Thermodynamic optimization of a Penrose process

An engineers' approach to black hole thermodynamics

[Phys. Rev. D 93, 064070 (2016)]

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with

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Contents

Energy extraction from BH
Mechanical perspective

Name	Year	Effic.	Reference
Penrose	1969	121%	Nuovo Cimento 1, 252
Piran	1975	130%	ApJ 196, L107
BZ	1977	143%	MNRAS 179, 433
BSW	2009	∞	PRL 103, 111102
Jacobson	2009	1000%	PRL 104, 021101
Schnittman	2014	1400%	PRL 113, 261102
Piran	2016	1400%	PRD 93, 043015

Mechanical efficiency: $\eta = E_p^{\text{out}}/E_p^{\text{in}}$!!

- Astrophysics: GRB, AGN
  (*several processes to extract work from BH*)
- Black holes analogues
  (*at hand in laboratory*)
  [*Linder, Schützhold, & Unruh, 1511.03900*]
- AdS/CFT correspondence
  (*processes in BH* $\rightarrow$ *in the dual CFT*)
  [*C. V. Johnson, CQG 31, 205002 (2014)*]

- Black holes thermodynamics (*BH are not black*)
- Thermodynamics $\rightarrow$ bounds (*limits for processes that extract work from BH*)
- Finite-time thermodynamics (*more realistic limits*) $\Rightarrow$ thermodynamic geometry

Energy extraction from BH

TD Penrose Process:

$$p_i = (M_i, J_i = M_i^2)$$
$$\downarrow$$
$$p_f = (M_f, J_f = 0)$$

Energy extraction from BH
Thermodynamic perspective

Isolated Kerr BH:

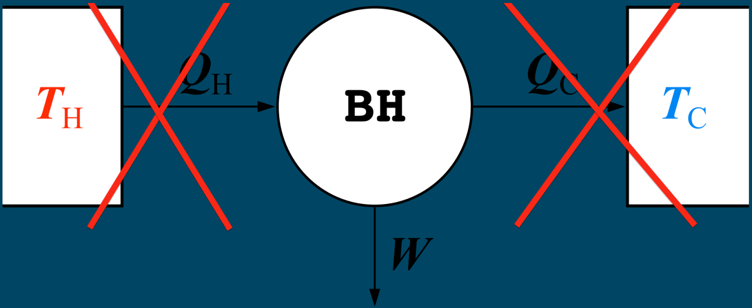
$$\mathrm{d}S = 0$$

$$M_f = M_{\text{irreducible}} = \frac{M_i}{\sqrt{2}}$$

$$W = -\Delta M = \left(1 - \frac{1}{\sqrt{2}}\right) M_i$$

$$\simeq 0.3 M_i$$

Energy extraction from BH
Isolated BH



Max. work: $W^{\max} = -\Delta M \simeq 0.3\, M_i$

TD efficiency: $\eta_1 = \dfrac{W^{\max}}{M_i} \simeq 30\%$!!

[Dolan, CQG 28, 235017 (2011)]

Energy extraction from BH considering the reservoir

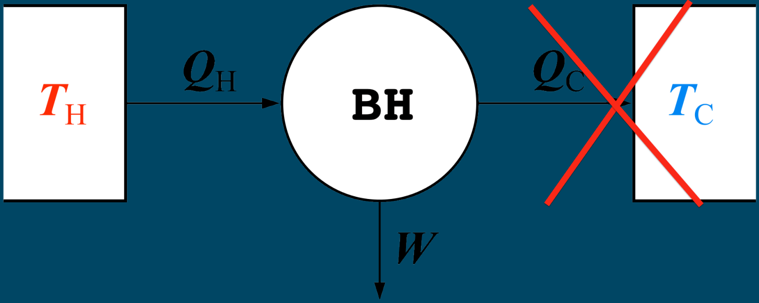
Energy extraction from BH
Thermodynamic perspective

Non-isolated Kerr BH:

$$\mathrm{d}S \neq 0$$
$$M_f = ?$$
$$\mathrm{d}W = ?$$

Energy extraction from BH

BH+Reservoir



Max. work: $W^{\max} = ?$

TD efficiency: $\eta_1 = \frac{W^{\max}}{E_i} = ?$

Black Hole with Reservoir
Maximum work

Availability [Gibbs]:

$$A = U - T_0 S + p_0 V$$

Max. Reversible Work [Landau & Lifshitz]:

$$W^{\max} = -\Delta A = -\Delta U + T_0 \Delta S - p_0 \Delta V$$

Maximum work in finite time:

$$W^{\max}(\tau) = W^{\max} - (\Delta A)_{\text{dest}}$$

Black Hole with Reservoir
Maximum work

Availability [Gibbs]:

$$A = U - T_0 S - \Omega_0 J$$

Max. Reversible Work [Landau & Lifshitz]:

$$W^{\max} = -\Delta A = -\Delta U + T_0 \Delta S + \Omega_0 \Delta J$$

Maximum work in finite time:

$$W^{\max}(\tau) = W^{\max} - (\Delta A)_{\mathrm{dest}}$$

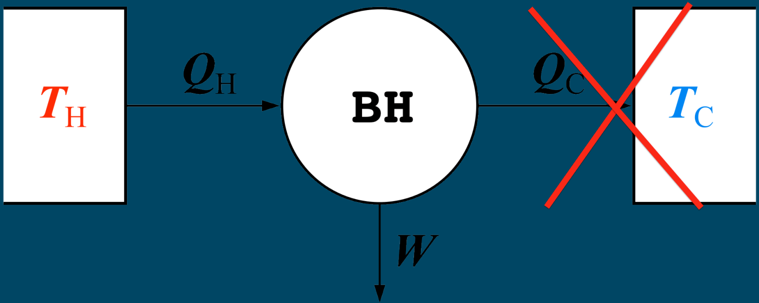
Black Hole with Reservoir
Optimum Penrose process

Non-isolated Kerr BH:

- extremal Kerr $\rightarrow$ Schwarzschild
- $T_i = 0 \rightarrow T_f = 1/8M_f$
- Minimization of dissipation

$$\boxed{M_f = \frac{1+\sqrt{2}}{2} M_i}$$

Black Hole with Reservoir
Maximum Work



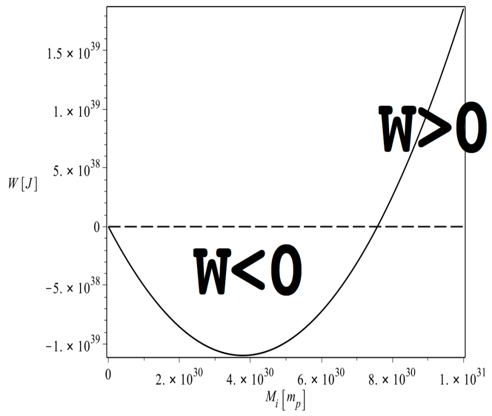
Main Result

$$W_\tau^{\max} = M_i[(1+\sqrt{2})T_0 M_i - (\sqrt{2}-1)(0.5+\tfrac{2\epsilon}{\tau})]$$

$$\eta_1 = \frac{W^{\max}(\tau)}{E_{in}} \simeq \left(1 - \frac{c_1 + c_2\epsilon/\tau}{T_0 M_i^2 + 1}\right)$$

Black Hole with Reservoir

Maximum Work



for $T_0 \sim$ CMB temperature

Black Hole with Reservoir
Maximum Work

Extracted energy:

Stellar mass BH:

$$M_i = M_\odot \to W \sim 10^{53} J$$

Efficiency:

$$\eta_1^{non-isol.} \simeq 99\%$$

Conclusions

Summary of results

- Kerr BH in a reservoir
  (*reservoir = universe*, $T_0 = T_{\rm CMB}$)
- Optimum Penrose process
  (*optimum = geodesic of TD metric*)
- Maximum work in finite time
  $$W^{\max}(\tau) = M_i[(1+\sqrt{2})T_0 M_i - (\sqrt{2}-1)(0.5 + 2\epsilon/\tau)]$$

- More black holes. Any horizon. $(\Lambda \neq 0?)$
- Black holes analogues (*predictions for the laboratory?*)
- AdS/CFT correspondence (*TD cycles in the dual CFT?*)

Thanks for your attention!

Any questions?