Synthetic dimensions and four-dimensional quantum Hall effect in photonics

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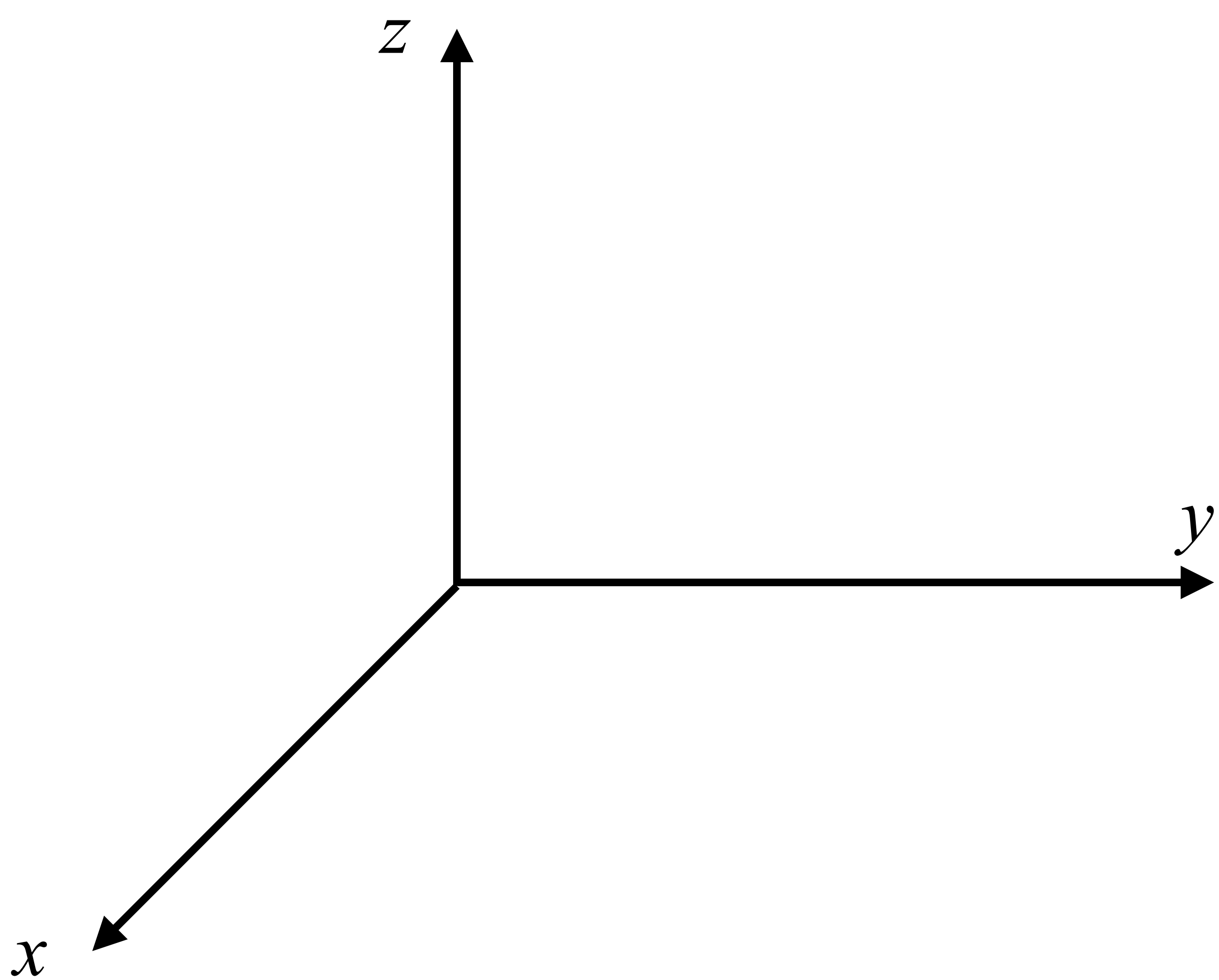
Dimension

- Dimension — Independent directions where a particle can move



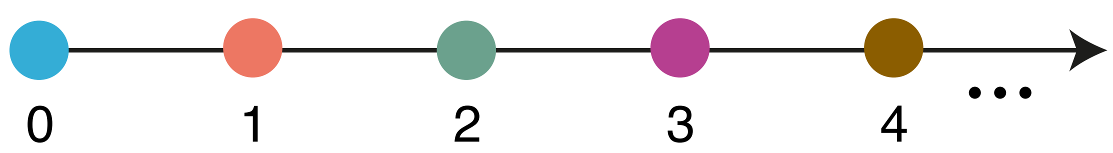
Outline

1. Introduction: synthetic dimensions for ultracold atoms
2. Synthetic dimensions for photons
3. Two & Four-dimensional quantum Hall effect and synthetic dimensions

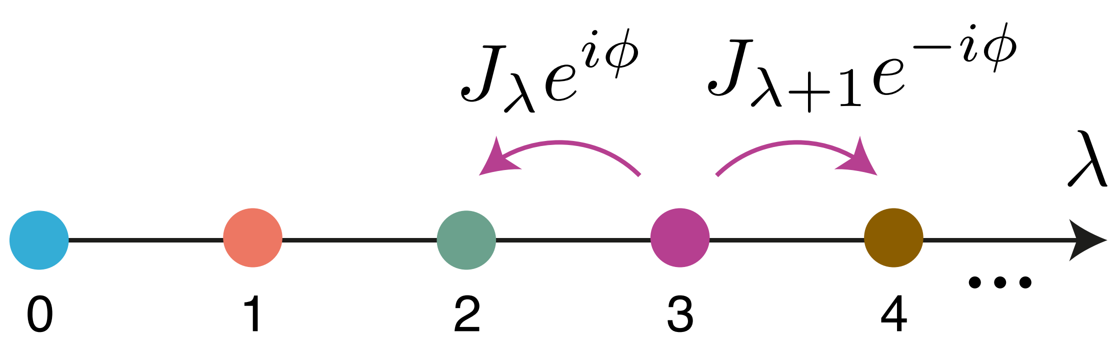
Synthetic dimensions

— A method to simulate higher-dimensional models —

- Identify a set of modes/states as lattice sites along a synthetic dimension
  e.g. Different spin or electronic states of an atom (in ultracold gases)
  e.g. Different angular momentum modes of a photon (in a photonic cavity)

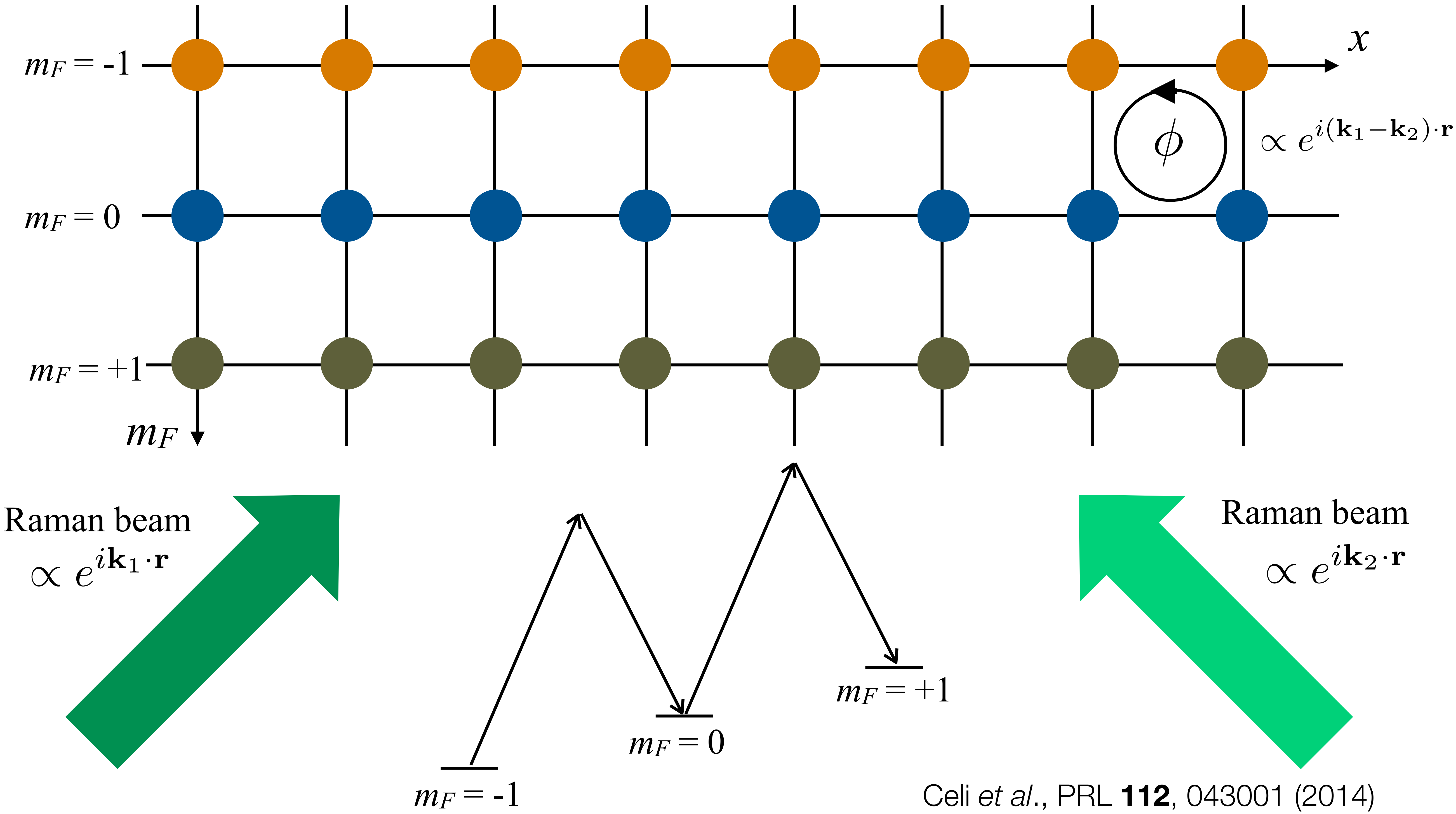


- Couple these modes to allow particles to move, or to simulate a tight-binding hopping



Celi *et al.*, PRL **112**, 043001 (2014)

Synthetic dimensions for ultracold atoms



Celi *et al.*, PRL **112**, 043001 (2014)

2D tight-binding model with complex hopping phase — Harper-Hofstadter model

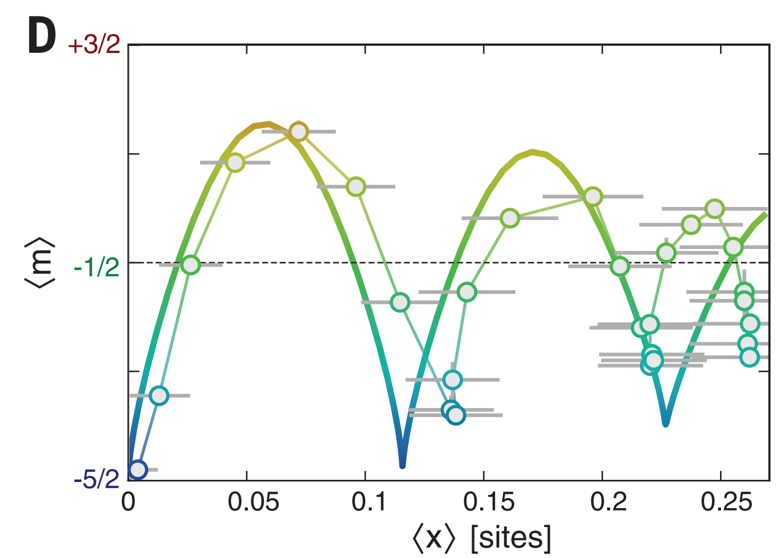
Experimental realization of synthetic dimensions

Florence group (Fallani & Inguscio) - ^{173}Yb (fermion)

Mancini et al., Science **349**, 1510 (2015); Livi, et al., PRL **117**, 220401 (2016)

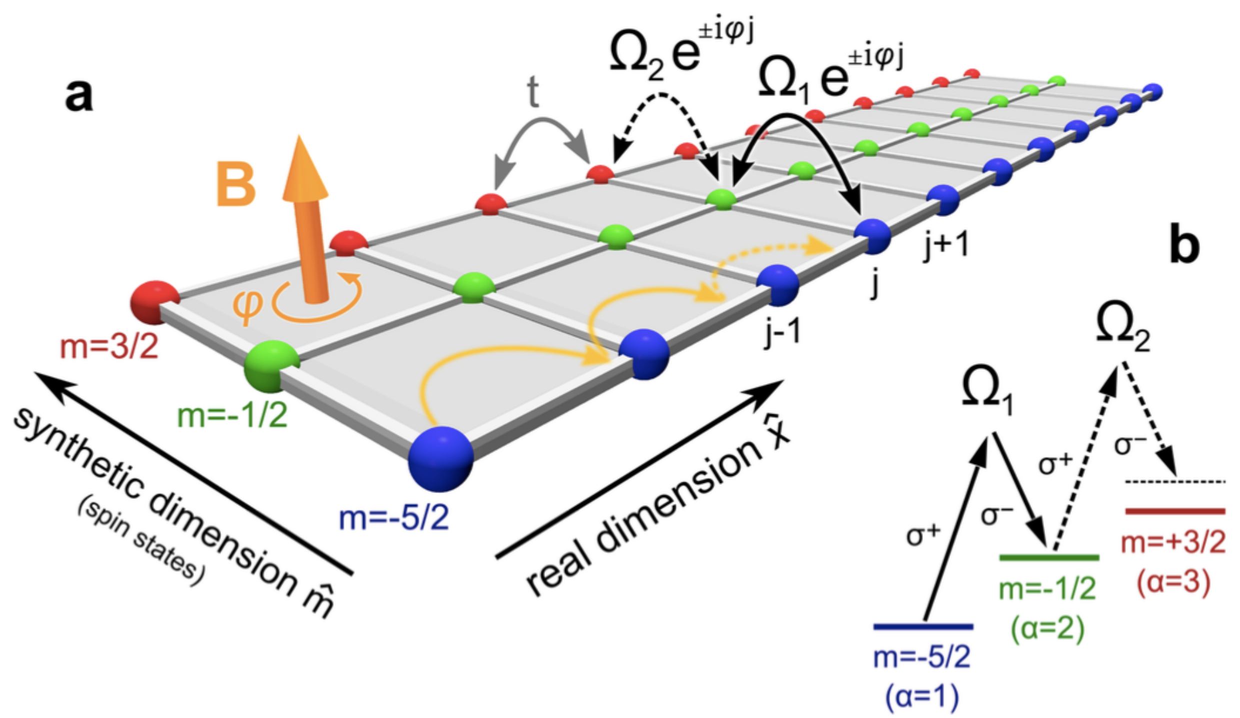
Maryland group (Spielman) - ^{87}Rb (boson)

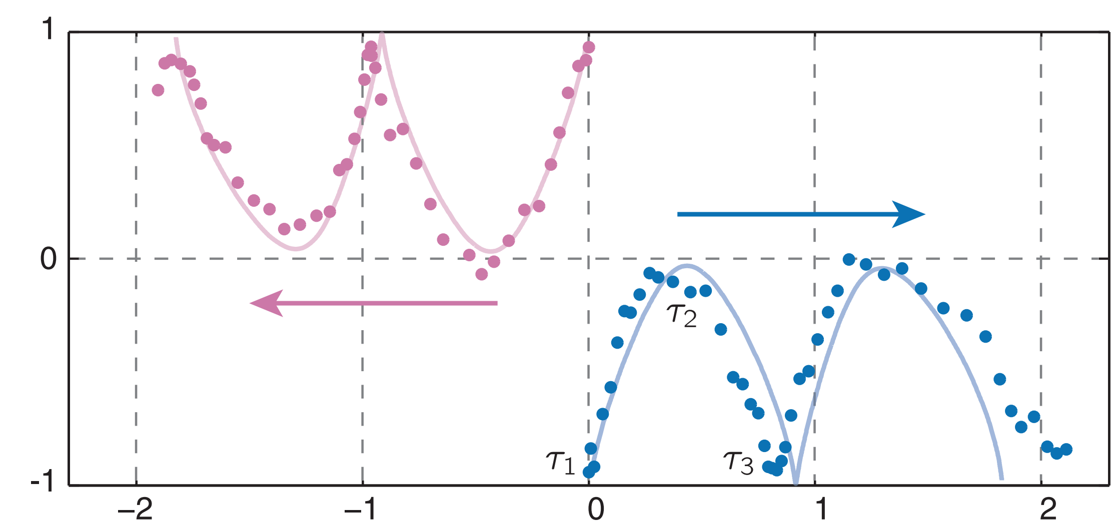
Stuhl et al., Science **349**, 1514 (2015)



Florence

- Three sites along the synthetic direction
- Chiral propagation of edge states observed

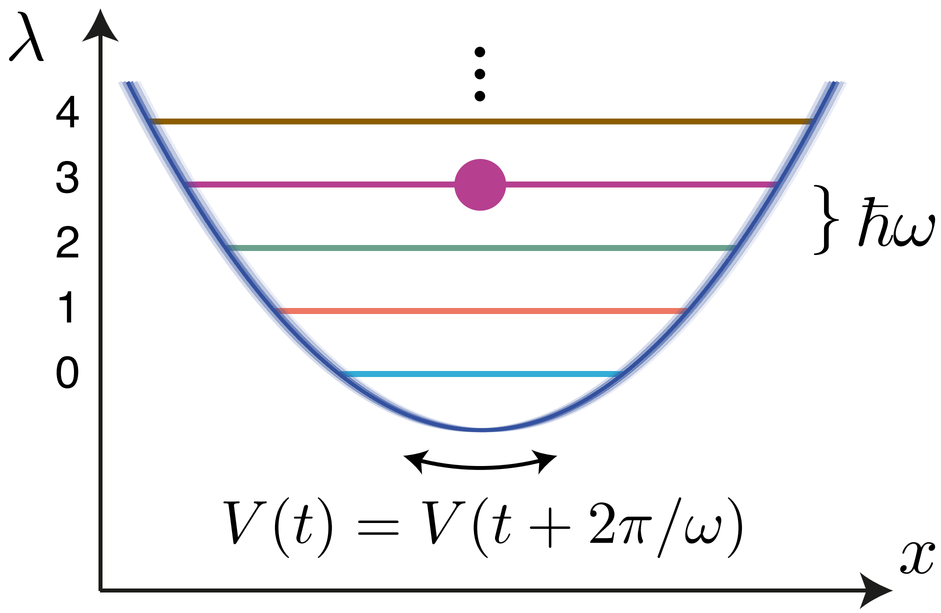


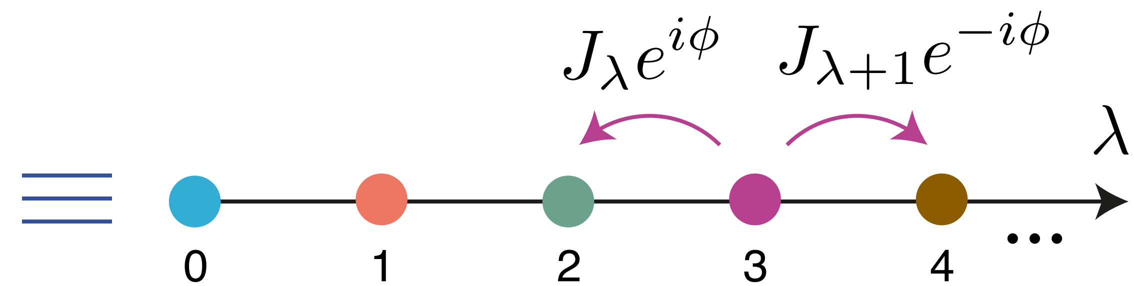


Maryland

Harmonic oscillator states as synthetic dimensions

Price, TO, & Goldman, PRA **95**, 023607 (2017)

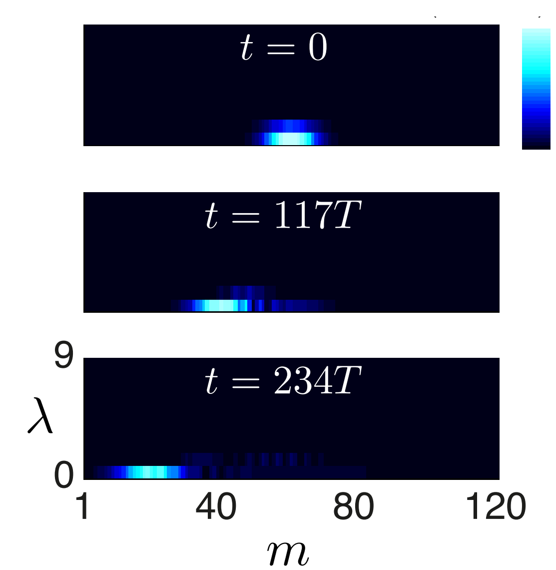




- Consider harmonic oscillator states as lattice sites in the synthetic direction
- Couple different states by shaking the potential resonantly with the level spacing

$$H_0 = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 = \sum_{\lambda=0}^{\infty} \omega\lambda |\lambda\rangle\langle\lambda|$$

$$H = H_0 + V(t) \to \sum_{\lambda} \kappa \sqrt{\frac{\lambda}{8m\omega}} \left(|\lambda - 1\rangle\langle\lambda| e^{i\phi} + h.c.\right)$$



Can in principle go up to 6D (3D optical lattice + 3 directions of harmonic potential)

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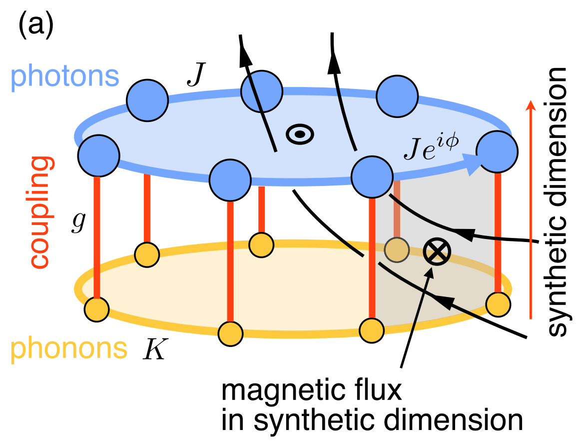
Synthetic dimensions in photonic systems

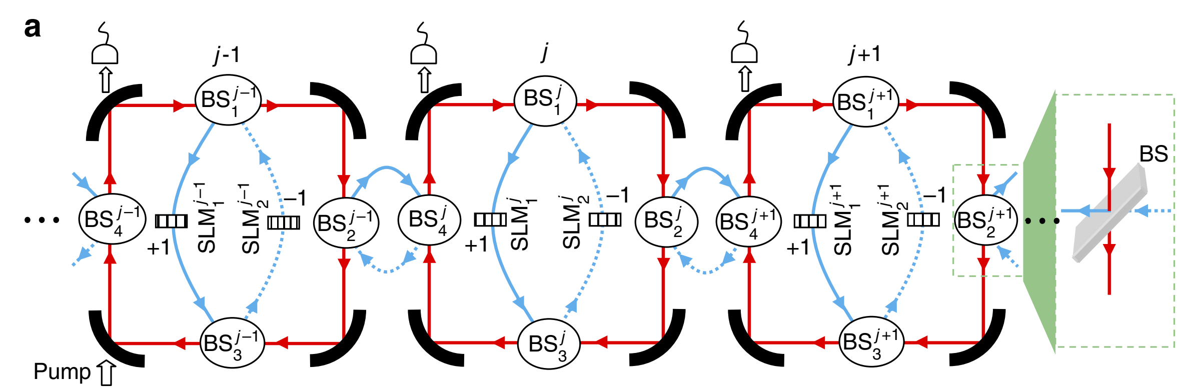
Optomechanics — photons & phonons

Schmidt et al., Optica **2**, 635 (2015)



Optical cavities — orbital angular momentum

Luo et al., Nature Comm. **6**, 7704 (2015)



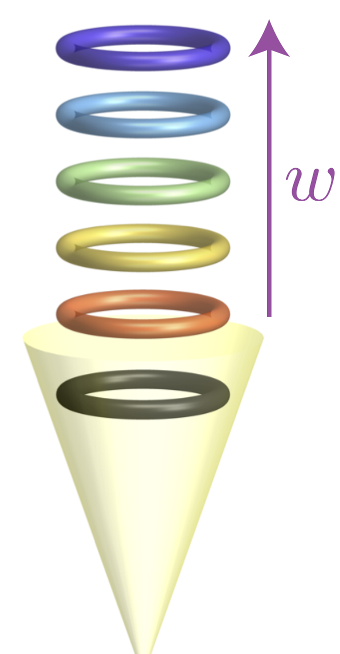
Ring resonator array — different modes of micro-ring resonators

TO, Price, Goldman, Zilberberg, & Carusotto, PRA **93**, 043827 (2016)

Yuan, Shi, & Fan, Opt. Lett. **41**, 741 (2016)

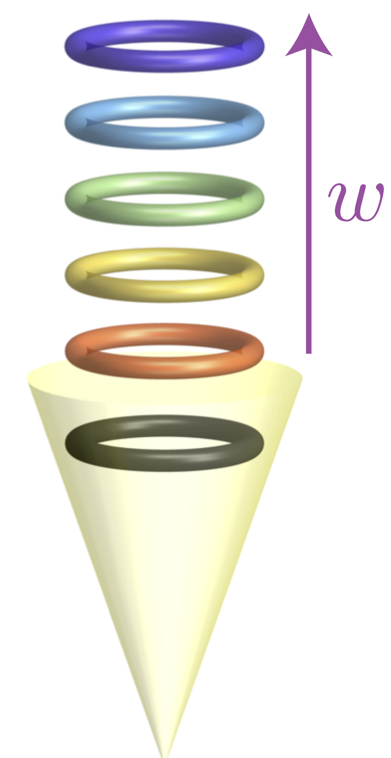
TO & Carusotto, Phys. Rev. Lett. **118**, 013601 (2017)

Hafezi et al, Nat. Photon. 7, 1001, (2013)



angular momentum
of mode around ring

Coupling different modes



Different modes can be coupled via some external modulation:

Nonlinearity with an external laser [TO, *et al.*, PRA **93**, 043827 (2016)]

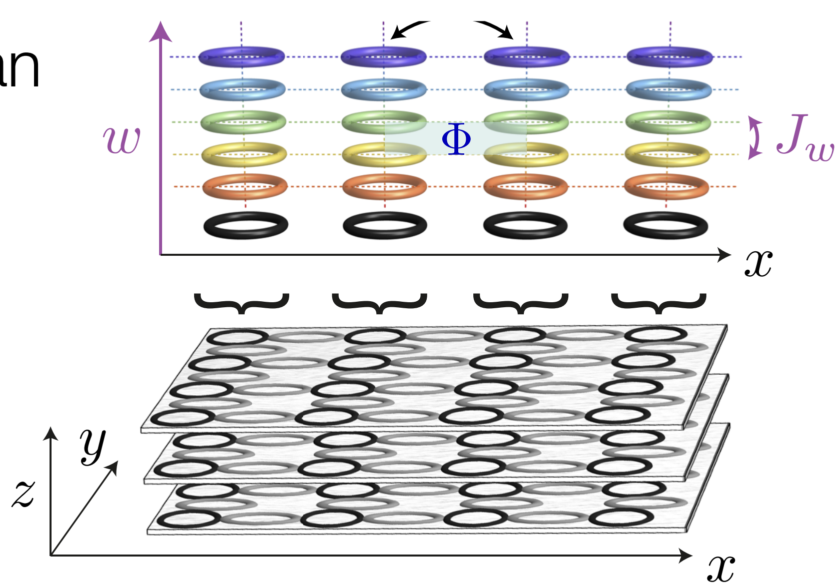
Electro-optic phase modulators [Yuan, *et al.*, Opt. Lett. **41**, 741 (2016)]

The effective Hamiltonian is

$$\boxed{H = -\sum_{w} \mathcal{J} e^{i\theta} b^\dagger_{w+1} b_w + h.c.}$$

- 1D tight-binding Hamiltonian with hopping phases -

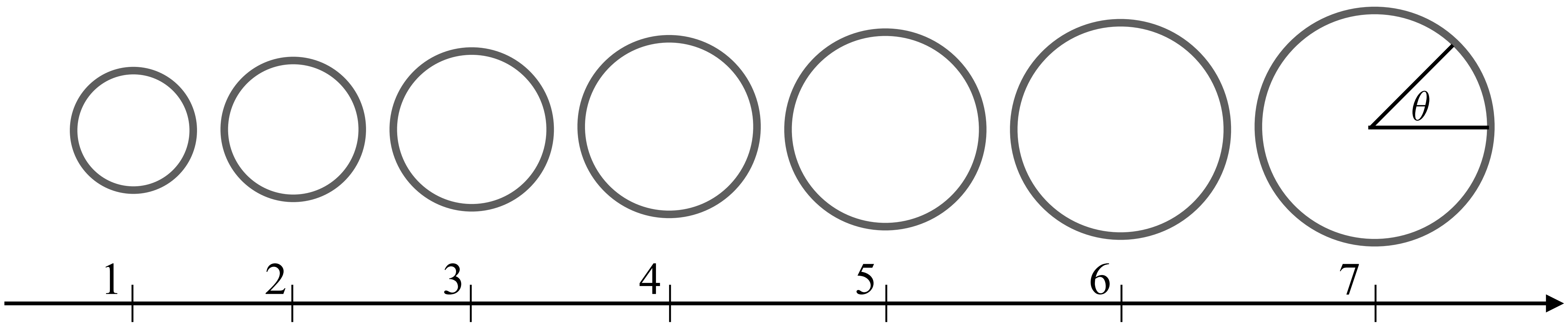
Spatially aligning resonators, one can build up to 4D Hamiltonian



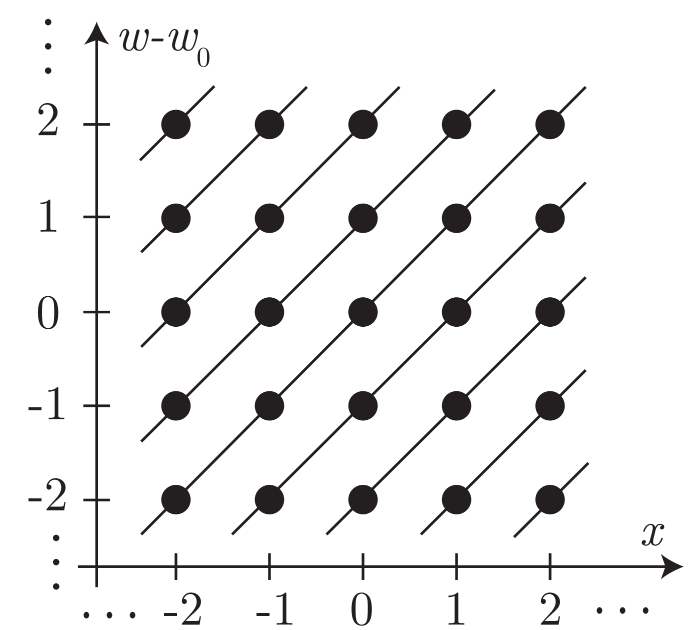
$$\boxed{\begin{aligned} H = \sum_{\mathbf{r},w} &- \mathcal{J}_x b^\dagger_{\mathbf{r}+\hat{e}_x,w} b_{\mathbf{r},w} - \mathcal{J}_y b^\dagger_{\mathbf{r}+\hat{e}_y,w} b_{\mathbf{r},w} \\ &- \mathcal{J}_z b^\dagger_{\mathbf{r}+\hat{e}_z,w} b_{\mathbf{r},w} - \mathcal{J}_w e^{i\theta(\mathbf{r})} b^\dagger_{\mathbf{r},w+1} b_{\mathbf{r},w} + h.c. \end{aligned}}$$

Angular coordinate as synthetic dimensions I

Align ring resonators with different sizes and shapes



$$\Omega_{1,w} \approx \Omega_0 + \Omega_{\rm FSR}(w - w_0) + D(w - w_0)^2/2 + \cdots$$
$$\Omega_{2,w} = \Omega_{1,w} - \Omega_{\rm FSR}$$
$$\Omega_{x+1,w} = \Omega_{x,w} - \Omega_{\rm FSR}$$



Then, neighboring resonators follow $\Omega_{x,w} \approx \Omega_{x+1,w+1}$

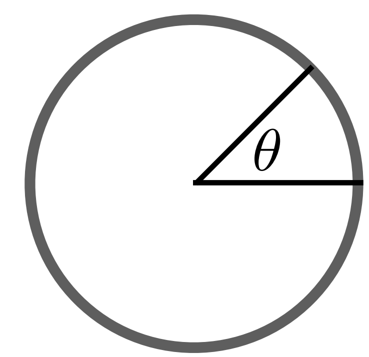
A photon with mode w at site x hops to mode w+1 at site x+1

TO & Carusotto, Phys. Rev. Lett. **118**, 013601 (2017)

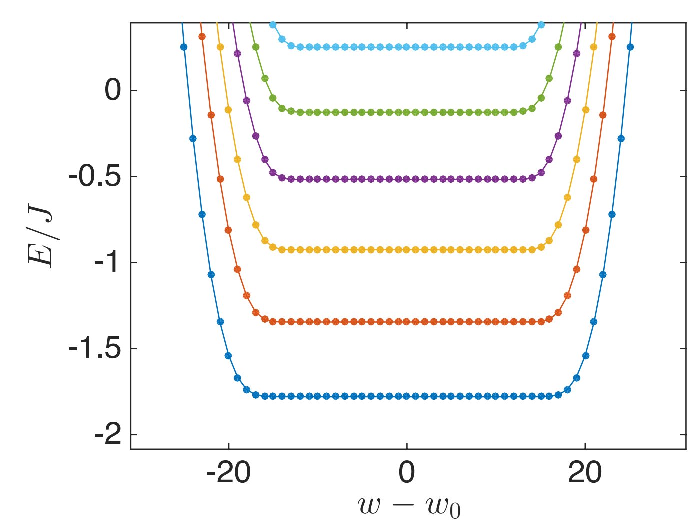
Angular coordinate as synthetic dimensions II

Effective tight-binding Hamiltonian in the space of θ (angular coordinate) is

$$\mathcal{H} = \sum_x \int_0^{2\pi} d\theta \left[\underbrace{\frac{D}{2} \left\{ i\nabla_\theta b_x^\dagger(\theta) \right\} \left\{ -i\nabla_\theta b_x(\theta) \right\}}_{\text{kinetic energy in synthetic dimension}} \boxed{- \underbrace{J \left\{ e^{i\theta} b_{x+1}^\dagger(\theta) b_x(\theta) + h.c. \right\}}_{\text{hopping with phase}}} + \underbrace{\frac{U}{2} b_x^\dagger(\theta) b_x^\dagger(\theta) b_x(\theta) b_x(\theta)}_{\text{zero-range interaction term}} \right]$$



- Synthetic dimension is continuous and periodic
- Real dimension is discrete
- Hopping along the real dimension is complex
- The interaction is zero-range in both dimensions
- Coupled wire setup



Angular coordinate to explore 4D quantum Hall effect:
Lu & Wang, arXiv:1611.01998

TO & Carusotto, Phys. Rev. Lett. **118**, 013601 (2017)

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Two & Four dimensional quantum Hall effect

2D Quantum Hall effect:
In a 2D system with a perpendicular magnetic field, Hall conductance is quantized

$$\boxed{j^x = \frac{e^2}{h}\nu_1^n E_y}$$

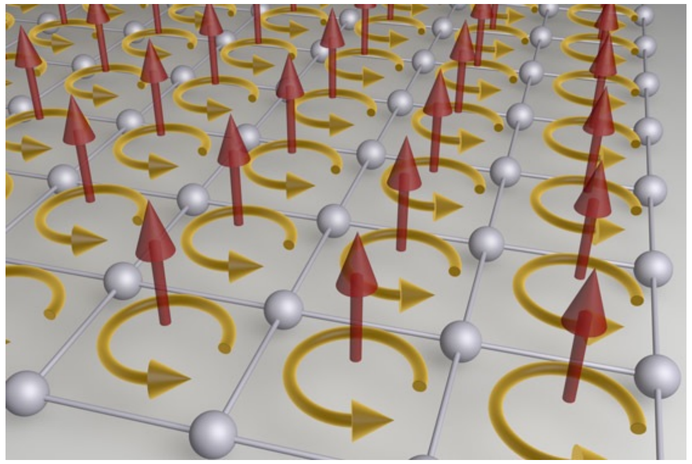


Image: http://www.quantum-munich.de/media/realization-of-the-hofstadter-hamiltonian/

Similar effects occur in any even dimensions!

4D Quantum Hall effect:
The current responds nonlinearly to external perturbing fields

$$j^\mu = \boxed{E_\nu \frac{1}{(2\pi)^4}\int_{\mathrm{BZ}} \Omega_n^{\mu\nu} d^4k} + \boxed{\frac{\nu_2^n}{(2\pi)^2}\epsilon^{\mu\nu\rho\sigma}E_\nu B_{\rho\sigma}} \qquad B_{\rho\sigma} \equiv \partial_\rho A_\sigma - \partial_\sigma A_\rho$$

2D Quantum Hall Contribution　　4D Quantum Hall Effect!

where, the 2nd Chern number is defined by

$$\nu_2^n \equiv \frac{1}{(2\pi)^2}\int_{\mathrm{BZ}} \{\Omega_n^{xy}\Omega_n^{zw} + \Omega_n^{wx}\Omega_n^{zy} + \Omega_n^{zx}\Omega_n^{yw}\}\, d^4k \in \mathbb{Z}$$

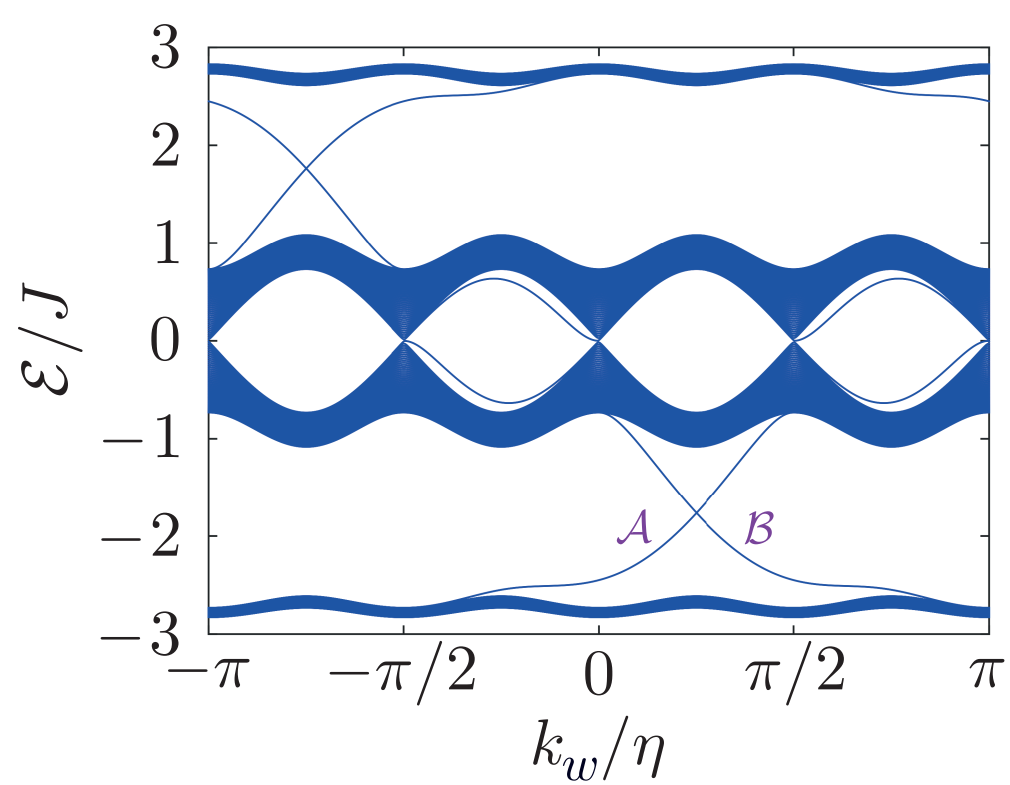
$\Omega_n^{\mu\nu}$: Berry curvature in $\mu\nu$ plane

Quantum Hall effect in driven-dissipative photonics

1D chain of resonators + 1 synthetic dimension
= 2D lattice model with effective magnetic fields

- In driven-dissipative systems, the Hall current is proportional to the center-of-mass shift of photonic fields — integer quantum Hall effect

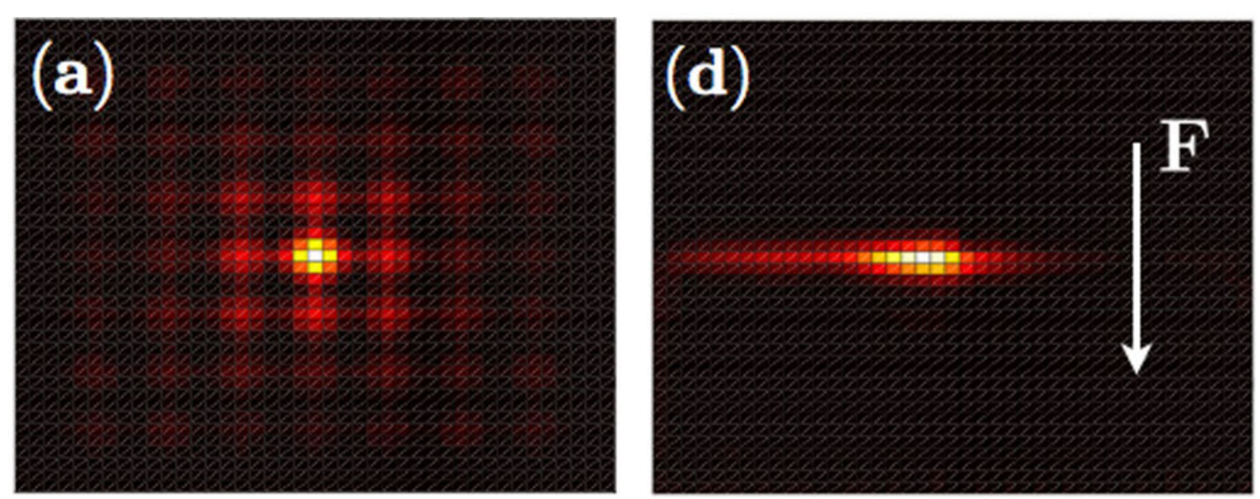
Band structure of ϕ = 1/4



1st Chern number
external force

$$\langle x \rangle \approx \frac{2\pi \overline{\nu_1}\, \overline{F}}{\underline{A_{\rm BZ}}\, \underline{\gamma}}$$

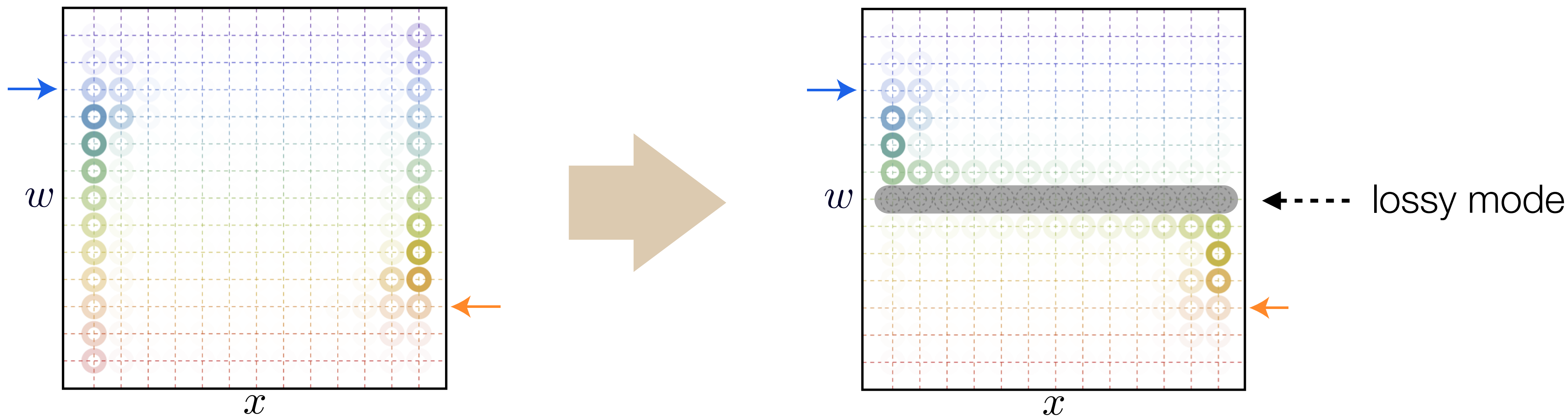
BZ volume
Loss



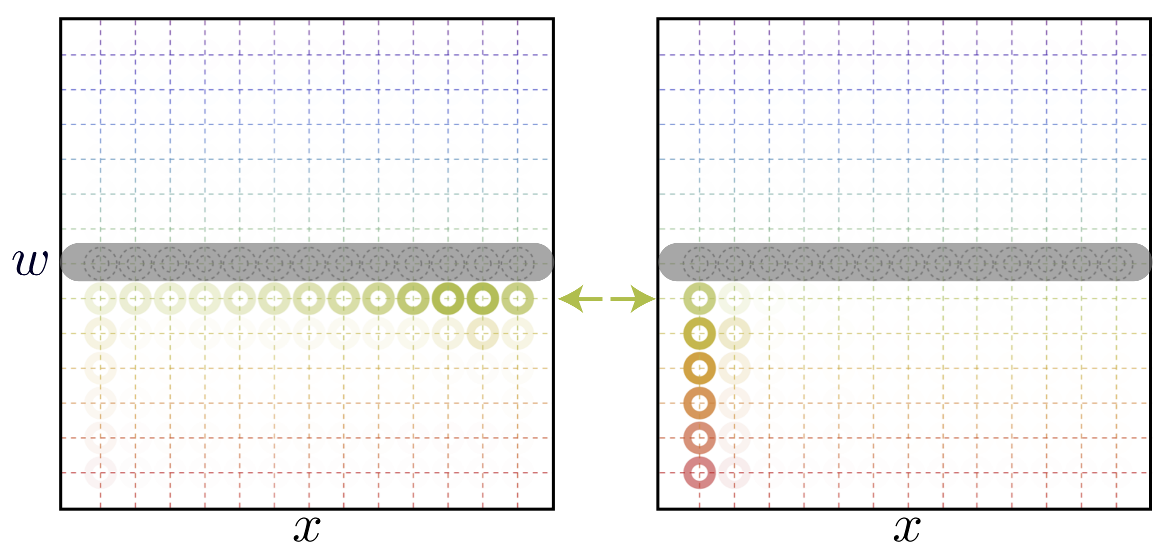
TO & Carusotto, PRL **112**, 133902 (2014)

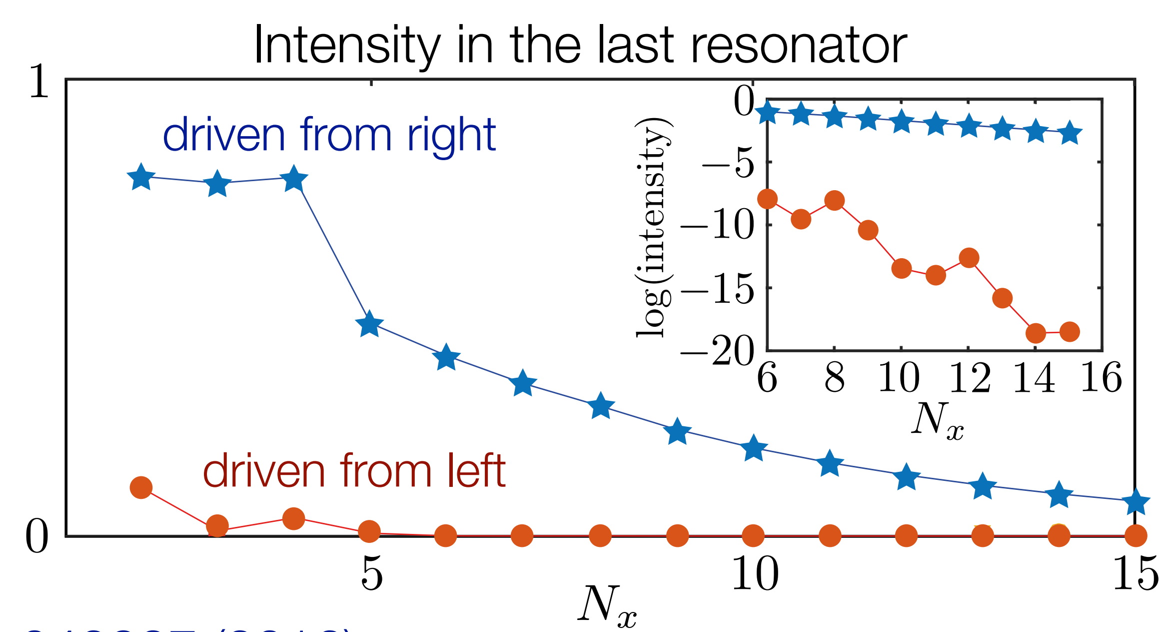
1+1D lattice as an optical isolator

One can introduce an artificial “edge” in w-direction by making one mode very lossy



The system can be used an optical isolator:





TO, Price, Goldman, Zilberberg, & Carusotto, PRA **93**, 043827 (2016)

4D quantum Hall effect in photonics

A minimal model to observe the 4D quantum Hall effect: TO, *et al.*, PRA **93**, 043827 (2016)

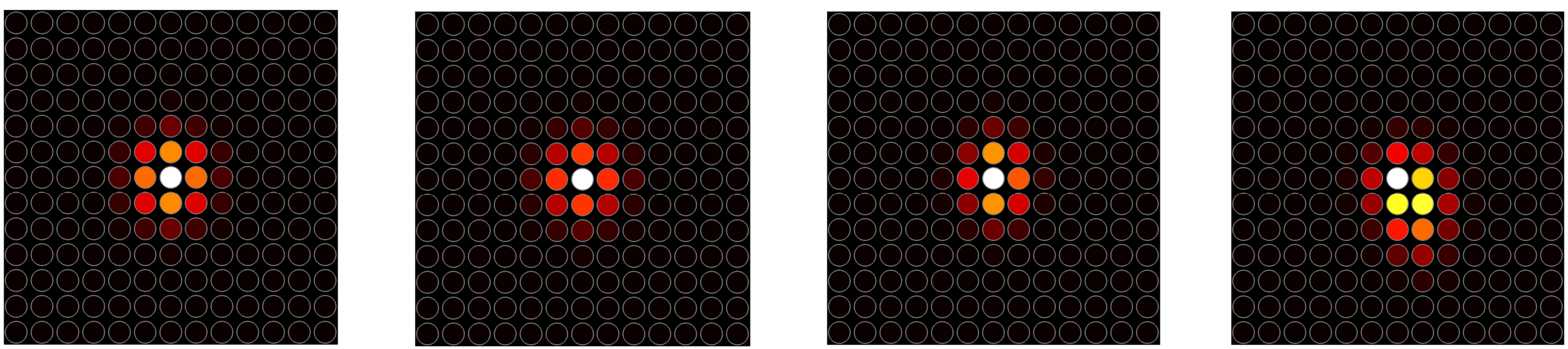
$$H = -J \sum_{x,y,z,w} \left(a^\dagger_{\mathbf{r}+\hat{e}_x} a_\mathbf{r} + e^{-iB_{yz}z} a^\dagger_{\mathbf{r}+\hat{e}_y} a_\mathbf{r} + a^\dagger_{\mathbf{r}+\hat{e}_z} a_\mathbf{r} + e^{iB_{xw}x} a^\dagger_{\mathbf{r}+\hat{e}_w} a_\mathbf{r} + \mathrm{H.c.} \right)$$

The steady-state distribution of photons also exhibits 4D quantum Hall effect:

$$\langle y \rangle \approx \frac{(2\pi)^4}{\gamma \underbrace{A_{\mathrm{BZ}}}_{\text{BZ volume}}} \underbrace{j^y}_{\text{Hall current}} = -\frac{(2\pi)^2}{\underbrace{\gamma}_{\text{Loss}} A_{\mathrm{BZ}}} \underbrace{\nu_2}_{\text{2nd Chern number}} \overbrace{\delta E_x \delta B_{zw}}^{\text{external electromagnetic field}}$$

Numerical simulation pumping the center: projection onto x-y plane

$\delta E_x = \delta B_{zw} = 0$ | $\delta E_x = 0,\ \delta B_{zw} \neq 0$ | $\delta E_x \neq 0,\ \delta B_{zw} = 0$ | $\delta E_x \neq 0,\ \delta B_{zw} \neq 0$

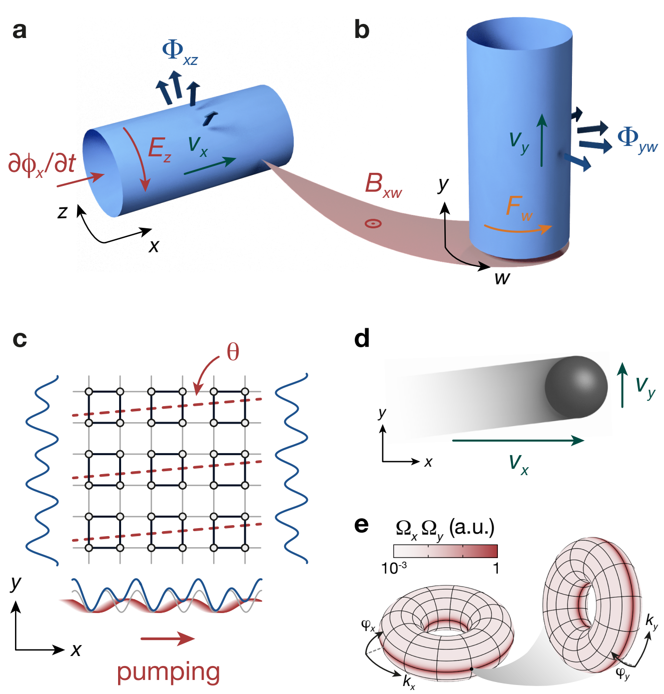


4D quantum Hall effect through charge pumping

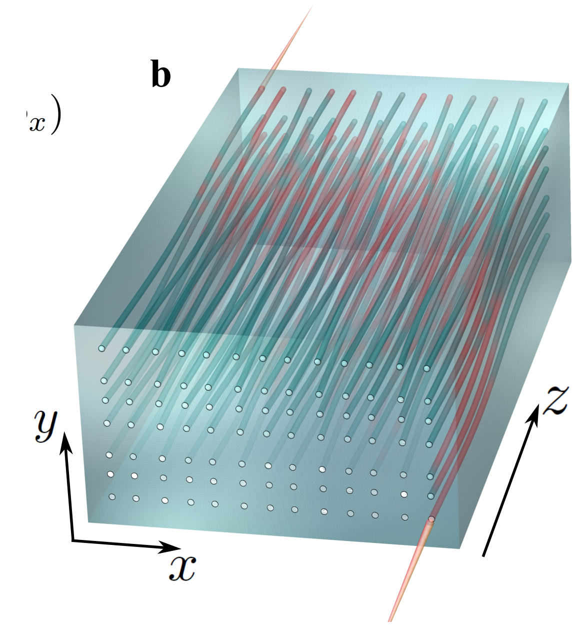
Lohse et al. (Munich, ETZ), arXiv:1705.08371

Exploring 4D Quantum Hall Physics with a 2D Topological Charge Pump

Michael Lohse[1,2], Christian Schweizer[1,2], Hannah M. Price[3], Oded Zilberberg[4] & Immanuel Bloch[1,2]



ultracold gases



photonic waveguides

Zilberberg et al. (Penn State, ETH), arXiv:1705.08361

Photonic topological pumping through the edges of a dynamical four-dimensional quantum Hall system

Oded Zilberberg,[1] Sheng Huang,[2] Jonathan Guglielmon,[3] Mohan Wang,[2]
Kevin Chen,[2] Yaacov E. Kraus,[4,†] and Mikael C. Rechtsman[3]

- Mapped 4D quantum Hall system to 2D models with 2 parameters
- Observed the 4DQH through charge pumping

Conclusions & Outlook

- Synthetic dimension: idea to simulate higher dimensional models using internal states
- There are proposals to
  - increase the number of sites in the synthetic dimension
  - make the interaction short-ranged
  - realize synthetic dimensions in photonics
  - realize 4D quantum Hall effect
- Many-body physics in higher dimensions?
- Higher dimensional topological defects?
- Edge states of four dimensional topological phases?
- Fractional Hall states in higher dimensions?

Collaborators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TO & Carusotto, PRL **112**, 133902 (2014)
Price, Zilberberg, TO, Carusotto, & Goldman, PRL **115**, 195303 (2015)
TO, Price, Goldman, Zilberberg, & Carusotto, PRA **93**, 043827 (2016)
Price, Zilberberg, TO, Carusotto, & Goldman, PRB **93**, 245113 (2016)
TO, & Carusotto, PRL **118**, 013601 (2017)
Price, TO, & Goldman, PRA **95**, 023607 (2017)