Space-Dependent RG, Anomalous Dimensions in a Hierarchical Model for 3d CFT and Connections to the AdS/CFT Correspondence

Abdelmalek Abdesselam
Mathematics Department, University of Virginia
Joint work with Ajay Chandra (Imperial)
and Gianluca Guadagni (UVa)

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1. Introduction
2. The hierarchical continuum
3. The rigorous hierarchical space-dependent renormalization group

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Used for generalizations of Zamolodchikov's c-"Theorem", study of scale vs. conformal invariance,...

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- Constructing explicit examples of holography or AdS/CFT correspondence.

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where $S[\phi]$ is an action for a field $\phi(x, x_{d+1})$ on AdS space and ϕ_{ext} makes it extremal for a boundary condition $\phi(x, x_{d+1}) \sim (x_{d+1})^{d-\Delta} j(x)$ when $x_{d+1} \to 0$.

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where m^2 is related to Δ and is allowed to be (not too) negative. This gives an expansion for connected CFT correlations in terms of tree-level Feynman diagrams (Witten diagrams). The simplest "Mercedes logo" 3-point Witten diagram reproduces the correct CFT prediction

$$\frac{O(1)}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2-x_3|^{\Delta_2+\Delta_3-\Delta_1}}$$

for $\langle\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\rangle$ by a calculation of Freedman, Mathur, Matusis and Rastelli 1999.

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The calculations of the last reference for scaling dimensions of Φ and Φ^2, for $N = 1$ in hierarchical case were made nonperturbatively rigorous in (ACG2013).

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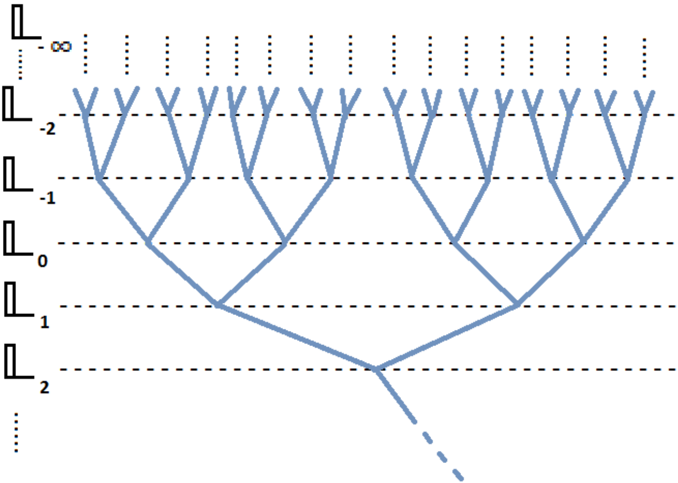
Hence $\mathbb{T} = \cup_{k\in\mathbb{Z}}\mathbb{L}_k$ naturally has the structure of a doubly infinite tree which is organized into layers or generations $\mathbb{L}_k$:



Picture for $d = 1$, $p = 2$

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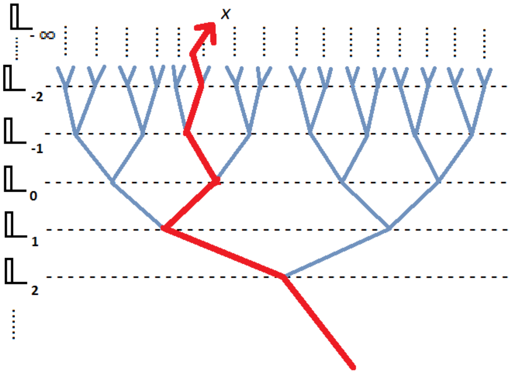
More precisely, these leafs at infinity are the infinite bottom-up paths in the tree.



A path representing an element $x \in \mathbb{Q}_p^d$

A point $x \in \mathbb{Q}_p^d$ is encoded by a sequence $(a_n)_{n\in\mathbb{Z}}$, $a_n \in \{0, 1, \dots, p-1\}^d$.
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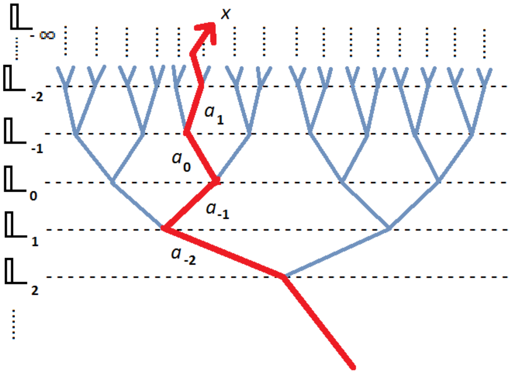

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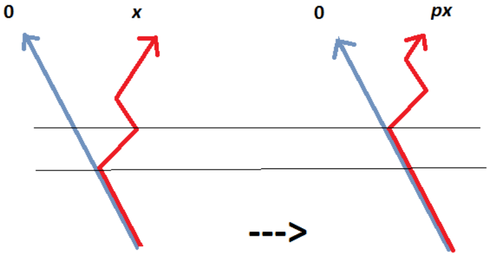

Likewise $p^{-1}x$ is downward shift, and so on for the definition of $p^k x$, $k \in \mathbb{Z}$.

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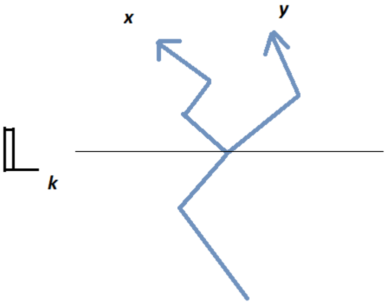

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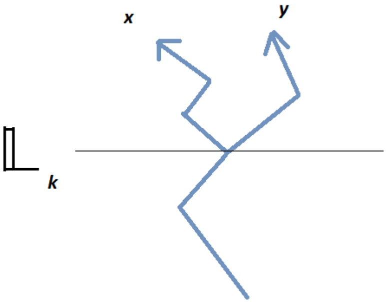

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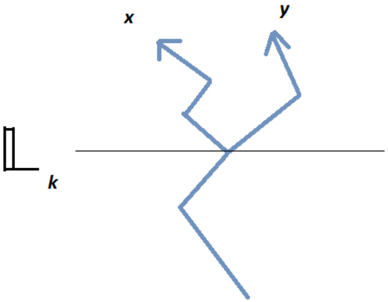

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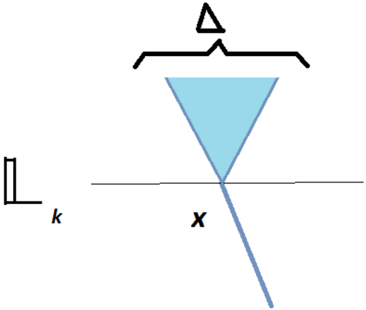

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Construction: take product of uniform probability measures on $(\{0, 1, \ldots, p-1\}^d)^{\mathbb{N}_0}$ for $\overline{B}(0, 1)$. Do the same for the other closed unit balls, and collate.

Lebesgue measure:

Metric space $\mathbb{Q}_p^d \to$ Borel σ-algebra $\to$ Lebesgue measure $d^d x$ which gives a volume p^{dk} to closed balls of radius p^k.

Construction: take product of uniform probability measures on $(\{0, 1, \ldots, p-1\}^d)^{\mathbb{N}_0}$ for $\overline{B}(0, 1)$. Do the same for the other closed unit balls, and collate.

The hierarchical unit lattice:

Truncate the tree at level zero and take $\mathbb{L} := \mathbb{L}_0$. Using the identification of nodes with balls, define the hierarchical distance as

$$d(\mathbf{x}, \mathbf{y}) = \inf\{|x - y|_p \mid x \in \mathbf{x}, \ y \in \mathbf{y}\} \ .$$

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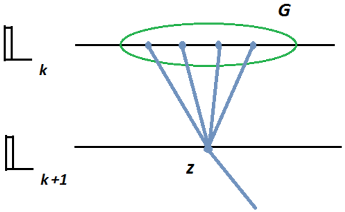

To every group of offsprings G of a vertex $\mathbf{z} \in \mathbb{L}_{k+1}$ associate a centered Gaussian random vector $(\zeta_{\mathbf{x}})_{\mathbf{x} \in G}$ with $p^d \times p^d$ covariance matrix made of $1 - p^{-d}$'s on the diagonal and $-p^{-d}$'s everywhere else. We impose that Gaussian vectors corresponding to different layers or different groups are independent.

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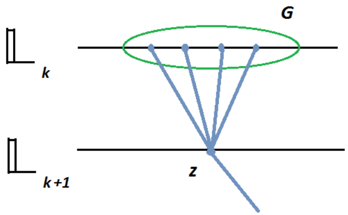

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The ancestor function: for $k < k'$, $\mathbf{x} \in \mathbb{L}_k$, let $\mathrm{anc}_{k'}(\mathbf{x})$ denote the ancestor in $\mathbb{L}_{k'}$.

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$$\phi(x) = \sum_{k \in \mathbb{Z}} p^{-k[\phi]} \zeta_{\mathrm{anc}_k(x)}$$

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I will now drop the p from $|\cdot|_p$.

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$$V_{r,s}(\phi) = \int_{\Lambda_s} \{g_r : \phi^4 :_r (x) + \mu_r : \phi^2 :_r (x)\} d^3x$$

where $: \phi^k :_r$ is Wick ordering using $d\mu_{C_r}$.
Define the probability measure

$$d\nu_{r,s}(\phi) = \frac{1}{\mathcal{Z}_{r,s}} e^{-V_{r,s}(\phi)} d\mu_{C_r}(\phi) \ .$$

Let $\phi_{r,s}$ be the random distribution in $S'(\mathbb{Q}_p^3)$ sampled according to $\nu_{r,s}$ and define the squared field $N_r[\phi_{r,s}^2]$ which is a deterministic function(al) of $\phi_{r,s}$, with values in $S'(\mathbb{Q}_p^3)$, given by

$$N_r[\phi_{r,s}^2](j) = (Z_2)^r \int_{\mathbb{Q}_p^3} \{Y_2 :\phi_{r,s}^2:_r (x) - Y_0 L^{-2r[\phi]}\} \, j(x) \, d^3x$$

for suitable parameters Z_2, Y_0, Y_2. We also need a Y_1.

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Our main result concerns the limit law of the pair $(Y_1\phi_{r,s}, N_r[\phi_{r,s}^2])$ in $S'(\mathbb{Q}_p^3) \times S'(\mathbb{Q}_p^3)$ when $r \to -\infty$, $s \to \infty$ (in any order).

For the precise statement we need the approximate fixed point value

$$\bar{g}_* = \frac{p^\epsilon - 1}{36 L^\epsilon (1 - p^{-3})} .$$

Theorems:

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Theorem 1: A.A.-Chandra-Guadagni 2013

$\exists\rho > 0$, $\exists L_0$, $\forall L \geq L_0$, $\exists\epsilon_0 > 0$, $\forall\epsilon \in (0,\epsilon_0]$, $\exists[\phi^2]>2[\phi]$, $\exists$ fonctions $\mu(g)$, $Y_0(g)$, $Y_2(g)$ on $(\bar{g}_* - \rho\epsilon^{\frac{3}{2}}, \bar{g}_* + \rho\epsilon^{\frac{3}{2}})$ such that if one lets $\mu = \mu(g)$, $Y_0 = Y_0(g)$, $Y_2 = Y_2(g)$ and $Z_2 = L^{-([\phi^2]-2[\phi])}$ then the joint law of $(Y_1\phi_{r,s}, N_r[\phi^2_{r,s}])$ converge weakly and in the sense of moments to that of a pair $(\phi, N[\phi^2])$ such that:

1. $\forall k \in \mathbb{Z}$, $(L^{-k[\phi]}\phi(L^k\cdot), L^{-k[\phi^2]}N[\phi^2](L^k\cdot)) \stackrel{d}{=} (\phi, N[\phi^2])$.
2. $\langle\phi(\mathbf{1}_{\mathbb{Z}_p^3}), \phi(\mathbf{1}_{\mathbb{Z}_p^3}), \phi(\mathbf{1}_{\mathbb{Z}_p^3}), \phi(\mathbf{1}_{\mathbb{Z}_p^3})\rangle^{\mathrm{T}} < 0$ i.e., ϕ is non-Gaussian. Here, $\mathbf{1}_{\mathbb{Z}_p^3}$ denotes the indicator function of $\overline{B}(0,1)$.
3. $\langle N[\phi^2](\mathbf{1}_{\mathbb{Z}_p^3}), N[\phi^2](\mathbf{1}_{\mathbb{Z}_p^3})\rangle^{\mathrm{T}} = 1$.
4. $\langle\phi(\mathbf{1}_{\mathbb{Z}_p^3})^2\rangle = 1$.

The mixed correlation functions satisfy, in the sense of distributions,

$$\langle\ \phi(L^{-k}x_1)\cdots\phi(L^{-k}x_n)\ N[\phi^2](L^{-k}y_1)\cdots N[\phi^2](L^{-k}y_m)\ \rangle$$

$$= L^{-(n[\phi]+m[\phi^2])k}\ \langle\ \phi(x_1)\cdots\phi(x_n)\ N[\phi^2](y_1)\cdots N[\phi^2](y_m)\ \rangle$$

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Not too far, if one boldly extrapolates to $\epsilon=1$, from the most precise available estimates concerning the short range 3D Ising model: $[\phi^2]-2[\phi]=0.376327\ldots$ (JHEP 2016 by Kos, Poland, Simmons-Duffin and Vichi, using conformal bootstrap).

We also proved the law $\nu_{\phi\times\phi^2}$ of $(\phi, N[\phi^2])$, is independent of g in the interval $(\bar{g}_* - \rho\epsilon^{\frac{3}{2}}, \bar{g}_* + \rho\epsilon^{\frac{3}{2}})$.

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$\nu_{\phi\times\phi^2}$ is fully scale invariant, i.e., invariant under the action of the scaling group $p^{\mathbb{Z}}$ instead of the subgroup $L^{\mathbb{Z}}$. Moreover, $\mu(g)$ and $[\phi^2]$ are independent of the arbitrary factor L.

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The two-point correlations are given in the sense of distributions by

$$\langle\phi(x)\phi(y)\rangle = \frac{c_1}{|x-y|^{2[\phi]}}$$

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Theorem 3: A.A., May 2015

Use ψ_i to denote the scaling limits ϕ or $N[\phi^2]$. Then, for all mixed correlation $\exists$ a smooth (i.e., locally constant) fonction $\langle\psi_1(z_1)\cdots\psi_n(z_n)\rangle$ on $(\mathbb{Q}_p^3)^n\backslash\mathrm{Diag}$ which is locally integrable (on the big diagonal Diag) and such that

$$\mathbb{E}\ \psi_1(f_1)\cdots\psi_n(f_n)\ =$$

$$\int_{(\mathbb{Q}_p^3)^n\backslash\mathrm{Diag}} \langle\psi_1(z_1)\cdots\psi_n(z_n)\rangle\ f_1(z_1)\cdots f_n(z_n)\ d^3z_1\cdots d^3z_n$$

for all test functions $f_1,\ldots,f_n \in S(\mathbb{Q}_p^3)$.

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This follows from the use of the SDRG to derive an explicit representation of pointwise correlations in terms of very close analogues of tree Witten diagrams. Hence, the emergent connection to the AdS/CFT correspondence.

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Take $RG(a, b) = \left(\frac{a+b}{2}, \sqrt{ab}\right)$.
(Landen-Gauss)

1st step: rescale to unit lattice/cut-off

$$\mathcal{S}^{\mathrm{T}}_{r,s}(f) := \log \mathbb{E}_{\nu_{r,s}} e^{\phi(f)} = \log$$

$$\frac{\int d\mu_{C_r}(\phi) \exp\left(-\int_{\Lambda_s}\{g_r : \phi^4 :_r (x) + \mu_r : \phi^2 :_r\}dx + \int \phi(x) f(x) dx\right)}{\int d\mu_{C_r}(\phi) \exp\left(-\int_{\Lambda_s}\{g_r : \phi^4 :_r (x) + \mu_r : \phi^2 :_r\}dx\right)}$$

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with

$$\mathcal{I}^{(r,r)}[f](\phi) = \exp\left(- \int_{\Lambda_{s-r}} \{ g : \phi^4 :_0 (x) + \mu : \phi^2 :_0 \} d^3x \right.$$

$$\left. + L^{(3-[\phi])r} \int \phi(x) f(L^{-r}x) d^3x \right)$$

2nd step: define inhomogeneous RG

Fluctuation covariance $\Gamma := C_0 - C_1$.

Associated Gaussian measure is the law of the fluctuation field

$$\zeta(x) = \sum_{0 \le k < \ell} p^{-k[\phi]} \zeta_{\mathrm{anc}_k(x)}$$

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$$\int \mathcal{I}^{(r,r)}[f](\phi)\, d\mu_{C_0}(\phi) = \int \int \mathcal{I}^{(r,r)}[f](\zeta + \psi)\, d\mu_\Gamma(\zeta) d\mu_{C_1}(\psi)$$

$$= \int \mathcal{I}^{(r,r+1)}[f](\phi)\, d\mu_{C_0}(\phi)$$

with new integrand

$$\mathcal{I}^{(r,r+1)}[f](\phi) := \int \mathcal{I}^{(r,r)}[f](\zeta + L^{-[\phi]}\phi(L\cdot))\, d\mu_\Gamma(\zeta)$$

Need to extract vacuum renormalization $\rightarrow$ better definition is

$$\mathcal{I}^{(r,r+1)}[f](\phi) = e^{-\delta b(\mathcal{I}^{(r,r)}[f])} \int \mathcal{I}^{(r,r)}[f](\zeta + L^{-[\phi]}\phi(L\cdot)) \ d\mu_\Gamma(\zeta)$$

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One must control

$$\mathcal{S}^{\mathrm{T}}(f) = \lim_{\substack{r\rightarrow-\infty \\ s\rightarrow\infty}} \sum_{r\le q<s} \left(\delta b(\mathcal{I}^{(r,q)}[f]) - \delta b(\mathcal{I}^{(r,q)}[0])\right)$$

limit of logarithms of characteristic functions.

Use a Brydges-Yau lift

$$
\begin{array}{ccc}
\vec{V}^{(r,q)} & \xrightarrow{RG_{\mathrm{inhom}}} & \vec{V}^{(r,q+1)} \\
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\mathcal{I}^{(r,q)} & \longrightarrow & \mathcal{I}^{(r,q+1)}
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$$

$$
\Big\{ \exp\left(-\beta_{4,\Delta} :\phi_\Delta^4:_{C_0} -\beta_{3,\Delta} :\phi_\Delta^3:_{C_0} -\beta_{2,\Delta} :\phi_\Delta^2:_{C_0} -\beta_{1,\Delta} :\phi_\Delta^1:_{C_0} \right)
$$

$$
\times \left(1 + W_{5,\Delta} :\phi_\Delta^5:_{C_0} + W_{6,\Delta} :\phi_\Delta^6:_{C_0} \right)
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Dynamical variable is $\vec{V} = (V_\Delta)_{\Delta\in\mathbb{L}_0}$ with

$$
V_\Delta = (\beta_{4,\Delta}, \beta_{3,\Delta}, \beta_{2,\Delta}, \beta_{1,\Delta}, W_{5,\Delta}, W_{6,\Delta}, f_\Delta, R_\Delta)
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RG_{inhom} acts on $\mathcal{E}_{\text{inhom}}$, essentially,

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Stable subspaces

$\mathcal{E}_{\mathrm{hom}} \subset \mathcal{E}_{\mathrm{inhom}}$: spatially constant data.
$\mathcal{E} \subset \mathcal{E}_{\mathrm{hom}}$: even potential, i.e., g, μ's only and R even function.
Let RG be induced action of RG_{inhom} on $\mathcal{E}$.

3rd step: stabilize bulk (homogeneous) evolution

Show that $\forall q \in \mathbb{Z}$, $\lim_{r\to-\infty} \vec{V}^{(r,q)}[0]$ exists, i.e.,

$$\lim_{r\to-\infty} RG^{q-r}\left(\vec{V}^{(r,r)}[0]\right)$$

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Tadpole graph with mass insertion

$$A_3 = 12 L^{3-2[\phi]} \int_{\mathbb{Q}_p^3} \Gamma(0,x)^2 \ d^3x$$

is main culprit for anomalous scaling $[\phi^2] - 2[\phi] > 0$.

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Thus

$$\forall q \in \mathbb{Z}, \qquad \lim_{r \to -\infty} \vec{V}^{(r,q)}[0] = v_*$$

Tangent spaces at fixed point: E^{s} and E^{u}.

$E^{\mathrm{u}} = \mathbb{C}e_{\mathrm{u}}$, with e_{u} eigenvector of $D_{v_*}RG$ for eigenvalue $\alpha_{\mathrm{u}} = L^{3-2[\phi]} \times Z_2 =: L^{3-[\phi^2]}$.

4th step: control deviation from homogeneous evolution
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For source term with ϕ^2 add

$$Y_2 Z_2^r \int :\phi^2:_{C_r}(x) j(x) d^3x$$

to potential. $\mathcal{S}^{\mathrm{T}}_{r,s}(f,j)$ now involves two test functions. After rescaling to unit lattice/cut-off

$$Y_2 \alpha_{\mathrm{u}}^r \int :\phi^2:_{C_0}(x) j(L^{-r}x) d^3x$$

to be combined with μ into $(\beta_{2,\Delta})_{\Delta\in\mathbb{L}_0}$ space-dependent mass.

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$\Psi(v, w)$ is holomorphic in v and w.

This is essential for probabilistic interpretation of $(\phi, N[\phi^2])$ as pair of random variables in $S'(\mathbb{Q}_p^3)$.

Thank you for your attention.