The Elementary Particle Angular Momentum Controversy: lessons from Laser Optics

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- **Fundamental structure of a relativistic dynamical theory; massless particles**

- **The elementary particle angular momentum controversy in Quantum Field Theory**

- **General classical Maxwell fields**

- **Lessons from Laser Optics; paraxial fields**

GENERAL APPROACH TO SPIN IN RELATIVISTIC QUANTUM MECHANICS

Fundamental: the **Poincare Group**: 10 generators:

Time translation $\Leftrightarrow$ Energy P^0

Space translation $\Leftrightarrow$ Momentum $\boldsymbol{P}$

Rotations $\Leftrightarrow$ AM $J_i = -\frac{1}{2}\epsilon_{ijk}M^{jk}$

Lorentz boosts $\Leftrightarrow K_i = M^{0i}$

Two **invariant** operators to define type of particle i.e whose eigenvaues specify its intrinsic properties:

$$P_\mu P^\mu = m^2$$

Pauli-Lubanski $W_\mu \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} M^{\nu\rho} P^\sigma$

$$W_\mu W^\mu = m^2 s(s+1) \quad s = 0,\, 1/2,\, 1 \cdots\cdot$$

For **massless particles** $W_\mu W^\mu$ does **not** fix s.

The spin vector in a relativistic theory: $s_i \equiv \frac{1}{m} W^i$

Acting on particle AT REST

$$[s_j, s_k]\, |m; \boldsymbol{p} = 0\rangle = i\epsilon_{jkl}\, s_l\, |m; \boldsymbol{p} = 0\rangle$$

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For a proton, for example, $|m; \boldsymbol{p}, \; s_z = 1/2\rangle$ MEANS it has spin along the Z-axis in its rest frame.

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Clearly none of this works for a massless particle e.g. for a photon.

Massless particles

Can label via eigenvalue of helicity.

$$\frac{\boldsymbol{J} \cdot \boldsymbol{P}}{|\boldsymbol{P}|} |\boldsymbol{p}, \lambda\rangle = \lambda \, |\boldsymbol{p}, \lambda\rangle$$

It can't be another Poincare invariant, but it is invariant in the subspace of massless states i.e. **only** when operating on massless states!

For photons $\lambda = \pm 1$.

Note that there is no state with $\lambda = 0$, which is equivalent to the classical statement that in a plane wave E and B are perpendicular to the momentum of the wave.

Summary: massless particles

From fundamental point of view of the Poincare Group there does NOT exist a spin vector with the standard expected commutation rules i.e. with

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The only genuine rotation operator is the **helicity**, which generates rotations about the momentum.

II: QUANTUM FIELD THEORY: THE CONTROVERSY

NB All fields in this Section are OPERATORS

In **Quantum Field Theory** one can start with the expressions from **Classical E and M Textbooks**:

Momentum density proportional to Poynting vector,

$$\boldsymbol{p}_{\,\mathsf{poyn}} = \boldsymbol{E} \times \boldsymbol{B}$$

Angular momentum density due to Belinfante

$$\boldsymbol{j}_{\,\mathsf{bel}} = \boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B}).$$

Has structure of an orbital AM, *i.e.* $\boldsymbol{r} \times \boldsymbol{p}$, but is the total photon angular momentum density.

In **Quantum Field Theory** more conventionally one starts with a Lagrangian, then from Noether's theorem obtains the **Canonical** densities which have a spin plus orbital part

$$\boldsymbol{j}_{\text{can}} = [\boldsymbol{l}_{\text{can}} + \boldsymbol{s}_{\text{can}}]$$

where the canonical densities are

$$\boldsymbol{s}_{\text{can}} = \boldsymbol{E} \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{l}_{\text{can}} = E^i(\boldsymbol{x} \times \boldsymbol{\nabla})A^i$$

but, clearly, each term is gauge non-invariant.

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Note that $\boldsymbol{p}_{\text{can}} = E^i \boldsymbol{\nabla} A^i$ so that $\boldsymbol{l}_{\text{can}} = \boldsymbol{x} \times \boldsymbol{p}_{\text{can}}$, as it should be for an orbital A.M.

The beginning of the controversy

Chen, Lu, Sun, Wang and Goldman: 2008

"We address and solve the long-standing gauge-invariance problem of the nucleon spin structure. Explicitly gauge-invariant spin and orbital angular momentum operators of photons and gluons are obtained. This was previously thought to be an impossible task ...."

THE CHEN et al PROCEDURE

Introduce fields $\boldsymbol{A}_{\text{pure}}$ and $\boldsymbol{A}_{\text{phys}}$, with

$$\boldsymbol{A} = \boldsymbol{A}_{\text{pure}} + \boldsymbol{A}_{\text{phys}}$$

where

$$\boldsymbol{\nabla} \times \boldsymbol{A}_{\text{pure}} = \boldsymbol{0}, \quad \text{and} \quad \boldsymbol{\nabla} \cdot \boldsymbol{A}_{\text{phys}} = 0$$

Exactly the same fields as in the Helmholz decomposition into longitudinal and transverse components!

$$\boldsymbol{A}_{\text{pure}} \equiv \boldsymbol{A}_{\|} \qquad\qquad \boldsymbol{A}_{\text{phys}} \equiv \boldsymbol{A}_{\perp}.$$

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Exactly the same fields as in the Helmholz decomposition into longitudinal and transverse components!

$$\boldsymbol{A}_{\text{pure}} \equiv \boldsymbol{A}_{\parallel} \qquad\qquad \boldsymbol{A}_{\text{phys}} \equiv \boldsymbol{A}_{\perp}.$$

Chen et al then obtain

$$\boldsymbol{J}_{\text{chen}} = \underbrace{\int \mathrm{d}^3x\, \boldsymbol{E} \times \boldsymbol{A}_{\perp}}_{\boldsymbol{S}_{\text{chen}}} + \underbrace{\int \mathrm{d}^3x\, E^i (\boldsymbol{x} \times \boldsymbol{\nabla}) A^i_{\perp}}_{\boldsymbol{L}_{\text{chen}}}$$

and since $\boldsymbol{A}_{\perp}$ and $\boldsymbol{E}$ are unaffected by gauge transformations, they claim to achieve the impossible.

But the **Chen et al** operators are exactly the same as those discussed in the textbook of **Cohen-Tannoudji, Dupont-Roc and Grynberg** and studied in detail by **van Enk and Nienhuis: 1994**, who state:
"Therefore we may write

$$[S_i, S_j] = 0. \quad !!!!!!!!!!!!!$$

This implies that $\boldsymbol{S}$ does NOT generate rotations of the polarization of the field"

Moreover van Enk-Nienhuis show that acting on a photon state with momentum $\boldsymbol{k}$ the eigenvalues of S_z are $\pm\frac{\hbar k_z}{k}$ i.e. are NOT quantized.

They state: "Thus $\boldsymbol{S}$ CANNOT be interpreted as spin angular momentum. .... this result does not seem to have been noticed before."

Consequently, van Enk-Nienhuis write "**spin**" and "**orbital angular momentum"** in inverted commas.

Chen et al:2008 CITE van Enk-Nienhuis:1994

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AMAZINGLY, NONE OF THE PARTICLE PHYSICISTS SEEM TO HAVE NOTICED THESE WARNINGS EITHER!!!

A KEY CHALLENGE FOR PARTICLE PHYSICS: UNDERSTAND THE A.M. OF THE GLUON

BETTER FIRST UNDERSTAND THE PHOTON!

WHICH VERSION OF A.M. IS MORE CORRECT OR MORE “PHYSICAL”?

CLASSICAL (BELINFANTE) VS CANONICAL VS CHEN et al

$$\boldsymbol{J}_{\text{bel}} = \int \mathrm{d}^3x\, \boldsymbol{j}_{\text{bel}}(x) \qquad \boldsymbol{J}_{\text{can}} = \int \mathrm{d}^3x\, \boldsymbol{j}_{\text{can}}(x)$$

$$\boldsymbol{J}_{\text{bel}} = \boldsymbol{J}_{\text{can}} + \text{surface term} = \boldsymbol{J}_{\text{chen}} + \text{surface term}.$$

Classically surface terms $= 0$ if fields vanish at infinity

BUT

QUANTUM FIELDS ARE OPERATORS

What does it mean to say **OPERATORS VANISH AT INFINITY?**

CONCLUDE: AS OPERATORS

$$\boldsymbol{J}_{\text{bel}} \neq \boldsymbol{J}_{\text{can}} \neq \boldsymbol{J}_{\text{chen}}.$$

FIRST ARGUMENT AGAINST $J_{\textbf{bel}}$: PROBLEMS with HELICITY $\mathcal{H} \equiv \boldsymbol{J} \cdot \boldsymbol{P}/|\boldsymbol{P}|$

1) The **Classical E and M Textbook expression due to Belinfante**:

Well known, for classical fields, $\mathcal{H}_{\text{bel}}$ gives zero for circularly polarized plane wave.

Similarly, with operators, $\mathcal{H}_{\text{bel}}$ acting on any quantum state gives zero, because

$$\boldsymbol{j}_{\text{bel}} \cdot \boldsymbol{p} = [\boldsymbol{r} \times (\boldsymbol{E} \times \boldsymbol{B})] \cdot (\boldsymbol{E} \times \boldsymbol{B}) = 0.$$

FIRST ARGUMENT AGAINST $J_{\textbf{bel}}$:
PROBLEMS with HELICITY $\mathcal{H} \equiv \boldsymbol{J} \cdot \boldsymbol{P}/|\boldsymbol{P}|$

2) Compare with the **Canonical and Chen expressions**:

Acting on any superposition of photon states $|\Psi\rangle$

$$\mathcal{H}_{\text{can}}|\Psi\rangle \equiv \boldsymbol{J}_{\text{can}} \cdot \boldsymbol{P}_{\text{can}}/|\boldsymbol{P}_{\text{can}}|\,|\Psi\rangle = \boldsymbol{S}_{\text{can}} \cdot \boldsymbol{P}_{\text{can}}/|\boldsymbol{P}_{can}|\,|\Psi\rangle$$

Can show that acting on a photon state with helicity $\lambda = \pm 1$

$$\mathcal{H}_{\text{chen}}\,|\boldsymbol{k}\,;\,\lambda\rangle = \mathcal{H}_{\text{can}}\,|\boldsymbol{k}\,;\,\lambda\rangle = \hbar\,\lambda\,|\boldsymbol{k}\,;\,\lambda\rangle$$

Quantum Field Theory Summary

- $\boldsymbol{J}_{\text{bel}}$ fails to give correct helicity

- $\boldsymbol{S}_{\text{can}}$ is a genuine A.M., but is not gauge invariant

- $\boldsymbol{S}_{\text{chen}}$ is gauge invariant but not a genuine A.M.

- Eigenvalues of $S_{\text{chen; z}}$ and $L_{\text{chen; z}}$ are not quantized

- Only the **helicities** $\mathcal{H}_{\text{can}} = \mathcal{H}_{\text{chen}}$ are both genuine A.M.s and gauge invariant.

NB Perhaps surprisingly, $\mathcal{H}_{\text{can}}$ IS gauge invariant!

III: GENERAL CLASSICAL FIELDS

NB All fields in this Section are CLASSICAL FIELDS

GENERAL CLASSICAL MAXWELL FIELD: SECOND ARGUMENT AGAINST J_{bel}

Just as for the operator case, $\boldsymbol{J}_{\text{bel}}$, for Classical MAXWELL Fields, fails to express helicity correctly.

1) For later reference a good way to see this is as follows: For the cycle average

$$\langle \boldsymbol{J}_{\text{bel}} \rangle = \frac{1}{2}\left[\int d^3r\,[(\boldsymbol{r}\cdot\boldsymbol{B})\boldsymbol{E}^* - (\boldsymbol{r}\cdot\boldsymbol{E}^*)\boldsymbol{B}] + \text{c.c.}\right]$$

Take general superposition

$$\boldsymbol{B}(\boldsymbol{r}) = \int d^3k\,\boldsymbol{\mathcal{B}}(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{r}}$$

$$(\boldsymbol{r}\cdot\boldsymbol{E}^*) = \int d^3k\,F(\boldsymbol{k})e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}$$

Then

$$\int d^3r\,(\boldsymbol{r}\cdot\boldsymbol{E}^*)\boldsymbol{B} = (2\pi)^3\int d^3k\,F(\boldsymbol{k})\,\boldsymbol{\mathcal{B}}(\boldsymbol{k}).$$

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$$\int d^3r\,(\boldsymbol{r}\cdot\boldsymbol{E}^*)\boldsymbol{B} = (2\pi)^3 \int d^3k\,F(\boldsymbol{k})\,\boldsymbol{\mathcal{B}}(\boldsymbol{k}).$$

Contribution to the helicity is then

$$(2\pi)^3 \int d^3k\,F(\boldsymbol{k})\,[\boldsymbol{\mathcal{B}}(\boldsymbol{k})\cdot\boldsymbol{k}]$$
$$= 0 \quad \text{because} \quad \boldsymbol{\nabla}\cdot\boldsymbol{B} = 0.$$

Similarly, other term gives zero contribution, because $\boldsymbol{\nabla}\cdot\boldsymbol{E} = 0$.

Summary: for an arbitrary superposition of Classical MAXWELL Fields $\boldsymbol{J}_{\text{bel}}$ fails to give correct helicity i.e.

$$\mathcal{H}_{\text{bel}} = 0$$

GENERAL CLASSICAL MAXWELL FIELD: SECOND ARGUMENT AGAINST J_{bel}:

(2) Compare with **the Canonical and Chen expressions**

Just as for the operator case, for any fields $\boldsymbol{E}$, $\boldsymbol{B}$,

$$
\begin{aligned}
\mathcal{H}_{\text{can}} &\equiv \boldsymbol{J}_{\text{can}} \cdot \boldsymbol{P}_{\text{can}}/|\boldsymbol{P}_{\text{can}}| = \boldsymbol{S}_{\text{can}} \cdot \boldsymbol{P}_{\text{can}}/|\boldsymbol{P}_{\text{can}} \\
&= \boldsymbol{S}_{\text{chen}} \cdot \boldsymbol{P}_{\text{chen}}/|\boldsymbol{P}_{\text{chen}}
\end{aligned}
$$

Take a general superposition of EITHER left- or right-circularly polarized plane waves

$$\boldsymbol{E}(\boldsymbol{r}) = \int d^3k\, E_0(\boldsymbol{k})\, [\boldsymbol{\epsilon}_1(\boldsymbol{k}) \pm i\boldsymbol{\epsilon}_2(\boldsymbol{k})] e^{i\boldsymbol{k}\cdot\boldsymbol{r}}.$$

where

$$\boldsymbol{\epsilon}_1(\boldsymbol{k})\,,\ \boldsymbol{\epsilon}_2(\boldsymbol{k})\,,\ \hat{\boldsymbol{k}}$$

form a three-dimensional orthogonal system of unit basis vectors.

For the cycle average, for the monochromatic case, one finds

$$\langle \boldsymbol{S}_{\text{chen}} \rangle = \pm \frac{(2\pi)^3 \,\epsilon_0}{\omega} \int d^3k |E_0(\boldsymbol{k})|^2 \, \hat{\boldsymbol{k}}$$

Thus

$$\langle \mathcal{H}_{\text{chen}} \rangle = \pm \frac{(2\pi)^3 \,\epsilon_0}{\omega} \int d^3k |E_0(\boldsymbol{k})|^2$$

N.B. My particle-physics-like definition of helicity coincides exactly with the expressions of Afanasiev-Stepanovsky and Trueba-Rañada.

Equating the total energy in the field to the number of photons $\times\ \hbar\omega$

$$\langle \mathcal{H}_{\text{can}} \rangle \Big|_{\text{photon}} = \langle \mathcal{H}_{\text{chen}} \rangle \Big|_{\text{photon}} = \pm\hbar$$

A BEAUTIFULLY INTUITIVE RESULT

Summary: Classical Superposition of Maxwell Fields

- $\boldsymbol{J}_{\text{bel}}$ fails to give correct helicity

- $\langle \mathcal{H}_{\text{can}} \rangle = \langle \mathcal{H}_{\text{chen}} \rangle$ give physically intuitive result

IV: LESSONS FROM LASER OPTICS

KEY QUESTION:TO WHAT EXTENT IS S_{chen} a GENUINE INTRINSIC A.M.?

(1) MACROSCOPIC PHYSICS

Torque $\boldsymbol{\tau}$ about the C.M.of a small neutral object, in electric dipole approximation, with complex polarizability $\alpha = \alpha_R + i\,\alpha_I$, acted on by field

$$\boldsymbol{\mathcal{E}} = Re(\boldsymbol{E}) \qquad \boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0(\boldsymbol{r})\, e^{-i\omega t}$$

For the cycle average, one finds

$$\begin{aligned} \langle \boldsymbol{\tau} \rangle &= \alpha_I [Re\boldsymbol{E}_0 \times Im\boldsymbol{E}_0] \\ &= \frac{\alpha_I \omega}{\epsilon_0} \langle \boldsymbol{s}_{\text{chen}} \rangle \end{aligned}$$

Thus $\langle \boldsymbol{s}_{\text{chen}} \rangle$ is measurable.

Let $\boldsymbol{S}_{\text{dipole}}$ be the internal A.M. of the induced dipole, about its C.M.

Then we expect

$$\langle \frac{d}{dt} \boldsymbol{S}_{\text{dipole}} \rangle = \langle \boldsymbol{\tau} \rangle$$

- Assume that the change of A.M. of the dipole is due to the average spin A.M. $\boldsymbol{S}_{\text{photon}}$ of each photon absorbed from the beam.

- Take the number of photons totally absorbed by the dipole per second to be given by $1/\hbar\omega$ times the rate of increase of the dipole's internal energy.

- Take photon density to be $N_\gamma = \frac{1}{\hbar\omega}$(Field Energy Density)

Then

$$\langle \frac{d}{dt} \boldsymbol{S}_{\text{dipole}} \rangle = \langle \boldsymbol{\tau} \rangle$$

holds provided we take the spin A.M. carried by each photon to be

$$\boldsymbol{S}_{\text{photon}} = \frac{1}{N_\gamma} \langle \boldsymbol{s}_{\text{chen}} \rangle$$

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holds provided we take the spin A.M. carried by each photon to be

$$\boldsymbol{S}_{\text{photon}} = \frac{1}{N_\gamma} \langle \boldsymbol{s}_{\text{chen}} \rangle$$

Thus, surprisingly, macroscopically, $\langle \boldsymbol{s}_{\text{chen}} \rangle$ seems to function as a measure of physical angular momentum carried by the photons.

KEY QUESTION:TO WHAT EXTENT IS S_{chen} a GENUINE INTRINSIC A.M.?

(2) ATOMIC PHYSICS

We already know that the commutation relations are wrong.

A further argument:

For generalized Maxwell Bessel beams the A.M. eigenvalues are **NOT QUANTIZED**:

$$J_{\text{chen, z}} = j\hbar \qquad S_{\text{chen, z}} = \pm\hbar\frac{k_z}{k} \qquad L_{\text{chen, z}} = j\hbar \mp \hbar\frac{k_z}{k}$$

Suppose an atom, in an eigenstate $|\, m_z \rangle$, absorbs one of these photons.

If $\boldsymbol{S}_{\text{chen, z}}$ were a genuine intrinsic A.M. then the atom's intrinsic m_z would change by this amount.

.

Suspose an atom, in an eigenstate $|\, m_z \,\rangle$, absorbs one of these photons.

If $\boldsymbol{S}_{\text{chen, z}}$ were a genuine intrinsic A.M. then the atom's intrinsic m_z would change by this amount.
But what you find is that the final atomic state is a superposition

$$|\text{final}\rangle = a\,|m_z + 1\rangle + b\,|m_z\rangle + c\,|m_z - 1\rangle.$$

What happens in paraxial approximation?

PARAXIAL FIELDS

GENERAL SOLUTION OF PARAXIAL WAVE EQUATION

$$\boldsymbol{E}(\boldsymbol{r}) = \left(u(\boldsymbol{r}),\, v(\boldsymbol{r}),\, \frac{-i}{k}\Big(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\Big)\right) e^{i(kz-\omega t)}$$

where

$$u(\boldsymbol{r}) = \int d^2k_\perp\, \tilde{u}(\boldsymbol{k}_\perp)\, e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{r}}\, e^{-ik_\perp^2 z/2k}$$

$$v(\boldsymbol{r}) = \int d^2k_\perp\, \tilde{v}(\boldsymbol{k}_\perp)\, e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{r}}\, e^{-ik_\perp^2 z/2k}$$

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$$v(\boldsymbol{r}) = \int d^2k_\perp\, \tilde{v}(\boldsymbol{k}_\perp)\, e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{r}}\, e^{-ik_\perp^2 z/2k}$$

KEY POINT: LASER PARAXIAL FIELDS: $k_\perp^2/k^2 \ll 1$.

Then $k_z/k \approx 1$ and

$$J_{\text{chen, z}} \approx j\hbar \qquad S_{\text{chen, z}} \approx \pm\hbar \qquad L_{\text{chen, z}} \approx j\hbar \mp \hbar$$

For atomic absorption of one photon one finds

$$|\text{final}\rangle \approx |m_z + 1\rangle \qquad \textbf{OR} \qquad |\text{final}\rangle \approx |m_z - 1\rangle$$

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Thus in paraxial approximation, for atomic absorption, $S_{\text{chen, z}}$, $L_{\text{chen, z}}$ behave approximately as genuine A.M.

$j_{\text{bel, z}}$ VS $j_{\text{chen, z}}$: LASER TESTS

In the foundation paper on laser angular momentum Allen, Beijersbergen, Spreeuw and Woerdman studied the Belinfante A.M. for a Laguerre-Gaussian mode in paraxial approximation, with $v(\boldsymbol{r}) = i\sigma u(\boldsymbol{r}) \qquad \sigma = \pm 1$. But same result holds for any field of the form, in cylindrical coordinates $(\rho,\ \phi,\ z)$,

$$u(\rho, \phi, z) = f(\rho, z)e^{il\phi}.$$

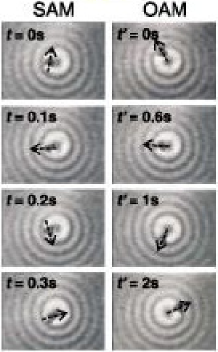
They obtain

$$\langle j_{\text{bel, z}} \rangle \approx \frac{\epsilon_0}{\omega} \left[l|u|^2 - \frac{\sigma}{2} \rho \frac{\partial |u|^2}{\partial \rho} \right]$$

which, surprisingly, looks like an orbital A.M. plus a spin A.M. term.
For the Chen et al version one obtains a different result:

$$\langle l_{\text{chen, z}} \rangle \approx \frac{\epsilon_0}{\omega} l|u|^2 \qquad \langle s_{\text{chen, z}} \rangle \approx \frac{\epsilon_0}{\omega} \sigma |u|^2$$

The first semi-quantitative test of the above was made by Garcés-Chávez, Mc Gloin, Padgett, Dulz, Schmitzer and Dholakia (GMPDSD) who succeeded in studying the motion of a tiny particle trapped at various radial distances ρ from the axis of a so-called Bessel beam. The transfer of orbital A.M. causes the particle to circle about the beam axis with a rotation rate Ω_{orbit} whereas the transfer of spin A.M. causes the particle to spin about its centre of mass with rotation rate Ω_{spin}.


SAM
OAM
t = 0s
t' = 0s
t = 0.1s
t' = 0.6s
t = 0.2s
t' = 1s
t = 0.3s
t' = 2s

For a Bessel beam, $|u|^2 \propto 1/\rho$, so for both Belinfante **and** Chen et al versions

$$\Omega_{\mathrm{orbit}} \propto 1/\rho^3 \qquad \text{and} \qquad \Omega_{\mathrm{spin}} \propto 1/\rho,$$

which is precisely the behaviour found experimentally.

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But the absolute rotation rates predicted are different!

Unfortunately the absolute rotation rates depend upon detailed parameters which, according to the authors, were beyond experimental control.

OTHER LASER TESTS?

Shift of the diffraction fringes, found by Ghai, Senthilkumaran and Sirohi, in single slit diffraction of optical beams with a phase singularity:

Experimentally seems to depend on l and not on σ.

Unpublished paper Chen and Chen 2012: Claim this implies the $\boldsymbol{J}_{chen}$ one is correct.

Recent review Bliokh and Nori: Canonical A.M. in the Coulomb gauge, i.e the Chen et al A.M. agrees with a wide range of experiments.

j_{bel} VS j_{chen} : SUMMARY

- On theoretical grounds, for the macroscopic case, requiring $\langle \frac{d}{dt}\boldsymbol{S}_{\text{dipole}} \rangle = \langle \boldsymbol{\tau} \rangle$, suggests that it is $\boldsymbol{j}_{\text{chen}}$ plays the role of a physical A.M.

- Seems that various Laser Optics experiments favour $\boldsymbol{j}_{\text{chen}}$.

- Laser Optics experiment, of the GMPDSD-type, offer the fantastic possibility of a direct check, if the absolute rotation rates could be determined.

THE ANGULAR MOMENTUM CONTROVERSY: SUMMARY

- The revolutionary claim, by Chen et al, that $\boldsymbol{J}_{\text{photon}}$ **CAN** be split, **contrary to all QED textbooks** , into $\boldsymbol{L}_{\text{photon}} + \boldsymbol{S}_{\text{photon}}$, in a gauge invariant way, is **WRONG**.

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- Particle physicists were apparently totally unaware that van Enk and Nienhuis had, long ago, shown that $\boldsymbol{L}_{\text{chen}}$ and $\boldsymbol{S}_{\text{chen}}$ are **NOT** genuine angular momentum operators.

- Laser physicists tend to use the Classical Electrodynamics Textbook $\boldsymbol{J}_{\text{bel}}$, perhaps being unaware that it gives $\mathcal{H}_{\text{bel}} = 0$ for a circularly polarized plane wave or a superposition of either left or right circularly plane waves.

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- That $\mathcal{H}_{\mathrm{bel}}^{\mathrm{paraxial}} \neq 0$ is purely a consequence of the paraxial approximation i.e. the fact that $\boldsymbol{\nabla} \cdot \boldsymbol{E}^{\mathrm{paraxial}} \neq 0$ and $\boldsymbol{\nabla} \cdot \boldsymbol{B}^{\mathrm{paraxial}} \neq 0$.

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- **N.B.** Laser optics can decide which of $\boldsymbol{j}_{\text{bel}}$ or $\boldsymbol{j}_{\text{chen}}$ correctly describes the physical A.M. carried by light.