6th Brazil, Chile and Mexico meeting on singularities

The conference took place at Casa Matemática Oaxaca, from August 28th to September 1st.

This was the 6th of a series of meetings held between Mexican, Brazilian and Chilean communities of singularity. They Started on August 2013, In Querétaro, Mexico, as a joint meeting in singularities between Mexico and Brazil. That first meeting was a satelite conference of the First Mathematical Congress of the Americas, held in Guanajuato, Mexico.

The second meeting took place in Salvador de Bahía, Brazil on july 2015, while the third one took place in Cuernavaca, Mexico, on August 2017. in that third meeting it was agreed that Chile will join the singularity communities of Brazil and Mexico.

The forth meeting was held in Talca, Chile, on March 2019. The COVID pandemic forced to pospone the following meeting to may 2022, held in Vittoria, Brazil, in hybrid way.

In order to maintain the frequency of the meetings it was decided to hold the 6th one on September 2023. It was carried-out as a CMO-BIRS hybrid meeting in Oaxaca, Mexico.

The conference consisted on 19 talks of one hour each and 5 talks of 20 minutes. The speakers were from Brazil (11), Mexico (9), Chile (4) and a special guest from Spain.

The list of speakers and participants is available at CMO web site.

The meeting was on hybrid way, 10 talks on-site and 14 on-line.

During the 5 days of the workshop many aspects of singularities were exposed as well as works in closely related areas such as algebraic geometry, commutative algebra and foliations.

We had different talks on equisingularity on real and complex singularities, talks on singularities of mappings and their fibers, from topological, homological and differential point of view. We had talks on T-varieties and T-singulirities,on toric varieties and Groebner fans. We also had a special day dedicated to foliations and their singularities.

A full list of titles and abstrat is attacched.

6th Brasil-Chile-Mexico Meeting on Singularities

Talks and abstracts

The talks will take place in the Conference Room San Felipe

Monday, August 28th

Morning activities

9:00–10:00 - Xavier Gómez–Mont Ávalos

A Closer Look at the Homology of the Milnor Fibre of a Germ of a Plane Curve Singularity

Abstract

As we all know, the rank of the 1 homology group $H_1(C_t, \mathbb{Z})$ of the Milnor fibre of a germ of a plane curve singularity may be computed algebraically as

$$dim_{\mathbb{C}} \frac{\mathcal{O}_{\mathbb{C}^2,0}}{(\frac{\partial f}{\partial z_1}, \frac{\partial f}{\partial z_2})}$$

We have obtained, together with Ch. Bonatti and M. Gónzalez-Villa, a precision to this statement. Using the factorization of a the equation of a generic polar curve into irreducible factors, say

$$\frac{\partial f}{\partial z_1} = p_1 \cdots p_s \qquad P_j := \{p_j = 0\} \subset (\mathbb{C}^2, 0)$$

we find disjoint subsurfaces $S_j \subset C_t$ of finite type such that:

1) $C_t - \cup_j S_j$ is a disjoint union of annulii.

2) The holonomy periodic part of the homology is

$$H_1(C_t, \mathbb{Z})^{per} = \sum_j H_1(S_t, \mathbb{Z}),$$

having the direct sum decomposition invariant by the algebraic monodromy map h_*.

3) We give an explicit construction of a basis of $H_1(S_t, \mathbb{Z})$ in such a way that the algebraic holonomy map h_* is a permutation matrix plus an invariant homology class, on using the projection to z_1 and constructing a graph with only points of valency 4 which is homotopic to S_j.

4) The intersection product is orthogonal with respect to this direct sum decomposition, and on each factor is a circulant matrix.

5) A power of the geometric monodromy map is a Dehn twist on each annulii of $C_t - \cup_j S_j$, which may be organized to produce a symmetric positive definite biliner form on

$$\frac{H_1(C_t, \mathbb{Z})}{H_1(C_t, \mathbb{Z})^{inv}}$$

where the amount of twists is encoded.

This decomposition is usefull in understanding the relationship between the topological anti-symmetric intersection product on $H_1(C_t, \mathbb{Z})$ and Grothendieck's bilinear form on

$$\frac{\mathcal{O}_{\mathbb{C}^2,0}}{(\frac{\partial f}{\partial z_1}, \frac{\partial f}{\partial z_2})} :$$

$$(g_1, g_2) \mapsto \frac{1}{2\pi i} \int_{|\frac{\partial f}{\partial z_1}|=|\frac{\partial f}{\partial z_2}|=\varepsilon} \frac{g_1 g_2 dz_1 \wedge dz_2}{\frac{\partial f}{\partial z_1} \frac{\partial f}{\partial z_2}}.$$

10:30–11:30 - Wagner Badilla Céspedes (Online CMO)

F-pure thresholds and F-Volumes of some non principal ideals

Abstract

We compute the F-pure threshold of non necessarily principal ideals which satisfy a geometric generic condition about their Newton polygons. We also contribute some evidence in favor of the conjectured equality between the F-pure threshold and the log canonical threshold of ideals. These results are obtained by generalizing the theory of splitting polytopes to the case of ideals. As applications of our results we obtain geometric lower bounds for the recently introduced F-volume of a collection of ideals.

11:45–12:45 - Barbara Karolline de Lima Pereira (Online CMO)

The Bruce-Roberts numbers of a function on an isolated complete intersection singularity

Abstract

In this talk I will present formulas for the Bruce-Roberts number of a function germ f with respect to an isolated hypersurface singularity (IHS) $(X, 0)$,

$$\mu_{BR}(f, X) = \mu(f) + \mu(X \cap f^{-1}(0), 0) + \mu(X, 0) - \tau(X, 0)$$

and for the relative Bruce-Roberts number of a function germ f with respect to an isolated complete intersection singularity (ICIS) $(X, 0)$,

$$\mu^{-}_{BR}(f, X) = \mu(f^{-1}(0) \cap X, 0) + \mu(X, 0) - \tau(X, 0),$$

where μ and τ are the Milnor and Tjurina numbers, respectively, of the ICIS.

Considering the first equality we prove that the logarithmic characteristic variety of an IHS is Cohen-Macaulay, and with the second one, we prove that its relative version is Cohen-Macaulay for any ICIS.

It is a joint work with J. J. Nuño-Ballesteros (Universitat de Valencia, SPAIN), B. Oréfice-Okamoto, (UFSCar, BRAZIL) and J.N. Tomazella, (UFSCar, BRAZIL).

Afternoon activities

15:00–16:00 - Miriam Garcia Manoel (Online CMO)

Synchronous singularities of Laplacian networks

Abstract

This talk is a glimpse of singularities and graph theory in the study of networks of coupled dynamical systems. In broad terms, a network is a graph whose vertices represent individual systems (cells) and whose edges represent couplings between those systems. Critical points are expected to be singularities lying on synchrony subspaces. Synchrony, a natural concept in network dynamics, is a partition of the cells into subsets (often called clusters) such that all cells in the same cluster are synchronous. Here we present recent findings of synchronous singularities and their stabilities for Laplacian networks. Our first motivation to study this case relies on the popularity of the Kuramoto network, which turns out to be a particular case. This is a joint work with Tiago Amorim (ICMC/USP).

16:30–17:30 - Manuel González Villa

F-thresholds and test ideals of Thom-Sebastiani type polynomials

Abstract

This is a joint work with Delio Jaramillo-Velez and Luis Núñez-Betancourt.We provide a formula for F-thresholds of a Thom-Sebastiani type polynomial over a perfect field of prime characteristic. We also compute the first test ideal of Thom-Sebastiani type polynomials. Finally, we apply our results to find hypersurfaces where the log canonical thresholds equal the F-pure thresholds for infinitely many prime numbers. For more details see Proc. Amer. Math. Soc. 150 (2022), 3739-3755.

Tuesday, August 29th

Morning activities

9:00–10:00 - Pedro Montero Silva

Smooth and singular del Pezzo quintics as equivariant compactifications of vector groups

Abstract

Del Pezzo varieties arise as a natural higher-dimensional generalization of the classical Del Pezzo surfaces. Over the complex numbers, they were extensively studied by T. Fujita in the 1980s, who classified them according to their degree. In degree 5, it follows from Fujita's classification that all of these manifolds are obtained as linear sections of the 6-dimensional Grassmannian $\mathrm{Gr}(2,5)$ with respect to the Plücker embedding, whose points parametrize 2-dimensional linear subspaces of a vector space of dimension 5.

In this talk, we will discuss the existence and uniqueness of $\mathbf{G}_a^n$-structures on these varieties, i.e., we will recall when and in how many ways one can obtain them as equivariant compactifications of the abelian unipotent group $\mathbf{G}_a^n$. To do so, we study the Hilbert schemes of certain linear subspaces on such varieties and we analyze some explicit equivariant Sarkisov links.

Our methods, based on the equivariant Minimal Model Program, allow us to consider mildly singular quintic del Pezzo surfaces and threefolds. As an application, we give some new results on k-forms of quintic del Pezzo varieties over an arbitrary field k of characteristic zero.

This is a joint work with Adrien Dubouloz (Dijon, France) and Takashi Kishimoto (Saitama, Japan).

10:30–11:30 - Maria Elenice Rodrigues Hernandes (Online CMO)

The delta invariant of map-germs

Abstract

A well-known invariant in the study of germs of parameterized curves $\phi : (\mathbb{C}, 0) \to (\mathbb{C}^n, 0)$ is the so-called delta invariant of ϕ, defined as $\delta_\phi = dim_{\mathbb{C}} \frac{\mathcal{O}_1}{\phi^*(\mathcal{O}_n)}$, where $\mathcal{O}_n$ is the local ring of function germs in $\mathbb{C}^n$ at the origin and $\phi^*(g) = g \circ \phi$, for $g \in \mathcal{O}_n$. This invariant can be calculated algebraically through the semigroup of ϕ, since δ_ϕ is the number of elements that do not belong to the semigroup. In particular, for parameterized plane curves it is known that $\delta_\phi = \frac{\mu}{2}$, where μ is the Milnor number of the defining equation of the curve.

We can naturally extend the definition of delta to map-germs $f : (\mathbb{C}^n, 0) \to (\mathbb{C}^p, 0)$ with $n < p$ as $\delta_f = dim_{\mathbb{C}} \frac{\mathcal{O}_n}{f^*(\mathcal{O}_p)}$. If $p \geq 2n$ then δ_f is an $\mathcal{A}$-invariant that characterizes $\mathcal{A}$-finitely determined map-germs. In this talk, we obtain the classification of finitely determined monomial map-germs for some pairs (n, p) using only the delta invariant, and explore some of its properties. This is a joint work with Maria Aparecida Soares Ruas.

11:45–12:45 - Edson Sampaio (Online CMO)

On the Fukui-Kurdyka-Paunescu Conjecture

Abstract

In this talk, I will prove the Fukui-Kurdyka-Paunescu Conjecture, which says that subanalytic arc-analytic bi-Lipschitz homeomorphisms preserve the multiplicities of real analytic sets. This is joint work with Alexandre Fernandes and Zbigniew Jelonek.

Afternoon activities

15:00–16:00 - Mirna Gómez (Online CMO)

Groebner fans and embedded resolutions of ideals on toric varieties

Abstract

Joint work with F. Aroca

16:30–16:50 - João Carlos Ferreira Costa (Online CMO)

Bi-Lipschitz and differentiable sufficiency of weighted jets

Abstract

In this work we study bi-Lipschitz and differential sufficiency in the set of weighted jets of map germs of weighted degree. Our main result improves the degree of sufficiency of jets in the classical jet space obtained by F. Takens and by L.A. Fávaro-J.C.C. Martins. Moreover, we give a condition for the bi-Lipschitz sufficiency in both the weighted and non-weighted cases. Bi-Lipschitz sufficiency was not considered in those autors cited above. This is a joint work with M.J. Saia and C.H. Soares Junior. This work has been partially supported by grant 2019/21181-0 São Paulo Research Foundation (Fapesp).

16:55–17:15 - Thaís Maria Dalbelo (Online CMO)

Invariants and Whitney equisingularity of Images of Finite Morphisms

Abstract

In this work, we present formulae to compute the local polar multiplicities of germs at zero of an analytic variety $Y^d \subset \mathbb{C}^p$, which is the image by a finite morphism $f : Z^d \to Y^d$, where Z is a reduced analytic variety in $\mathbb{C}^n$. In particular, when $Z \subset \mathbb{C}^n$ is a complete intersection singularity of dimension d whose the singular set has dimension 1, these formulae are obtained using Milnor numbers of appropriate germs at zero of ICIS which are in the source, fact that makes this computation much easier than the usual computation of polar multiplicities. We apply these results to show how to compute the local Euler obstruction of such varieties Y^d.

Joint work with Marcelo J. Saia and Miriam S. Pereira.

17:20–17:40 - Eder Leandro Sanchez Quiceno (Online CMO)

Strong partial non-degeneracy of mixed functions

Abstract

Let $f : \mathbb{C}^2 \to \mathbb{C}$ be a mixed function, i.e., a complex polynomial of variables $(u, \bar{u}, v, \bar{v})$ such that $f(0) = 0$. In this talk, we discuss the strong partial non-degeneracy of mixed functions, which generalizes a previous non-degeneracy for holomorphic functions defined by Mondal. We also discuss criteria that transform the strong partial non-degeneracy into a necessary and sufficient condition for a mixed function to have an isolated singularity at the origin. Joint work with Benjamin Bode.

The research has been supported by São Paulo Research Foundation (FAPESP), grants 2017/25902-8 and 2019/11415-3.

Wednesday, August 30

Morning activities

9:00–10:00 - Giancarlo Urzúa

Optimal bounds for many T-singularities in stable surfaces

Abstract

T-singularities, defined by Kollár-Shepherd-Barron 1988, are the 2-dimensional quotient singularities that admit a smoothing with constant K^2. These singularities are relevant in the Kollár–Shepherd-Barron-Alexeev compactification of the moduli space of surfaces of general type. In a recent joint project with Fernando Figueroa (PhD student at Princeton) and Julie Rana (Lawrence University), we effectively bound T-singularities on non-rational projective surfaces W with an arbitrary amount of T-singularities, and K_W ample. This generalizes a previous work with Julie, passing from 1 to arbitrarily many singularities. We classify all possible geometric situations for which our bound is higher than expected. Our bounds are asymptotically optimal, and we work out the case of two singularities as an application.

10:30–11:30 -Álvaro Liendo (Online CMO)

Singularities of T-varieties of complexity one

Abstract

Joint work with H. Sub, A. Laface and J. Moraga

11:45–12:45 - Javier Fernández de Bobadilla

Equimultiplicity of families with constant Milnor number

Abstract

I will summarize the main steps of our proof of the isolates singularity family case of Zariski's multiplicity conjecture. It is based on the construction of symplectic representatives of the monodromy with special dynamics, and a Floer theoretic computation. This is a joint work with T. Pelka

Free afternoon

Thursday, August 31

Morning activities

9:00–10:00 - Jessica Angélica Jaurez Rosas

Geometric interpretation of invariants of vector fields

Abstract

It is well known that, under suitable conditions, holomorphic vector fields are locally linearizable at their singular point, whereas when these conditions fail different behaviors may occur. In this direction, certain classes of holomorphic degenerated vector fields in $(C^2, 0)$ have been studied by Ortiz, Rosales and Voronin. For such vector fields, they proved that each and every one of the different behaviors can be identified from a pair of invariants. Each pair consists of a "parametric invariant" and a "functional invariant".

In this talk we will work with such a pair of invariants. We will see that the "functional invariant" has an intrinsic relation with the geometry of the foliations generated by the vector fields, and on the other hand, we will address the problem of giving a geometric interpretation to the "parametric invariant". To this end, we will present a work in collaboration with L. Ortiz, about the realization of tangency curves of pairs of foliations from parametric invariants.

10:30–11:30 - Oziel Gómez Martínez

Jacobian curves of dicritical foliations and the analytic invariants of separatrices of foliations

Abstract

The study of the analytical classification of plane branches was introduced by O. Zariski in the 1960's. The aim of Zariski work's was to provide a complete description of the moduli space of an equisingularity class modulo analytic equivalence. In his work Zariski introduces the first analytic invariant λ_1 of classification of plane branches known as the Zariski invariant. Zariski gave the complete description of the moduli space for many equisingularity classes, however the problem remained open until 2011. The complete solution of this problem was given by A. Hefez and M.E. Hernandes. The key in the solution of this problem is the semimodule of the branch Λ which is obtained as the set of Kähler differential values of the branch and the fact that it has a unique basis $\{\lambda_{-1}, \lambda_0, \lambda_1, ..., \lambda_s\}$, that is to say, the semimodule Λ is generated by $\{\lambda_{-1}, \lambda_0, \lambda_1, ..., \lambda_s\}$.

In this talk we study the Zariski invariant λ_1 of the separatrices of a family of dicritical foliations of $(\mathbb{C}^2, 0)$. In particular, we recover the Zariski's invariant in terms of the equation defining the foliation and we give conditions to ensure that

the Zariski invariant is the same for all separatrices. For the previous result, the locus of tangencies between two foliations, known as the Jacobian curve or polar curve, plays an important role. Finally we give some ideas about a work in progress that allow us to ensure under certain assumptions about the Jacobian curves of foliations, that all the separatrices have the same semimodule.

11:45–12:45 - Arturo Fernández Pérez (Online CMO)

On Briançon-Skoda theorem for foliations

Abstract

In this talk, I present a generalization of a Mattei's result relative to the Briançon-Skoda theorem for foliations to the family of foliations of second type. Using this generalization, we obtain a relationship between the Milnor and Tjurina numbers of foliations of second type. This work is joint with Evelia García Barroso and Nancy Saravia Molina.

Afternoon activities

15:00–16:00 - Petra Rubí Pantaleón Mondragón

Foliation Stability

Abstract

The Geometric Invariant Theory (GIT) introduced by Munford is a powerful tool that allows us to classify objects. During the talk, we will explore the use of GIT on foliations of the complex projective plane with the aim of characterizing them through certain invariants. We will illustrate examples and mention some results.

16:30–16:50 - Gibran Rodrigo Espejo Ramos

Neighborhoods of elliptic curves

Abstract

The classification of neighborhoods of regular elliptic curves with zero self-intersection has been studied, in the formal and analytic settings, by Grauert, Arnold, Ilyashenko, Mishustin, Loray, Thom, Touzet and Voronin among others. The goal of this talk is to present the formal classification of neighborhoods of certain singular elliptic curves given as the union of five Riemann spheres, each with self-intersection -2.

17:00–17:20 - Yerko Torres-Nova (Online CMO)

Asymptotic aspects of n–root covers threefolds

Abstract

In this talk, we will see how the study of an asymptotic phenomenon relating the Chern numbers of n–th root covers threefolds leads us to the study of asymptotic properties of partial desingularizations of threefold cyclic quotient singularities. Finally, we state some questions and conjectures about this topic.

Friday, September 1st

Morning activities

9:00–10:00 - Alex Carlucci Rezende (Online CMO)

Quadratic differential systems possessing invariant ellipses

Abstract

In this talk, we consider the class **QS** of all non-degenerate planar quadratic systems and we provide necessary and sufficient conditions for a system in **QS** to have at least one invariant ellipse. We give the global "bifurcation" diagram of the family **QS** which indicates where an ellipse is present or absent and in case it is present, the diagram indicates if the ellipse is or it is not a limit cycle. The diagram is expressed in terms of affine invariant polynomials and it is done in the 12-dimensional space of parameters. This diagram is also an algorithm for determining for each quadratic system if it possesses an invariant ellipse and whether or not this ellipse is a limit cycle. Joint work with Regilene D.S. Oliveira, Dana Schlomiuk and Nicolae Vulpe.

These results are part of the article [1] . This work was partially supported by Fapesp grant 2019/21181-0.

[1]Oliveira, R., Rezende, A.C., Schlomiuk, D., Vulpe, N. *Characterization and bifurcation diagram of the family of quadratic differential systems with an invariant ellipse in terms of invariant polynomials*, Revista Matemática Complutense, 35, (2022), pp 361–413.

10:30–11:30 - Otoniel Nogueira da Silva (Online CMO)

On invariants of map germs from n-space to 2n-space

Abstract

In this talk we consider $\mathcal{A}$-finite map germs f from $(\mathbb{C}^n, 0)$ to $(\mathbb{C}^{2n}, 0)$. First, we show that the number of double points that appears in a stabilization of f, denoted by $d(f)$, can be calculated as the length of the local ring of the double point set $D^2(f)$ of f, given by the Mond's ideal. In the case where $n \leq 3$ and f is quasihomogeneous, we also present a formula to calculate $d(f)$ in terms of the weights and degrees of f. Finally, we consider an unfolding $F(x,t) = (f_t(x), t)$ of f and we find a set of invariants whose constancy in the family f_t is equivalent to the Whitney equisingularity of F. As an application, we present a formula to calculate the Euler obstruction of the image of f. This is a joint work with Nuño-Ballesteros and Tomazella.