A short proof of the discontinuity of phase transition in the planar random-cluster model with q>4

Yinon Spinka Joint with Gourab Ray

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Banff, November, 2019

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The random-cluster model ●0000	The BKW coupling	The height function	Proof of discontinuity O	More 000	Thanks ○
The random-c	luster mode	Definition			

- Finite graph G = (V, E)
- Two parameters: $p \in [0,1]$ and q > 0
- Configurations: $\omega \in \{0,1\}^E$

 $\mathbb{P}_{p,q}(\omega) \ \propto \ p^{\#\{\text{open edges}\}} (1-p)^{\#\{\text{closed edges}\}} q^{\#\text{clusters}}$

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The random-cluster model on \mathbb{Z}^d :

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The random-cluster model on \mathbb{Z}^d :

• Consider weak limits of measures on finite graphs.

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- Consider weak limits of measures on finite graphs.
- **Two** extreme limits:



The random-cluster model on \mathbb{Z}^d :

- Consider weak limits of measures on finite graphs.
- Two extreme limits:

1 The free measure



- Closed boundary conditions
- The "smallest" measure



- Consider weak limits of measures on finite graphs.
- Two extreme limits:



- Closed boundary conditions
- The "smallest" measure

2 The wired measure



- Open boundary conditions
- The "largest" measure

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The random-c	luster mode	Phase trar	nsition		

Does ω have an infinite open cluster?

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Does ω have an infinite open cluster?

There exists $p_c = p_c(q, d) \in (0, 1)$ such that 1 No if $p < p_c$ 2 Yes if $p > p_c$

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$$\begin{split} \theta^{\text{free}}(p) &:= \mathbb{P}_{p,q}^{\text{free}}(\text{the origin is in an infinite open cluster of } \omega) \\ \theta^{\text{wired}}(p) &:= \mathbb{P}_{p,q}^{\text{wired}}(\text{the origin is in an infinite open cluster of } \omega) \end{split}$$

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1
$$\theta^{\text{wired}}(p) = \theta^{\text{free}}(p) = 0 \text{ if } p < p_c$$

2 $\theta^{\text{wired}}(p) \ge \theta^{\text{free}}(p) > 0 \text{ if } p > p_c$



The random-cluster model on \mathbb{Z}^d – one of two possibilities:

1 Continuous phase transition



2 Discontinuous phase transition $\oint \theta$





The random-cluster model on \mathbb{Z}^d – one of two possibilities:

1 Continuous phase transition





The random-cluster model	The BKW coupling	The height function	Proof of discontinuity	More	Thanks
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The random-cluster model on \mathbb{Z}^2 :

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The random-c	cluster mode	History			

The random-cluster model on \mathbb{Z}^2 :



[Duminil-Copin –Beffara 2011]

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 q ≥ 25.72 via Pirogov–Sinai theory and entropy techniques [Kotecký–Shlosman 82, Laanait–Messager–Ruiz 86, +Miracle-Solé 91]



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- **2** Discontinuous phase transition for q > 4
 - q ≥ 25.72 via Pirogov–Sinai theory and entropy techniques [Kotecký–Shlosman 82, Laanait–Messager–Ruiz 86, +Miracle-Solé 91]
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 - Short proof [Ray–Spinka 2019+]

The random-cluster model	The BKW coupling	The height function	Proof of discontinuity	More	Thanks
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Main tools					

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Main tools					

1 The Baxter-Kelland-Wu (BKW) coupling

The random-cluster model	The BKW coupling	The height function	Proof of discontinuity	More	Thanks
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Main tools					

1 The Baxter-Kelland-Wu (BKW) coupling

- The random-cluster model
- The six-vertex model

[Temperley-Lieb 71, BKW 76, Glazman-Peled 2019]

The random-cluster model	The BKW coupling	The height function	Proof of discontinuity	More	Thanks
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Main tools					

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[Temperley-Lieb 71, BKW 76, Glazman-Peled 2019]

2 Height function representation for the six-vertex model

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The random-cluster model	The BKW coupling	The height function	Proof of discontinuity	More	Thanks
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The six-vertex	model				

• Arrow configurations satisfying the ice rule: 2 in, 2 out

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- Arrow configurations satisfying the ice rule: 2 in, 2 out
- **Six** possible types:



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Six possible types:



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• Split into loop segments:



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• Split into loop segments:



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<u>The BKW coupling</u>



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The random-cluster model	The BKW coupling	The height function ●	Proof of discontinuity O	More 000	Thanks ○
The height fu	nction				

• A six-vertex config is the **gradient** of a height function:



• The height function is defined up to a global additive constant



The random-cluster model	The BKW coupling	The height function ○	Proof of discontinuity 	More 000	Thanks ○
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• Fix q > 4 and consider the random-cluster model.



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Droof of disco	ntinuity by	contradictic	ND .		

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- Fix q > 4 and consider the random-cluster model.
- Suppose it undergoes a **continuous** phase transition.



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- All clusters (primal and dual) are finite.



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- Suppose it undergoes a **continuous** phase transition.
- All clusters (primal and dual) are finite.
- Every vertex is surrounded by infinitely many loops.



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- All clusters (primal and dual) are finite.
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- Height function distribution does not depend on the sign of λ .



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The spin representation of the six-vertex model

• Height function gives spin configuration via mod 4.



The random-cluster model

The BKW coupling

The height function

Proof of discontinuity More ○ 0●0 Thanks

The FK-spin representation of the six-vertex model

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

The random-cluster model

The BKW coupling

The height function

Proof of discontinuity

Thanks

More

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The FK-spin representation of the six-vertex model



The random-clus	ter model	The BKW coupling	The height fun O	ction Proof of discontinuity O	More ○○●	Thanks ○
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The gradient as a finitary factor of iid



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The rando	m-cluster model	The BKW coupling	The height function	Proof of discontinuity	More	Thanks

The gradient as a finitary factor of iid





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Thank you!

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