# Neostability 13 July 2015 – 17 July 2015

#### MEALS

\*Breakfast: 7:00 9:00 am, Restaurant Hotel Hacienda Los Laureles, Monday–Friday \*Lunch: 13:30 15:00 pm, Restaurant Hotel Hacienda Los Laureles, Monday–Friday \*Dinner: 19:00 21:00 pm, Restaurant Hotel Hacienda Los Laureles, Sunday–Thursday \*Continuous Coffee Breaks: Conference Room San Felipe, Hotel Hacienda Los Laureles

#### MEETING ROOMS

All lectures will be held in the Conference Room San Felipe at Hotel Hacienda Los Laureles. An LCD projector, laptop, document camera and blackboards are available for presentations.

#### SCHEDULE

Sunday 16:00 19:00–21:00 20:00	Check-in begins (front desk at your assigned hotel - open 24 hours) Dinner, Restaurant Hotel Hacienda Los Laureles Informal gathering Hotel Hacienda Los Laureles (if desired) A welcome drink will be served by the hotel.
Monday	
7:00-8:45	Breakfast
8:45 - 9:00	Introduction and Welcome
9:00-9:50	Artem Chernikov
10:00-10:40	Nick Ramsey
10:40 - 11:40	Coffee Break
	Unstructured working time
11:40 - 12:20	Assaf Hasson
12:30 - 13:10	Itay Kaplan
13:10-13:30	Coffee break
	Group Photo
13:30 - 15:00	Lunch
15:00 - 15:50	Krzysztof Krupiński
15:50 - 16:40	Unstructured working time
16:40 - 17:10	Coffee Break
17:10 - 17:50	Unstructured working time
18:00 - 18:50	Tomasz Rzepecki
19:00-21:00	Dinner

Tuesday	
7:00-9:00	Breakfast
9:00-9:50	Will Johnson
10:00-10:40	Vince Guingona
10:40 - 10:50	Coffee Break
10:50 - 12:10	Unstructured working time
12:10 - 12:50	Silvain Rideau
13:00-13:30	Will Boney
13:30 - 15:00	Lunch
15:00 - 15:50	Pierre Simon
15:50 - 16:50	Unstructured working time
16:50 - 17:10	Coffee Break
17:10 - 18:00	Unstructured working time
18:00 - 18:50	Anand Pillay
19:00-21:00	Dinner

# Wednesday

7:00-9:00	Breakfast
9:00 - 13:30	Free Morning / Excursion to Monte Albán
13:30 - 15:00	Lunch
15:00 - 15:50	Chris Laskowski
15:50 - 16:40	Alex Berenstein
16:40 - 17:30	Coffee Break & Unstructured working time
17:30 - 18:10	Amador Martin Pizarro
18:10 - 18:50	Gabe Conant
19:00-21:00	Dinner

# Thursday

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7:00 - 9:00	Breakfast
9:00-9:50	Dugald Macpherson
10:00 - 10:40	Darío García
10:40 - 11:00	Coffee Break
11:00 - 12:50	Unstructured working time
12:50-13:00	Coffee break
13:00-13:30	Nadja Hempel
13:30 - 15:00	Lunch
15:00 - 15:50	Enrique Casanovas
15:50 - 16:50	Unstructured working time
16:50 - 17:10	Coffee Break
17:10 - 18:00	Byunghan Kim
18:00 - 18:50	Frank Wagner
19:00-21:00	Dinner

Friday	
7:00-9:00	Breakfast
9:00 - 9:50	Isaac Goldbring
10:00 - 10:50	Sergei Starchenko
10:50-11:10	Coffee break
11:10-13:30	Unstructured working time
13:30 - 15:00	Lunch
Checkout	
$(by \ 12 \ noon)$	

\*\* 5-day workshop participants are welcome to use Hotel Hacienda Los Laureles facilities until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*

# Neostability 13 July 2015 – 17 July 2015

# ABSTRACTS

# Speaker: Alex Berenstein (Andes)

Title: H-structures and preservation of  $NTP_2$ 

Abstract: A complete first order theory is called geometric if it eliminates the quantifier exists infinity and for all models M of T the algebraic closure satisfies the exchange property.

An *H*-structure associated to *T* is an expansion of a model *M* of *T* with a new predicate *H* which is algebraically independent, dense and condense in *M*. Density in this setting means if  $\varphi(x, a)$  is an infinite formula in a single variable, then  $\varphi(x, a)$  intersects *H*.

All *H*-structures have the same theory  $T^{ind}$  and, conversely, all sufficiently saturated models of  $T^{ind}$  are H-structures.

In this talk we study the preservation of some properties from T to  $T^{ind}$ .

In particular we show (joint work with Vassiliev) that if is Stable or NIP then  $T^{ind}$  is Stable or NIP respectively.

Then we show (joint work with Hyeungjoon Kim) that if T is NTP2 or strong, then  $T^{ind}$  is NTP<sub>2</sub> or strong, respectively.

# Speaker: Will Boney (UIC)

Title: Stability in Nonelementary Classes

Abstract: We discuss some changes in stability and other dividing lines when working in a nonelementary context. For instance, a nonelementary class can have a definable order and still look like a stable or even superstable class (in terms of having a nonforking notion satisfying certain properties), provided that the order is of bounded size. We focus on contextualizing these in terms of concrete classes.

# Speaker: Enrique Casanovas (Barcelona)

Title: Stable forking and imaginaries

Abstract: The stable forking conjecture for T is equivalent to the stable forking conjecture for  $T^{eq}$ . Joint work with Joris Potier.

# Speaker: Artem Chernikov (Paris and UCLA)

# Title: Title: Zarankiewicz problem for (hyper-)graphs and counting types

Abstract: Zarankiewicz problem in extremal graph theory asks for the largest possible number of edges in a bipartite graph that has a given number of vertices but has no complete bipartite subgraphs of a given size. We discuss how bounds for the hypergraph version of it given by Erdős can be used to count (partial) types in n-dependent and NTP<sub>2</sub> theories, and how improved bounds may be obtained for graphs definable in various NIP structures.

# Speaker: Gabe Conant (Notre Dame)

# Title: An axiomatic approach to free amalgamation

Abstract: We consider complete theories equipped with a ternary notion of independence satisfying certain axioms, which mimic free amalgamation of relational structures. Examples of structures with such a theory include the random graph (and several other Fraïssé limits of graphs), as well as certain Hrushovski constructions defined from a predimension and "good" control function. Using this axiomatic framework, we give a relatively straightforward proof of weak elimination of imaginaries for any theory with such a ternary relation. We then discuss connections with thorn-forking and the class of rosy theories.

# Speaker: Darío García (Andes and Lyon)

# Title: Pseudofinite structures and simplicity

Abstract: The concept of pseudofinite dimension for ultraproducts of finite structures was introduced by Hrushovski and Wagner. In this talk I will present joint work with Macpherson and Steinhorn in which we explored conditions on the (coarse) pseudofinite dimension that guarantee simplicity or supersimplicity of the theory of the underlying theory, and that a drop in pseudofinite dimension is equivalent to forking.

Under a suitable assumption, a measure-theoretic condition is shown to be equivalent to local stability.

# Speaker: Isaac Goldbring (UIC)

# Title: On a sumset conjecture of Erdős

Abstract: Erdős conjectured that a set of natural numbers of positive lower density contains the sum of two infinite sets. In this talk I will describe progress on the conjecture.

# Speaker: Vince Guingona (Ben-Gurion and Wesleyan)

# Title: VC-density in VC-minimal theories

Abstract: I discuss various methods of computing VC-density in strongly dependent theories, focusing primarily on VC-minimal theories (which include, for example, algebraically closed valued fields). Specifically, I outline a method to show that, in any VC-minimal theory, the VC-density of formulas with two parameter variables is bounded above by two. Under certain stronger conditions, this argument gives a similar bound for all formulas.

# Speaker: Assaf Hasson (Ben-Gurion)

Title: Nonlocally modular reducts of ACF. Abstract: (This is joint work with D. Sustretov.) We give a new proof (and generalise) Rabinovich's theorem that a non-locally modular reduct of ACF interprets an algebraically closed field. This work uses classical stability theory together with quite a lot of algebrogeometric machinery.

# Speaker: Nadja Hempel (Lyon)

# Title: Groups definable in NTP<sub>2</sub> theories (joint work with Alf Onshuus)

Abstract: This talk centers around finding definable groups around groups with a certain property. Poizat showed that for any nilpotent/solvable subgroup of a stable group there is a definable nilpotent/solvable group of the same class which contains the given group. Furthermore, Shelah proved that one can find a definable abelian group around any abelian subgroup of an NIP group, and moreover Aldama generalized Poizat's result to nilpotent/ normal solvable subgroups of NIP group. In groups definable in simple theories, this no longer holds. However, Milliet showed that an abelian subgroup of a group defined in a simple is contained in a definable finite-by-abelian subgroup and any n-nilpotent/n-solvable subgroup is "almost contained" (up to finite index) in a definable nilpotent/soluble subgroups of groups definable in an NTP<sub>2</sub> theory.

# Speaker: Will Johnson (Berkeley)

# Title: dp-minimal fields

Abstract: We classify dp-minimal fields showing in particular that every dp-minimal field possibly considered in an expansion of the language of rings, is either a strongly minimal expansion of an algebraically closed field, is real closed, or possesses a definable valuation with respect to which it is henselian. Moreover, in the last case we classify which theories of valued fields are dp-minimal.

# Speaker: **Itay Kaplan** (Hebrew University)

Title: Distal theories and exact saturation

Abstract: (This is joint work with Saharon Shelah and Pierre Simon.)

Say that a model M is saturated exactly at  $\kappa$  if M is  $\kappa$  saturated but not  $\kappa^+$  saturated.

I will present the following result, generalizing a result of Shelah: Assume that  $\kappa$  is singular and  $\kappa^+ = 2^{\kappa}$ . Then for any NIP theory T of cardinality less than  $\kappa$ , T is not distal iff T has a model which is saturated exactly at  $\kappa$ .

#### Speaker: Byunghan Kim (Yonsei)

Title: The Lascar groups and  $H_1$  in rosy theories

Abstract: This is a survey talk on the 1<sup>st</sup> homology groups  $(H_1)$  of strong types and also delivering recent works by Junguk Lee. Let p be a strong type in a rosy theory T. We work in  $\mathcal{M}^{eq}$  of T.

Recent results on  $H_1(p)$  (computed over acl) are as follows:

- 1. If p is a Lascar type then  $H_1(p) = 0$ . In particular, if T is o-minimal, or simple having ehi, then  $H_1$  is trivial.
- 2. Even if  $H_1(p) = 0$ , there is no uniform upper bound, when p varies, on the lengths of minimal 2-chains having the same 1-shell boundary in p. This result comes from a classification of 2-chains.
- 3. There is a canonical epimorphism from  $\operatorname{Gal}_L(p)$ , the Lascar group of p, to  $H_1(p)$  (by J. Lee using the classification).
- 4. In a new sequent work (by J. Dobrowolski, B. Kim, and J. Lee) it is shown that  $H_1(p) = \operatorname{Aut}(p)/K$ , where  $K := \{f \in \operatorname{Aut}(p) : \text{ for any } a \models p, \text{ there is } g \in \operatorname{Aut}(p)' \text{ such that } f(a) = g(a)\}.$

#### Speaker: Krzysztof Krupiński (Wrocław)

Title: Topological dynamics and the complexity of strong types

Abstract: (This is a joint work with Anand Pillay and Tomasz Rzepecki.)

Some methods and ideas from topological dynamics were introduced to the context of definable groups by Ludomir Newelski. In my last paper with Anand Pillay, we proved some results which relate the so-called generalized Bohr compactification of a given definable group to its quotients by various model-theoretic connected components. I will discuss more recent (analogous) results for the group of automorphisms of the monster model of a given theory, relating notions from topological dynamics to various Galois groups of the theory in question. One of the main outcomes of this approach is a natural presentation of the Lascar Galois group of any given theory as a quotient of a compact Hausdorff group by a dense subgroup.

As an application, I will present a very general theorem concerning the complexity of bounded, invariant equivalence relations (whose classes are sometimes called strong types) in countable theories, generalizing a theorem of Kaplan, Miller and Simon concerning Borel cardinalities of Lascar strong types and also later extensions of this result to certain bounded,  $F_{\sigma}$  equivalence relations (which were obtained in a paper of Kaplan and Miller and, independently, in a paper of Rzepecki and myself). Our general theorem yields answers to some open questions from the last two papers. In a simplified form, the theorem says that a bounded, invariant equivalence relation defined on the set of realizations of a single complete  $\emptyset$ -type is either type-definable or non-smooth. As a corollary, we get that if the relation in question is Borel, then it is type-definable or otherwise it has  $2^{\aleph_0}$ -classes.

We have also obtained a variant of the last corollary for theories in an arbitrary (not necessarily countable) language, which is a wide generalization of a similar theorem of Newelski for  $F_{\sigma}$  relations.

#### Speaker: Chris Laskowski (Maryland)

#### Title: Beginning a classification theory for classes of atomic models

Abstract: Inspired by some absoluteness questions for  $L_{\omega_1,\omega}$  we are led to investigate the class of atomic models for a complete first-order theory in a countable language. Using the existence of  $2^{\aleph_1}$  non-isomorphic atomic models of size  $\aleph_1$  as 'chaos', we show that atomic classes with few atomic models of size  $\aleph_1$  exhibit superstable-like behavior. In order to see this, we define and investigate pseudo-algebraicity, which seems to be the correct analogue of algebraicity in the context of atomic models. This is joint work with John Baldwin and Saharon Shelah.

# Speaker: Dugald Macpherson (Leeds)

#### Title: Profinite groups with NIP theory

Abstract: In joint work with Katrin Tent, we consider profinite groups as 2-sorted structures (G, I) where the sort I acts as an index system for a basis of open subgroups of the profinite group G (so the basic open subgroups indexed by I are uniformly definable). We call (G, I) a FULL profinite group if I codes ALL open subgroups of G. The main theorem is that a full profinite group (G, I) has NIP theory if and only if G has a finite index (open) subgroup which is a direct product of finitely many compact p-adic analytic groups (for finitely many primes p).

# Speaker: Anand Pillay (Notre Dame)

#### Title: On compactifications of pseudofinite groups

Abstract: I will try to give precise versions of some questions posed by Zilber around homomorphic images of ultraproducts of finite groups. And then give a few observations, including results of Vojdani on pseudofinite cyclic groups in Pressburger.

#### Speaker: Amador Martin Pizarro (Lyon)

#### Title: On bounded automorphisms of fields with operators

Abstract: Together with T. Blossier and C. Hardouin, we give an alternative proof, valid in all characteristics, of a result of Lascar characterising the bounded automorphisms of an algebraically closed field. We generalise this method to various fields equipped with operators. In particular, there are no non-trivial bounded automorphisms of an algebraically closed field with free operators as introduced by Moosa and Scanlon.

#### Speaker: Nick Ramsey (Berkeley)

#### Title: Model-theoretic tree properties

Abstract: A remarkable theorem of Shelah says that an unsimple theory must be unsimple in at least one of two ways: either it has the tree property of the first kind  $(TP_1)$  or the tree property of the second kind  $(TP_2)$ . In the wake of simplicity theory, there have been a number of tree properties introduced between the tree property, the absence of which characterizes simplicity, and the tree property of the first kind. Among them are SOP<sub>1</sub>, introduced by Džamonja and Shelah, as well as weak k-TP<sub>1</sub> and k-TP<sub>1</sub>, studied by Kim and Kim. With Artem Chernikov, we studied these properties, showing, among other things, that SOP<sub>1</sub> can be characterized by a version of independent amalgamation of types. This gives rise to a very useful criterion for establishing that a structure is NSOP<sub>1</sub>. Leveraging work of Granger and Chatzidakis, this allowed us to show that the two-sorted theory of vector spaces over an algebraically closed field with a generic bilinear form and  $\omega$ -free PAC fields of characteristic zero are NSOP<sub>1</sub>.

# Speaker: Silvain Rideau (Orsay and Berkeley)

#### Title: Definable and invariant types in enrichments of NIP theories

Abstract: (This is joint work with Pierre Simon.) Using the theory of honest definitions, we show how under certain weak hypotheses invariant types in a reduct may be obtained from definable types in an expansion.

# Speaker: Tomasz Rzpecki (Wrocław)

Title: Topological dynamics and the complexity of strong types, part II

Abstract: This is a joint work with Krzysztof Krupiński and Anand Pillay.

In his talk, Krzysztof Krupiński will introduce some relations between Galois groups and a certain compact Hausdorff group related to the topological dynamics of the action of the automorphism group of a monster model on its type space.

He will also describe how these methods give a generalisation of previous results about type-definability and smoothness of certain bounded  $F_{\sigma}$  equivalence relations (including the Lascar strong type) to (restrictions to type-definable sets of) Borel equivalence relations on a single type, under the assumption that the theory is countable (similarly to those previous results). In my talk, I intend to explain how these methods can be generalised to the case of uncountable language, where the notions of Borel cardinality are very restricted. In particular, our study leads to the conclusion that for the class of "analytic" (or "Souslin") equivalence relations on a single type, having less than continuum many classes in a given type-definable (and saturated with respect to the relation in question) set implies that the relation is type-definable, and, under some further assumptions, it actually implies that the relation is relatively definable. This result applies immediately to the case of  $F_{\sigma}$  equivalence relations. In that case, the first part (about type-definability) was previously known as a corollary of a theorem of Newelski, while the second is still a new result.

This theorem, although it does not require countability of the language, yields an elegant corollary if we do assume it: suppose E is a bounded, Borel equivalence relation on a single type. Then exactly one of the following holds:

- E is relatively definable and has finitely many classes,
- E is type-definable and smooth and has continuum many classes, or
- E is not type-definable and non-smooth (and thus has continuum many classes).

These results immediately transfer to the case of connected components of definable groups.

If time permits, I will also briefly discuss a more technical conjecture in the uncountable case – in the spirit of those obtained before by Kaplan, Miller and Simon, as well as myself and Krupiński – highlighting the difficulties we have in proving it.

#### Speaker: **Pierre Simon** (Lyon)

#### Title: Decomposing types in NIP theories

Abstract: The intuition that NIP theories are somehow combinations of stable and order-like components has been stated by several authors, probably starting with Shelah. Taking order-like to mean distal, I will make this intuition explicit by showing that one can actually decompose any type in an NIP theory into a stable part and an order-like "quotient". This yields in particular a stronger form of honest definitions.

#### Speaker: Sergei Starchenko (Notre Dame)

Title: Ramsey-type statement for polynomially bounded o-minimal expansions of the real field Abstract: In the paper "Erdős-Szekeres-type statements: Ramsey function and decidability in dimension 1", B. Bukh, Boris and J. Matousek proved the following theorem :

**Theorem.** Let  $\varphi(x_1, \ldots, x_r)$ , with all  $x_i$  singletons, be a semi-algebraic formula over the reals. Then there is a constant  $C = C_{\varphi}$  such that for every  $n \in \mathbb{N}$  setting  $N = 2^{2^{Cn}}$  every sequence of real numbers of length N contains a  $\varphi$ -indiscernible subsquence of length n.

In this talk we give a model-theoretic proof of this theorem that also works for any polynomially bounded o-minimal expansion of the real field.

#### Speaker: Frank Wagner (Lyon)

#### Title: Thorny questions and simple answers

Abstract: The existence of big definable abelian subgroups is one of the cornerstones of the study of superstable groups: First established by Reinecke for minimal groups, it was generalized by Chantal Berline to the superstable case, while the lack of a similar theorem for stable groups is one of the main obstacles for their analysis. In the supersimple (even rank 1) case, the existence of extraspecial *p*-groups shows that we have to weaken "abelian" to at least "finite-by-abelian", but even then the question whether an SU-rank 1 group is finite-by-abelian-by-finite embarrassingly remains open. Elwes and Ryten have shown that a pseudofinite (or more generally "functionnally unimodular") group of SU-rank 1 is finite-by-abelian-by-finite. I shall show that a pseudofinite supersimple group of SU-rank  $\geq \omega^{\alpha}$  has a finite-by-abelian definable subgroup of SU-rank  $\geq \omega^{\alpha}$ , and that a pseudofinite supersimple group of SU-rank  $\omega^{\alpha} \cdot 2$  is soluble-by-finite. I shall also discuss the possbible generalization to the functionally unimodular case, and to superrosy groups.