ROWMOTION AND THE COXETER TRANSFORMATION

October 26, 2020

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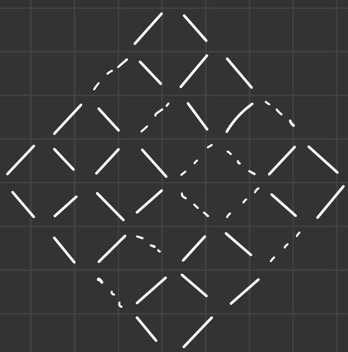
– Dynamical Algebraic Combinatorics –

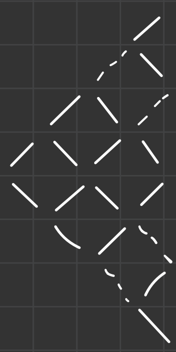
BIRS (online)

* This talk is being recorded *



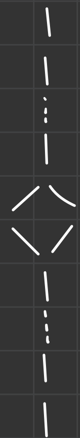
$m \times n$ grid

C_{I}



$m \times m$ triangular

C_{II}

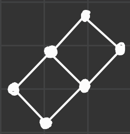


C_{III}

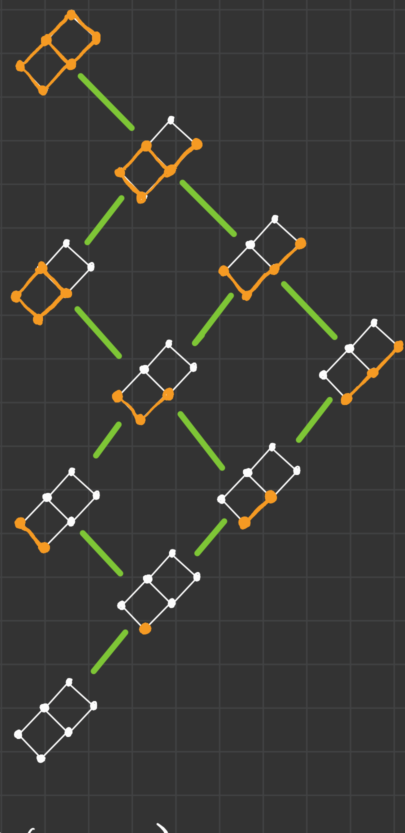
Cominuscule Posets

Let $J(P)$ be the order ideal lattice for a poset P.

Example:



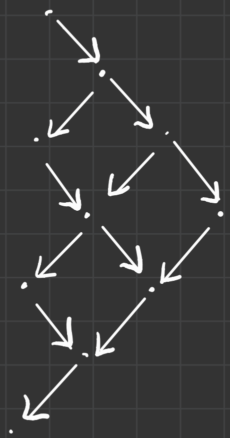
2x3 grid $C_{2\times 3}$



$J(C_{2\times 3})$

• Let $\mathcal{A}$ be the incidence algebra of $J(P)$.

Ex:



$\mathcal{A}$ = Span of all paths with commutativity relations.

• Write a matrix $C_A = (c_{ij})$ where

$$c_{ij} = \begin{cases} 1 & \text{if there is a path from } i \text{ to } j; \\ 0 & \text{otherwise.} \end{cases}$$

• The Coxeter transformation τ_A

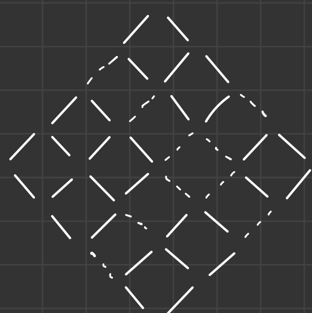
$$\tau_A = - C_A^{t} C^{-1}$$

THEOREM (Y.)

The Coxeter transformation

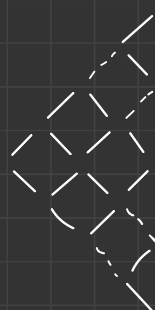
$$\tau_A^{(h+1)} = \pm \mathrm{id}$$

for the incidence algebra of $J(C_{I})$, $J(C_{III})$ and exceptionals $J(E_6)$ and $J(E_7)$ where h is the corresponding Coxeter number.



$m \times n$ grid

C_{I}



$m \times n$ triangular

C_{II}

C_{III}

Cominuscule Posets

THEOREM (Rush & Shi '13)

The order of Rowmotion on cominuscule posets is h.

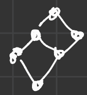
QUESTION:

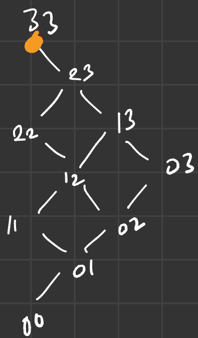
Rowmotion "Row" ? The Coxeter transformation τ_A

$\mathrm{Row}^h = \mathrm{id}$

$\tau_A^{h+1} = \pm \mathrm{id}$

e.g.

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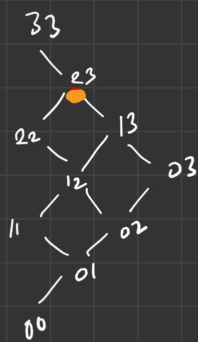
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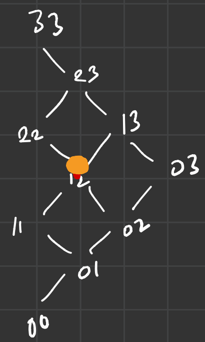
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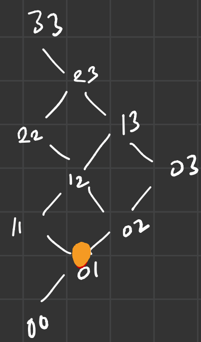
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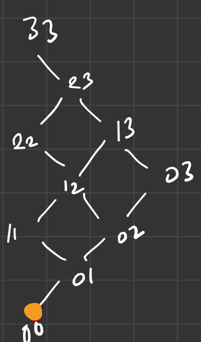
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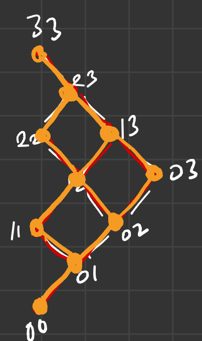
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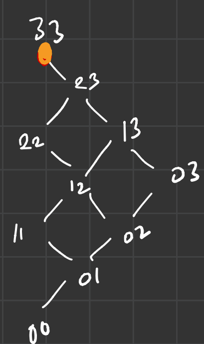
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Example:

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 & 22 \arrow[dr] & & 13 \arrow[dl] \arrow[dr] & \\
 & & 12 \arrow[dl] \arrow[dr] & & 03 \arrow[dl] \\
 & 11 \arrow[dr] & & 02 \arrow[dl] & \\
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 & 00 & & &
\end{tikzcd}
$$

$$0 \to P_{12} \to P_{13} \oplus P_{22} \to P_{23} \to 0$$

$$\downarrow \tau$$

$$0 \to P_{01} \to P_{02} \oplus P_{11} \to P_{12} \to 0$$

$$\downarrow \tau$$

$$0 \to P_{00} \to P_{01} \to 0$$

$$\downarrow \tau$$

$$0 \to P_{00} \to 0$$

$$\downarrow \tau$$

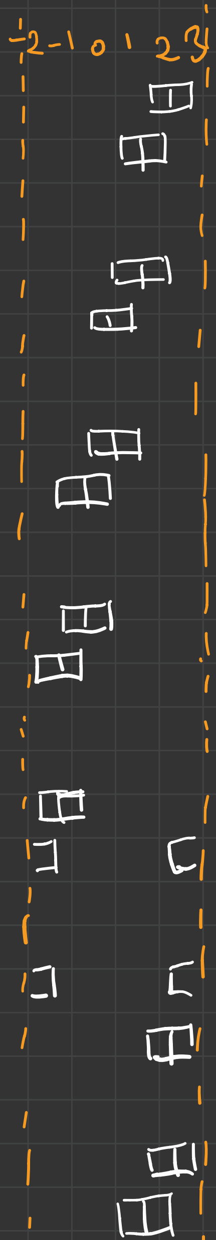
$$0 \to P_{33} \to 0$$

$$\downarrow \tau$$

$$0 \to P_{23} \to P_{33} \to 0$$

$$\downarrow \tau$$

$$0 \to P_{12} \to P_{13} \oplus P_{22} \to P_{23} \to 0$$



$0 \to P_{12} \to P_{13} \oplus P_{22} \to P_{23} \to 0$

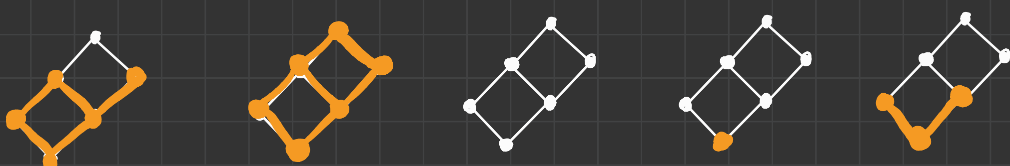
$P_{01} \to P_{02} \oplus P_{11} \to P_{12} \to 0$

$0 \to P_{00} \to P_{01} \to 0$

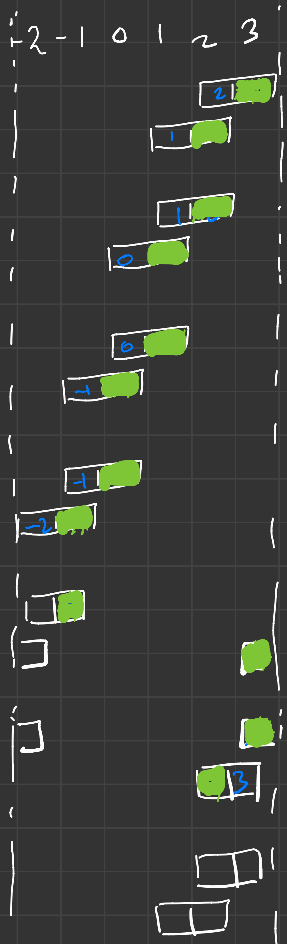
$0 \to P_{00} \to 0$

$0 \to P_{33} \to 0$

$0 \to P_{23} \to P_{33} \to 0$



$23 \leftarrow 33 \leftarrow 00 \leftarrow 01 \leftarrow 12$



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FUTURE WORK:

More Conjectures:

- (F. Chapoton)

$$\tau_{\mathcal{A}}^{(h+1)} = \pm \,\mathrm{id} \quad \text{for } J(\text{root posets } \Phi^+)$$

- (D. Panyushev)

The order of Row on $J(\Phi^+(A_n))$, $J(\Phi^+(D_n))$ for n odd and $J(\Phi^+(E_6))$ is $2h$, and the order is h of all other types.

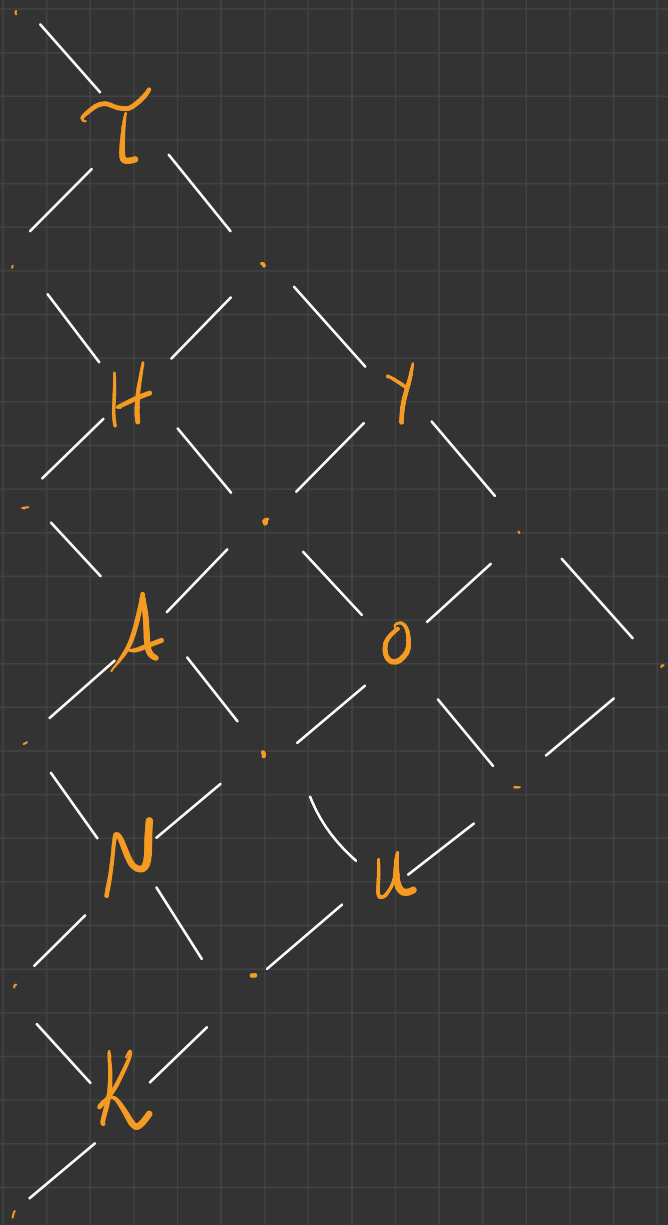
Uniformly Proved by D. Armstrong, C. Stump, H. Thomas.

- (S. Hopkins)

$$(\mathrm{Row}, \tau)^2 = 2 \text{ for any distributive lattice}$$

of order ideals in a poset P.

Proved by EY, H. Thomas during this conference!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