Maximum Scatter TSP in Doubling Metrics

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BIRS

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Maximum Scatter TSP

Picture: Jordan Wooley, CC BY-ND

Cost Function

Minimize

- Overall length (TSP)
- Maximal edge length (bottleneck TSP)

Maximize

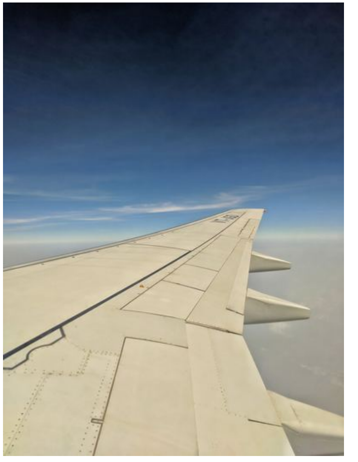
- Overall length (Max-TSP)
- Minimum length (Max-Scatter TSP; MSTSP)

Applications

Riveting:

Aim to place rivets

far from each other

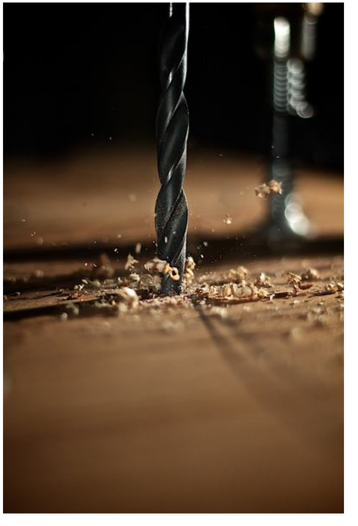


Drilling:

Workpiece heats up

Consecutive holes

far from each other



Background

Maximum Scatter TSP introduced by
[Arkin–Chiang–Mitchell–Skiena–Yang SODA'97]

In metric graphs:

2-approximation

APX hard, lower bound 2

$\Rightarrow$ tight if P $\neq$ NP

They asked: What happens for instances in the Euclidean plane?

Known Results

	Metric	Euclidean (fixed dimension)
min TSP	$1.081 < \alpha \leq 1.5$ [Karpinski–Lampis–Schmied 2015] [Christofides 1976]	$\alpha = 1 + \epsilon$ [Arora 1996] [Mitchell 1996]
max TSP	$1 + \epsilon < \alpha \leq 1.14$ [Papadimitriou–Yannakakis 1993] [Kowalik–Mucha 2009]	$\alpha = 1 + \epsilon$ [Barvinok 1996]
min max TSP (bottleneck)	$\alpha = 2$ [Parker–Rardin 1982] [Doroshko–Sarvanov 1981]	$\alpha = 2$ [Sarvanov 1995]
max min TSP (max. scatter)	$\alpha = 2$ [Arkin–Chiang–Mitchell–Skiena–Yang 1997]	

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Our Results [SODA 2017]

$(1+\epsilon)$-approximation algorithm for d-dimensional doubling metrics with running time

$$\tilde{O}(n^3 + 2^{O(K \log K)}),\ K = (13/\epsilon)^d$$

Corollary:

$(1+\epsilon)$-apx for $(\log\log n)/c$-dimensional doubling metrics (for some constant c)

Matching hardness result:

$(4/3)^{1/p} - \epsilon$ lower bound for ℓ_p distances in $\mathbb{R}^{c \cdot \log n}$

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Simplified Setup

There is a PTAS for MSTSP in the Euclidean plane

$(1+\epsilon)$-approximation
for arbitrarily small constant $\epsilon > 0$

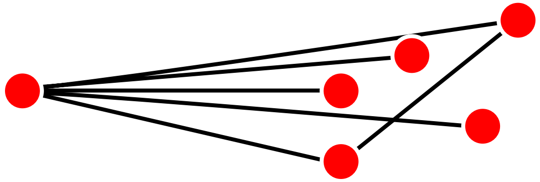
Answers question of Arkin et al. [SODA 1997]

Step One

Formulate as decision problem
by guessing optimum solution value ℓ

Remaining Problem:

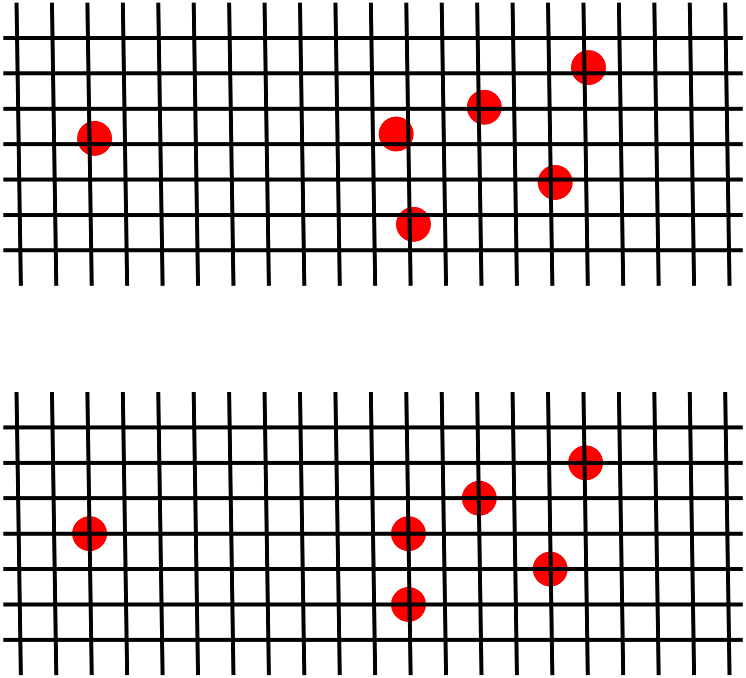
Find Hamiltonian tour in unweighted graph G



Structural property: high degrees if vertices far apart

Step Two

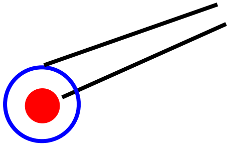
Idea: Snap vertices to grid



Step Two Cont'd

$\epsilon \cdot \ell$ grid:

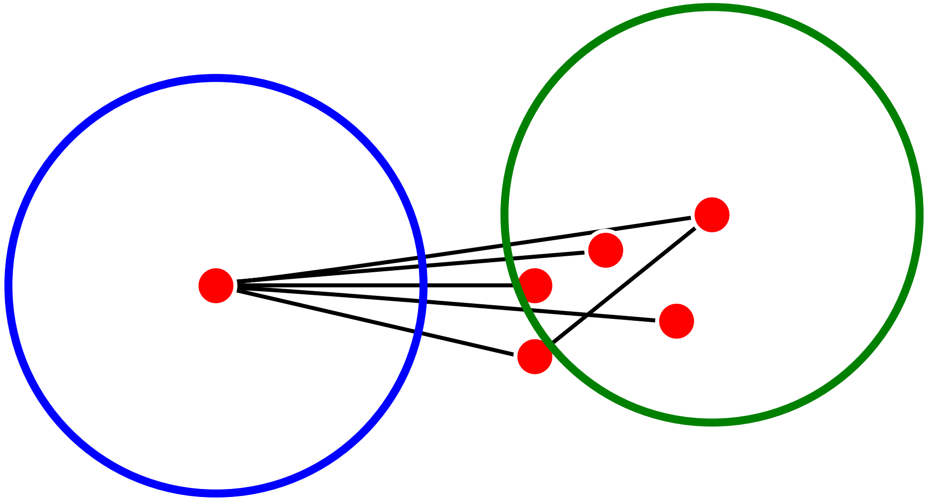
No point farther than $\epsilon\ell \cdot \sqrt{2}/2$ from next grid point



Length changes by at most $O(\epsilon)\ell$

Step Three

Consider balls of radius ℓ



If no ℓ-ball contains $> n/2$ vertices, all degrees $\geq n/2$

Step Three Cont'd

Dirac's Theorem

If all degrees $\geq n/2$, we can efficently find Hamiltonian tour

$\Rightarrow$ If no ℓ-ball contains $> n/2$ vertices, we can efficiently find the tour

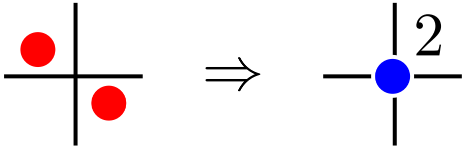
Assume in the following:

The graph contains ℓ-ball with $> n/2$ vertices

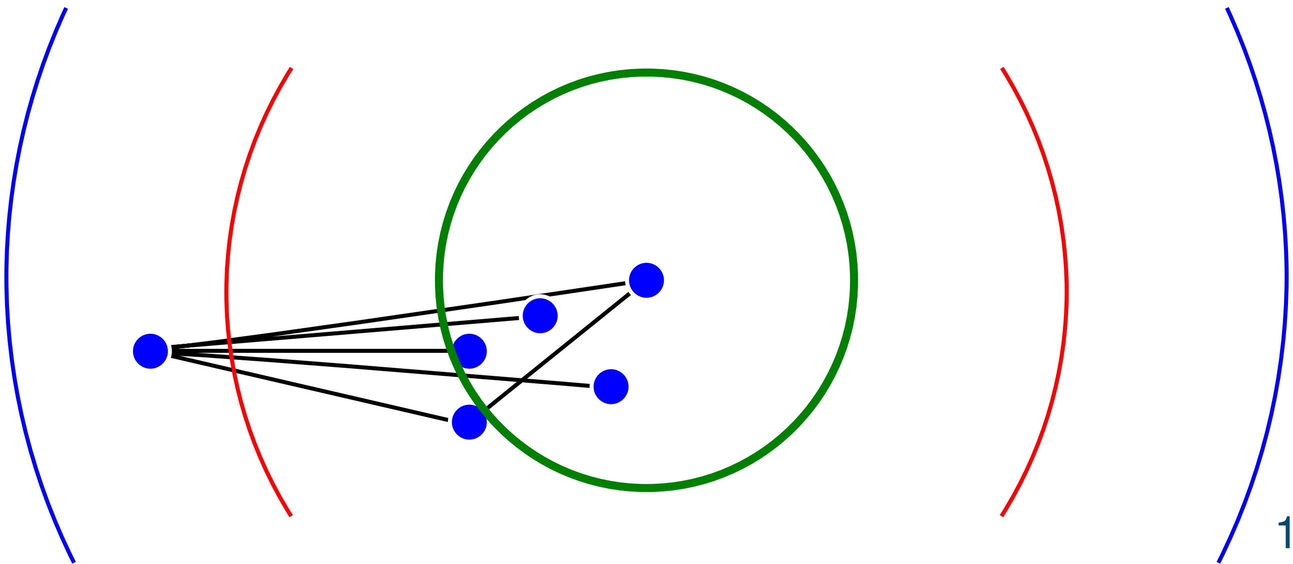
Remaining Instance

Several vertices at one grid point:

Super-vertex with multiplicity



Constantly many super-vertices in 3ℓ-ball

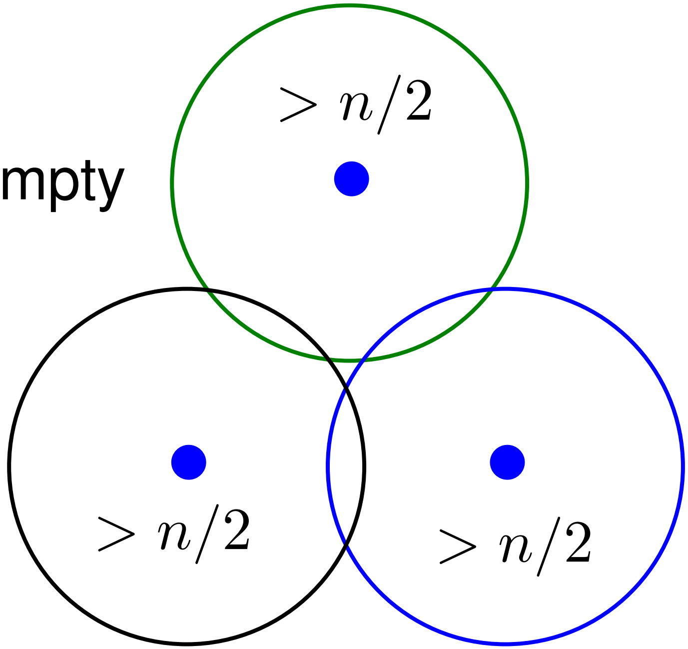


Low Degree Vertices

If deg$(v) \leq n/2$, $> n/2$ vertices in ℓ-ball around v

All these balls have non-empty
pairwise intersection

$\Rightarrow$ they fit into 2ℓ-Ball



Bondy-Chvátal Thm

If

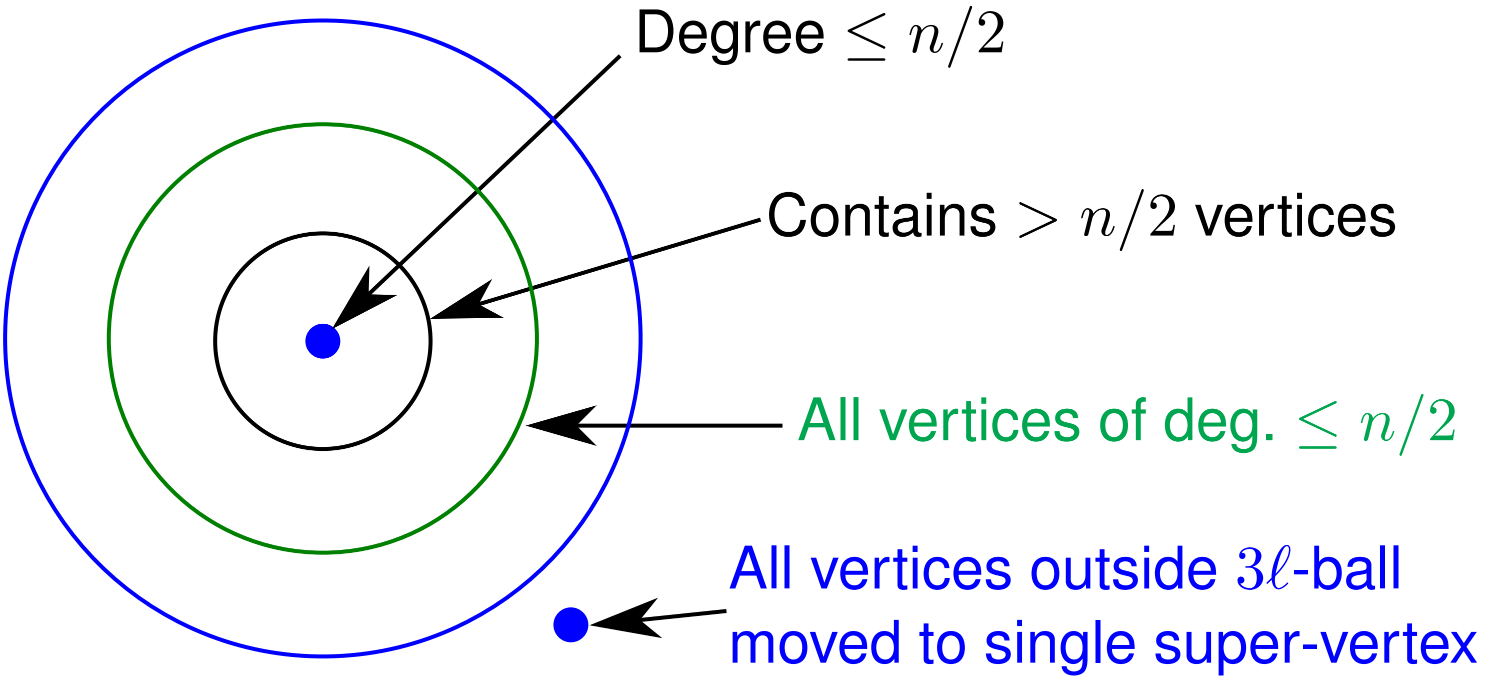
$$\text{deg}(u) + \text{deg}(v) \geq n \text{ in } G \text{ and}$$

$$G' = G + \{u, v\} \text{ Hamiltonian,}$$

then G Hamiltonian

Bondy-Chvátal closure: apply theorem iteratively

Single Vertex Outside



Solve many-visits TSP

Many Visit TSP

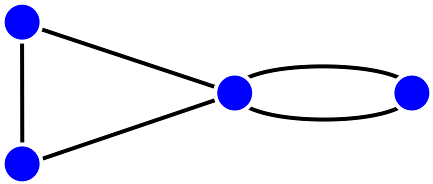
[Berger, Kozma, Mnich, Vincze 2018]

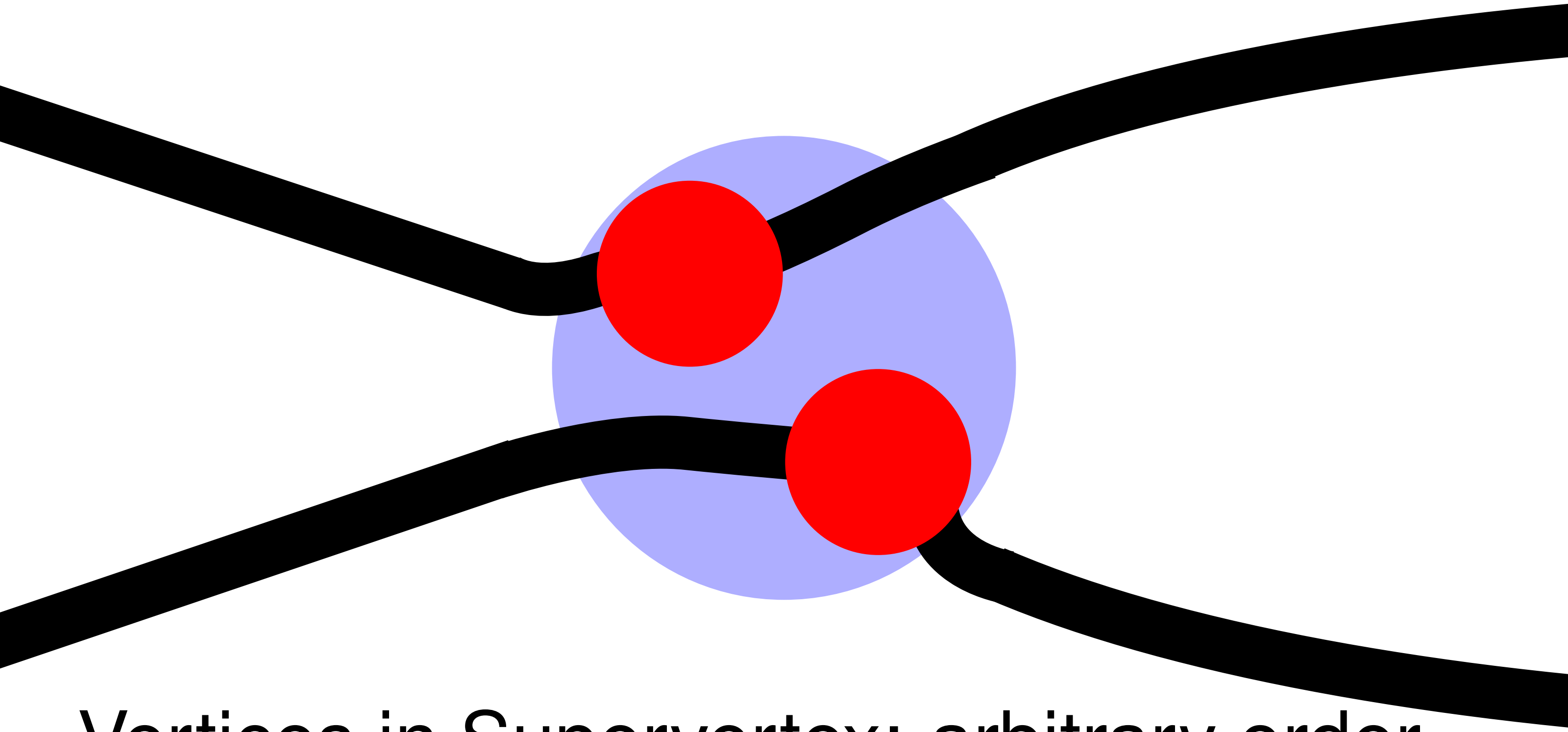
Many-visits TSP exact in time $2^{O(k)} + \text{poly}(n, k)$

Number of vertices: n
Number of super-vertices: k

Last Step

Compute Eulerian tour





Vertices in Supervertex: arbitrary order

$\Rightarrow$ Eulerian implies Hamiltonian tour

High Dimensions

Runtime dominated by 2^{2^d}
$\Rightarrow$ Polynomial time for $d = \log \log n / c$
for some constant c

What about higher dimensions?

APX hard for $\geq \log n$ dimensions

Proof: Along the lines of [Trevisan STOC 1997],
but needs changes in encoding

Non-Euclidean Metric

Hardness due to dimensionality

Doubling Dimension k:

Each ball $B_r(\cdot)$ can be covered with 2^k balls $B_{r/2}(\cdot)$

Property of arbitrary metric space (not restricted to Euclidean space)

Definition matches the properties needed for a “good” ϵ-net

Conclusion

Solved:

Basically tight result for all dimensions ≥ 3

Open:

Sitation not clear for 2 dimensions
- Could still be in P
- FPTAS?

Related: MaxTSP in $2D$
- Issues with real values:
  sum of square roots