The graviton with delocalized interactions

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Motivation for nonlocal gravity theories

Several lines of argument indicate some kind of nonlocality underlying gravitational interactions.

One such line is through higher derivative (HD) gravity theories.
General HD (physical transverse) graviton propagator pole structure:

$$\frac{1}{k^2(1+P_n(\kappa^2 k^2))} = \frac{r_o}{k^2} + \sum_{i=1}^{n} \frac{r_i}{k^2 - M_i^2} \quad .$$

Hence, at least one the residues r_i must be negative.

The same type of argument may be given also for the dressed propagator in the Källen-Lehmann representation to conclude that the spectral function must contain negative contributions and satisfy a superconvergence relation.

Unitarity Issue of HD theories.

Note: 4-th order HD theory appears rather special.

This suggests one take infinite number of derivatives replacing polynomial P_n by infinite series so that graviton denominator possesses no poles beyond graviton, i.e., so that

$$(1 + P_n(\kappa^2 k^2)$$

is an entire function possessing no zeros.

This amounts to a nonlocal action.

General (minimal) form of such actions:

$$\mathscr{L} = \sqrt{-g}\, \left\{ \frac{\beta}{\kappa^2} R + R_{\mu\nu}\, h_1(-\frac{\nabla^2}{\Lambda^2})\, R^{\mu\nu} + R\, h_2(-\frac{\nabla^2}{\Lambda^2})\, R \right\}$$

TT (1997), Modesto (2012), Mazumdar et al (2012), ...

Cf. talks (Mazumdar, Modesto, Frolov, ...) in this conference.

The nonlocality in these models arises from the insertion in the action of operators of the form

$$\hat{\mathscr{F}} = \exp\big(f(\ell^2\nabla^2)\big) \tag{1}$$

Historically, insertion of (1) characterizes most nonlocal QFTs in the literature going back to 1940-50's.
Example: String field theory vertices of general schematic form $[\exp(\ell\partial^2)\phi]^3$

Note: *This cannot be implemented with just an EH term to produce nonlocal gravity theory*.

The action of an operator (1) can be represented by an integral kernel

$$(\hat{\mathscr{F}}\phi)(x) = \int d^d y\, F(x,y)\phi(y) \tag{2}$$

An integral kernel representation exists provided the operator is well-defined on some domain.

The class of integral kernels representing possible nonlocal interactions is wider than the class of just those representing differential operators (1).

Defining nonlocal interactions by integral kernels provides then a unified framework for studying such interactions.

Issues with nonlocal QFT

Long history of attempts at nonlocal QFT. Many issues very inadequately investigated.

- UV behavior
- Unitarity - Structure in the complex energy plane
- Classical IVP - Causality
  Simplest models:

$$L = \frac{1}{2}\partial^\mu \Phi \partial_\mu \Phi - V(\tilde{\Phi})$$

$$\tilde{\Phi}(x) = \int d^4y \, F(x-y)\Phi(y)$$

(TT, PRD 92, 125037 (2015)).

Graviton interactions in GR

$$S[g] = \frac{1}{\kappa^2} \int dx^4 \sqrt{-g} R(g)$$

Expansion about a background metric $\bar{g}^{\mu\nu}$: $\quad g^{\mu\nu} = \bar{g}^{\mu\nu} + h^{\mu\nu}$

$$S[\bar{g}+h] = S[\bar{g}] + S_2[\bar{g},h] + \sum_{n=3}^{\infty} S_n[\bar{g},h]$$

with

$$S_n[\bar{g},h] = \frac{1}{n!} \left(\partial_s^n S[\bar{g}+sh] \right)_{|s=0} .$$

The entire expansion can then be generated from the $S_2[\bar{g},h]$ partial action by recursion

$$S_n[\bar{g},h] = \frac{2}{n!} \left[\int d^4x \, h^{\alpha\beta}(x) \frac{\delta}{\delta \bar{g}^{\alpha\beta}(x)} \right]^{n-2} S_2[\bar{g},h] , \qquad n \geq 3 .$$

(7) actually holds for any metric gravity action.

In the case of the EH action the second variation

$$S_2[\bar{g}, h] = \frac{-1}{2\kappa^2} \int d^4x \, \sqrt{-\bar{g}} \, h^{\mu\nu} \hat{G}_{\mu\nu\kappa\lambda} h^{\kappa\lambda}$$

for background $\bar{g}^{\alpha\beta}$ obeying the classical equation of motion

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{R}\,\bar{g}_{\mu\nu} = 0\,.$$

Operator $\hat{G}$ is given by:

$$\begin{aligned}\hat{G}_{\mu\nu\kappa\lambda}(\bar{g}, \bar{\nabla}) \;\; &\equiv \;\; \frac{1}{2}(\bar{g}_{\mu(\kappa}\bar{g}_{\lambda)\nu} - \bar{g}_{\mu\nu}\bar{g}_{\kappa\lambda})\bar{\nabla}^2 + \frac{1}{2}\bar{g}_{\mu\nu}\bar{\nabla}_{(\kappa}\bar{\nabla}_{\lambda)} \\ &\quad + \frac{1}{2}\bar{g}_{\kappa\lambda}\bar{\nabla}_{(\mu}\bar{\nabla}_{\nu)} - \bar{\nabla}_{(\kappa}\bar{g}_{\lambda)(\mu}\bar{\nabla}_{\nu)}\,.\end{aligned}$$

Here $\bar{\nabla} \equiv \nabla(\bar{g})$. For a Minkowski background $\bar{g}^{\alpha\beta} = \eta^{\alpha\beta}$ (8) reduces to the Fierz-Pauli action for a free massless spin-2 field.

The action for the n-point self-interaction S_n is generated by the coupling of the graviton field $h^{\mu\nu}$ to the source provided by the energy-momentum tensor

$$t^{(n)}_{\mu\nu} = \frac{1}{\sqrt{-\bar{g}}} \frac{\delta S_n[\bar{g}, h]}{\delta \bar{g}^{\mu\nu}} .$$

of the action S_{n-1} for the $(n-1)$-point interaction:

$$S_n[\bar{g}, h] = \frac{1}{n} \int d^4x \sqrt{-\bar{g}}\, h^{\mu\nu} t^{(n-1)}_{\mu\nu} ,$$

and

$$\hat{G}_{\mu\nu\kappa\lambda} h^{\kappa\lambda} = T_{\mu\nu} = \sum_{n=2}^{\infty} t^{(n)}_{\mu\nu}$$

This then implies the fact that the expansion (7) cannot be consistently terminated at any point: the entire action (7) must be recovered.

Delocalized graviton fields

Introduce a metric $\gamma^{\mu\nu}$ (eventualyy to be taken to be flat).

A delocalized graviton field $\tilde{h}^{\mu\nu}$ is now defined by

$$\tilde{h}^{\mu\nu}(x) \equiv \int d^4y \sqrt{-\gamma(y)}\, F^{\mu\ \nu}_{\ \kappa\ \lambda}(x,y)\, h^{\kappa\lambda}(y)\,.$$

The delocalization kernel $F^{\mu\ \nu}_{\ \kappa\ \lambda}(x,y)$ is a bitensor:

$$F'^{\mu\ \nu}_{\ \kappa\ \lambda}(x',y') = \frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x'^{\nu}}{\partial x^{\delta}} F^{\alpha\ \delta}_{\ \beta\ \zeta}(x,y) \frac{\partial y^{\beta}}{\partial y'^{\kappa}} \frac{\partial y^{\zeta}}{\partial y'^{\lambda}}$$

May be taken to be of the general form

$$F^{\mu\ \nu}_{\ \kappa\ \lambda}(x,y) = \mathscr{U}^{\mu}_{\ \kappa}(x,y)\, \mathscr{U}^{\nu}_{\ \lambda}(x,y) F(x,y)$$

$\mathscr{U}$ denotes the parallel transport matrix of the γ-metric connection $\Gamma^{\kappa}_{\rho\sigma}(\gamma)$:

$$\mathscr{U}(x,y) = P\exp\left(-\int_y^x \Gamma(\gamma)\cdot dz \right),$$

F is a biscalar: $F'(x,y) = F(\xi^{-1}(x), \xi^{-1}(y))$.

In particular, in coordinates where $\gamma_{\mu\nu} = \eta_{\mu\nu}$, $\mathscr{U}$ reduces to the constant unit matrix $\mathscr{U}_\lambda^\kappa(x,y) = \delta_\lambda^\kappa$; and one may set $F(x,y) = F(x-y)$.

Delocalization requires, in general, specifying a (bi)scalar function and a metric (γ) on the manifold.

Types of delocalization

I) Delocalization kernels representing action of transcendental differential operators - form most often used in the literature.

$$F^{\mu\ \nu}_{\ \kappa\ \lambda}(x,y) = \delta^{\mu}_{\kappa}\,\delta^{\nu}_{\lambda}\,e^{f(\ell^2\nabla^2)}\delta(x,y)$$

where f is a smooth (entire) function. For example, for $f(z) = z^2$, in coordinates where $\gamma = \eta$, this has the kernel representation

$$F(x-y) = \left(\frac{\sqrt{\pi}}{\ell}\right)^4 \int d^4k\, e^{-(\ell^2 k^2)^2}\, e^{-ik\cdot(x-y)}\,.$$

Note: This type of delocalization is strictly nonlocal: any two spacetime points connected.

II) Delocalization kernels represented by smooth functions of strictly bounded support.

Generic example:

$$F(x,y) = \frac{1}{\ell^4} \exp\left[- \frac{\ell}{(\ell - \sigma(x,y))} \right] \qquad \text{for} \quad \sigma(y,x) < \ell$$

$$F(x,y) = 0 \qquad \text{for} \quad \sigma(x,y) \geq \ell\,.$$

ℓ is a fixed length scale characterizing the delocalization scale.

$\sigma(x,y) = \int_\lambda |\gamma(\partial_s, \partial_s)|\, ds$ is geodesic distance along some specified type of (polygonal) path λ connecting events x, y.

Support given by the (invariant) set of spacetime points that can be reached from starting point by all such paths.

Different specific examples given by specification of type of path - but qualitatively equivalent for sufficiently small ℓ: x, y within a normal neighborhood (w.r.t. γ).

Properties

1. Functions of bounded support have Fourier transforms $\hat{F}(k)$ which are of rapid decay.
2. Functions of bounded support have Fourier transforms $\hat{F}(k)$ which are entire analytic functions in each of their arguments k_μ.

General nonlocal kernels (type II above) which are of rapid decay satisfy 1. above.
They do not automatically satisfy 2.

Theory of delocalized graviton interactions

With background γ:

$$S[\gamma, h] = S_2[\gamma, h] + \sum_{n=3}^{\infty} S_n[\gamma, \tilde{h}]$$

where S_2 is given by

$$S_2[\gamma, h] = \frac{-1}{2\kappa^2} \int d^4x \sqrt{-\gamma}\, h^{\mu\nu} \hat{G}_{\mu\nu\kappa\lambda}(\gamma, \nabla) h^{\kappa\lambda}$$

Same relations as in GR except with delocalized fields in all graviton interaction vertices S_n, $n \geq 3$, e.g.,

$$\hat{G}_{\mu\nu\kappa\lambda} h^{\kappa\lambda}(x) = \kappa^2 \int dy^4 \sqrt{-\gamma(y)} \left(\sum_{n=2}^{\infty} \tilde{t}^{(n)}_{\alpha\beta}(y) \right) F^{\alpha\ \beta}_{\ \mu\ \nu}(y, x) \equiv \kappa^2\ \tilde{T}_{\mu\nu}\,,$$

with

$$\tilde{t}^{(n)}_{\mu\nu} = \frac{1}{\sqrt{-\gamma}} \frac{\delta S_n[\gamma, \tilde{h}]}{\delta \gamma^{\mu\nu}}\,, \qquad S_n[\gamma, \tilde{h}] = \frac{1}{n} \int d^4x \sqrt{-\gamma}\, \tilde{h}^{\mu\nu} \tilde{t}^{(n-1)}_{\mu\nu}\,.$$

Gauge invariance

Restore local invariance - nonlinear realization:

$$\gamma^{\mu\nu}(\omega) = (\exp \mathscr{L}_\omega)\bar{\gamma}^{\mu\nu}$$

with reference metric $\bar{\gamma}$. Action now invariant under

$$\begin{aligned} h'^{\mu\nu} &= (\exp \mathscr{L}_\xi) h^{\mu\nu} \\ \omega'^{\mu} &= \omega'^{\mu}(\xi,\omega) \end{aligned}$$

with $\omega'(\xi,\omega)$ given by

$$(\exp \mathscr{L}_\xi)(\exp \mathscr{L}_\omega)\bar{\gamma}^{\mu\nu} = (\exp \mathscr{L}_{\omega'(\xi,\omega)})\bar{\gamma}^{\mu\nu}$$

and background $\bar{\gamma}^{\mu\nu}$ kept fixed. Explicitly

$$\omega'(\xi,\omega) = \omega + \xi + \frac{1}{2}[\xi,\omega] - \frac{1}{12}[\omega,[\xi,\omega]] + \frac{1}{12}[\xi,[\xi,\omega]] \cdots ,$$

Note: $S[\gamma(\omega), h] = S[\bar{\gamma}, (\exp \mathscr{L}_{-\omega})h]$
Thus, the ω field couples as a gauge degree of freedom

Background field formalism - Quantization

Given (flat) background $\bar{\gamma}$, define field $\sigma^{\mu\nu}$ through

$$\begin{aligned} \gamma^{\mu\nu}(\omega) &= \bar{\gamma}^{\mu\nu} + \big((\exp \mathscr{L}_\omega) - 1\big)\bar{\gamma}^{\mu\nu} \\ &\equiv \bar{\gamma}^{\mu\nu} + \sigma^{\mu\nu}(\omega, \bar{\gamma}) \end{aligned}$$

Graviton field shifted by $\varphi + h$.

Backgrounds: φ, $\bar{\gamma}$; quantum: h, ω.

The action

$$S[\bar\gamma+\sigma,\tilde\varphi+\tilde h] = S_2[\bar\gamma+\sigma,\varphi+h] + \sum_{n\geq 3} S_n[\bar\gamma+\sigma,\tilde\varphi+\tilde h]$$

is now invariant under the finite field transformations

$$\begin{aligned} h'^{\mu\nu} &= (\exp\mathscr{L}_\xi)(\varphi^{\mu\nu}+h^{\mu\nu}) - \varphi^{\mu\nu} \\ \sigma'^{\mu\nu}(\omega,\bar\gamma) &= (\exp\mathscr{L}_\xi)(\bar\gamma^{\mu\nu}+\sigma^{\mu\nu}) - \bar\gamma^{\mu\nu} . \end{aligned}$$

The latter is, of course, just a restatement of the transformation for ω:

$$\begin{aligned} \sigma'^{\mu\nu}(\omega,\bar\gamma) = \Big((\exp\mathscr{L}_\xi)(\exp\mathscr{L}_\omega) - 1\Big)\bar\gamma^{\mu\nu} &= \Big((\exp\mathscr{L}_{\omega'}) - 1\Big)\bar\gamma^{\mu\nu} \\ &= \sigma^{\mu\nu}(\omega',\bar\gamma) , \end{aligned}$$

with $\omega' = \omega'(\xi,\omega)$ as above.

Invariance fixed by gauge-fixing function $C^\mu(\bar{\gamma}, \varphi, \sigma, h)$ transforming covariantly w.r.t. the backgrounds $\bar{\gamma}$, φ, e.g.,

$$C^\mu(\bar{\gamma}, \sigma, h) = \frac{1}{\sqrt{\alpha}} \bar{\nabla}_\kappa h^{\kappa\mu} \, .$$

Performing change of variables in the form of gauge transformations in the functional integral one derives the corresponding WST identities implying the decoupling of gauge degrees of freedom from physical amplitudes.

In particular, the WST identities imply the decoupling of ω in physical amplitudes.

The theory contains only a massless physical graviton.

Perturbation theory

The Feynman rules are now essentially the rules for the EH action with a couple of crucial differences.

The flat background $\bar{\gamma}$ is fixed. External graviton legs in graphs are represented by φ and internal lines represent h.

Each vertex prong contributing a kernel factor $\hat{F}(k)$*.* The vertices S_n additionally contain the at most two powers of momenta of standard GR.

The presence of a delocalization kernel for each leg of a vertex amounts to the effective replacement

$$\frac{iP^{(2)}_{\mu\nu\kappa\lambda}(q)}{q^2+i\varepsilon} \longrightarrow \frac{\hat{F}(q)\, iP^{(2)}_{\mu\nu\kappa\lambda}(q)\, \hat{F}(q)}{q^2+i\varepsilon}$$

for each internal line of momentum q in a graph. Note: Simpler than other nonlocal actions. s

- For appropriate choice of kernel (e.g., rapid decay) all loops are now UV finite - the rapid decay $\hat{F}^2$ factors in each internal line killing all (standard GR) power divergences.
- For appropriate choice of kernel (e.g., bounded support) perturbatively unitary theory - no additional singularities (beyond the standard Landau graviton (and matter) generated ones) anywhere in the complex energy plane.
- Nonlocal kernels induce acausal effects - they can be isolated as non-vanishing right hand side to the Bogoliubov Causality Condition equation (TT, PRD 92, 125037 (2015)).

For kernels of bounded support the acausal effects are confined within ℓ-sized region of support.

Resulting picture

Construct effective action $\Gamma[\bar{\gamma}, \varphi]$
By construction $\Gamma[\bar{\gamma}, \varphi]$ is background invariant under

$$\bar{\gamma}' = (\exp(\mathscr{L}_\xi)\bar{\gamma}, \qquad \varphi' = (\exp(\mathscr{L}_\xi)\varphi$$

and

$$\frac{\delta\Gamma[\bar{\gamma}, \varphi]}{\delta\varphi} = 0$$

Long distance ($k < 1/\ell$, $\hat{F}(k) \to 1$) behavior identical to GR on background $\bar{\gamma} + \varphi$.

Short distance ($k > 1/\ell$, $\hat{F}(k) \to 0$) essentially free gravitons on $\bar{\gamma}$.

Effective change of scale of unit of length may result between short and long scales.

Coupling to matter

Matter coupling only via delocalized graviton

- Simplest: Point particles coupled to $\tilde{h}$ - Equation of geodesic deviation (Raychaudhuri equation)
- Matter fields with global symmetries:

$$S[\gamma, h, \Phi] = S_2[\gamma, h] + \sum_{n=3}^{\infty} S_n[\gamma, \tilde{h}] + S_M[\gamma, \Phi] + \sum_{n=1}^{\infty} S_{M,n}[\gamma, \tilde{h}, \tilde{\Phi}] \,.$$

Conclusions - Outlook

- Simplest nonlocal gravity actions - delocalization scale ℓ-
  - graviton: fat but massless
- For appropriate delocalization kernels (rapid decay, entire FT):
  - UV finite
  - perturbatively unitary (normal Cutkowski rules)

  Long distance GR - short distance free gravitons
- Acausal effects present
  For bounded support kernels effects confined within ℓ-sized region - Early universe applications?
- Coupling to matter via delocalized gravitons -
  Avoidance of geodesic focusing -singularities?
- Cosmological and Black Hole solutions? Simplest context within nonlocal actions.
- Cosmological constant problem - where does one place ℓ?

- Delocalization kernel (a (bi)scalar function) specified as part of the theory
  Obtaining delocalization from possible physical substrate or mechanism requires deeper theory.