Disruption Risk Mitigation in Supply Chains – The Risk Exposure Index Revisited

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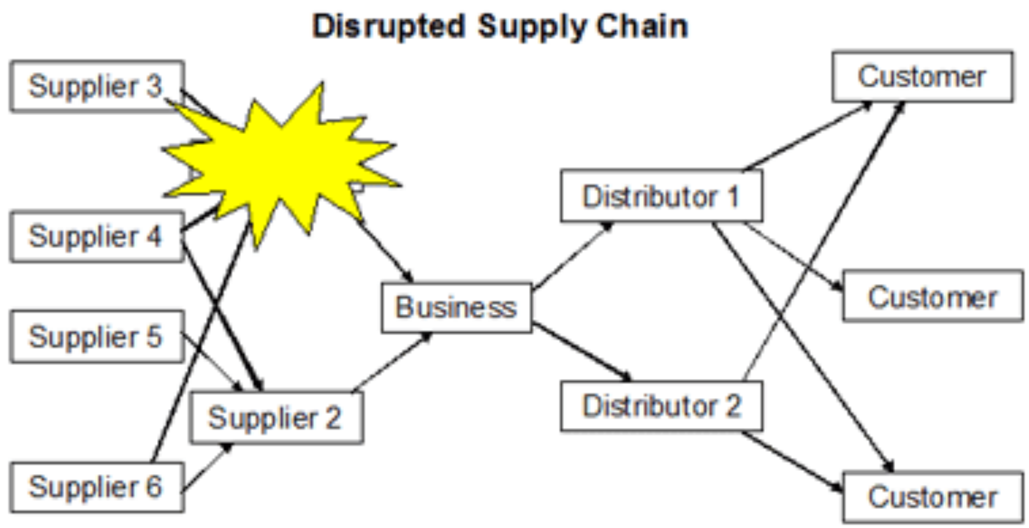
Acknowledgement

Thanks to Sun De-Feng, Toh Kim Chuan for their software SDPNAL+ to solve large scale Doubly Nonegative Problem.

Supply Chain Risk Management



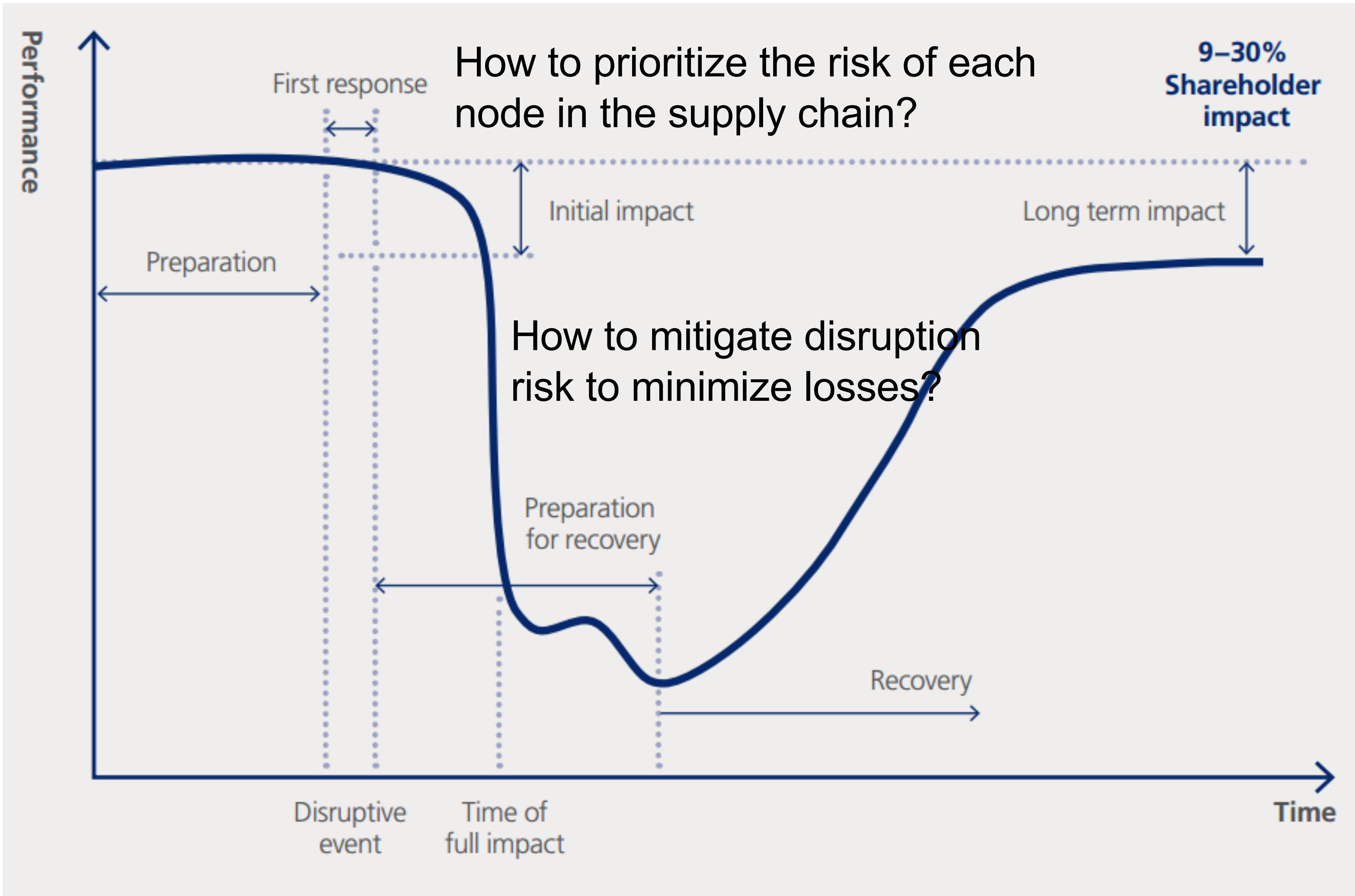
How to reduce the impact of disruption to supply chain?

Resilinc: Supply Chain Disruptions Nearly Doubled in 2017

	Factory Fire/Explosion	Merger & Acquisition	Business Sale or Spin-off	Factory shutdown/ Disruption	Hurricane/ Typhoon/Cyclone
2017	343	324	135	118	116
2016	194	293	102	66	82

The following chart provides the top 5 supply chain event types for 2016 and 2017. Factory Fire/Explosion increased in 2017, representing 18% of all EventWatch bulletins issued up from 13% in 2016. Merger & Acquisition dropped slightly from 20% of bulletins issued in 2016 to 17% issued in 2017, though the number of bulletins issued for the event type actually rose last year. While Business Sale or Spin-off maintained its

Supply Chain Risk Management



RISK PRIORITIZATION

RISK MITIGATION

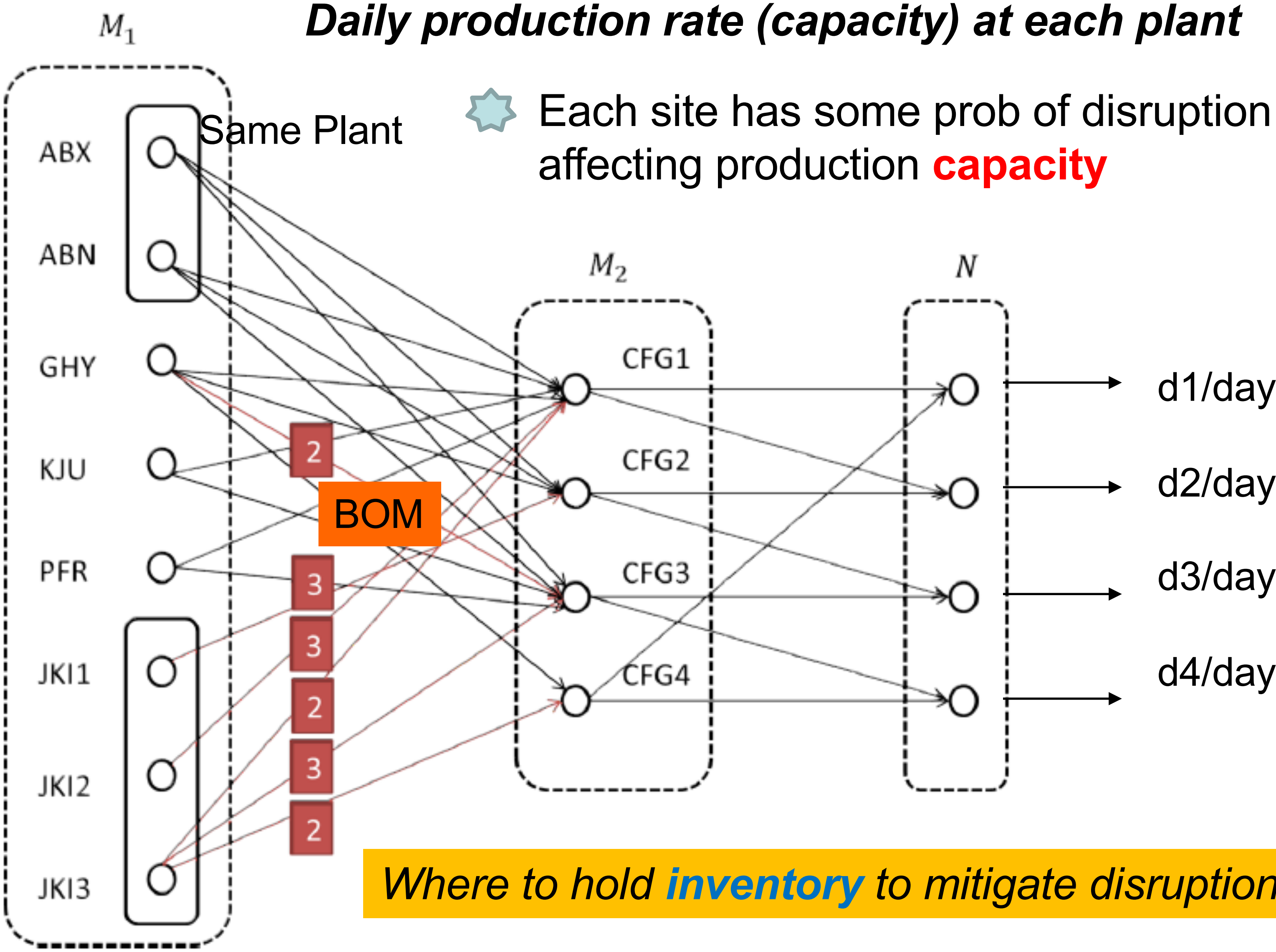
Plan

- Motivation:
  - Time-To-Recovery and Supply Chain Disruption
- Mitigation:
  - Distributionally Robust Model using worst case CVAR
- Prioritization:
  - Sensitivity Analysis on Supply Chain Mitigation

Research Question

Daily production rate (capacity) at each plant

Each site has some prob of disruption affecting production **capacity**



Depends on TTR(k):

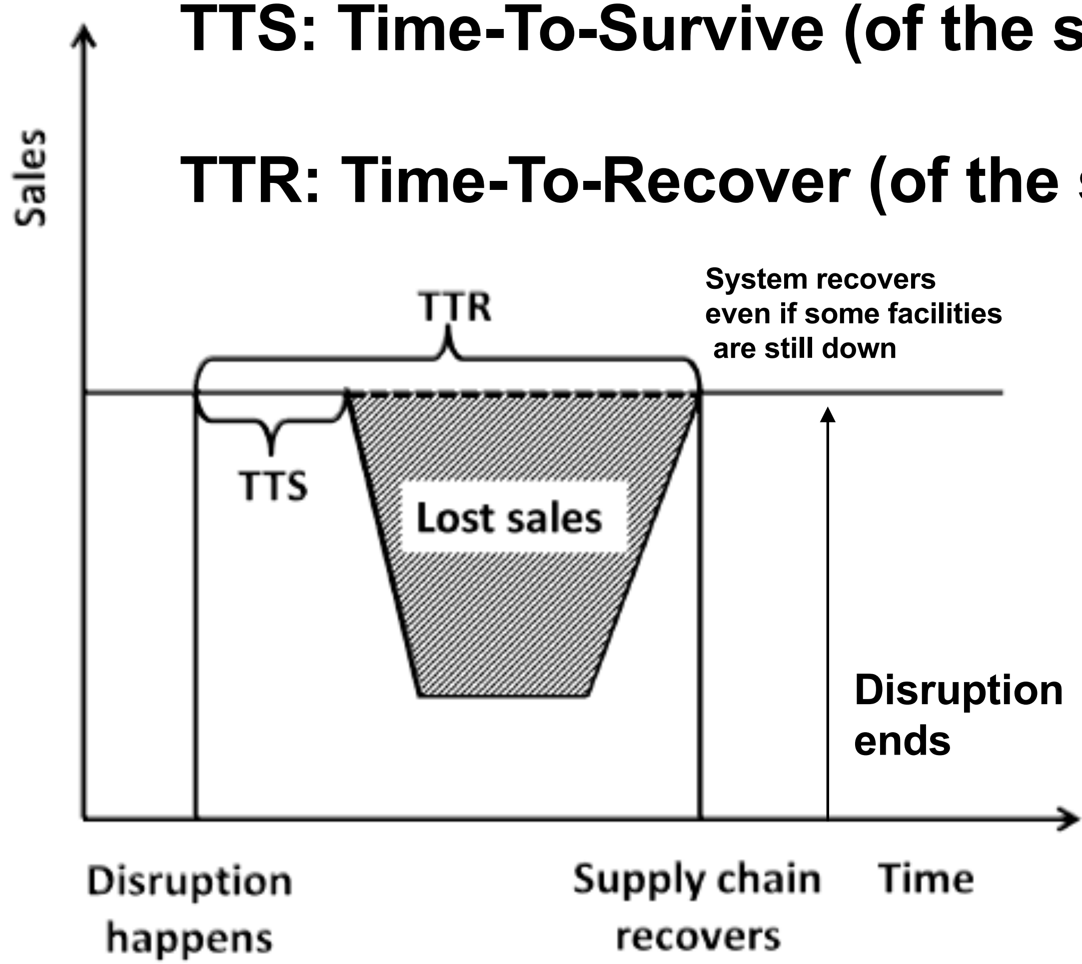
Time-To-Recovery of node k

*Where to hold **inventory** to mitigate disruption?*

Measuring the Supply Chain Disruption Impact

TTS: Time-To-Survive (of the supply chain)

TTR: Time-To-Recover (of the supply chain)



Supply Chain Mitigation to minimize the impact of Lost Sales?

Model

Lost sales cost

$$Z(\boldsymbol{v},\boldsymbol{r}) = \min_{(x_{ij},\boldsymbol{u},\boldsymbol{l})} \sum_{j\in\mathcal{N}} f_j l_j$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} x_{ij} + l_j \geq d_j T^R, \quad \forall j \in \mathcal{N}$$

$$\sum_{j\in\mathcal{M}\cup\mathcal{N},(i,j)\in\mathcal{G}} x_{ij} - u_i \leq r_i, \quad \forall i \in \mathcal{M}$$

$$\sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} \frac{x_{ij} I_{it}^{PT}}{B_{tj}} - u_j \geq 0, \quad \forall j \in \mathcal{M}, t \in \mathcal{T}_j$$

$$\sum_{i\in\mathcal{A}_k} u_i \leq (T^R - T_k^r(1 - v_k))c_k, \quad \forall k \in \mathcal{P}$$

$$x_{ij} \geq 0, \boldsymbol{u}, \boldsymbol{l} \geq \boldsymbol{0}$$

Computes lost sales for given disruption scenario **v** and *inventory* strategy **r**

Time of Disruption

Nodes in the same plant

TTR(node k)

Vk = 1 if no disruption

Disruption to capacity only!

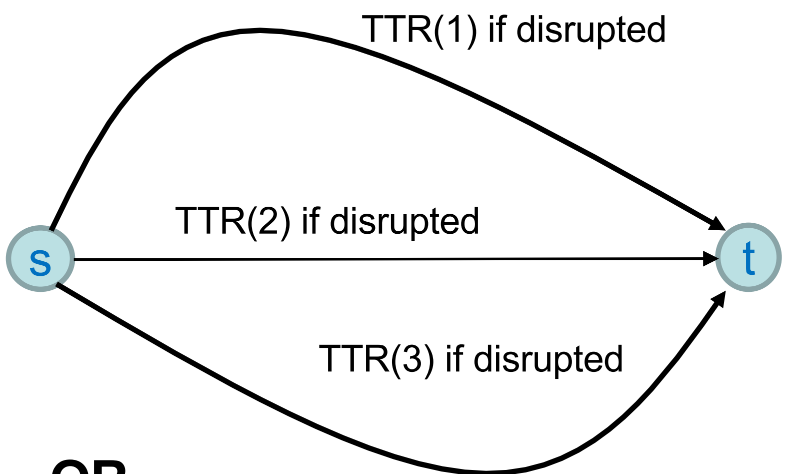
Minimize inventory **r** investment s.t. CVaR(Z(**v**,**r**)) acceptable

HOW TO MODEL TIME-TO-RECOVERY?

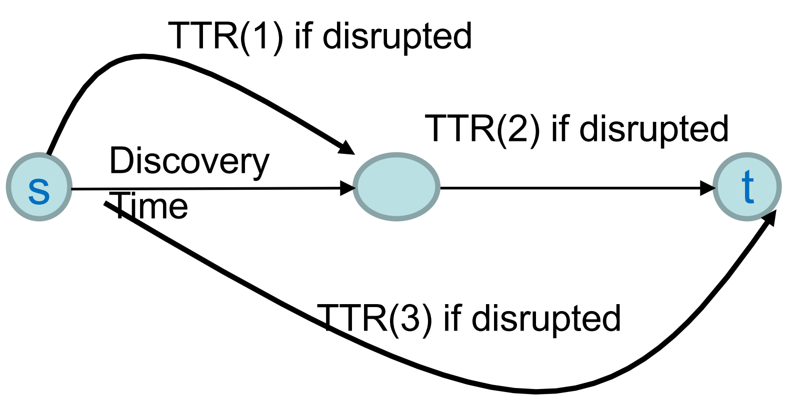
Complex function of random events

Building DRO Model difficult

LONGEST PATH MODEL



OR



Time of Disruption

$$
\begin{aligned}
\min_{(x_{ij},\mathbf{u},\mathbf{l})} \quad & \sum_{j \in \mathcal{N}} f_j l_j \\
s.t. \quad & \sum_{i \in \mathcal{M},(i,j)\in \mathcal{G}} x_{ij} + l_j \geq d_j T^R, \\
& \sum_{j \in \mathcal{M}\cup\mathcal{N},(i,j)\in \mathcal{G}} x_{ij} - u_i \leq r_i, \\
& \sum_{i \in \mathcal{M},(i,j)\in \mathcal{G}} \frac{x_{ij} I_{it}^{PT}}{B_{tj}} - u_j \geq 0, \\
& \sum_{i \in \mathcal{A}_k} u_i \leq (T^R - T_k^r(1-v_k))c_k, \\
& x_{ij} \geq 0, \mathbf{u}, \mathbf{l} \geq \mathbf{0}
\end{aligned}
$$

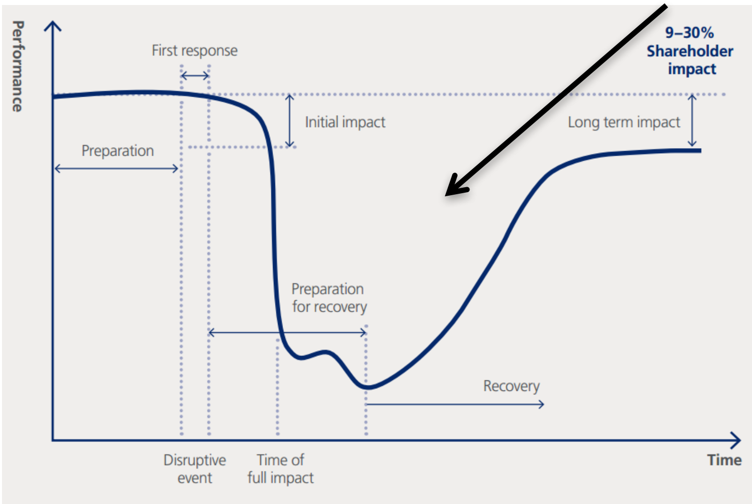
Available capacity within time of disruption

Benchmark #1
Risk-Exposure-Index: At most one node disrupted

Find min cost inventory strategy to ensure zero lost sales in all disruption scenarios

$$
\begin{array}{llcll}
\min & \boldsymbol{h}^\top \boldsymbol{r} & & & \\
s.t. & \sum_{i \in \mathcal{M}:(i,j)\in\mathcal{G}} x_{ij}^{(w)} & \geq & d_j T^{R(w)}, & \forall j \in \mathcal{N}, w \in \{1,2,..,p\} \\
& \sum_{j \in \mathcal{M}\cup\mathcal{N}:(i,j)\in\mathcal{G}} x_{ij}^{(w)} - u_i^{(w)} & \leq & r_i, & \forall i \in \mathcal{M}, w \in \{1,2,..,p\} \\
& \sum_{i \in \mathcal{M}:(i,j)\in\mathcal{G}} \frac{x_{ij}^{(w)} I_{it}^{PT}}{B_{tj}} - u_j^{(w)} & \geq & 0, & \forall j \in \mathcal{M}_2, t \in \mathcal{T}_j, w \in \{1,2,..,p\} \\
& \sum_{i \in A_k} u_i^{(w)} & \leq & (T^{R(w)} - T_k^r(1 - v_k^{(w)}))c_k, & \forall k \in \mathcal{P}, w \in \{1,2,..,p\} \\
& v_w^{(w)} & = & 1 & \forall w \in \{1,2,..,p\} \\
& v_k^{(w)} & = & 0 & \forall k \neq w, \forall w \in \{1,2,..,p\} \\
& x_{ij}^{(w)} \geq 0, u_j^{(w)} \geq 0, & & & \forall w \in \{1,2,..,p\} \\
& \boldsymbol{r} \geq \boldsymbol{0} & & &
\end{array}
$$

Risk Measure: Worst Case CVaR



$$
\begin{aligned}
Z(\boldsymbol{v},\boldsymbol{r}) = \min_{(x_{ij},\boldsymbol{u},\boldsymbol{l})} \quad & \sum_{j\in\mathcal{N}} f_j l_j \\
s.t. \quad & \sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} x_{ij} + l_j \geq d_j T^R, & \forall j \in \mathcal{N} \\
& \sum_{j\in\mathcal{M}\cup\mathcal{N},(i,j)\in\mathcal{G}} x_{ij} - u_i \leq r_i, & \forall i \in \mathcal{M} \\
& \sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} \frac{x_{ij} I_{it}^{PT}}{B_{tj}} - u_j \geq 0, & \forall j \in \mathcal{M}, t \in \mathcal{T}_j \\
& \sum_{i\in\mathcal{A}_k} u_i \leq (T^R - T_k^r(1-v_k))c_k, & \forall k \in \mathcal{P} \\
& x_{ij} \geq 0, \boldsymbol{u}, \boldsymbol{l} \geq \boldsymbol{0}
\end{aligned}
$$

Optimize inventory strategy

$$
WCVaR_{1-\eta}(\boldsymbol{r}) := \max_{p(\tilde{v})\in\mathcal{P}} \left\{ \min_{\theta} \theta + \frac{1}{\eta} E[(Z(\boldsymbol{r},\tilde{\boldsymbol{v}}) - \theta)^+] \right\}
$$

↑

Robust to disruption probabilities

Benchmark #2
Stochastic Programming using CVaR

$$\begin{aligned} \min\ & \theta + \frac{1}{\eta}\sum_{w\in S} p^{(w)} Q^{(w)}(\boldsymbol{r},\theta) \\ s.t.\ & \boldsymbol{h}^{\top}\boldsymbol{r} \le br \\ & \boldsymbol{r} \ge \boldsymbol{0} \end{aligned}$$

where for any $w = 1, \ldots, |S|$,

$$\begin{aligned} Q^{(w)}(\boldsymbol{r},\theta) = \min\ & y^{(w)} \\ s.t.\ & y^{(w)} \ge \sum_{j\in\mathcal{N}} f_j^{(w)} l_j^{(w)} - \theta \\ & \sum_{i\in\mathcal{M}:(i,j)\in\mathcal{G}} x_{ij}^{(w)} + l_j^{(w)} \ge d_j T^{R(w)}, && \forall j\in\mathcal{N} \\ & \sum_{j\in\mathcal{M}\cup\mathcal{N}:(i,j)\in\mathcal{G}} x_{ij}^{(w)} - u_i^{(w)} \le r_i, && \forall i\in\mathcal{M} \\ & \sum_{i\in\mathcal{M}:(i,j)\in\mathcal{G}} \frac{x_{ij}^{(w)} I_{it}^{PT}}{B_{tj}} - u_j^{(w)} \ge 0, && \forall j\in\mathcal{M}_2, t\in\mathcal{T}_j \\ & \sum_{i\in A_k} u_i^{(w)} \le (T^{R(w)} - T_k^r(1 - v_k^{(w)}))c_k, && \forall k\in\mathcal{P} \\ & y^{(w)} \ge 0 \\ & x_{ij}^{(w)}, u_j^{(w)}, l_j^{(w)} \ge 0, \forall i\in\mathcal{M}, j\in\mathcal{N} \end{aligned}$$

Plan

- Motivation:
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Conic Formulation for Worst Case CVaR

$$
\begin{array}{llll}
Z(\boldsymbol{v},\boldsymbol{r}) = \min\limits_{(x_{ij},\boldsymbol{u},\boldsymbol{l})} & \sum\limits_{j\in\mathcal{N}} f_j l_j & & \text{Dual Variables} \\
\text{s.t.} & \sum\limits_{i\in\mathcal{M},(i,j)\in\mathcal{G}} x_{ij} + l_j \geq d_j T^R, & \forall j \in \mathcal{N} & \boldsymbol{\alpha} \\
& \sum\limits_{j\in\mathcal{M}\cup\mathcal{N},(i,j)\in\mathcal{G}} x_{ij} - u_i \leq r_i, & \forall i \in \mathcal{M} & \boldsymbol{\beta} \\
& \sum\limits_{i\in\mathcal{M},(i,j)\in\mathcal{G}} \frac{x_{ij} I_{it}^{PT}}{B_{tj}} - u_j \geq 0, & \forall j \in \mathcal{M}, t \in \mathcal{T}_j & \boldsymbol{\gamma} \\
& \sum\limits_{i\in\mathcal{A}_k} u_i \leq (T^R - T_k^r(1-v_k))c_k, & \forall k \in \mathcal{P} & \boldsymbol{\delta} \\
& x_{ij} \geq 0, \boldsymbol{u}, \boldsymbol{l} \geq \boldsymbol{0} & &
\end{array}
$$

Dual:

$$
\begin{array}{ll}
\max & \sum\limits_{j\in\mathcal{N}} d_j \alpha_j T^R - \sum\limits_{i\in\mathcal{M}} r_i \beta_i - \sum\limits_{k\in\mathcal{P}} c_k \delta_k (T^R - T_k^r(1-v_k)) \\
\text{s.t.} & (\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{s^1}, \boldsymbol{s^2}, \boldsymbol{s^3}, \boldsymbol{s^4}, \boldsymbol{s^5}) \in \mathcal{F}
\end{array}
$$

$$
\mathcal{F} = \left\{
\begin{array}{ll}
\alpha_j - \beta_i + s_l^1 = 0, & \forall j \in \mathcal{N}, (i,j) \in \mathcal{G}_1 \\
-\beta_i + \sum\limits_{t\in\mathcal{T}_j} \frac{\gamma_t^j I_{it}^{PT}}{B_{tj}} + s_l^2 = 0, & \forall i \in \mathcal{M}, \forall j \in \mathcal{M}_2, (i,j) \in \mathcal{G} \\
-\delta_k + \beta_{i(k)} - \sum\limits_{t\in\mathcal{T}_{i(k)}} \gamma_t^{i(k)} + s_l^3 = 0, & \forall k \in \mathcal{P}, i(k) \in \mathcal{A}_k, i(k) \in \mathcal{M}_2 \\
-\delta_k + \beta_{i(k)} + s_l^4 = 0, & \forall k \in \mathcal{P}, i(k) \in \mathcal{A}_k, i(k) \in \mathcal{M}_1 \\
\alpha_j + s_j^5 = f_j & \forall j \in \mathcal{N} \\
\boldsymbol{\alpha} \in \mathbb{R}_+^n, \boldsymbol{\beta} \in \mathbb{R}_+^m, \boldsymbol{\gamma} \in \mathbb{R}_+^{tp}, \boldsymbol{\delta} \in \mathbb{R}_+^p & \\
\boldsymbol{s^1} \in \mathbb{R}_+^{|\mathcal{G}_2|}, \boldsymbol{s^2} \in \mathbb{R}_+^{|\mathcal{G}_1|}, \boldsymbol{s^3} \in \mathbb{R}_+^{m_2} & \\
\boldsymbol{s^4} \in \mathbb{R}_+^{m_1}, \boldsymbol{s^5} \in \mathbb{R}_+^n &
\end{array}
\right\}
$$

Conic Formulation for Linear Constraints and Given Moments

Sam Burer (2009)

The Model (CPCMM)

Starting Point

On the copositive representation of binary and continuous nonconvex quadratic programs

Journal	Mathematical Programming
Publisher	Springer Berlin / Heidelberg
ISSN	0025-5610 (Print) 1436-4646 (Online)
Category	FULL LENGTH PAPER
DOI	10.1007/s10107-008-0223-z
Subject Collection	Mathematics and Statistics
SpringerLink Date	Tuesday, April 29, 2008

Online First

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Received: 27 November 2006 **Accepted:** 26 March 2008 **Published online:** 29 April 2008

Completely positive programs (CPP) for deterministic optimization problems

Natarajan-Teo-Zheng (2009):

$$
\text{(P)} \quad \sup_{\tilde{\mathbf{c}} \sim (\mu, Q)} \mathbf{E}\left[Z(\tilde{\mathbf{c}})\right]
$$

$$
\text{where } Z(\tilde{\mathbf{c}}) = \begin{array}{ll} max & \tilde{\mathbf{c}}^T \mathbf{x} \\ s.t. & \mathbf{a}_i^T \mathbf{x} = b_i, \ \forall i \in M \\ & \mathbf{x} \geq \mathbf{0} \\ & x_j \in \{0,1\}, \ \forall j \in B \end{array}
$$

$$
||
$$

$$
\begin{array}{lll} \text{(C)} & max & \sum_{j \in N} Y_{jj} \\ & s.t. & \mathbf{a}_i^T \mathbf{p} = b_i, \ \forall i \in M \\ & & \mathbf{a}_i^T X \mathbf{a}_i = b_i^2, \ \forall i \in M \\ & & X_{jj} = p_j, \ \forall j \in B \\ & & \begin{pmatrix} 1 & \mu^T & \mathbf{p}^T \\ \mu & Q & Y \\ \mathbf{p} & Y^T & X \end{pmatrix} \succ_{cp} 0 \end{array}
$$

Conic Formulation for Quadratic Constraints and Given Moments

$$
\begin{aligned}
Z(\tilde{\boldsymbol{v}}) = \max \ & \boldsymbol{c}_1^\top \boldsymbol{x} + \tilde{\boldsymbol{v}}^\top C_2 \boldsymbol{x} + \boldsymbol{x}^\top C_3 \boldsymbol{x} \\
s.t. \ \ & A_1 \boldsymbol{x} = \boldsymbol{b}_1 \\
& A_2 \boldsymbol{x} = \boldsymbol{b}_2 - M\tilde{\boldsymbol{v}} \\
& (A_3 \boldsymbol{x}) \circ (A_4 \boldsymbol{x}) = 0 \\
& x_j \in \{0, 1\} \qquad \forall j \in \mathcal{B} \\
& \boldsymbol{x} \geq 0
\end{aligned}
$$

Conic Formulation for Quadratic Constraints

THEOREM 1. *Problem (2) is equivalent to the following completely positive program.*

$$
\begin{array}{llll}
Z^m = & \max\ \mathbf{c_1}^\top \boldsymbol{p}^x + C_2 \cdot Y^x + C_3 \cdot X^x & & \\
& \text{s.t. } \underline{\textit{Constraints on Decision Variables}} & & \underline{\textit{Dual Variables}} \\
& A_1 \boldsymbol{p}^x = \boldsymbol{b}_1 & & \boldsymbol{\phi}_x \\
& diag(A_1 X^x A_1^\top) = \boldsymbol{b}_1 \circ \boldsymbol{b}_1 & & \boldsymbol{\epsilon}_x \\
& A_2 \boldsymbol{p}^x + M\mathbf{w} = \boldsymbol{b}_2 & & \boldsymbol{\phi}_{xv} \\
& diag\Big(\begin{pmatrix} A_2 & M \end{pmatrix} \begin{pmatrix} \boldsymbol{X}^x & Y^x \\ Y^{x\top} & X^w \end{pmatrix} \begin{pmatrix} A_2 & M \end{pmatrix}^\top \Big) = \boldsymbol{b}_2 \circ \boldsymbol{b}_2 & & \boldsymbol{\epsilon}_{xv} \\
& diag(A_3 X^x A_4^\top) = \mathbf{0} & & \boldsymbol{\lambda} \\
& p_j^x = X_{jj}^x, & \forall j \in \mathcal{B} & \boldsymbol{\psi}_x \\
& & & \\
& \underline{\textit{Constraints on Random Variables}} & & \\
& M_1 \boldsymbol{w} = \boldsymbol{b} & & \boldsymbol{\phi}_v \\
& diag(M_1 X^w M_1^\top) = \boldsymbol{b} \circ \boldsymbol{b} & & \boldsymbol{\epsilon}_v \\
& w_j = X_{jj}^w, & \forall j \in \mathcal{U}^{\mathcal{B}} & \boldsymbol{\psi}_v \\
& \boldsymbol{w}_j = \mu_j, & \forall j \in \mathcal{U} & \boldsymbol{\nu}_v \\
& \boldsymbol{X}_{ij}^w = \Sigma_{ij}, & \forall i,j \in \mathcal{U} & \boldsymbol{\Theta}_v \\
& & & \\
CP = & \begin{pmatrix} 1 & \boldsymbol{w}^\top & \boldsymbol{p}^{x\top} \\ \boldsymbol{w} & X^w & Y^{x\top} \\ \boldsymbol{p}^x & Y^x & X^x \end{pmatrix} \succcurlyeq_{cp} 0 & & \rho
\end{array}
\qquad (3)
$$

where $\mathcal{U}$ denotes the set of random variables with specified moments. We assume there is a partition of principal sub-matrices of X^w specified with moments. $\mathcal{U}^{\mathcal{B}}$ denotes the set of Bernoulli random variable.

Conic Formulation for Quadratic Constraints

DUAL

$$\min\ \rho + \mu^\top \nu_v + \Sigma \bullet \Theta_v + \boldsymbol{b}_1^\top \boldsymbol{\phi}_x + \boldsymbol{b}_1^\top (\boldsymbol{\epsilon}_x \circ \boldsymbol{b}_1) + \boldsymbol{b}_2^\top \boldsymbol{\phi}_{xv} + \boldsymbol{b}_2^\top (\boldsymbol{\epsilon}_{xv} \circ \boldsymbol{b}_2) + \boldsymbol{b}^\top \boldsymbol{\phi}_v + \boldsymbol{b}^\top (\boldsymbol{\epsilon}_v \circ \boldsymbol{b})$$

$$s.t. \begin{pmatrix} \rho & \frac{1}{2}(\nu_v + \boldsymbol{\psi}_v + M^\top \boldsymbol{\phi}_{xv} + M_1^\top \boldsymbol{\phi}_v)^\top & \frac{1}{2}(B^\top \boldsymbol{\psi}_x + {A_1}^\top \boldsymbol{\phi}_x + A_2^\top \boldsymbol{\phi}_{xv} - \boldsymbol{c}_1)^\top \\ \frac{1}{2}(\nu_v + \boldsymbol{\psi}_v + M^\top \boldsymbol{\phi}_{xv} + M_1^\top \boldsymbol{\phi}_v) & \Theta_v + M^\top \Lambda(\boldsymbol{\epsilon}_{xv}) M + M_1^\top \Lambda(\boldsymbol{\epsilon}_v) M_1 - \Lambda(\boldsymbol{\psi}_v) & (A_2^\top \Lambda(\boldsymbol{\epsilon}_{xv}) M - \frac{C_2}{2})^\top \\ \frac{1}{2}(B^\top \boldsymbol{\psi}_x + {A_1}^\top \boldsymbol{\phi}_x + A_2^\top \boldsymbol{\phi}_{xv} - \boldsymbol{c}_1) & A_2^\top \Lambda(\boldsymbol{\epsilon}_{xv}) M - \frac{C_2}{2} & {A_1}^\top \Lambda(\boldsymbol{\epsilon}_x) A_1 + A_2^\top \Lambda(\boldsymbol{\epsilon}_{xv}) A_2 + A_4^\top \Lambda(\boldsymbol{\lambda}) A_3 + A_3^\top \Lambda(\boldsymbol{\lambda}) A_4 - B^\top \Lambda(\boldsymbol{\psi}_x) B - C_3 \end{pmatrix}$$

$$\succcurlyeq_{co} 0 \qquad (4)$$

LEMMA 1 **(Co-positive Schur Complement (Hanasusanto and Kuhn (2017)))**. *Consider a symmetric matrix*

$$D = \begin{pmatrix} A & B \\ B^\top & C \end{pmatrix}$$

with $A \succ 0$. *Then* $D \succ_{co} 0$ *if* $C - B^\top A^{-1} B \succ_{co} 0$.

PROPOSITION 2. *Consider the completely positive program (3) and its dual co-positive program (4). If* $(A_1^\top A_1 - C_3) \succ_{co} 0$, *then there is no duality gap between the two problem.*

Conic Formulation for Worst Case CVaR

$Z(\tilde{\boldsymbol{v}}, r)$

$$\begin{array}{ll} \max & \sum_{j\in\mathcal{N}} d_j\alpha_j T^R - \sum_{i\in\mathcal{M}} r_i\beta_i - \sum_{k\in\mathcal{P}} c_k\delta_k(T^R - T_k^r(1-v_k)) \\ s.t. & (\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{s^1}, \boldsymbol{s^2}, \boldsymbol{s^3}, \boldsymbol{s^4}, \boldsymbol{s^5}) \in \mathcal{F} \end{array}$$

$$\mathcal{F} = \left\{ \begin{array}{ll} \alpha_j - \beta_i + s_l^1 = 0, & \forall j \in \mathcal{N}, (i,j) \in \mathcal{G}_1 \\ -\beta_i + \sum_{t\in\mathcal{T}_j} \frac{\gamma_t^j I_{it}^{PT}}{B_{tj}} + s_l^2 = 0, & \forall i \in \mathcal{M}, \forall j \in \mathcal{M}_2, (i,j) \in \mathcal{G} \\ -\delta_k + \beta_{i(k)} - \sum_{t\in\mathcal{T}_{i(k)}} \gamma_t^{i(k)} + s_l^3 = 0, & \forall k \in \mathcal{P}, i(k) \in \mathcal{A}_k, i(k) \in \mathcal{M}_2 \\ -\delta_k + \beta_{i(k)} + s_l^4 = 0, & \forall k \in \mathcal{P}, i(k) \in \mathcal{A}_k, i(k) \in \mathcal{M}_1 \\ \alpha_j + s_j^5 = f_j & \forall j \in \mathcal{N} \\ \boldsymbol{\alpha} \in \mathbb{R}_+^n, \boldsymbol{\beta} \in \mathbb{R}_+^m, \boldsymbol{\gamma} \in \mathbb{R}_+^{tp}, \boldsymbol{\delta} \in \mathbb{R}_+^p & \\ \boldsymbol{s^1} \in \mathbb{R}_+^{|\mathcal{G}_2|}, \boldsymbol{s^2} \in \mathbb{R}_+^{|\mathcal{G}_1|}, \boldsymbol{s^3} \in \mathbb{R}_+^{m_2} & \\ \boldsymbol{s^4} \in \mathbb{R}_+^{m_1}, \boldsymbol{s^5} \in \mathbb{R}_+^n & \end{array} \right\}$$

$$\begin{array}{lll} (Z(\tilde{\boldsymbol{v}}, r) - \theta)^+ = \max & \sum_{j\in\mathcal{N}} d_j\alpha_j T^R - \sum_{i\in\mathcal{M}} r_i\beta_i - \sum_{k\in\mathcal{P}} c_k\delta_k(T^R - T_k^r(1-\tilde{v}_k)) - \theta y & \\ s.t. & (\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{s^1}, \boldsymbol{s^2}, \boldsymbol{s^3}, \boldsymbol{s^4}, \boldsymbol{s^5}) \in \mathcal{F} & \\ & (1-y)(\sum_{j\in\mathcal{N}} \alpha_j + \sum_{i\in\mathcal{M}} \beta_i + \sum_{k\in\mathcal{P}} \delta_k) & = \quad 0 \\ & y \in \{0, 1\} & \end{array}$$

Strengthening the Formulation

LEMMA 3. *The optimal dual variables* $(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*, \gamma^*, \boldsymbol{\delta}^*)$ *in Problem (14) satisfies:* $\sum_{j\in\mathcal{N}} d_j\alpha_j^* \leq \sum_{k\in\mathcal{P}} c_k\delta_k^*$

LEMMA 4. *We have following constraints as valid cuts to the problem.*

$$\beta_j + s_j^8 = \left(\max_{i=1}^{n} f_i\right) y, \forall j = 1,\ldots,m$$

$$\delta_k + s_k^9 = \left(\max_{i=1}^{n} f_i\right) y, \forall k = 1,\ldots,p$$

$$y + s^{10} = 1$$

$$\alpha_i s_i^5 = 0, \forall i = 1,\ldots,n$$

$$\alpha_j + s_j^{11} = f_j y$$

$$\boldsymbol{s^8}, \boldsymbol{s^9}, s^{10}, \boldsymbol{s^{11}} \geq \boldsymbol{0}$$

Conic Formulation for Worst Case CVaR

$$
\begin{aligned}
(Z(\boldsymbol{v}, T^R(\tilde{\boldsymbol{v}}), \boldsymbol{r}) - \theta)^+ = \max\ & (\sum_{j\in\mathcal{N}} d_j\alpha_j - \sum_{k\in\mathcal{P}} c_k\delta_k)T^R - \sum_{i\in\mathcal{M}} r_i\beta_i + \sum_{k\in\mathcal{P}} c_k\delta_k T_k^r(1-\tilde{v}_k) - \theta y \\
s.t.\ & (\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{s^1}, \boldsymbol{s^2}, \boldsymbol{s^3}, \boldsymbol{s^4}, \boldsymbol{s^5}) \in \mathcal{F} \\
& (1-y)(\sum_{j\in\mathcal{N}} \alpha_j + \sum_{i\in\mathcal{M}} \beta_i + \sum_{k\in\mathcal{P}} \delta_k) = 0 \\
& y \in \{0, 1\}
\end{aligned}
$$

$$T^R(\boldsymbol{v}) - s_k^6 = T_k^r(1 - v_k), \forall k \in \mathcal{P}$$

$$\boldsymbol{s^6} \geq 0$$

…..has a completely positive conic programming formulation

Conic Formulation for Worst Case CVaR

$$
\begin{aligned}
(Z(\boldsymbol{v}, T^R(\tilde{\boldsymbol{v}}), \boldsymbol{r}) - \theta)^+ = \max \;& (\sum_{j\in\mathcal{N}} d_j\alpha_j - \sum_{k\in\mathcal{P}} c_k\delta_k)T^R - \sum_{i\in\mathcal{M}} r_i\beta_i + \sum_{k\in\mathcal{P}} c_k\delta_k T_k^r(1-\tilde{v}_k) - \theta y \\
s.t. \;& (\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{s^1}, \boldsymbol{s^2}, \boldsymbol{s^3}, \boldsymbol{s^4}, \boldsymbol{s^5}) \in \mathcal{F} \\
& (1-y)(\sum_{j\in\mathcal{N}} \alpha_j + \sum_{i\in\mathcal{M}} \beta_i + \sum_{k\in\mathcal{P}} \delta_k) = 0 \\
& y \in \{0,1\}
\end{aligned}
$$

…..has a completely positive conic programming formulation

$$
\left\{
\begin{array}{ll}
\sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} x_{ij}^0 > d_j, & \forall j \in \mathcal{N} \\
-\sum_{j\in\mathcal{M}\cup\mathcal{N},(i,j)\in\mathcal{G}} x_{ij}^0 + u_i^0 > 0, & \forall i \in \mathcal{M} \\
\sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} \frac{x_{ij}^0 I_{it}^{PT}}{B_{tj}} - u_j^0 > 0, & \forall j \in \mathcal{M}_2, t \in \mathcal{T}_j \\
\sum_{i\in\mathcal{A}_k} u_i^0 < (1+\epsilon)c_k, & \forall k \in \mathcal{P} \\
x_{ij}^0 > 0, \boldsymbol{u}^0 > \boldsymbol{0} &
\end{array}
\right\}
$$

has a strict interior solution ⇒ Strong duality holds

Plan

- Motivation:
  - Time-To-Recovery and Supply Chain Disruption
- Mitigation:
  - Distributionally Robust Model using worst case CVAR
- Prioritization:
  - Sensitivity Analysis on Supply Chain Mitigation

Which company's recovery ability matters the most?

Considering decreasing firm k's TTR by one unit, how will the worst-case expected lost sales change?

$$
\begin{aligned}
Z(\mathbf{v},\mathbf{r}) = \min_{(x_{ij},\mathbf{u},\mathbf{l})} \quad & \sum_{j\in\mathcal{N}} f_j l_j \\
\text{s.t.} \quad & \sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} x_{ij} + l_j \geq d_j T^R(\mathbf{v}), && \alpha \\
& \sum_{j\in\mathcal{M}\cup\mathcal{N},(i,j)\in\mathcal{G}} x_{ij} - u_i \leq r_i, && \beta \\
& \sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} \frac{x_{ij} I_{it}^{PT}}{B_{tj}} - u_j \geq 0, && \gamma \\
& \sum_{i\in\mathcal{A}_k} u_i \leq (T^R(\mathbf{v}) - T_k^r(1-v_k))c_k, && \delta \\
& x_{ij} \geq 0, \mathbf{u}, \mathbf{l} \geq \mathbf{0}
\end{aligned}
$$

$E[c_k \delta_k (1 - v_k)]$

An optimal solution of the completely positive program equivalent to the worst-case expected lost sale problem.

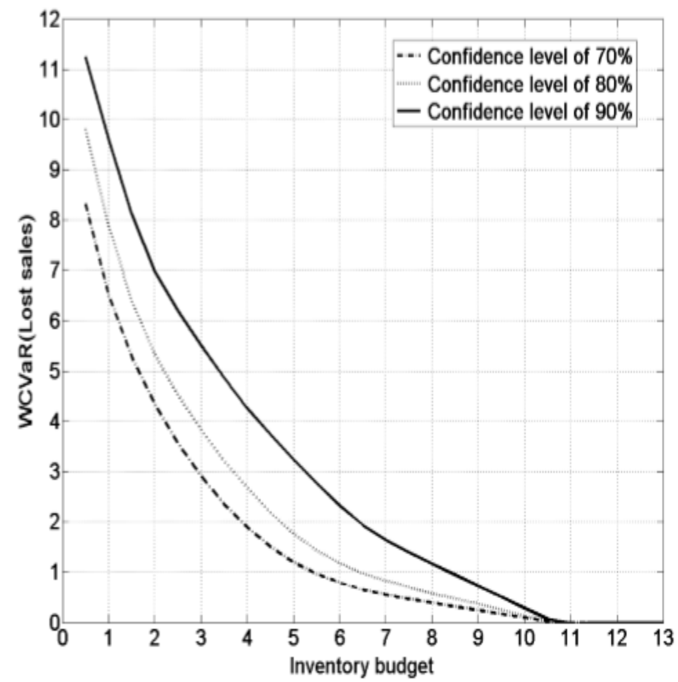
Which company's capacity or inventory level matters the most?

Considering decreasing firm k's capacity or inventory level by one unit, how will the worst-case expected lost sales changes?

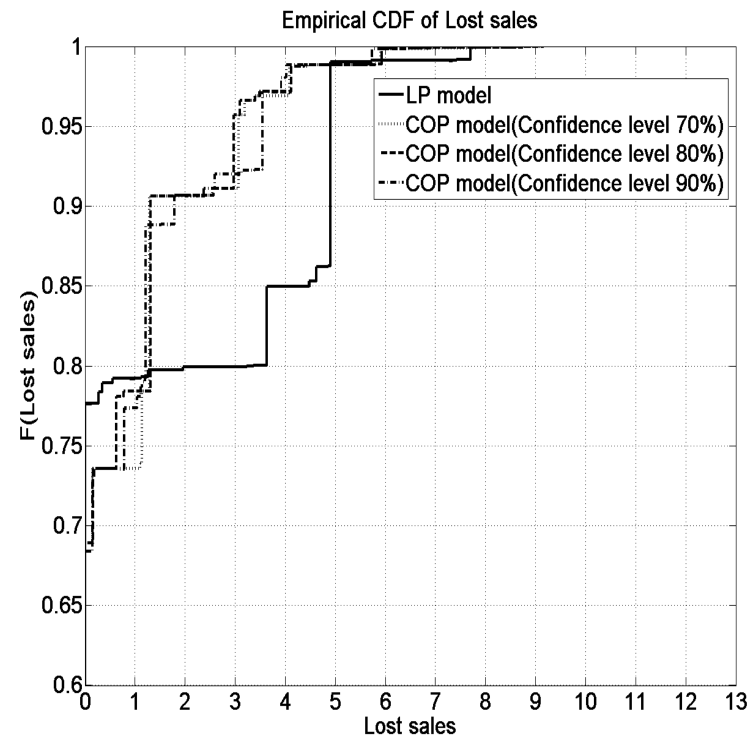
$$
\begin{aligned}
Z(\mathbf{v},\mathbf{r}) = \min_{(x_{ij},\mathbf{u},\mathbf{l})} \quad & \sum_{j\in\mathcal{N}} f_j l_j \\
s.t. \quad & \sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} x_{ij} + l_j \geq d_j T^R(\mathbf{v}), && \alpha \\
& \sum_{j\in\mathcal{M}\cup\mathcal{N},(i,j)\in\mathcal{G}} x_{ij} - u_i \leq r_i, && \boldsymbol{\beta} && E[\beta_k] \\
& \sum_{i\in\mathcal{M},(i,j)\in\mathcal{G}} \frac{x_{ij} I_{it}^{PT}}{B_{tj}} - u_j \geq 0, && \gamma \\
& \sum_{i\in\mathcal{A}_k} u_i \leq (T^R(\mathbf{v}) - T_k^r(1-v_k))c_k, && \delta && E[\delta_k(T^R - T_k^r(1-v_k))] \\
& x_{ij} \geq 0, \mathbf{u}, \mathbf{l} \geq \mathbf{0}
\end{aligned}
$$

An optimal solution of the completely positive program equivalent to the worst-case expected lost sale problem.

Computational Results

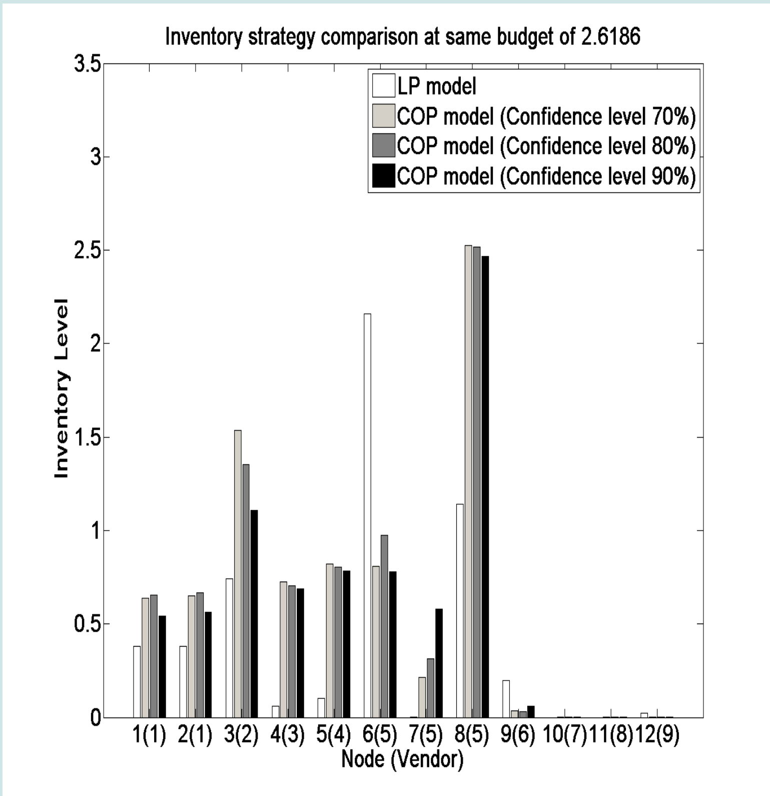


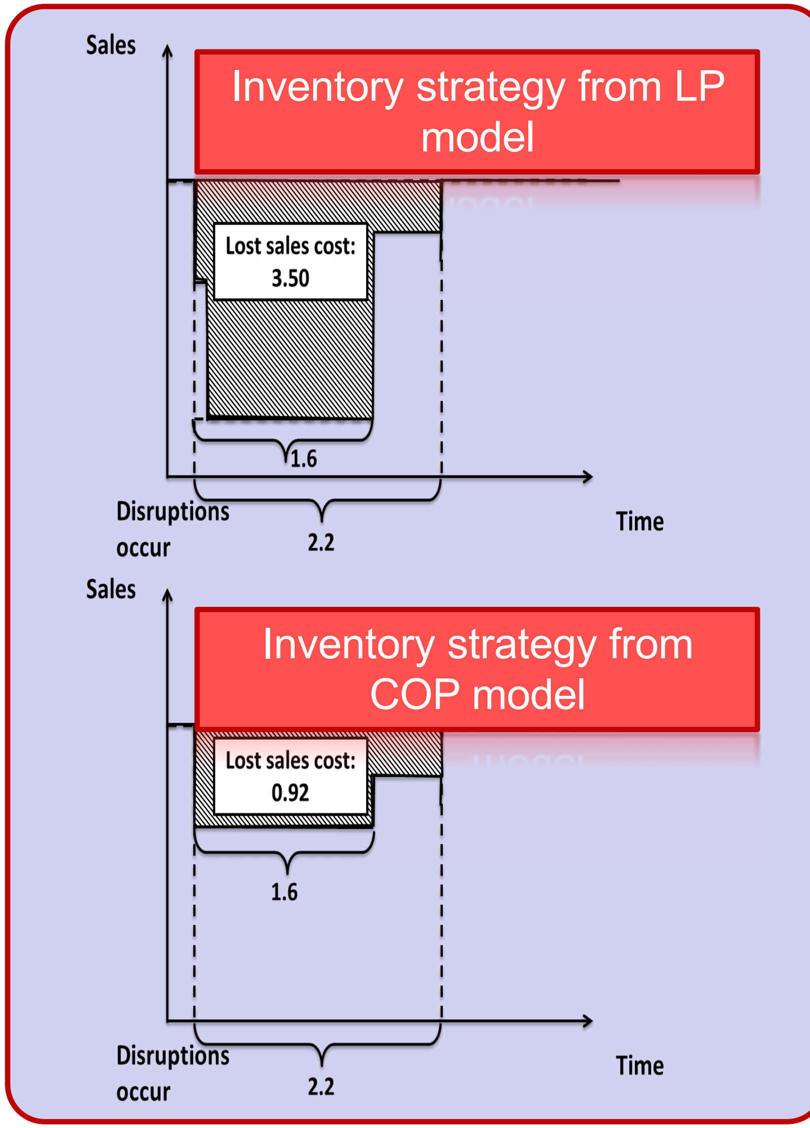
WCVaR of lost sales for different inventory budgets



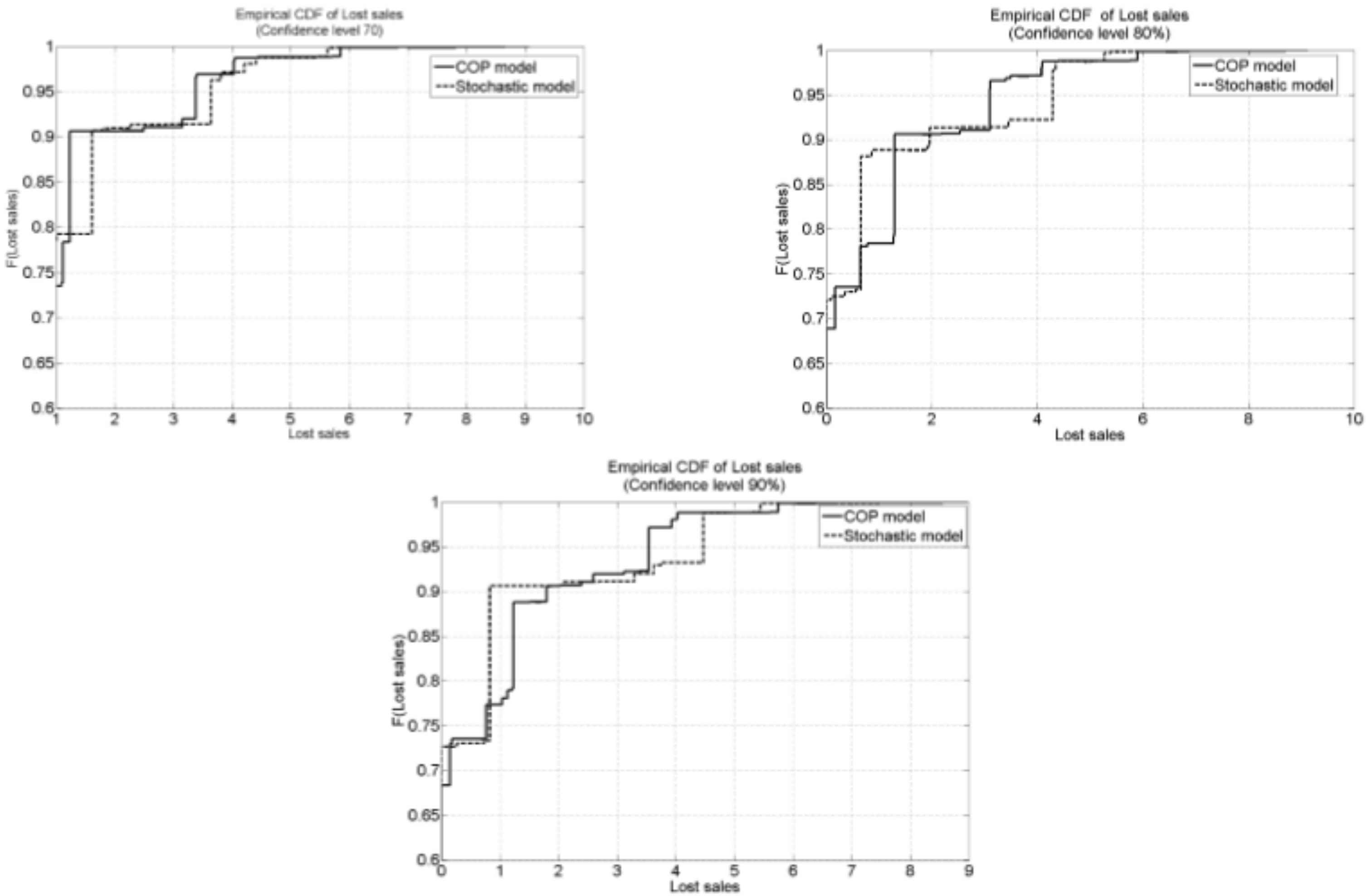
Benchmark #1

Inventory Strategy





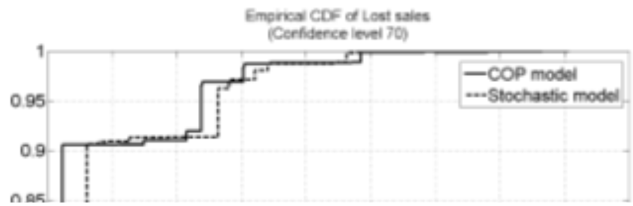
Computational Results



Simulated Lost Sales CDFs Comparison

Benchmark #2

Computational Results



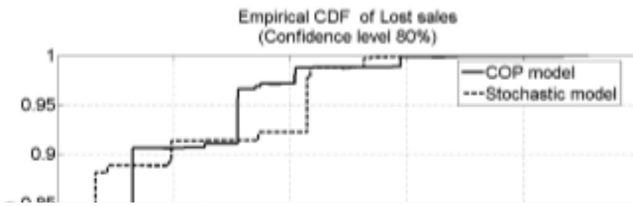
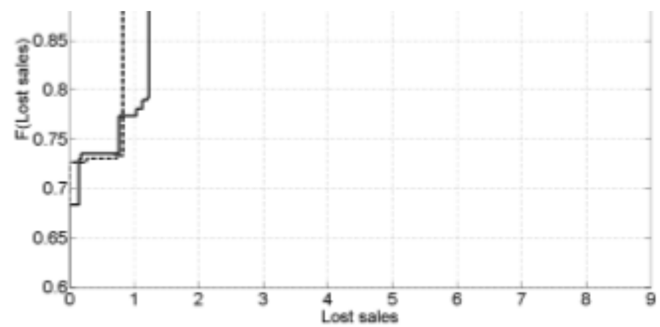


Table 5 Statistics of simulated lost sales under the same budget

	Mean	STD	70% CVaR	80% CVaR	90% CVaR
COP Model (Confidence level 70%)	0.5673	1.1675	1.8851	2.4305	3.6178
Stochastic Model(Confidence level 70%)	0.5623	1.2343	1.8745	2.6999	3.7901
COP Model (Confidence level 80%)	0.5383	1.1339	1.7884	2.3937	3.4969
Stochastic Model(Confidence level 80%)	0.5341	1.2635	1.7803	2.4339	4.0535
COP Model (Confidence level 90%)	0.5605	1.1626	1.8603	2.4669	3.6391
Stochastic Model(Confidence level 90%)	0.5476	1.2681	1.8254	2.4472	4.0676

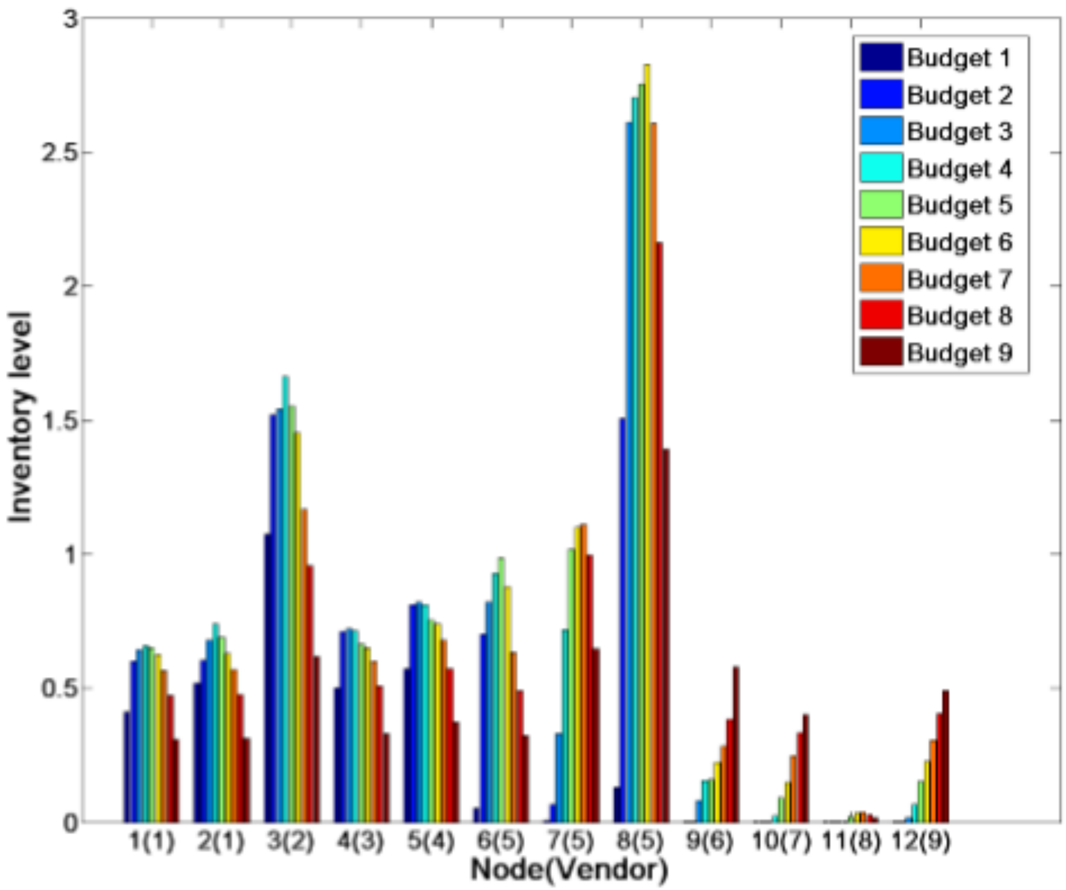


Simulated Lost Sales CDFs Comparison

The computation time to obtain the inventory strategy from stochastic model is about 45mins, compared with less than 5 mins in the case of running the COP model.

Benchmark #2

Inventory Strategy: non-monotone in budget



Inventory levels under different inventory budgets

Disruption Risk Mitigation in Supply Chains – The Risk Exposure Index Revisited

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THANK YOU

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