta^{(1,1)}, ..., \theta^{(1,N)} \sim WN_q(\tilde{A}, \tilde{m}, \tilde{M}^*, \tilde{F})$, where $\tilde{m} = m + N, \tilde{F} = F + N, \tilde{M}^* = \left(FM^* + \sum_{j=1}^N \theta^{(1,j)}\right)/\tilde{F}, \tilde{A} = A + FM^*M^{*T} + \sum_{j=1}^N \theta^{(j)}\theta^{(j)T} - \tilde{F}\tilde{M}^*\tilde{M}^{*T}$.
- Metropolis update for $\theta^{(1,1)}, ..., \theta^{(1,N)}$ given $M_1, D_1$ and $\mathbf{Y}$'s
- Metropolis update for $\theta^{(1,0)}$ based on conditional density

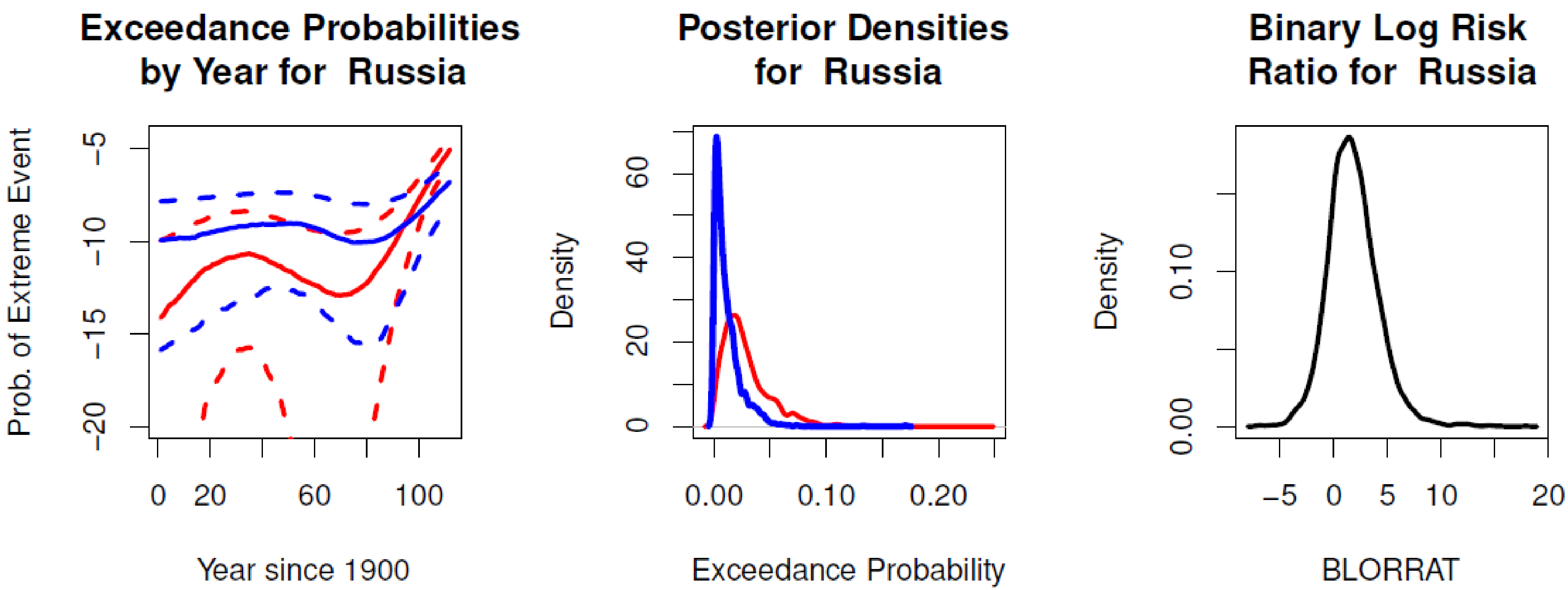
  $$\exp\left\{-\frac{\psi}{2}\left(\theta^{(1,0)} - M_1\right)^T D_1 \left(\theta^{(1,0)} - M_1\right)\right\} \cdot L\left(\theta^{(1,0)}; \mathbf{Y}^{(\mathrm{obs})}\right)$$

  where $L$ is likelihood for $\theta^{(1,0)}$ given data $\mathbf{Y}^{(\mathrm{obs})}$ and $\Xi = 1$
- Similar updates for $\Xi = 0$ side of picture; up to 1,000,000 iterations

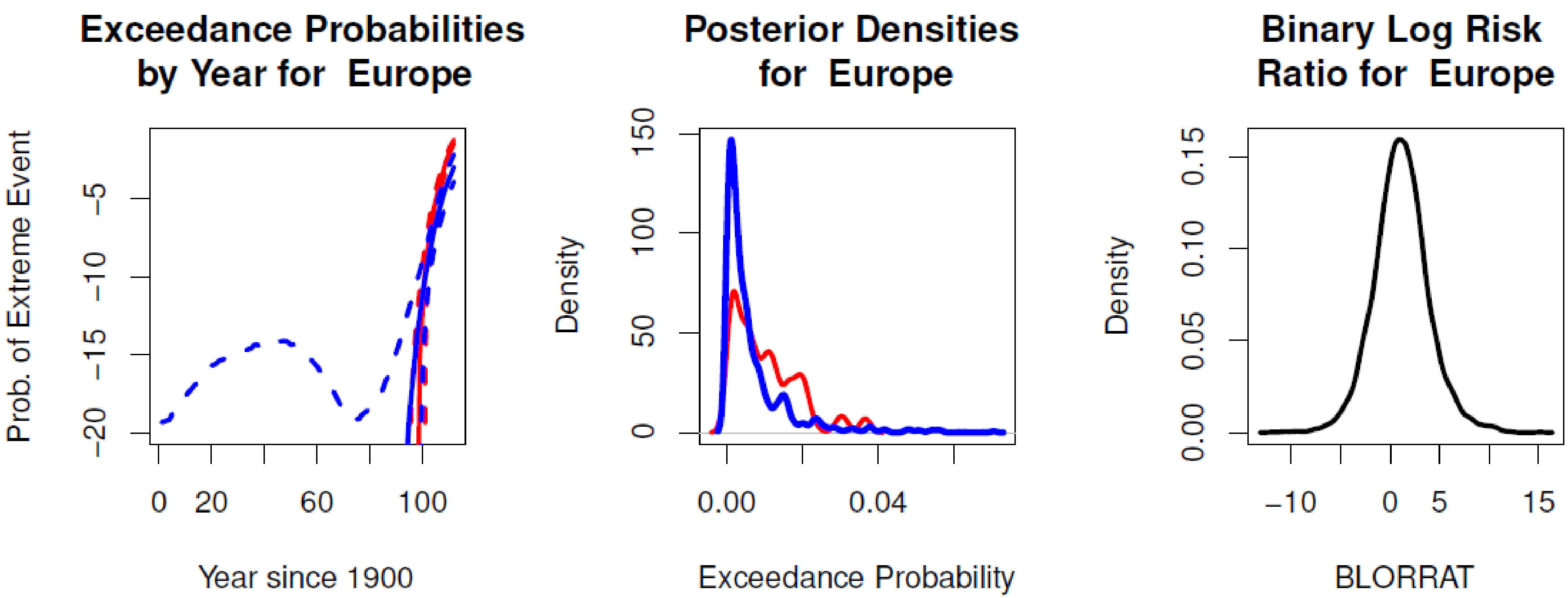
# *Posterior Densities (Central USA)*



# *Posterior Densities (Russia)*



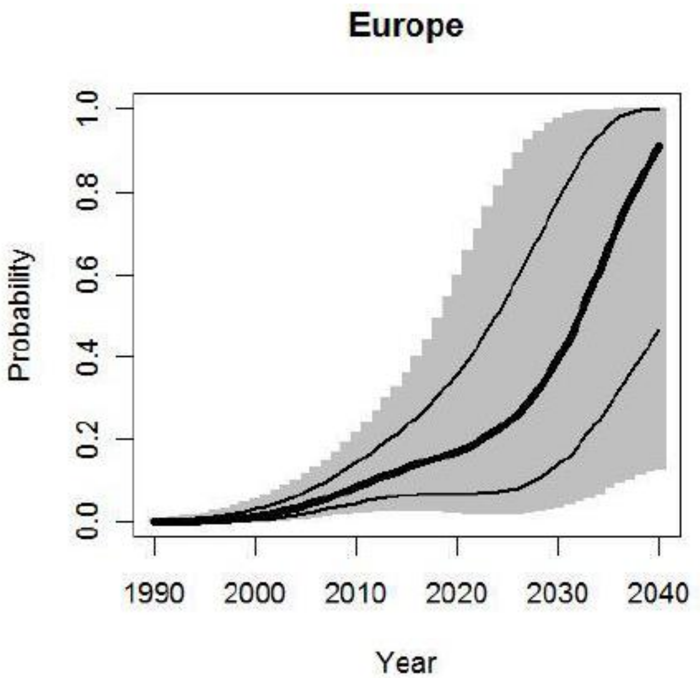
# *Posterior Densities (Europe)*

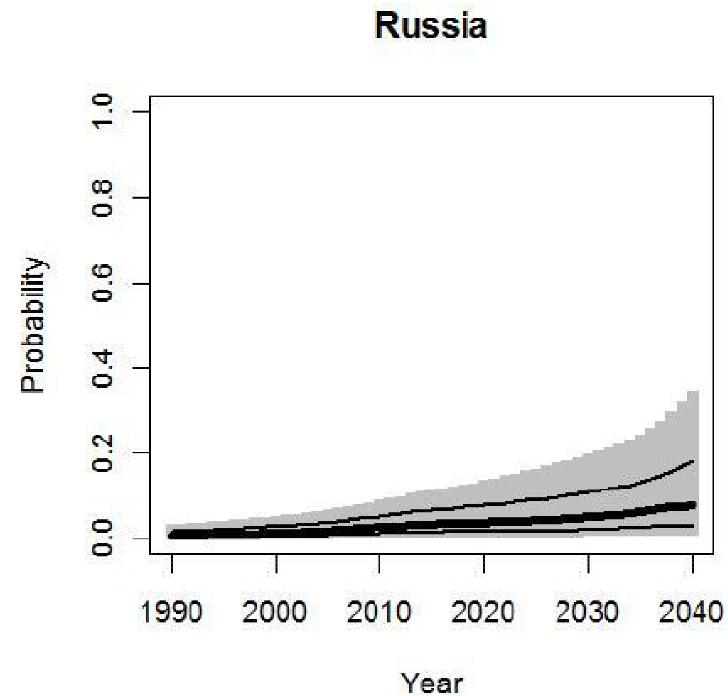


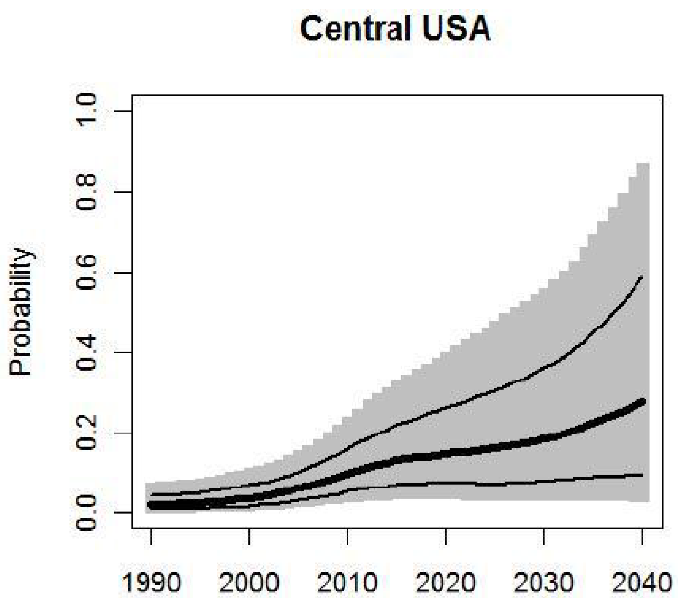
# Quantiles of the Posterior Density of the Binary Log Risk Ratio (BLORRAT)

| Quantile | 5% | 10% | 25% | 33% | 50% | 66% | 75% | 90% | 95% |
|---|---|---|---|---|---|---|---|---|---|
| Europe | 0.11 | 0.22 | 0.68 | 1.03 | 2.22 | 4.34 | 7.03 | 23.15 | 56.24 |
| Russia | 0.30 | 0.51 | 1.18 | 1.64 | 3.07 | 5.65 | 8.85 | 26.95 | 57.27 |
| CentUSA | 0.85 | 1.26 | 2.55 | 3.36 | 5.48 | 9.18 | 12.79 | 30.97 | 54.84 |

# *Changes in Projected Extreme Event Probabilities Over Time*




Central Solid Curve: Posterior Median
Thin Outer Curves: Posterior Quartiles
Outer Limits of Shaded Region:
Posterior 10$^{th}$ and 90$^{th}$ percentiles

# *Future Work*

- This is still very much a "vanilla MCMC" algorithm: apart from needing more tuning, possibilities for treating K or the threshold as unknowns, alternatives to multivariate normal (e.g. Dirichlet process) for prior distributions
- Bivariate extensions, e.g. joint distributions of temperature and precipitation
- Fully spatial model – possibility of hierarchical models based on max-stable processes – (need a "full likelihood" – problematic but see Wadsworth and Tawn (2014), Thibaud et al. (2016, in review) for recent developments)