Asynchronous Robot Gathering

(A Tiny Tutorial on Distributed Computing Through Combinatorial Topology)

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Gathering

A collection of robots start on vertices of a connected graph. They can move on vertices.

- **Termination.** Correct robots decide a vertex.
- **Validity.** If participating robots start on the same vertex, they stay there.
- **Agreement.** All final vertices are the same

Asynchronous Luminous Robots (ALR)

- n **fully asynchronous** robots with low memory (the less the better).
- **Luminous:** Each robot has a light to communicate information (the less the better).
- Two available **atomic** operations:
    1. **Look:** Takes a snapshot of the whole environment (position and lights colors).
    2. **Move:** Robot moves to an adjacent or same vertex and changes light color.
- Up to n-1 **crash failures** (robots stop or disappear).

Asynchronous Luminous Robots (ALR)

Computation proceed in a sequence of **asynchronous** rounds.

```
Algorithm choose(v, G):
    r = non-negative integer
    view = empty
    undecided = true
    While undecided do
        Move(v, r)
        view = Look(G) U view
        (v, r, undecided) = Compute(view)
    endWhile
    return v
```

Robots can **start at distinct times** or **do not even start**.

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Algorithm choose(v, G):
    r = non-negative integer
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    While $undecided$ **do**
        Move(v, r)
        $view$ = **Look**(G) U $view$
        (v, r, $undecided$) = **Compute**($view$)
    endWhile
    return v

Becomes visible

Robots can **start at distinct times** or **do not even start**.

Related Work

- Gathering has been studied a lot, usually without failures and in continuous space.
  [Flocchini et al. 2012, Agmon and Peleg 2006, Bramas and Tixeuil 2015, Cieliebak et al. 2012, Klasing et al. 2008, …]

- ALR model introduced in the past, without failures and without distinct starting times.
  [Shantanu Das et al. 2016]

- First time gathering is studied considering full asynchrony+failures.

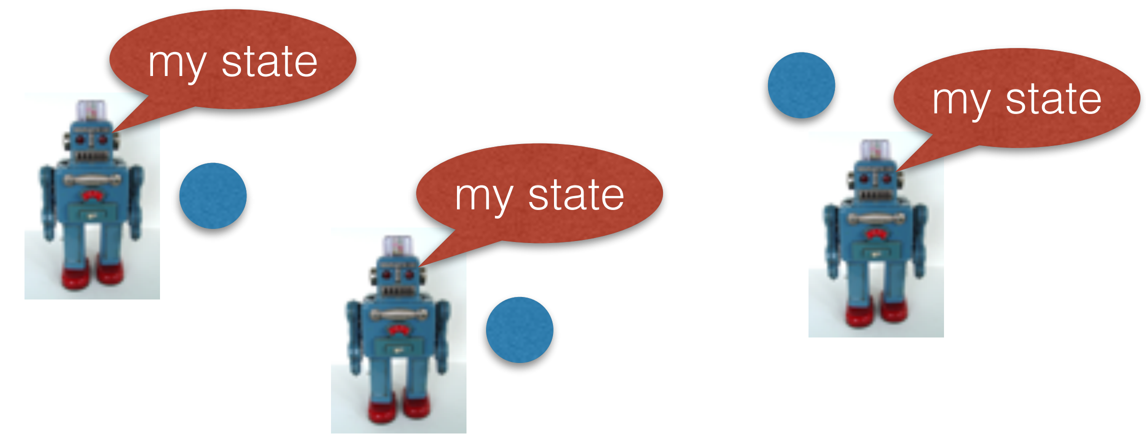
Impossibility of Gathering

For $n > 1$, the gathering is solvable if and only if the base graph $\mathcal{G}$ is a single vertex

- Proof based on the topology approach to distributed computing.
- Enough to analyze the case $n=2$.

Distributed Computing and Topology

- Vertices represent robot/ process states.



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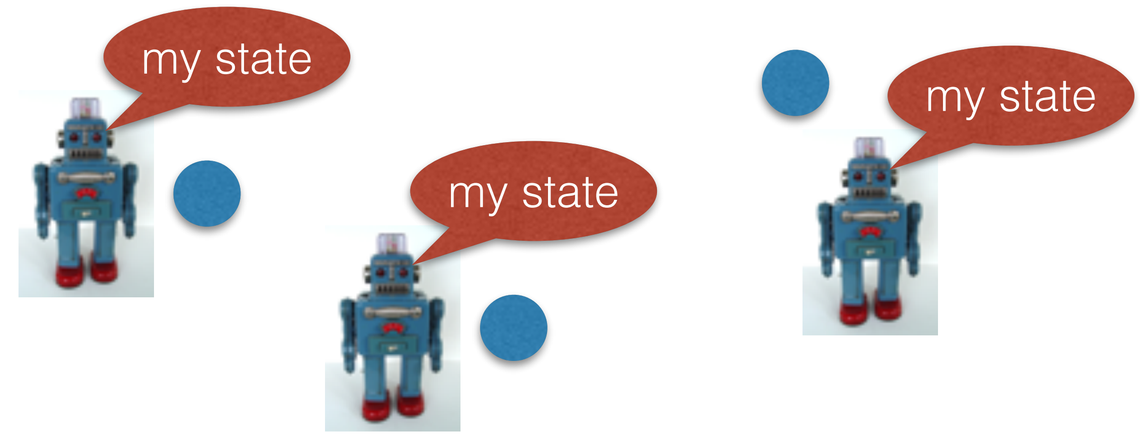
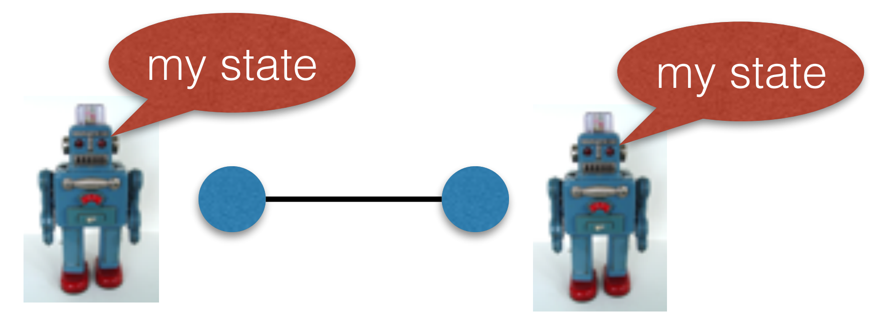


- Simplices (vertices, edges, triangles…) represent mutually compatible system states.



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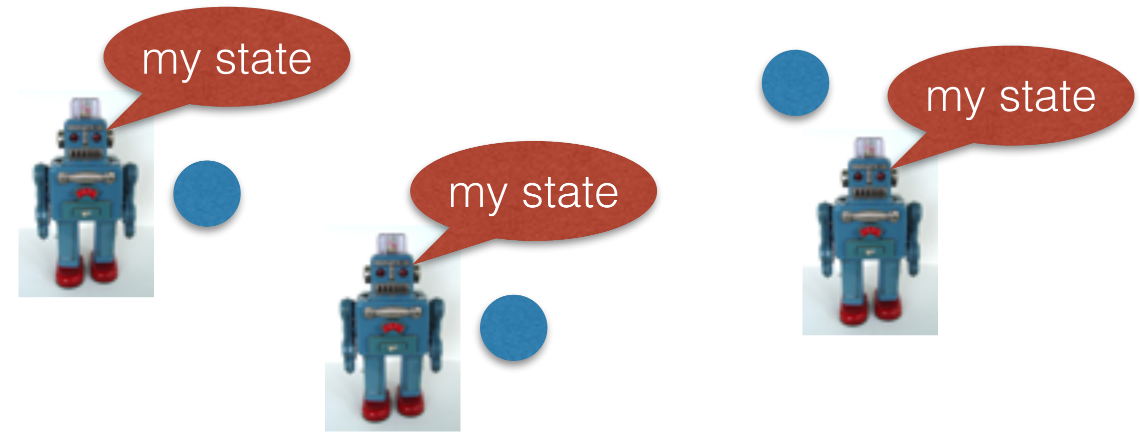

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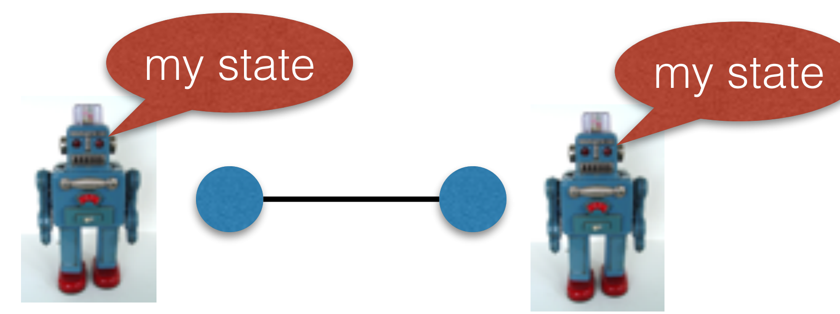
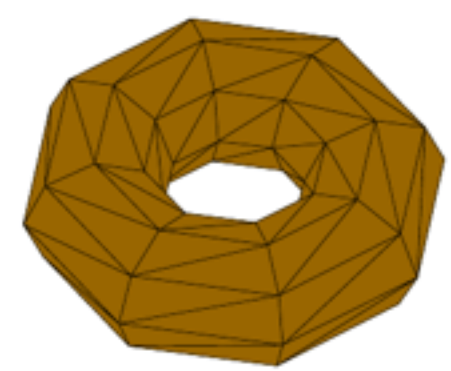


- Complexes put all together.



- Target: Understand properties of the complexes.

Colorless Tasks

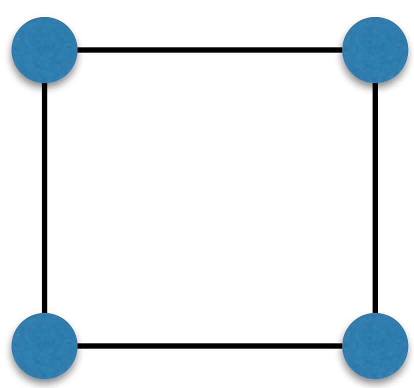
- A task is a triple $(\mathcal{I}, \mathcal{O}, \mathcal{F})$:

1. $\mathcal{I}$: input complex with valid input sets. Each set represent several configurations.
2. $\mathcal{O}$: Output complex with valid output sets. Each set represent several configurations.
3. $\mathcal{F}$: Input/Output function from input simplexes to output sub complexes.

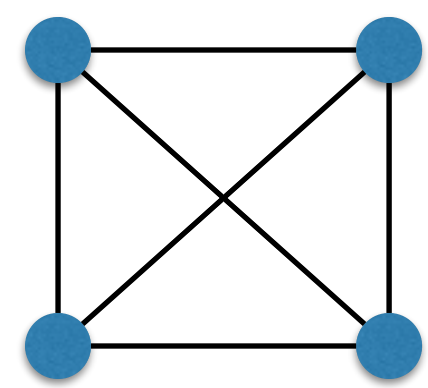
2-Robot Gathering Task

- Input Complex: Graph modeling possible input sets.

$\mathcal{G}$



$\mathcal{I}$

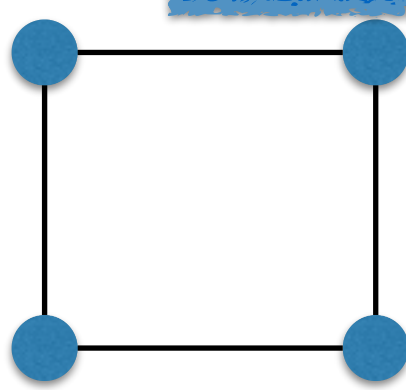


2-Robot Gathering Task

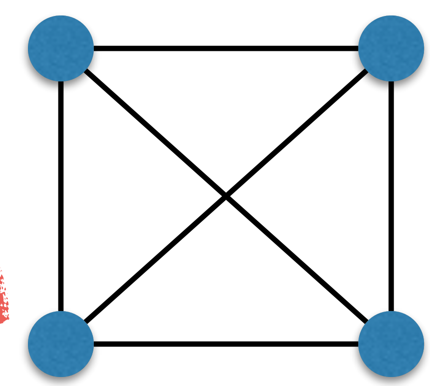
- Input Complex: Graph modeling possible input sets.

Robots start on the vertex

$\mathcal{G}$



$\mathcal{I}$



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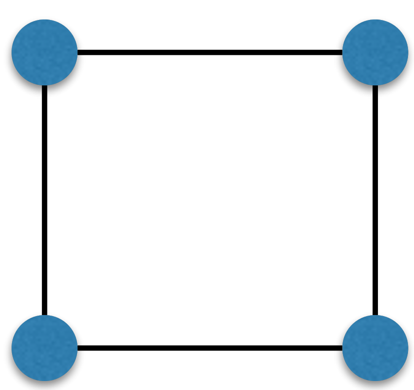
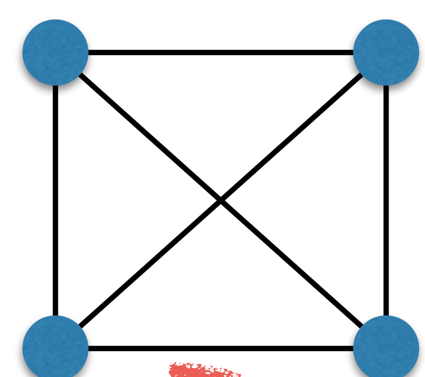


$\mathcal{I}$



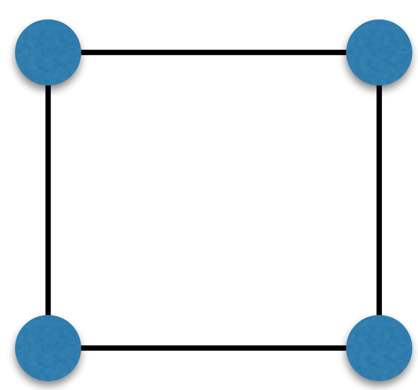
Robots start on the vertices
(it does not matter which one*)

*Colorless task

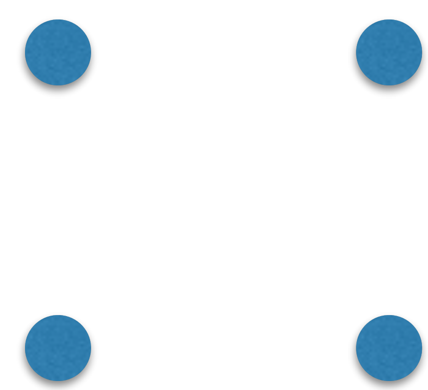
2-Robot Gathering Task

- <u>Output Complex:</u> Graph modeling possible output sets.

$\mathcal{G}$

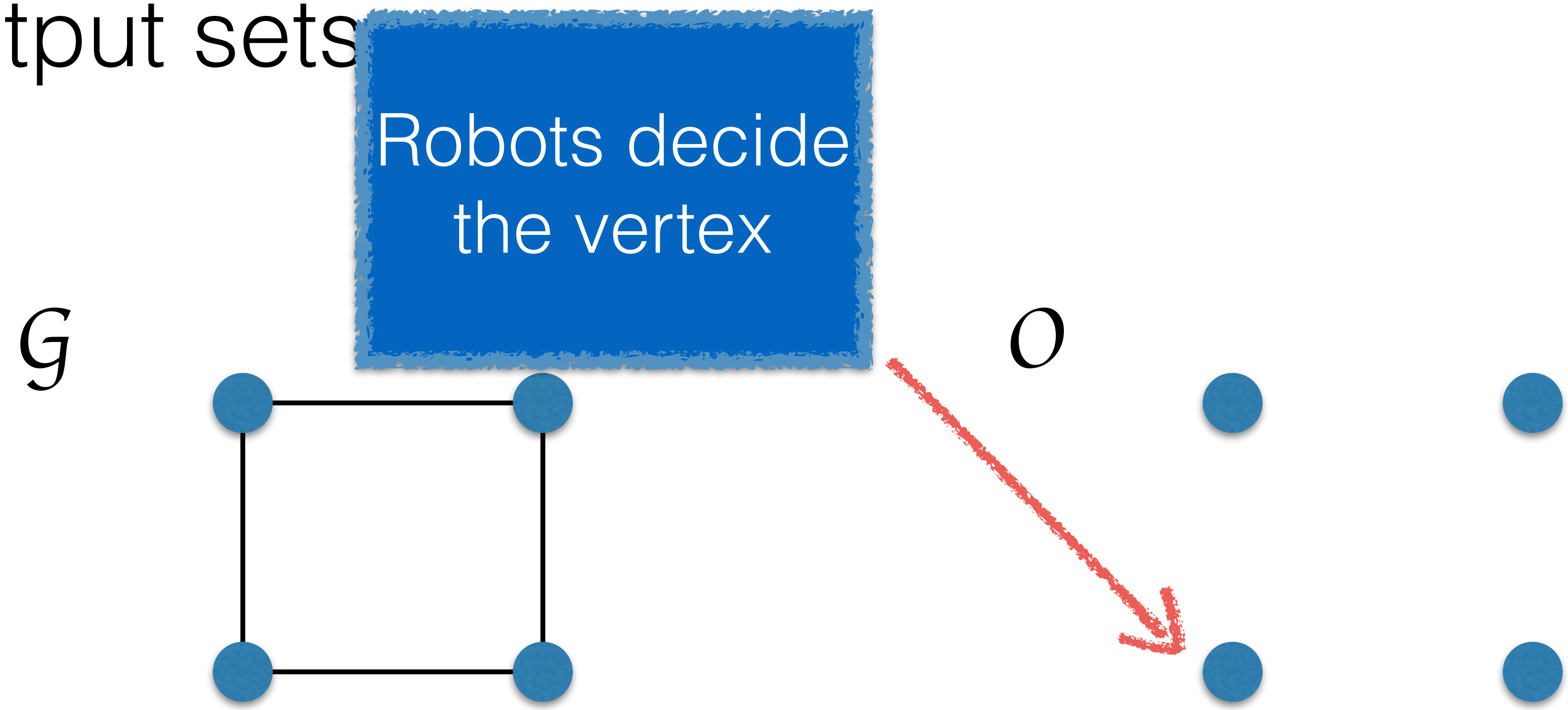


$\mathcal{O}$



2-Robot Gathering Task

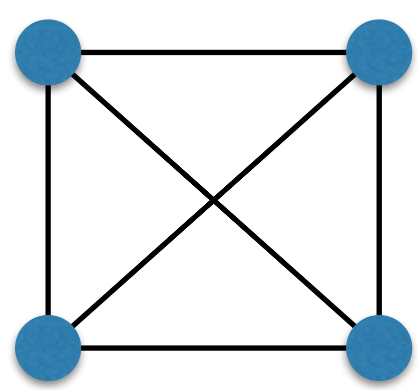
- <u>Output Complex:</u> Graph modeling possible output sets



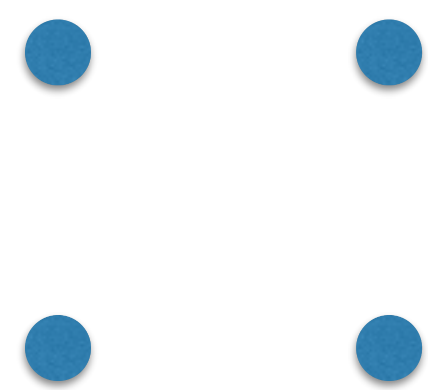
2-Robot Gathering Task

- <u>Input/Output Function:</u> Relation between inputs and outputs.

$\mathcal{I}$



$\mathcal{O}$



2-Robot Gathering Task

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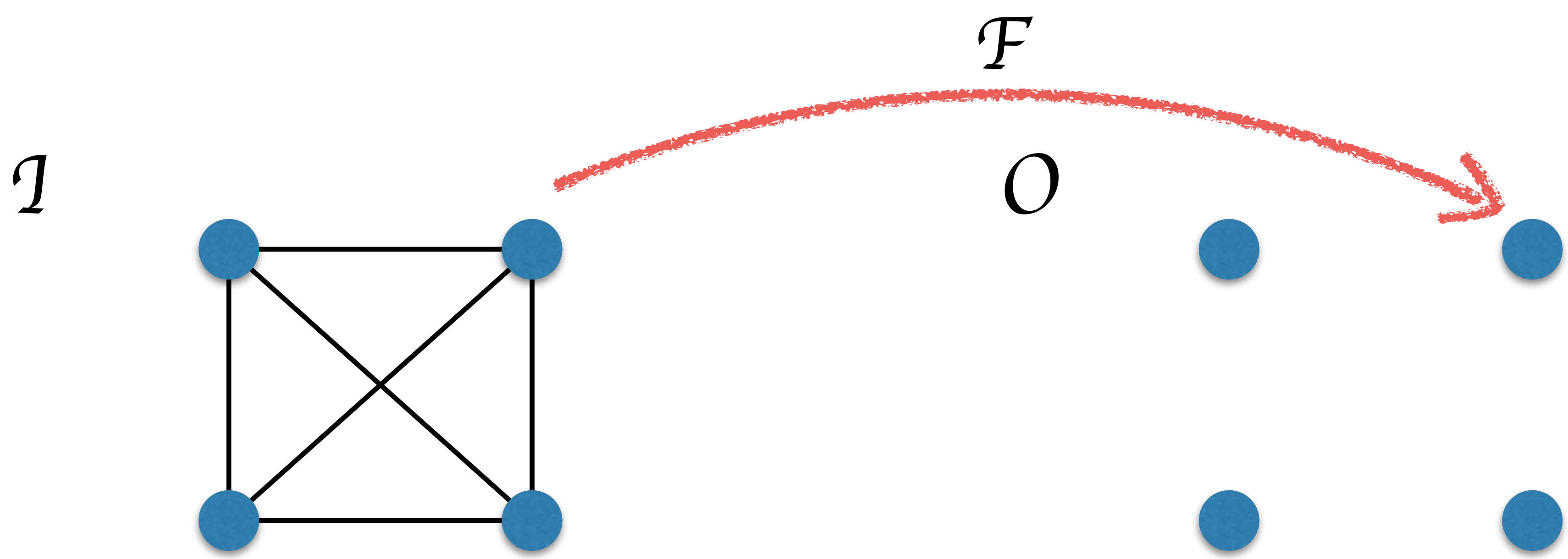

2-Robot Gathering Task

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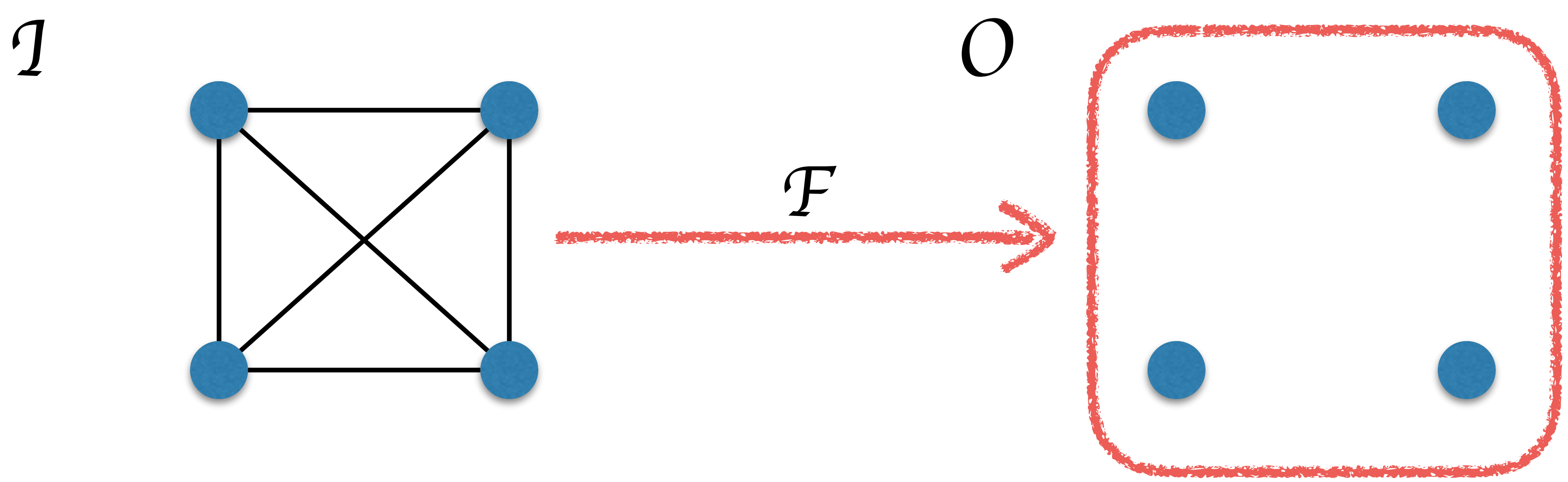


2-Robot Gathering Task

- For a given graph $\mathcal{G}$, the 2-robot gathering task is the triple $(\mathcal{I}, \mathcal{O}, \mathcal{F})$:

    1. $\mathcal{I}$: Complete graph with vertices $\mathcal{V}(\mathcal{G})$.
    2. $\mathcal{O}$: Empty graph with vertices $\mathcal{V}(\mathcal{G})$.
    3. $\mathcal{F}$: For every vertex v, $\mathcal{F}(v) = v$;
    For every edge e, $\mathcal{F}(e) = \mathcal{O}$.

Protocol Complex

- Complex $\mathcal{P}$ modeling all (or some relevant subset of) executions.
- Vertices: Local states.
- Simplices: Mutually compatible states (state of robots at the end of an execution).
- For every input configuration simplex s, $\mathcal{E}(s)$ = sub complex of $\mathcal{P}$ with all executions with initial state s.

Protocol Complex

- Complex $\mathcal{P}$ m … levant subset of) executi…

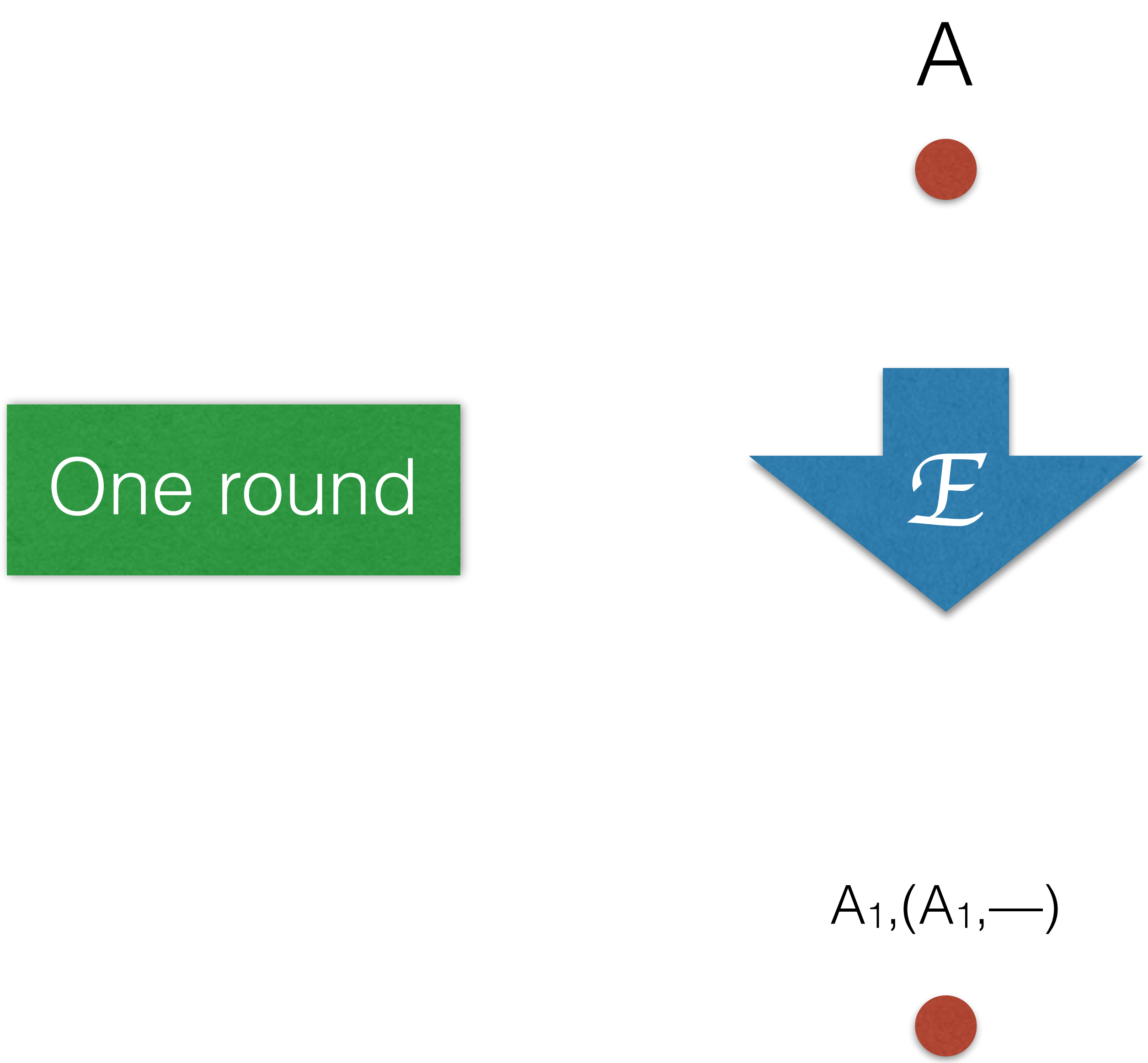
Tiny detail:
full-information (roughly)

Algorithm choose(v, $\mathcal{G}$):
  r = non-negative integer
  $view$ = empty
  $undecided$ = true
  While $undecided$ **do**
    Move(v, r)
    $view$ = **Look**($\mathcal{G}$) U $view$
    (v, r, $undecided$) = **Compute**($view$)
  endWhile
  return v

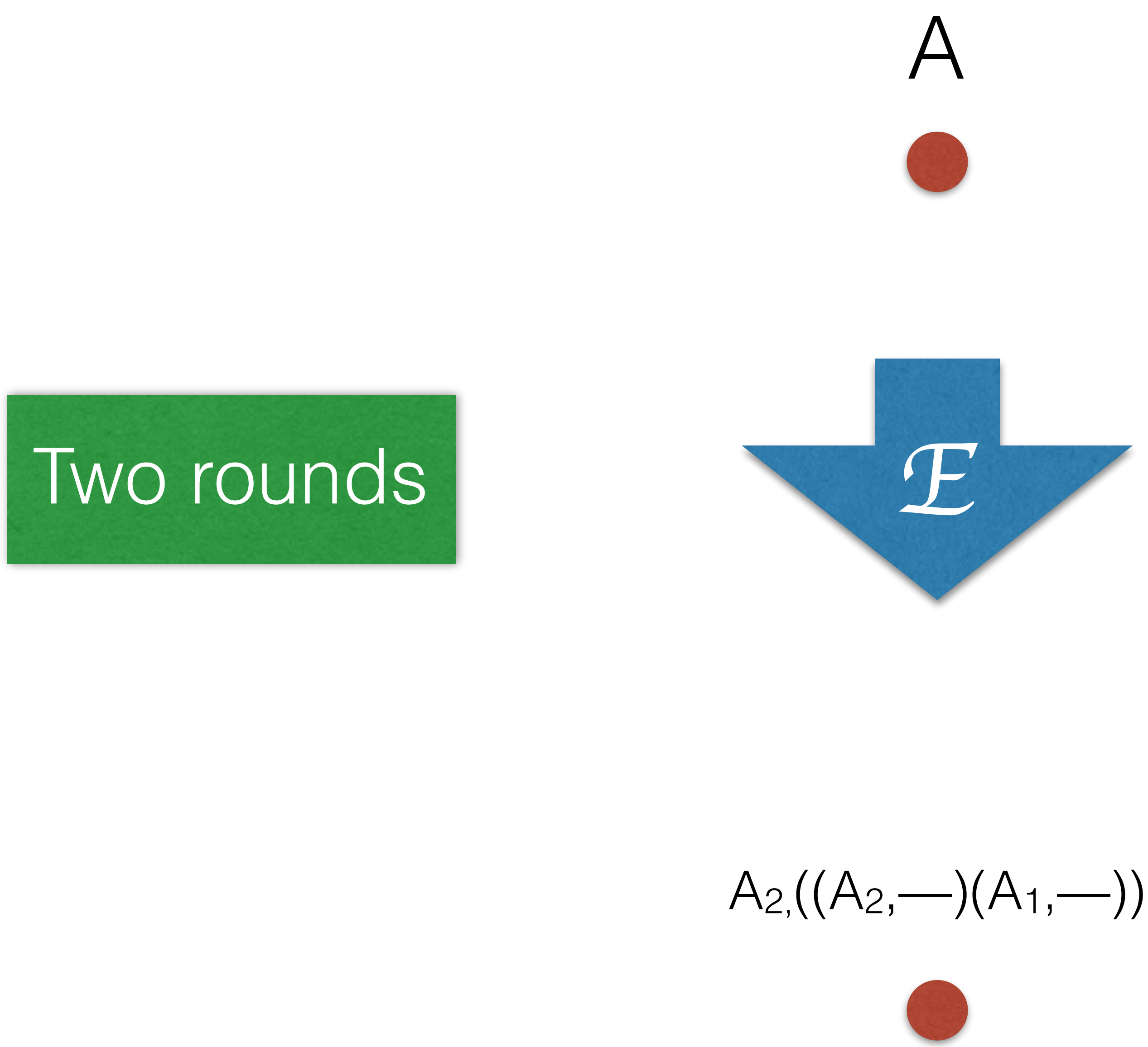
Encode all it has seen

- Vertices: L…
- Simplices … robots at t…
- For every … , $\mathcal{E}(s)$ = sub complex of $\mathcal{P}$ with all executions with initial state s.

2-Robot Protocol



2-Robot Protocol



2-Robot Protocol

A

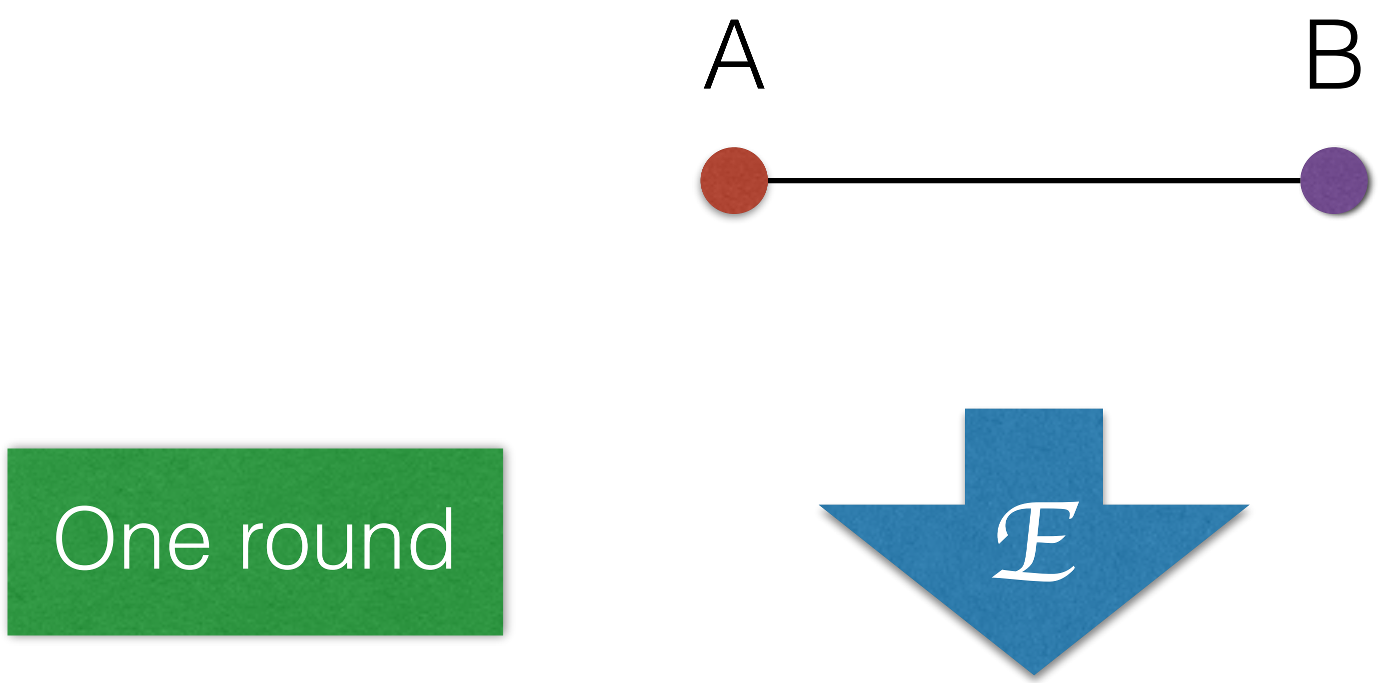
Two rounds

And so on …

$\mathcal{E}$

$A_2, ((A_2, —)(A_1, —))$

2-Robot Protocol

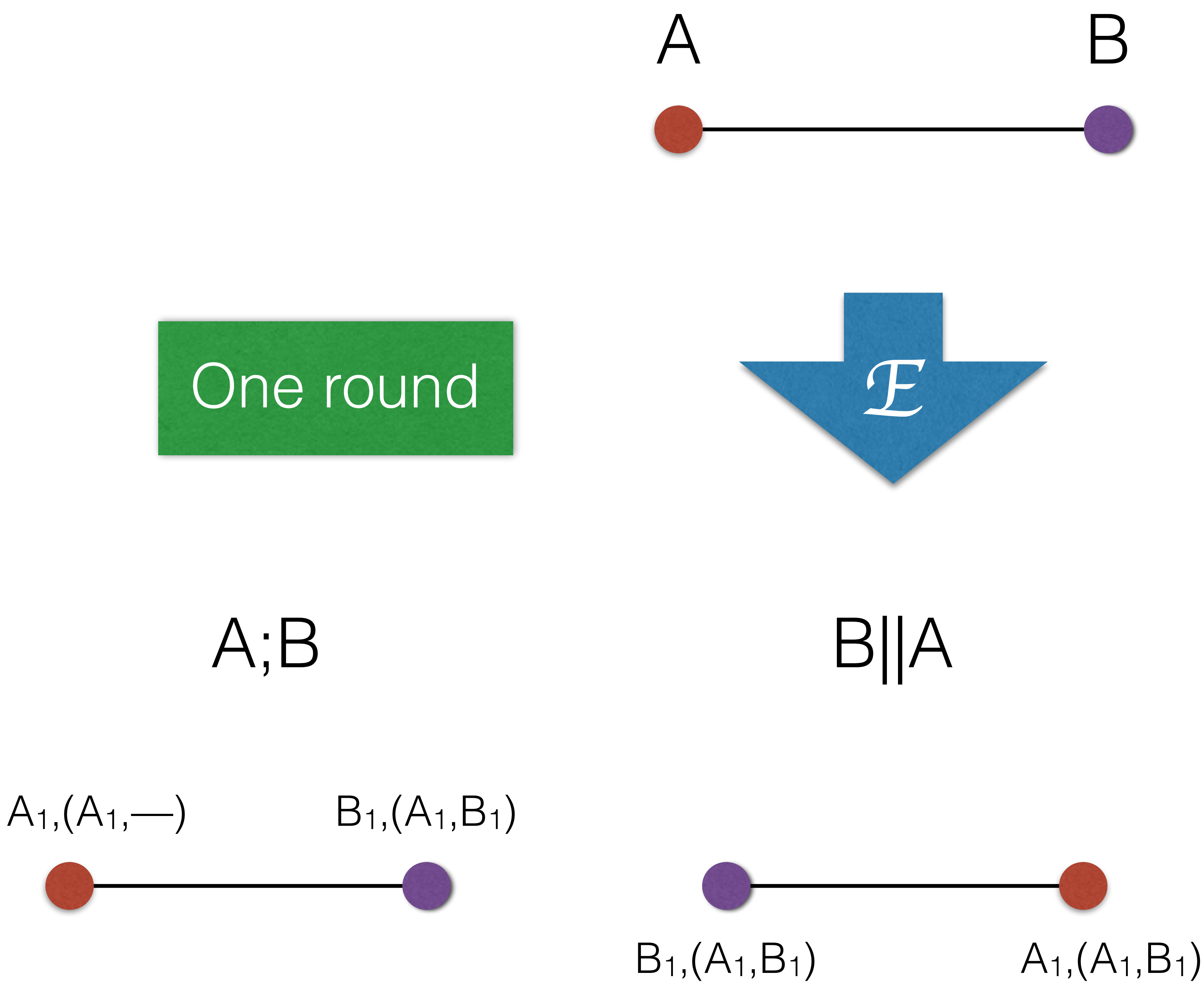


2-Robot Protocol

One round

A;B

2-Robot Protocol



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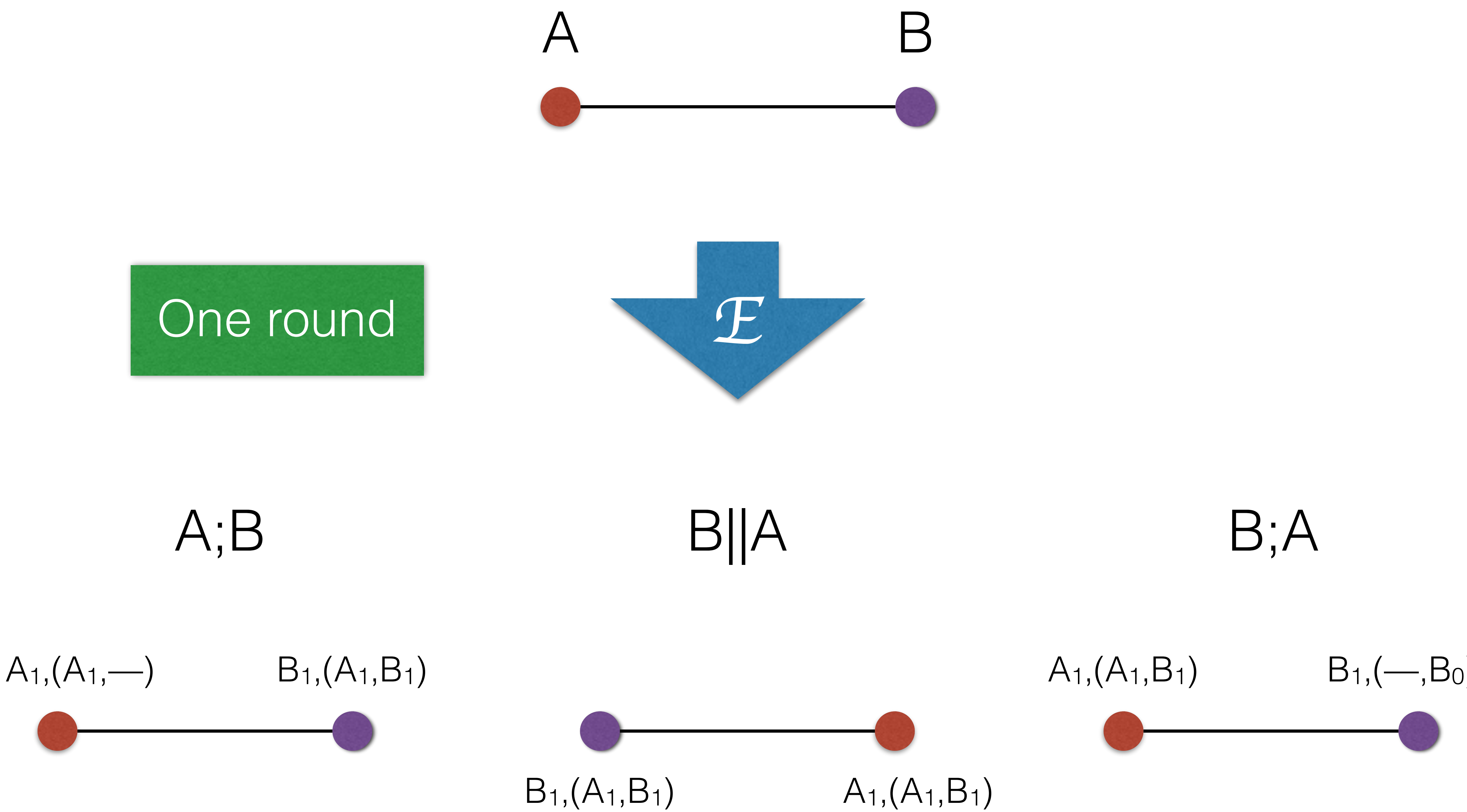

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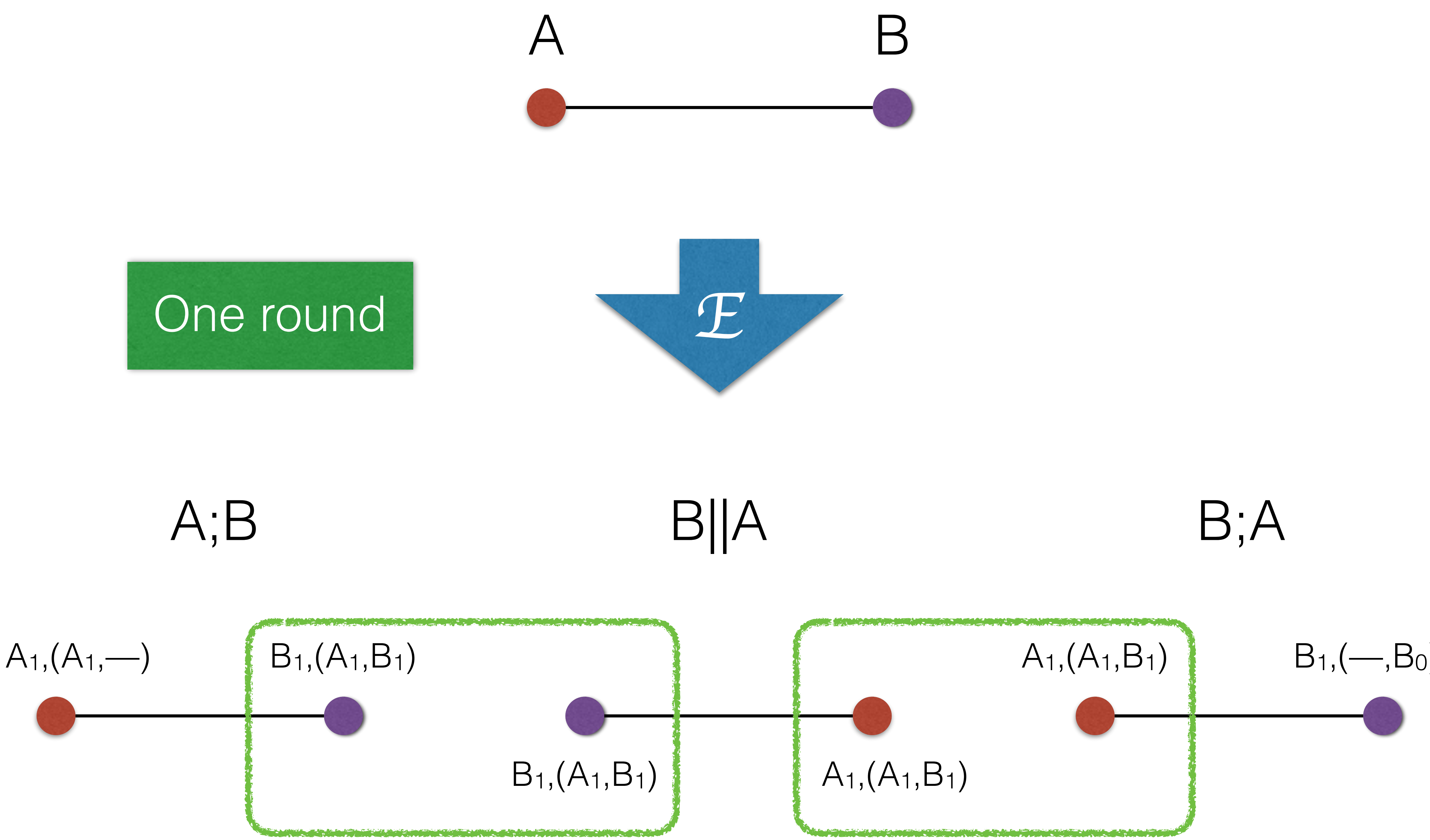

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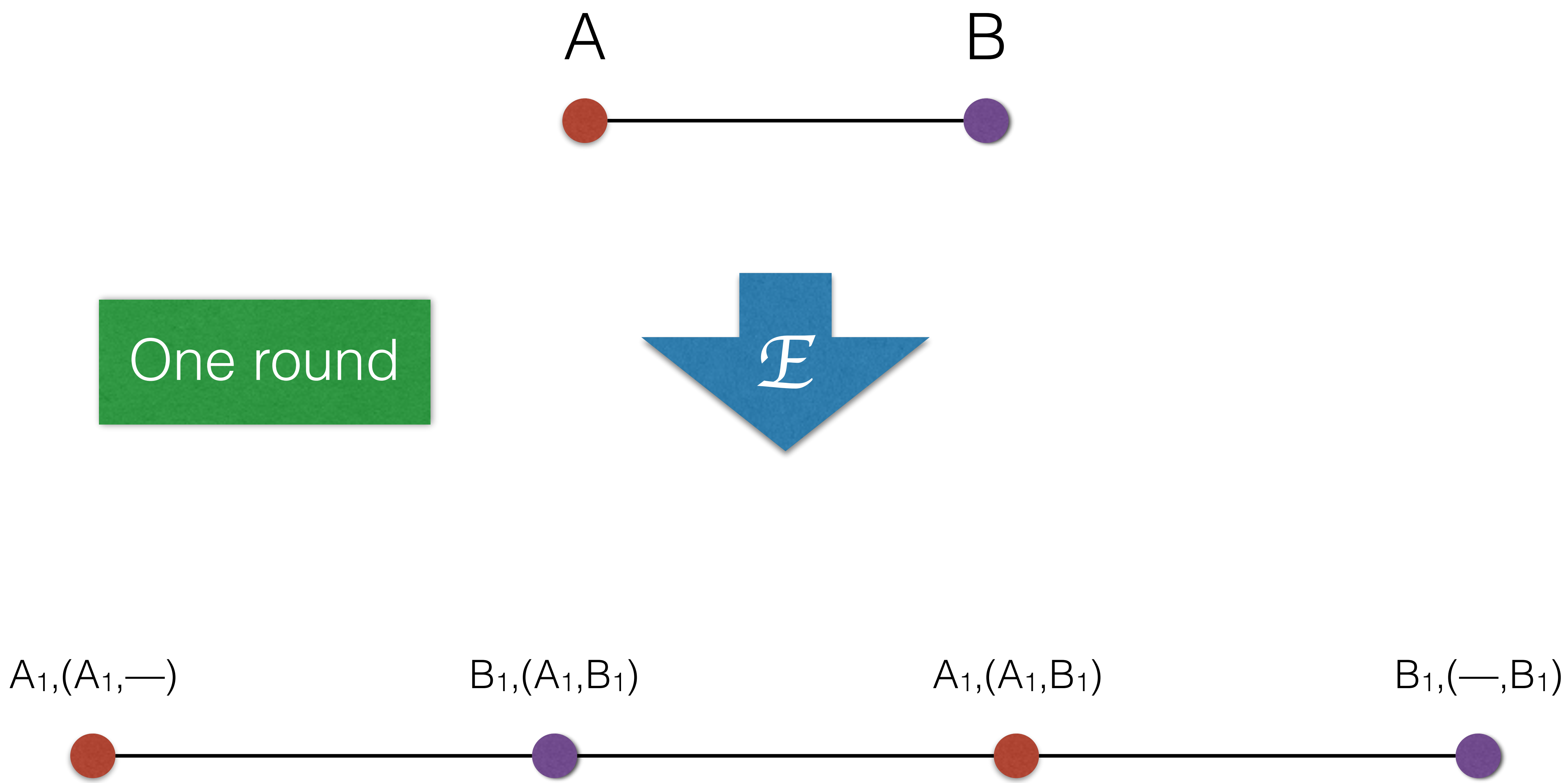

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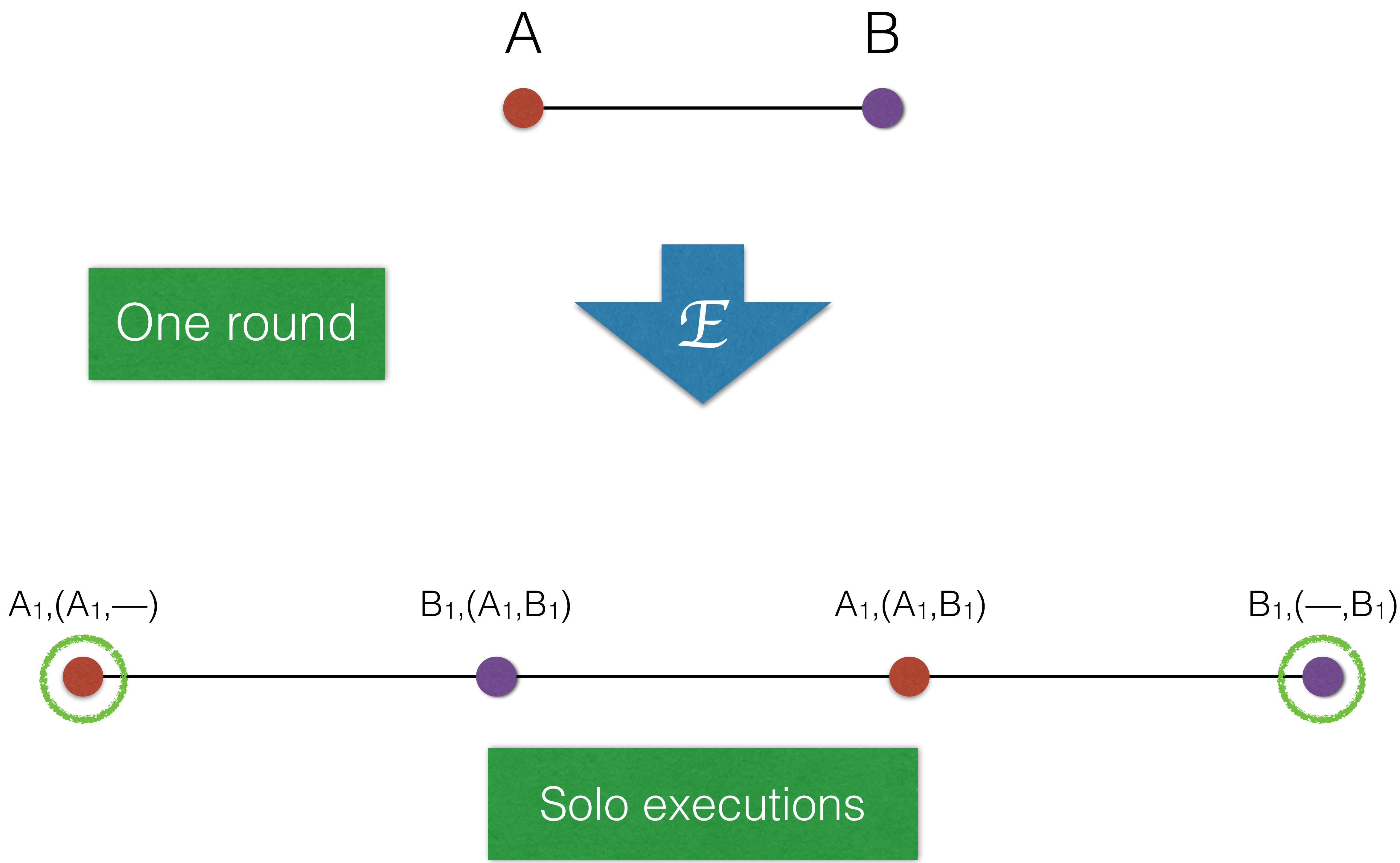

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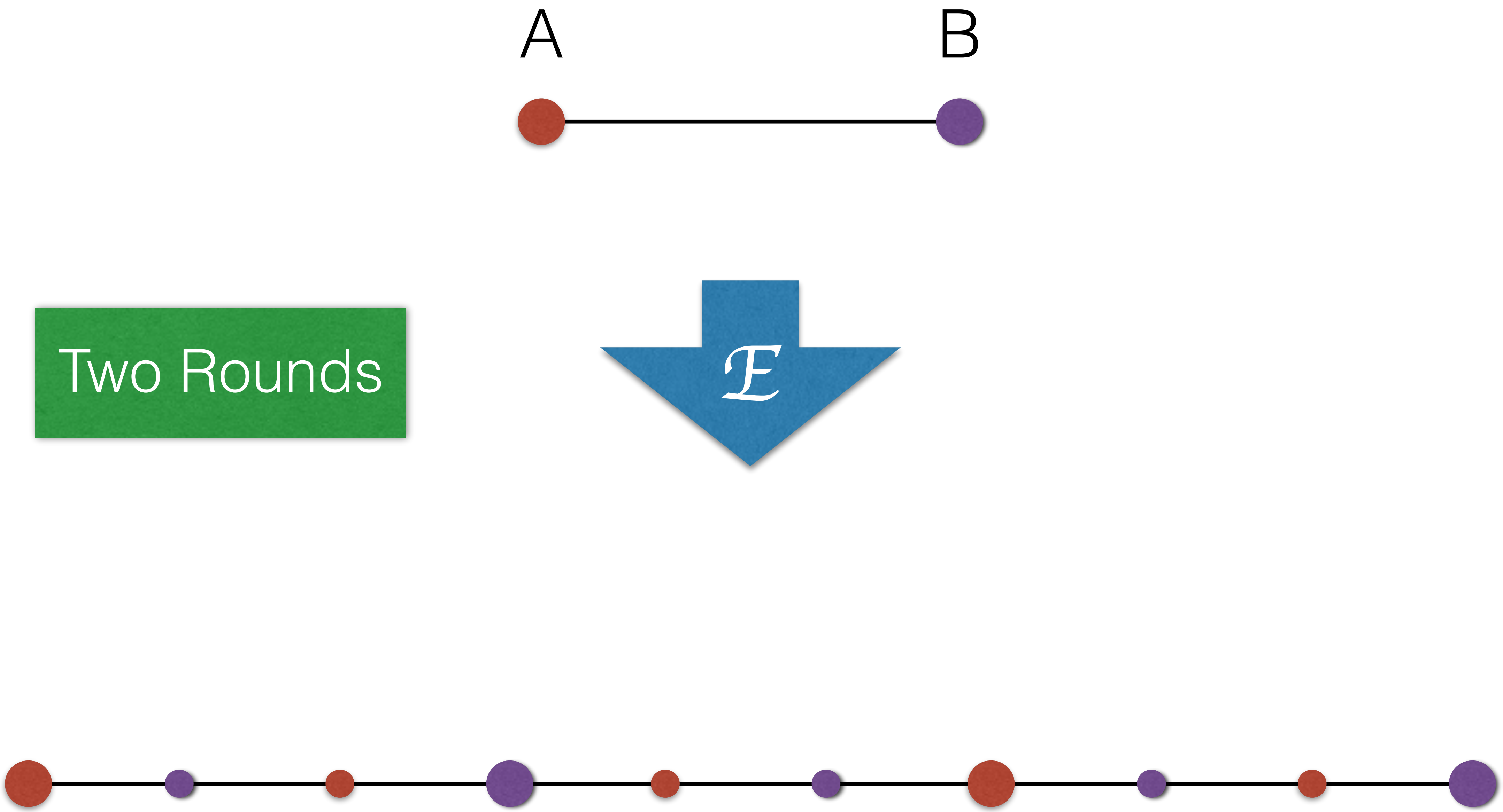

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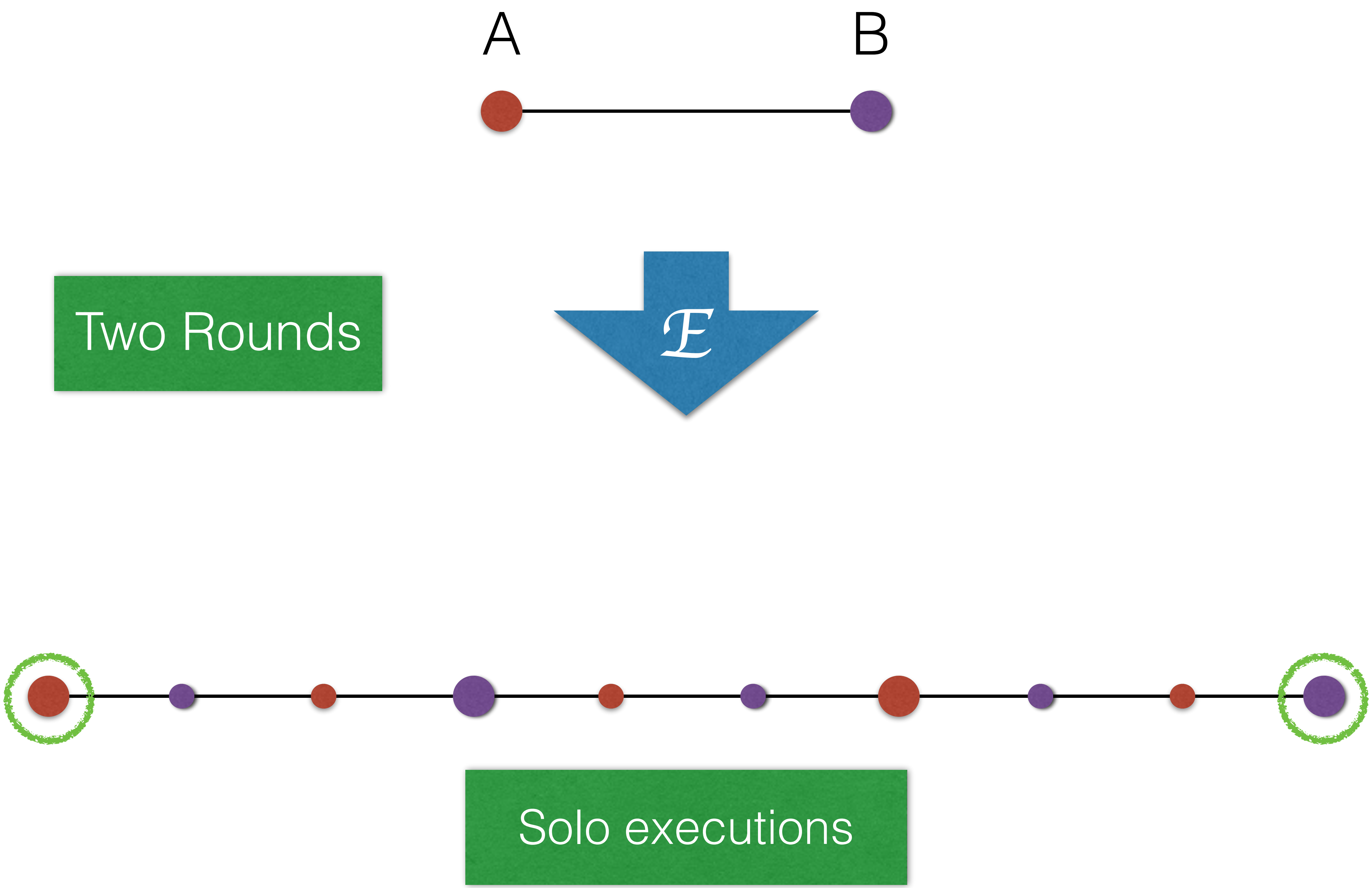
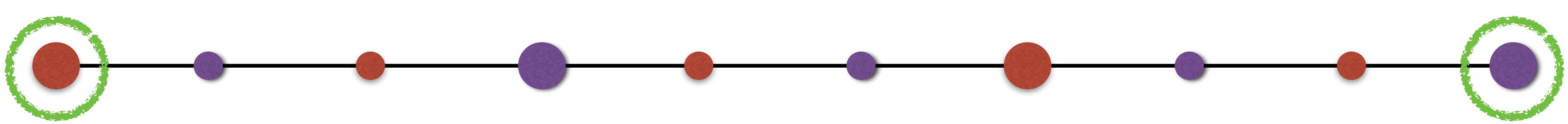


2-Robot Protocol

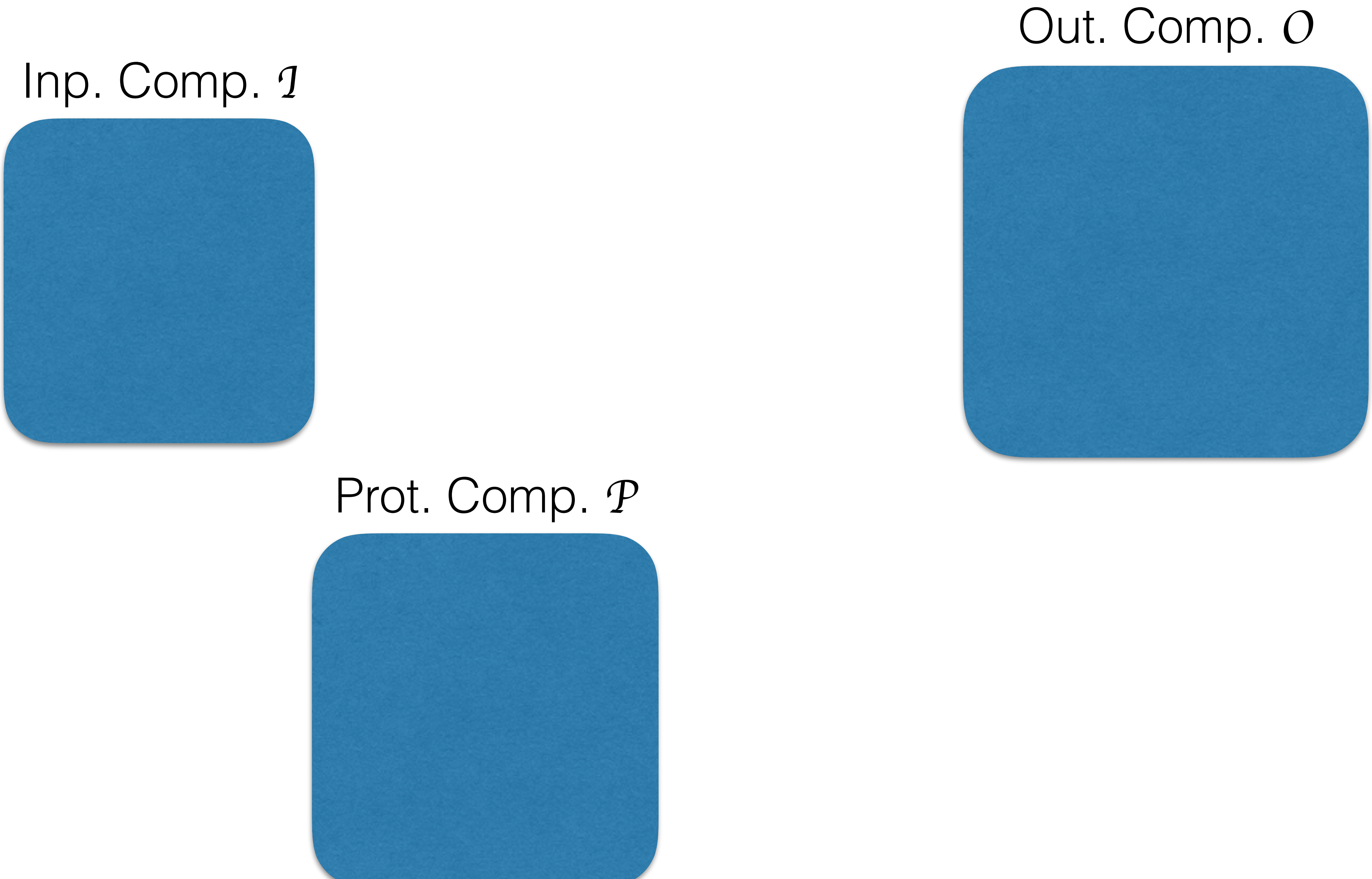
Two Rounds

And so on …



Solo executions

Solvability Condition



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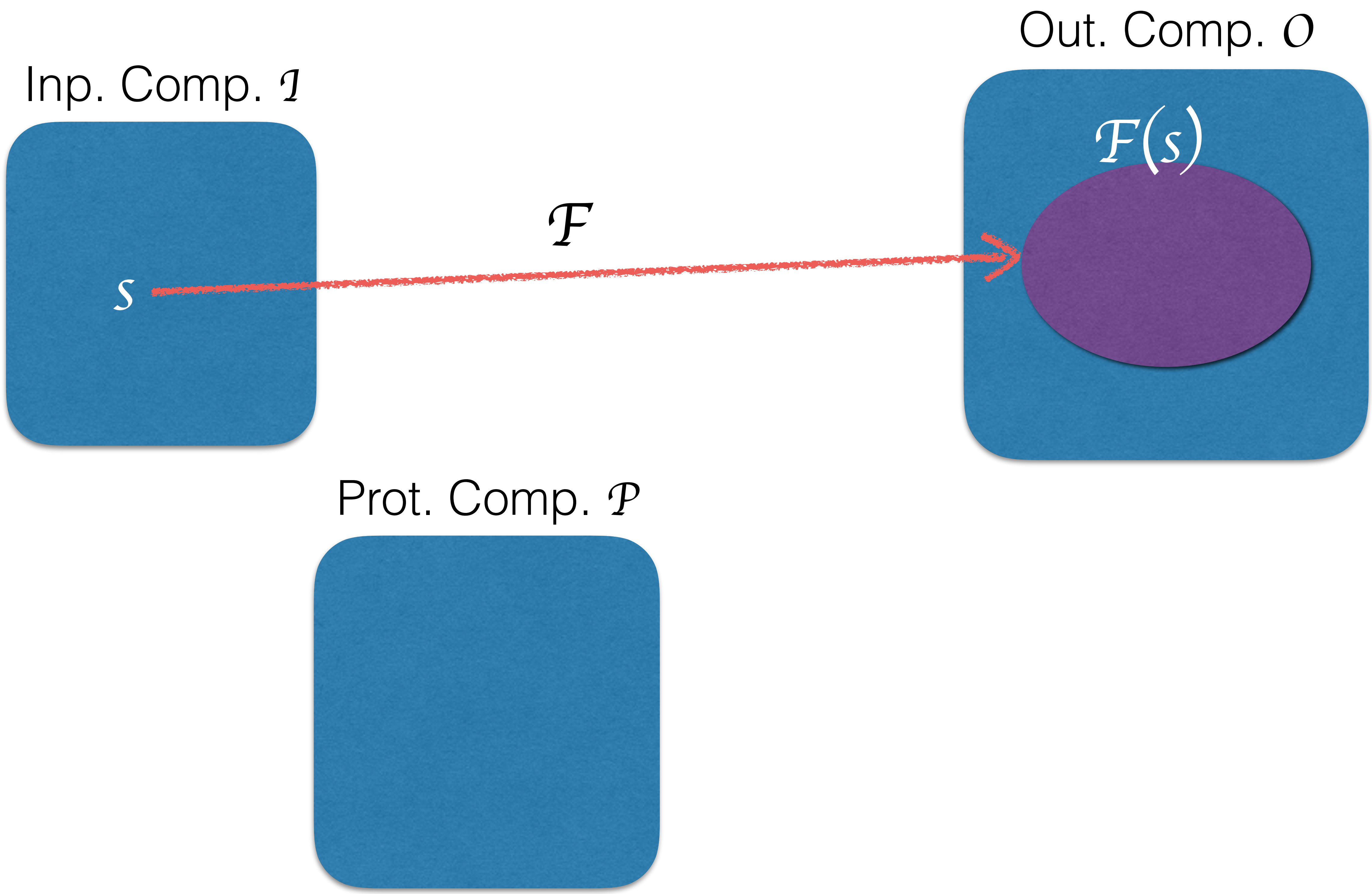

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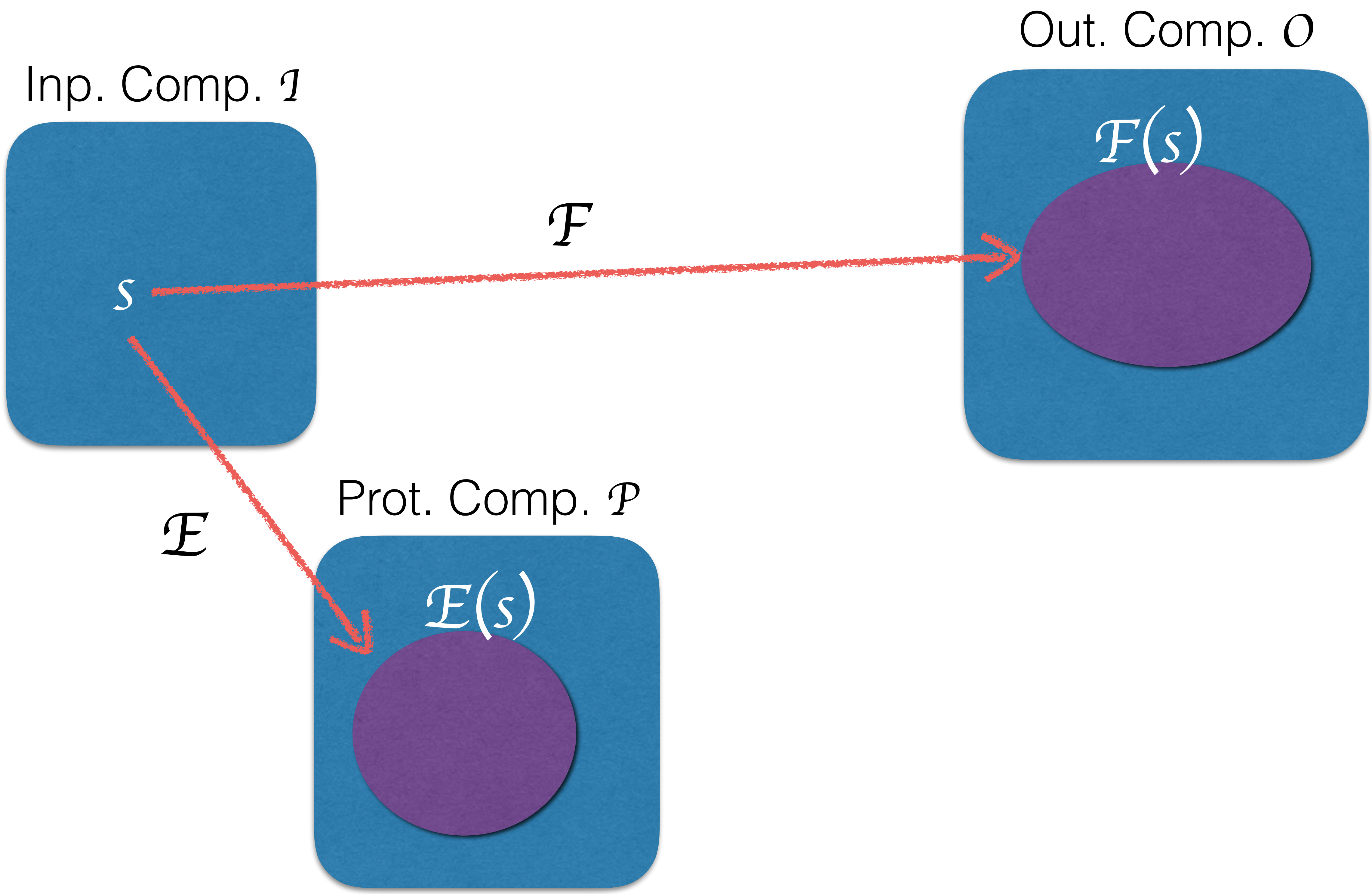

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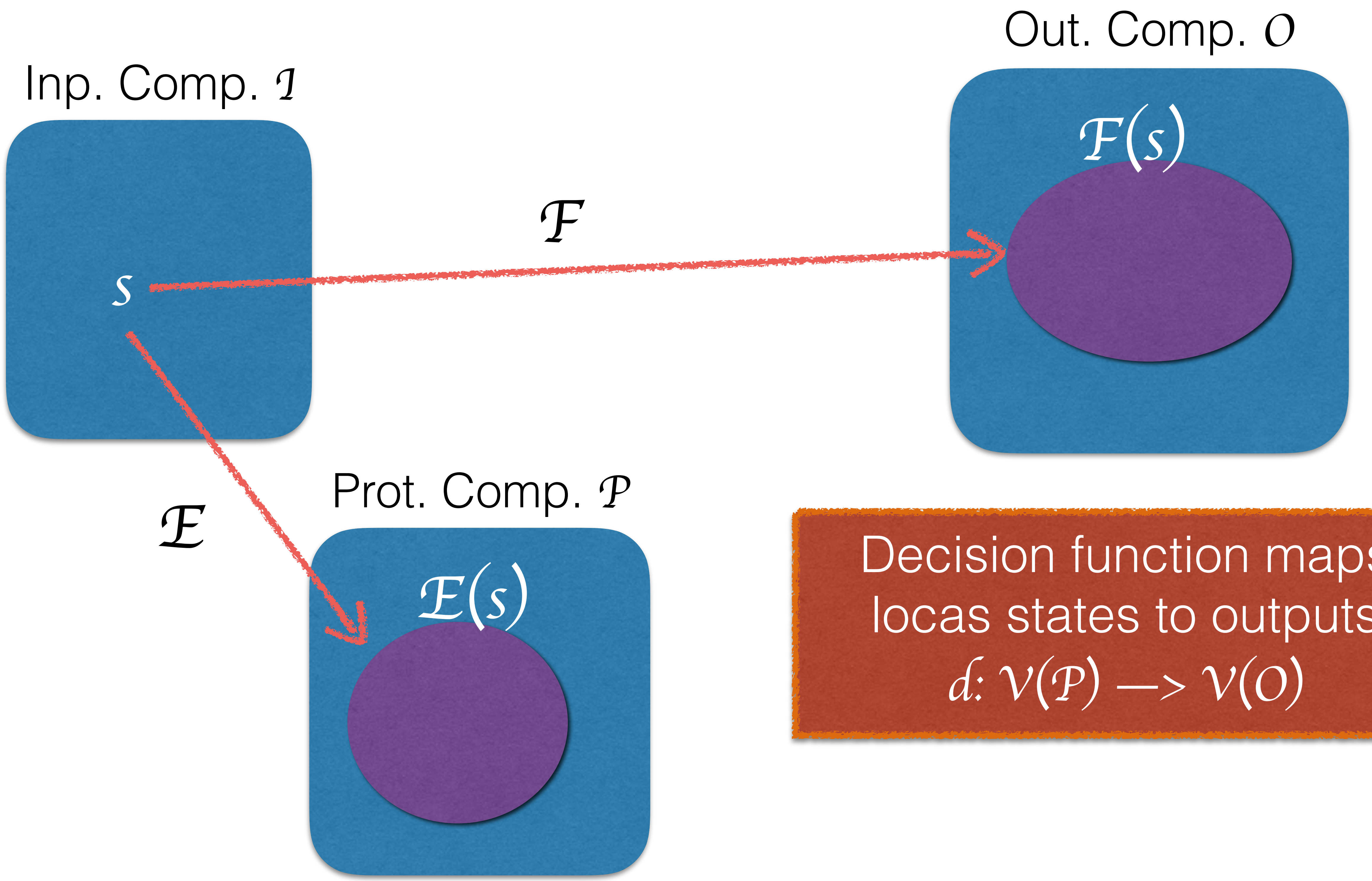
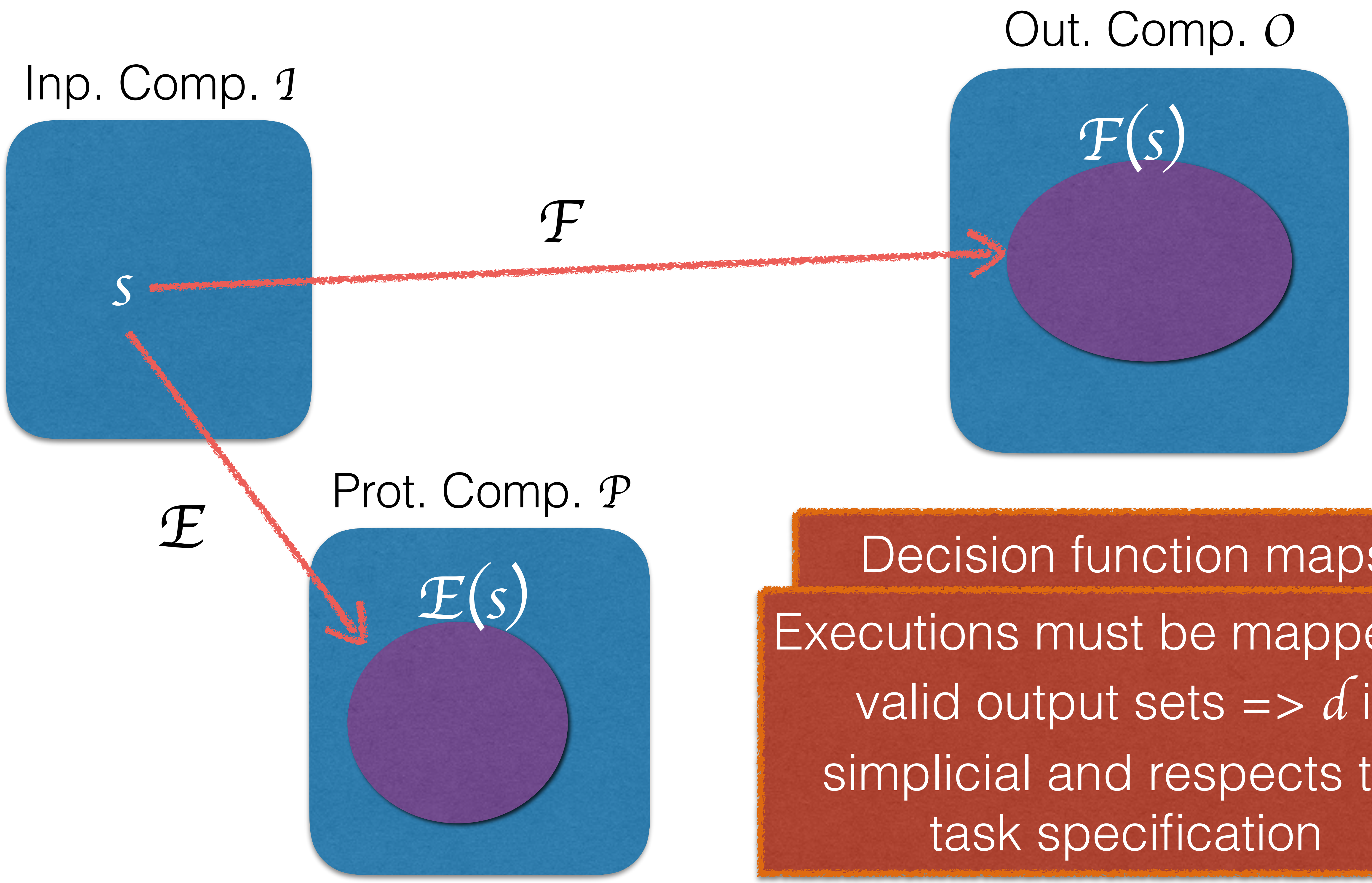


Decision function maps
locas states to outputs
$d: \mathcal{V}(\mathcal{P}) \text{—>} \mathcal{V}(\mathcal{O})$

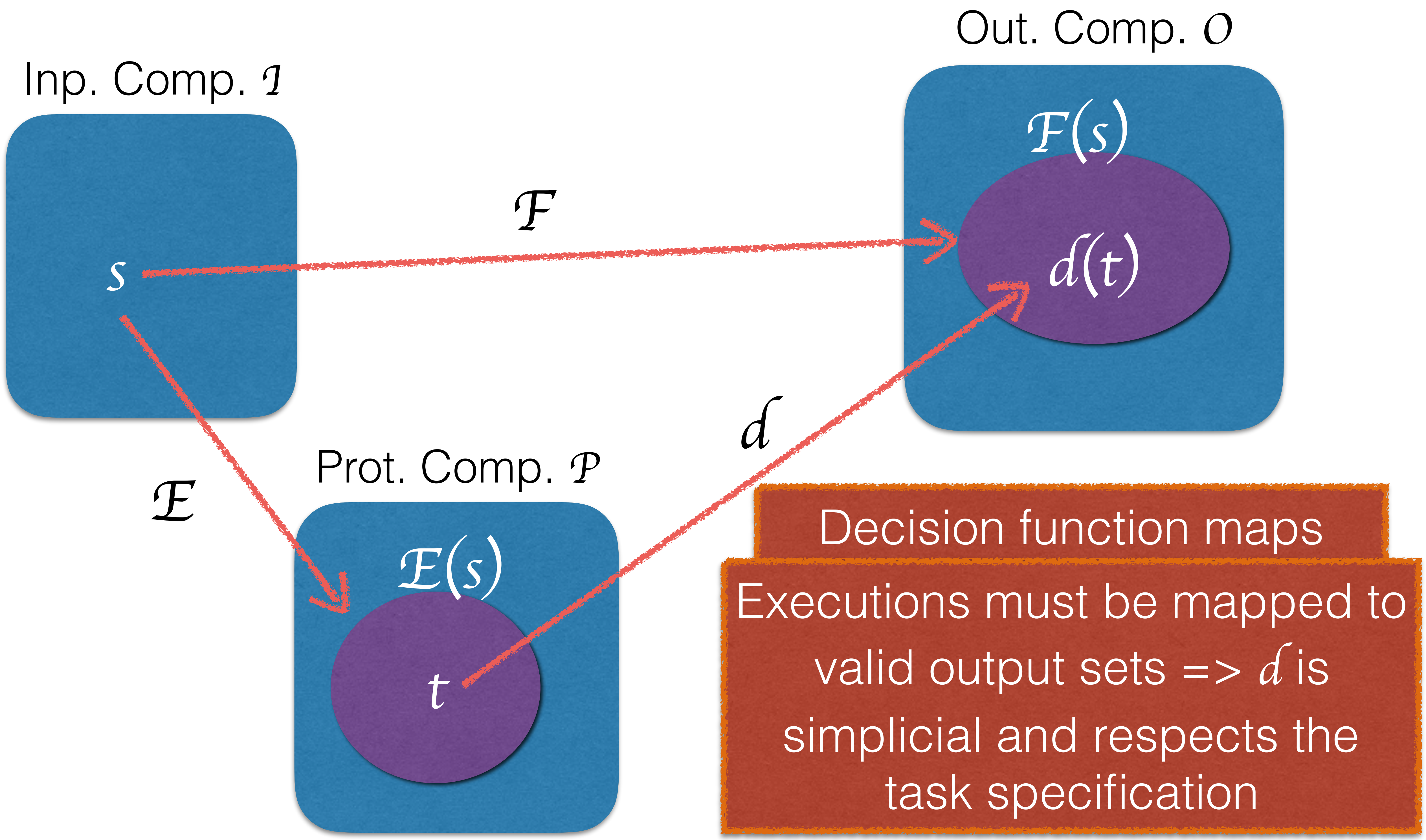
Solvability Condition



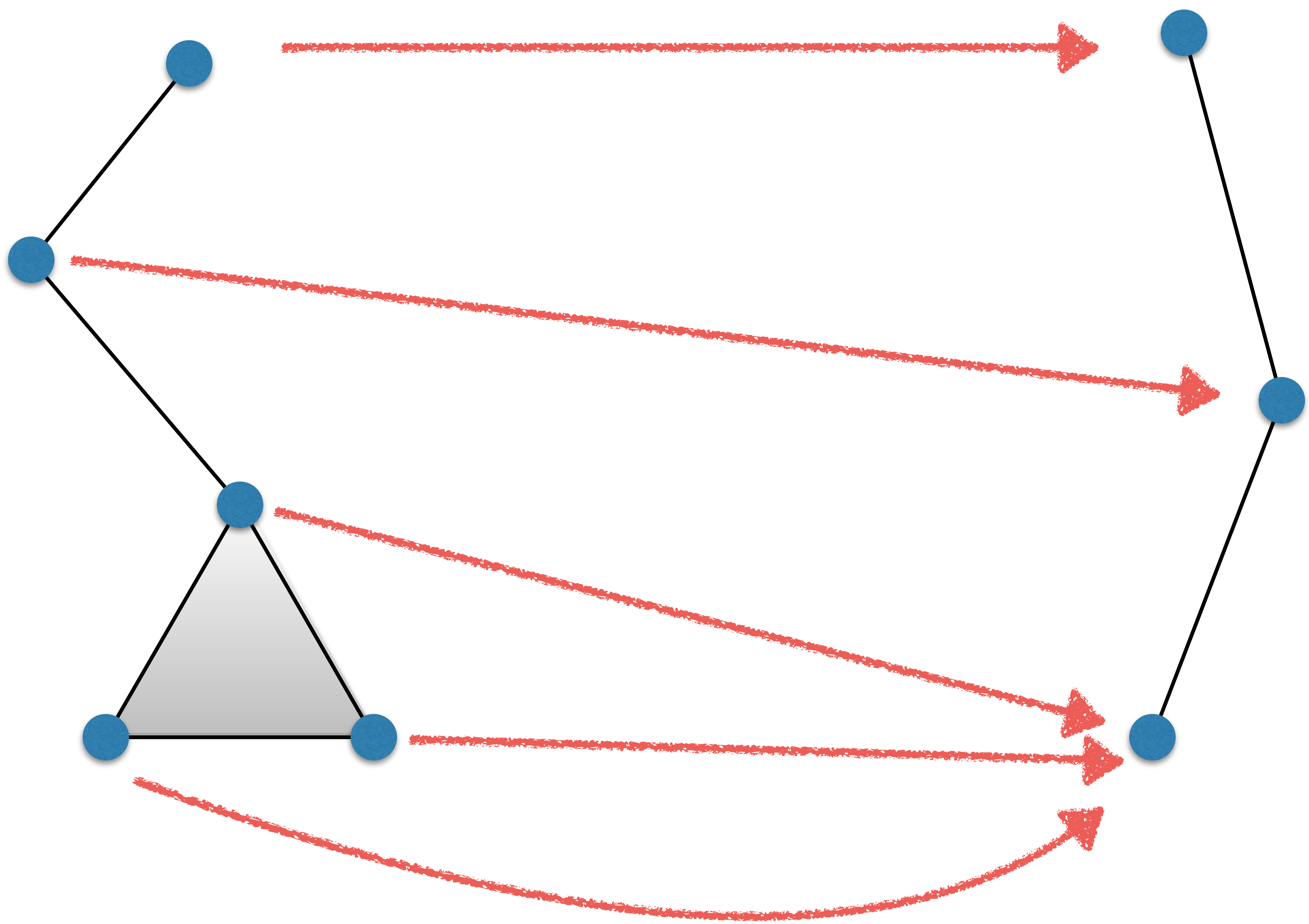
Decision function maps

Executions must be mapped to valid output sets => d is simplicial and respects the task specification

Solvability Condition



Simplicial Maps



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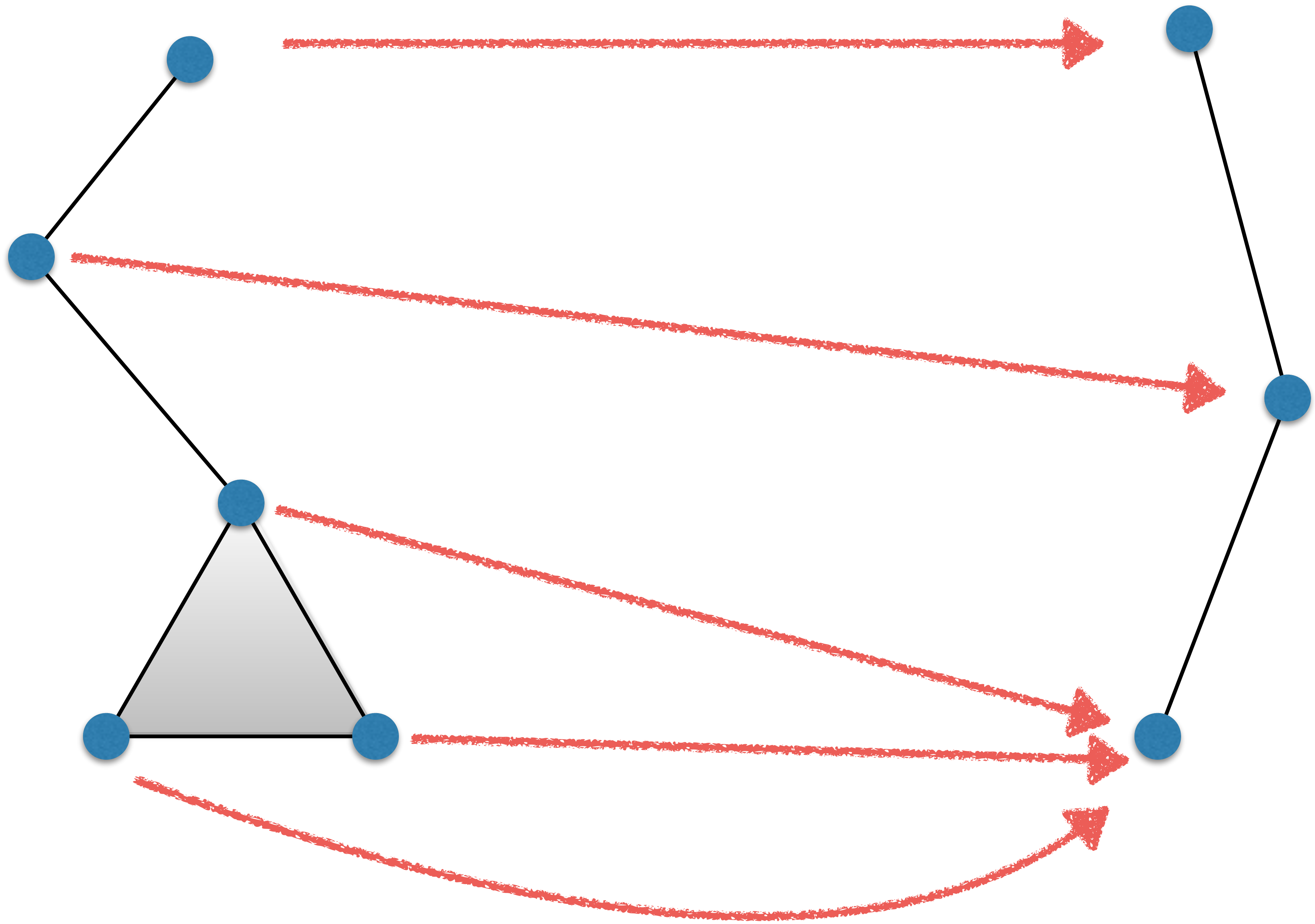

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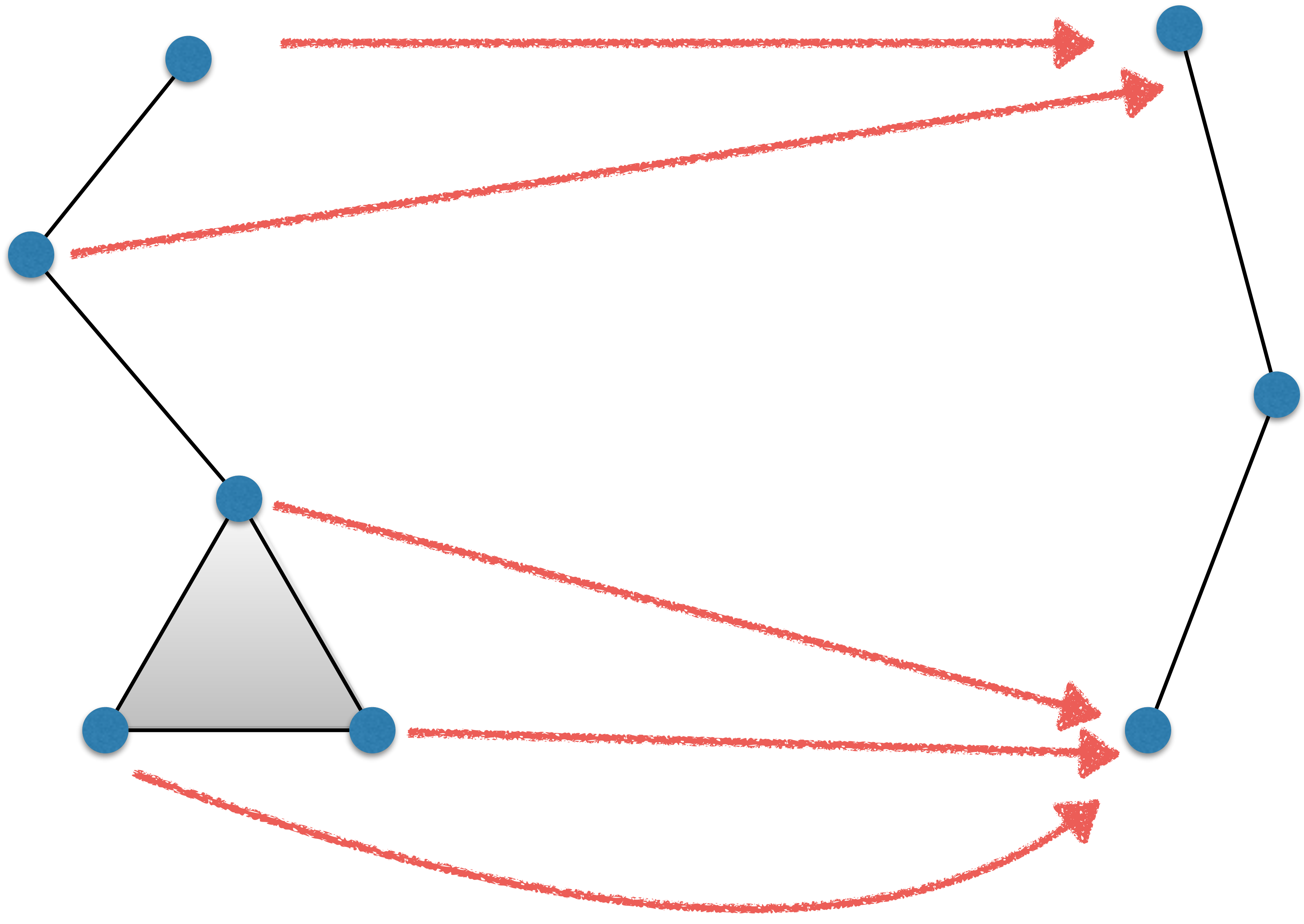

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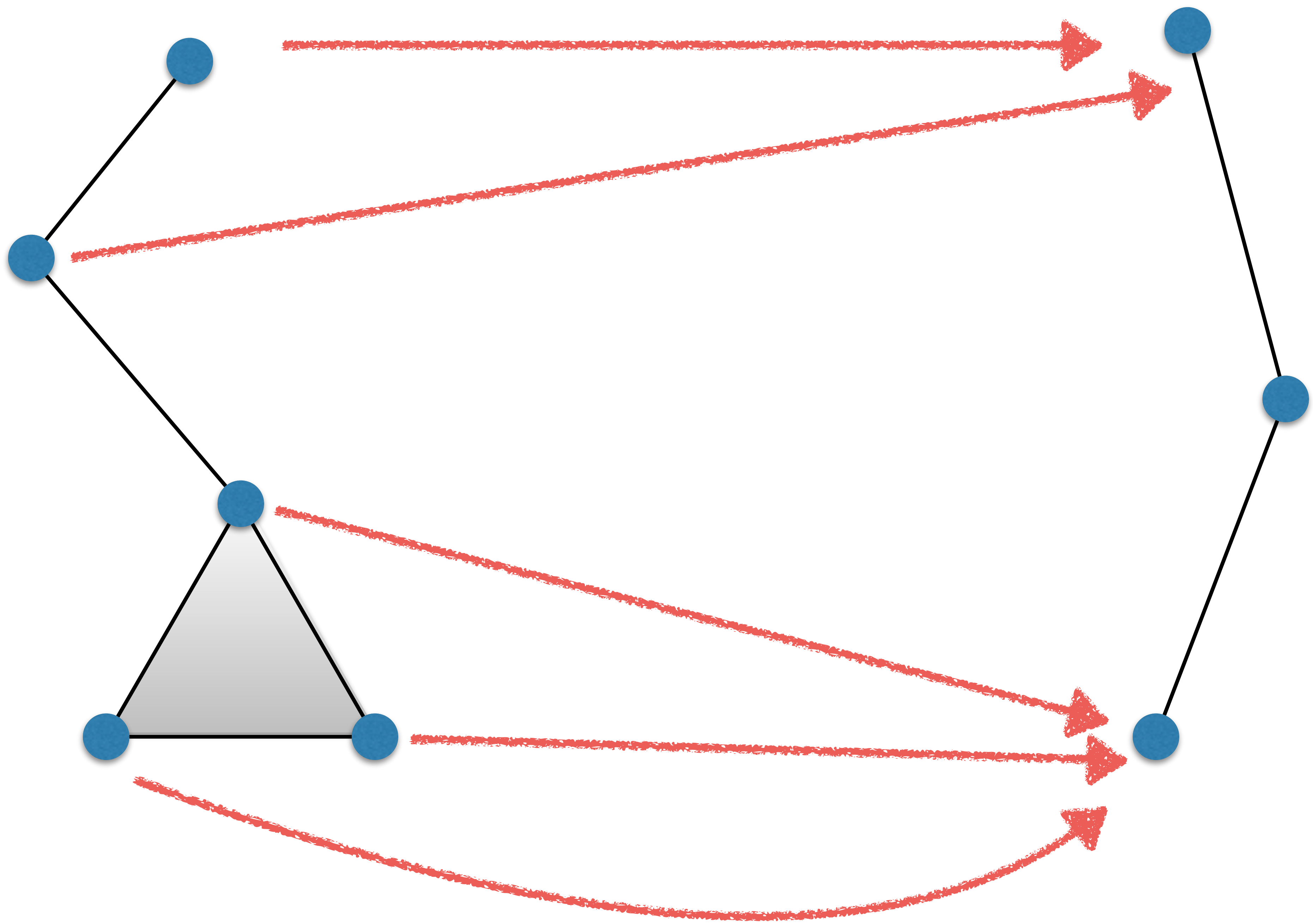

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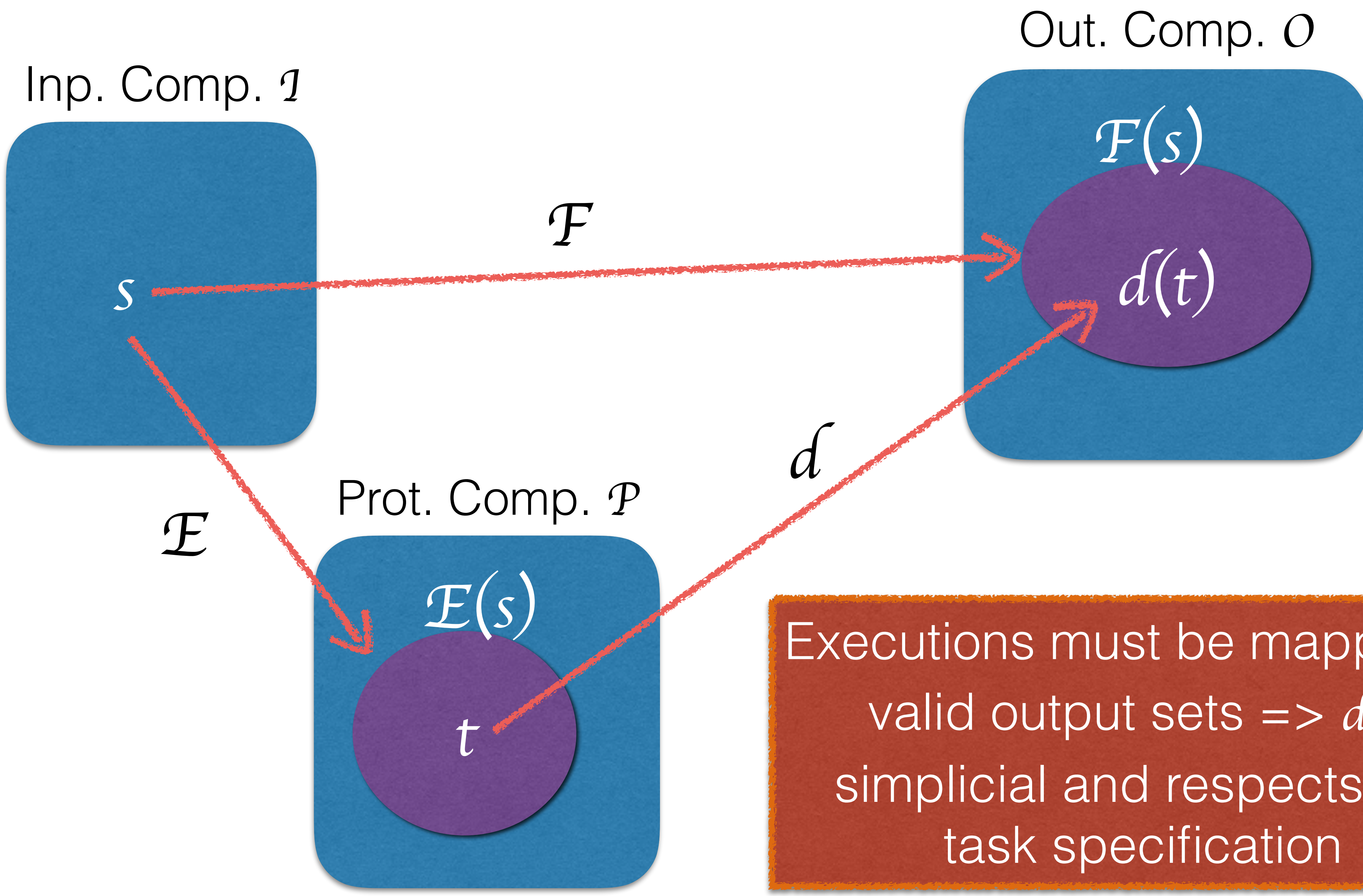

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Gathering is Impossible

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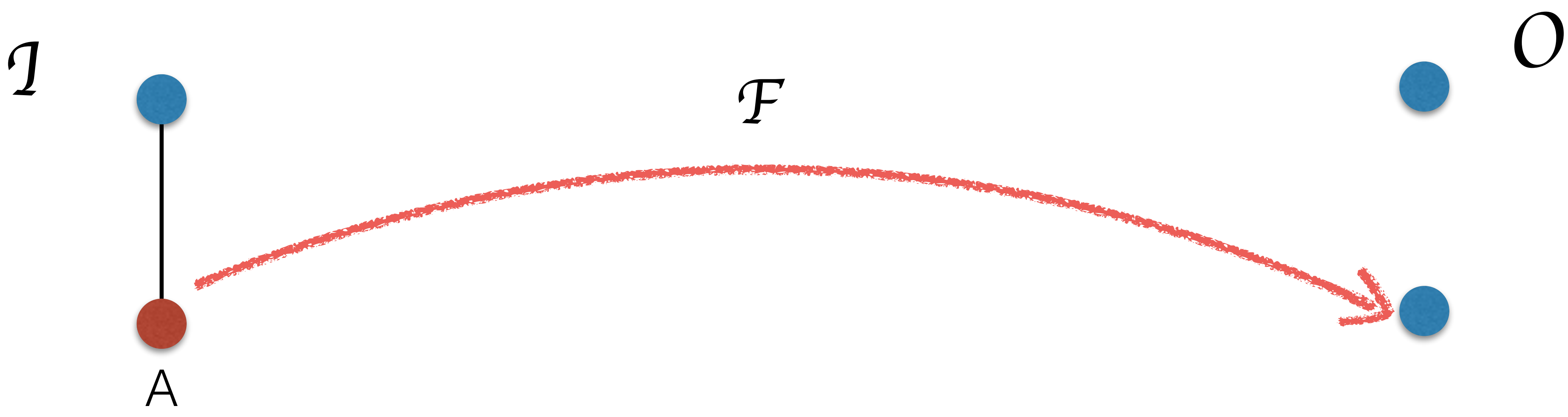

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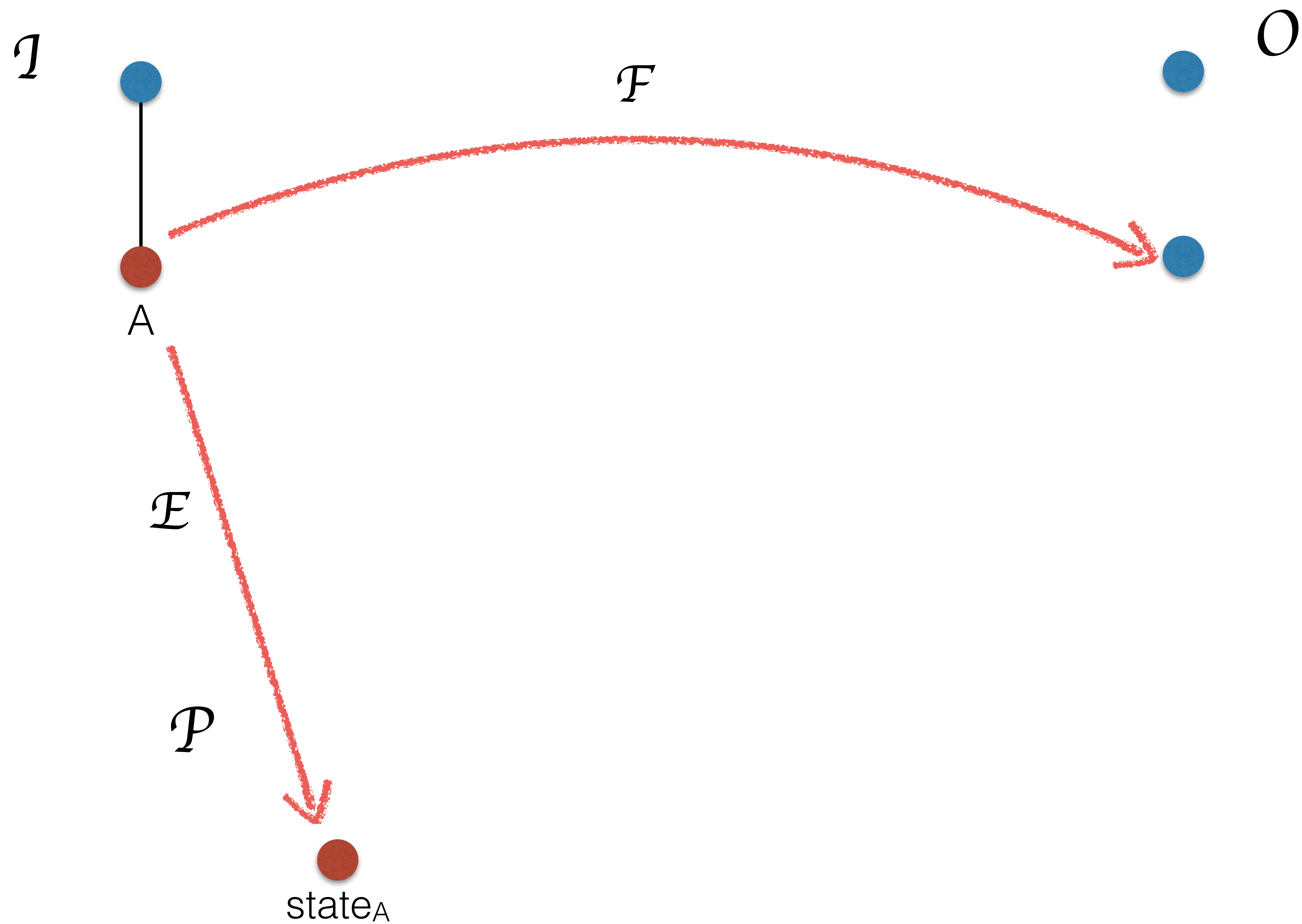

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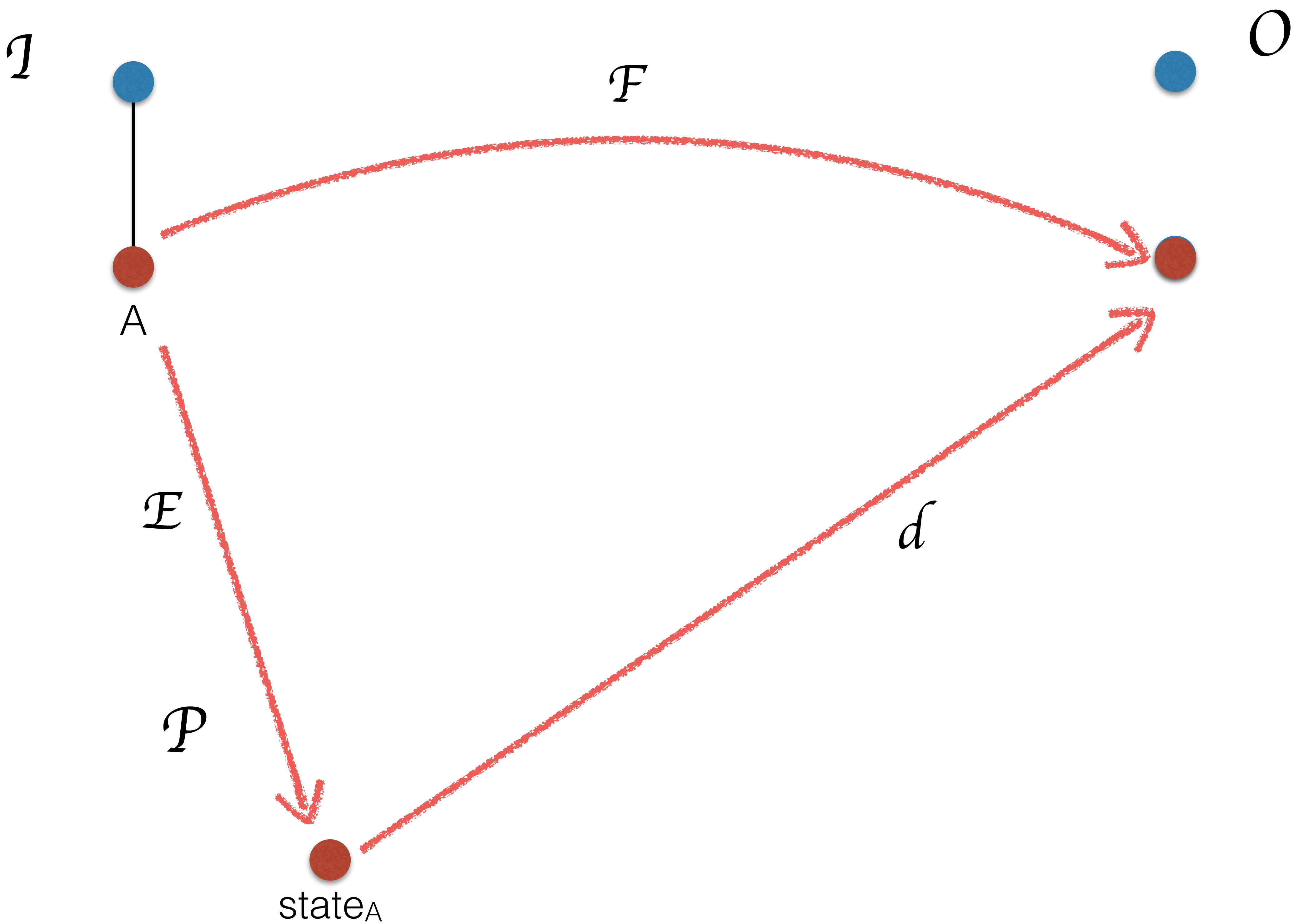

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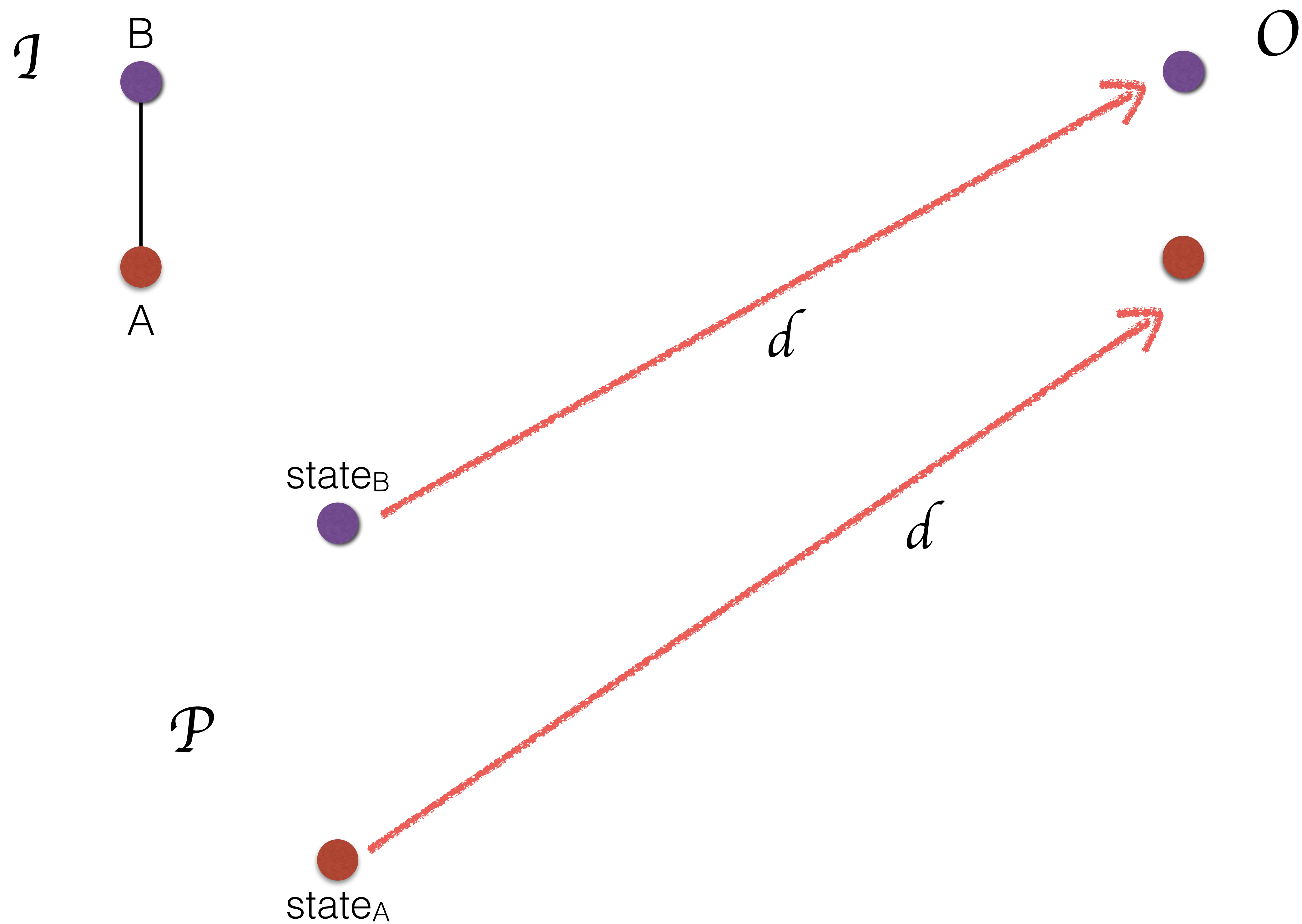

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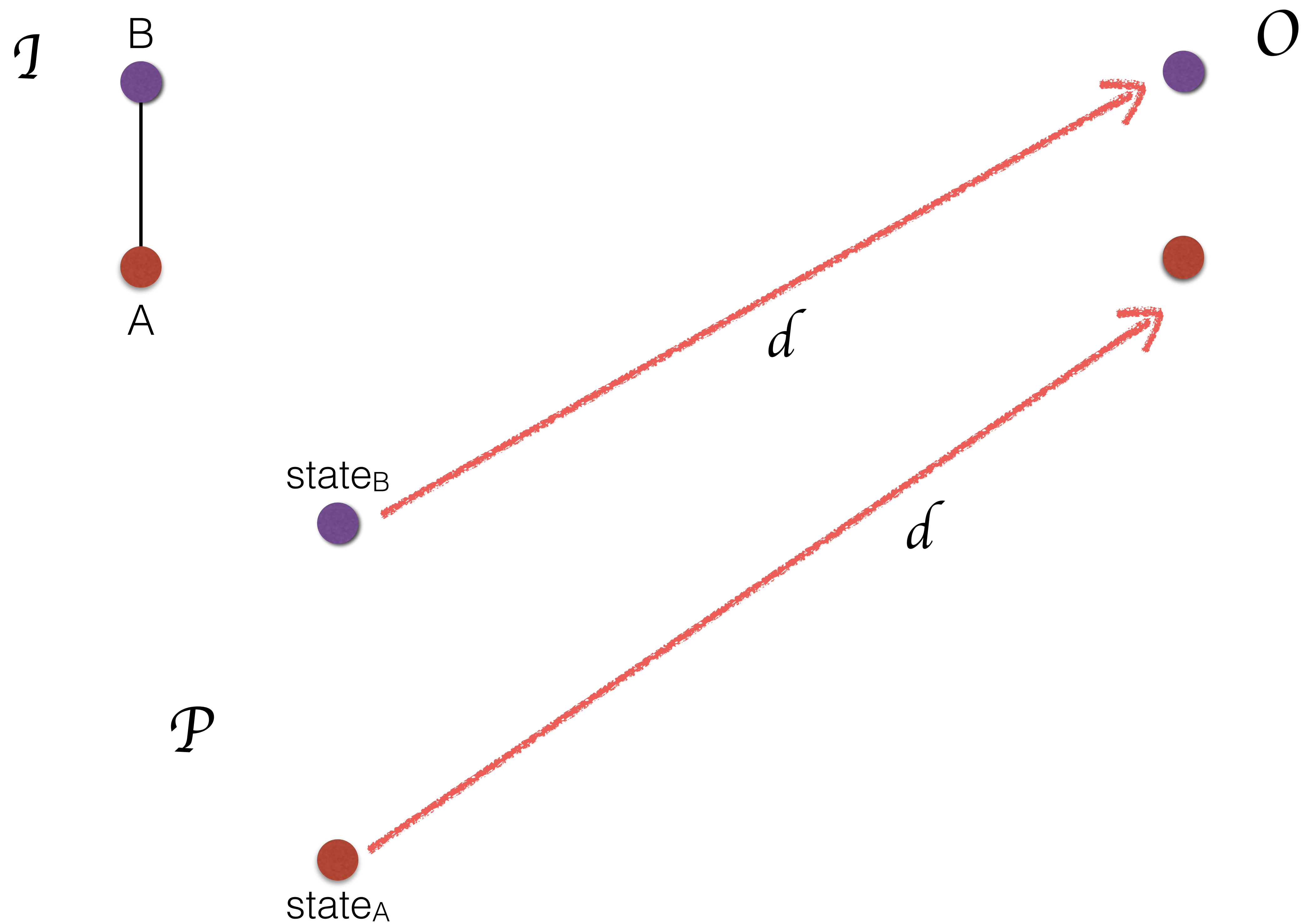

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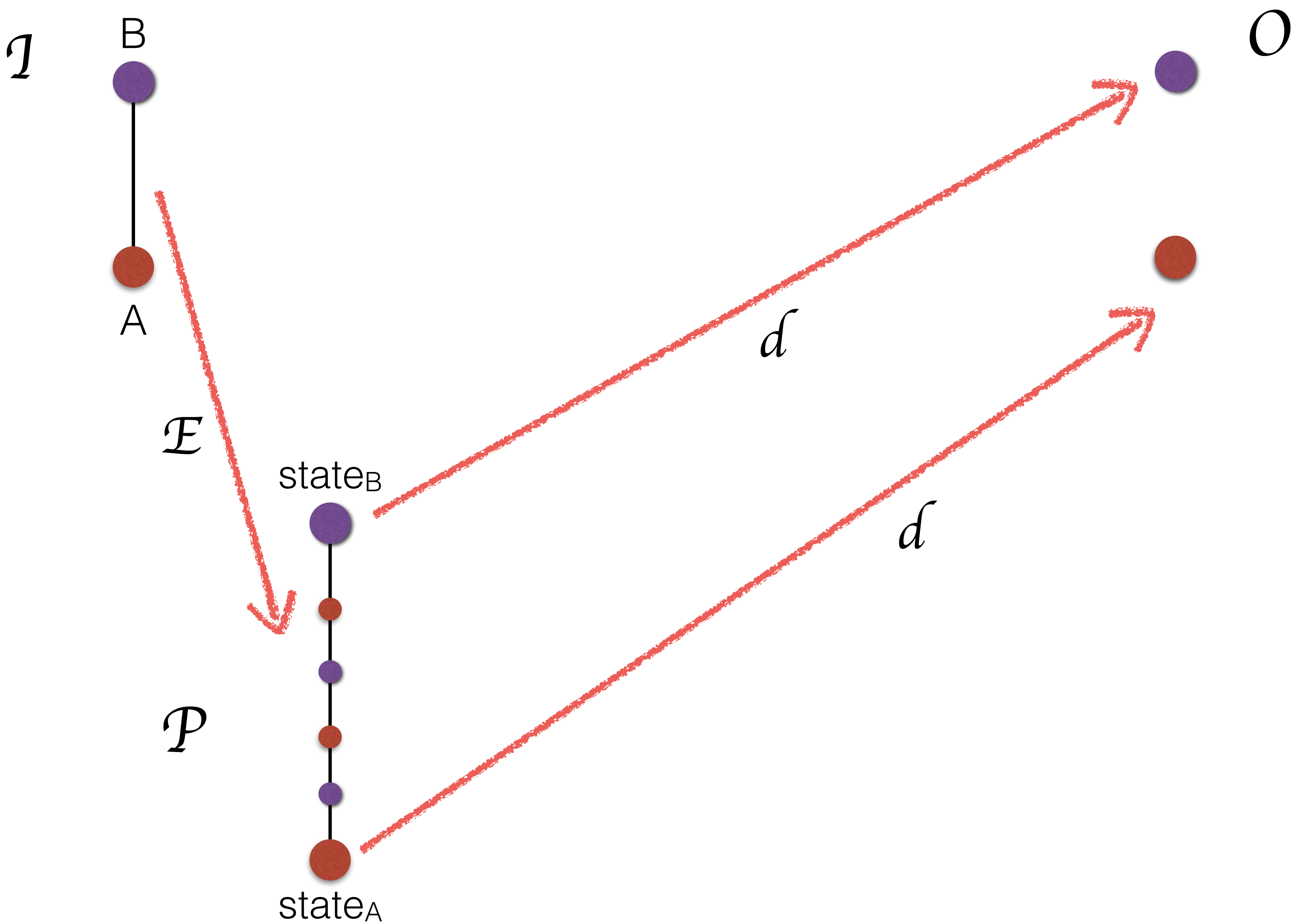

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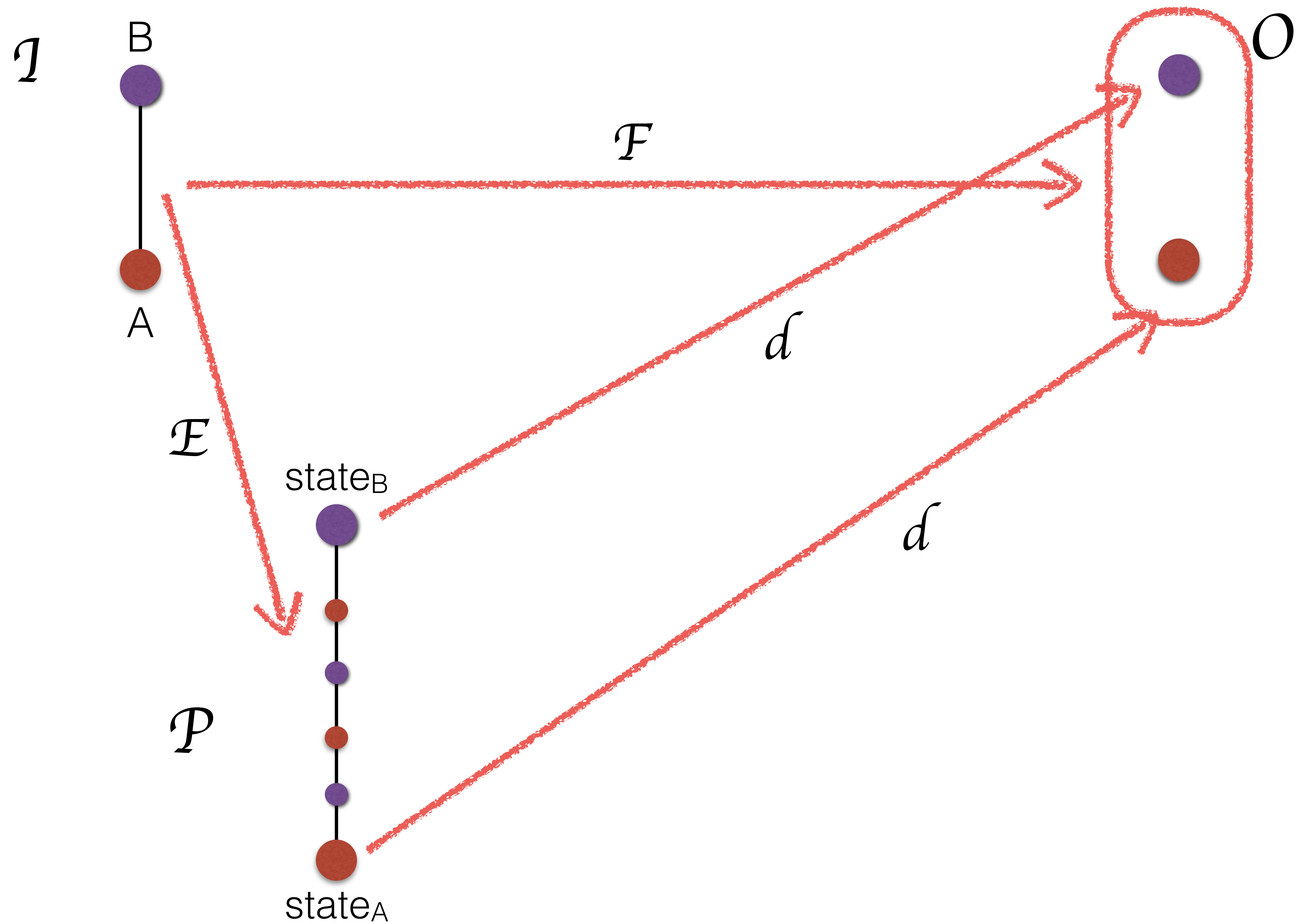

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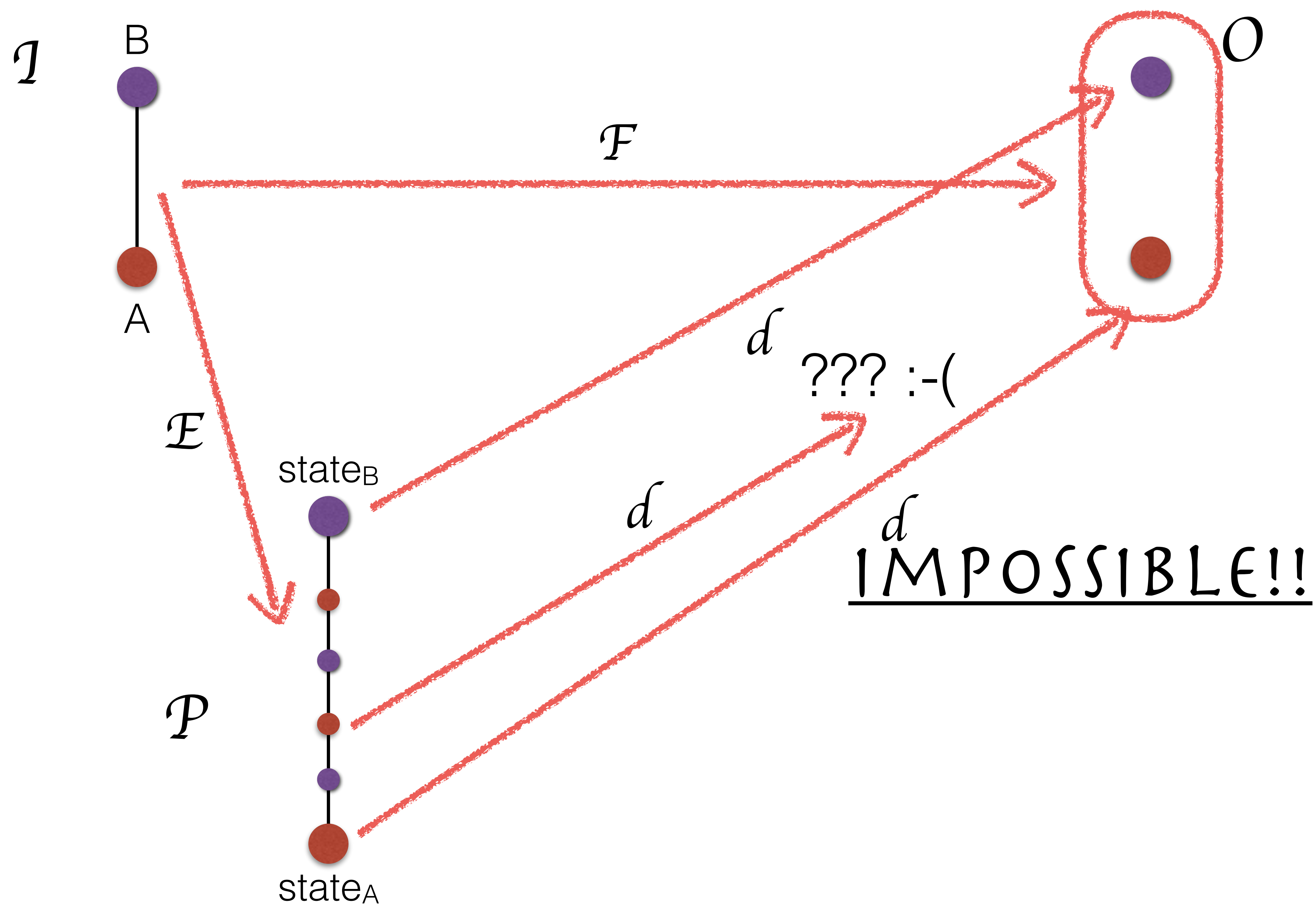

So … Relax

- What about gathering on an edge?
- **Edge Gathering**:
  - **Termination.** Correct robots decide a vertex.
  - **Validity.** If participating robots start on the same vertex, they stay there. If start on an edge, decide vertices of the edge.
  - **Edge Agreement.** Decided vertices belong to an edge (it could be the same vertex).

Solvability of Edge Gathering

For $n = 2$, edge gathering is solvable on any connected base graph $\mathcal{G}$

For $n > 2$, edge gathering is solvable if and only if the base graph $\mathcal{G}$ is acyclic

Edge Gathering on Trees

Algorithm 1 Edge Gathering for $N \geq 2$ robots on any tree $T = (V, E)$. Code for robot p_i.

Function GatheringTree(v_i, T)

Move$(v_i, 0)$ % p_i becomes visible to the others
for $r_i \leftarrow 1$ **to** $diam(T) - 1$ **do**
    $view_i \leftarrow$ **Look**(T) % positions and lights states of the others
    $max\_round_i \leftarrow \max\{r_j : (*, r_j) \in view_i\}$
    $S_i \leftarrow \{v_j : (v_j, max\_round_i) \in view_i \vee v_j = v_i\}$ % max round position and position of p_i
    $T_i \leftarrow$ smallest subtree of T spanning all vertices in S_i % subtree induced by positions in S_i
    if v_i is leaf of $T_i \wedge diam(T_i) > 0$ **then**
        $v_i \leftarrow$ vertex of T_i that is adjacent to v_i
    end if
    Move(v_i, r_i) % p_i makes visible its new position and updates its lights
end for
return v_i

Edge Gathering on Trees

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The farthest two robots might be

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Edge Gathering on Trees

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Function GatheringTree(v_i, T)

```
Move(v_i, 0) % p_i becomes visible to the others
for r_i ← 1 to diam(T) − 1 do
    view_i ← Look(T) % positions and lights states of the others
    max_round_i ← max{r_j : (*, r_j) ∈ view_i}
    S_i ← {v_j : (v_j, max_round_i) ∈ view_i ∨ v_j = v_i} % max round p
    T_i ← smallest subtree of T spanning all vertices in S_i % subtree
    if v_i is leaf of T_i ∧ diam(T_i) > 0 then
        v_i ← vertex of T_i that is adjacent to v_i
    end if
    Move(v_i, r_i) % p_i makes visible its new position and updates its lights
end for
return v_i
```

Positions of leaders and mine

Edge Gathering on Trees

Algorithm 1 Edge Gathering for $N \geq 2$ robots on any tree $T = (V, E)$. Code for robot p_i.

Function GatheringTree(v_i, T)

Move$(v_i, 0)$ % p_i becomes visible to the others
for $r_i \leftarrow 1$ **to** $diam(T) - 1$ **do**
    $view_i \leftarrow$ **Look**(T) % positions and lights states of the others
    $max\_round_i \leftarrow \max\{r_j : (*, r_j) \in view_i\}$
    $S_i \leftarrow \{v_j : (v_j, max\_round_i) \in view_i \vee v_j = v_i\}$ % max round pos
    $T_i \leftarrow$ smallest subtree of T spanning all vertices in S_i ~~% subtree~~ in
    if v_i is leaf of $T_i \wedge diam(T_i) > 0$ **then**
        $v_i \leftarrow$ vertex of T_i that is adjacent to v_i
    end if
    Move(v_i, r_i) % p_i makes visible its new position and updates its lights
end for
return v_i

Tree with leaders and me

Edge Gathering on Trees

Algorithm 1 Edge Gathering for $N \geq 2$ robots on any tree $T = (V, E)$. Code for robot p_i.

Function GatheringTree(v_i, T)

```
Move(v_i, 0) % p_i becomes visible to the others
for r_i ← 1 to diam(T) − 1 do
    view_i ← Look(T) % positions and lights states of the others
    max_round_i ← max{r_j : (*, r_j) ∈ view_i}
    S_i ← {v_j : (v_j, max_round_i) ∈ view_i ∨ v_j = v_i} % max round p
    T_i ← smallest subtree of T spanning all vertices in S_i % subtre
    if v_i is leaf of T_i ∧ diam(T_i) > 0 then
        v_i ← vertex of T_i that is adjacent to v_i
    end if
    Move(v_i, r_i) % p_i makes visible its new position and updates
end for
return v_i
```

Need to move?

Edge Gathering on Trees

Algorithm 1 Edge Gathering for $N \geq 2$ robots on any tree $T = (V, E)$. Code for robot p_i.

```
Function GatheringTree(v_i, T)
Move(v_i, 0) % p_i becomes visible to the others
for r_i ← 1 to diam(T) − 1 do
    view_i ← Look(T) % positions and lights states of the others
    max_round_i ← max{r_j : (*, r_j) ∈ view_i}
    S_i ← {v_j : (v_j, max_round_i) ∈ view_i ∨ v_j = v_i} % max round position and position of p_i
    T_i ← smallest subtree of T spanning all vertices in S_i % subtree induced by positions in S_i
    if v_i is leaf of T_i ∧ diam(T_i) > 0 then
        v_i ← vertex of T_i that is adjacent to v_i
    end if
    Move(v_i, r_i) ~~% p_i makes visible its new~~ p ... ats
end for
return v_i
```

Move and update light

Edge Gathering on Trees

Algorithm 1 Edge Gathering for $N \geq 2$ robots on any tree $T = (V, E)$. Code for robot p_i.

Function GatheringTree(v_i, T)

Move$(v_i, 0)$ % p_i becomes visible to the others
for $r_i \leftarrow 1$ **to** $diam(T) - 1$ **do**
    $view_i \leftarrow$ **Look**(T) % positions and lights states of the others
    $max\_round_i \leftarrow \max\{r_j : (*, r_j) \in view_i\}$
    $S_i \leftarrow \{v_j : (v_j, max\_round_i) \in view_i \vee v_j = v_i\}$ % max round position and position of p_i
    $T_i \leftarrow$ smallest subtree of T spanning all vertices in S_i % subtree induced by positions in S_i
    if v_i is leaf of $T_i \wedge diam(T_i) > 0$ **then**
        $v_i \leftarrow$ vertex of T_i that is adjacent to v_i
    end if
    Move(v_i, r_i) % p_i makes visible its new position and updates its lights
end for
return v_i

Edge Agreement. For every prefix of an execution:
dist(pos(i), pos(j)) <= diam(T) - min{round(i),round(j)}

2-Robot Edge Gathering

1. **Precompute** a spanning tree $\mathcal{T}$ of $\mathcal{G}$
2. **Algorithm** $\mathcal{A}$: Algorithm for trees.
3. **For** $r=1$ to $diam(\mathcal{G})$ **do**
4.   **Look**($\mathcal{G}$)
5.   **If** distance of current positions on $\mathcal{G}$ > 1 **then**
6.   **Simulate** a round of $\mathcal{A}$ on $\mathcal{T}$
7.   **Move** to next vertex
8. **Return** current position

Cycles are Obstacles

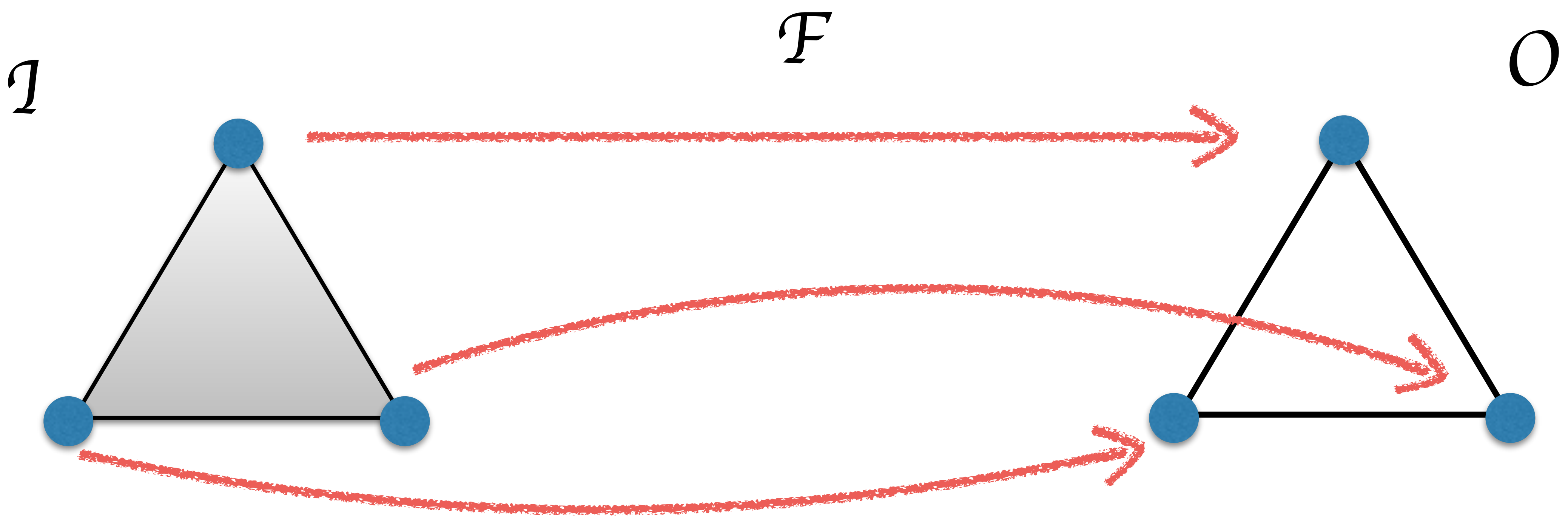
For n > 2, if the base graph $\mathcal{G}$ is has cycles, then edge gathering is unsolvable

Proof:

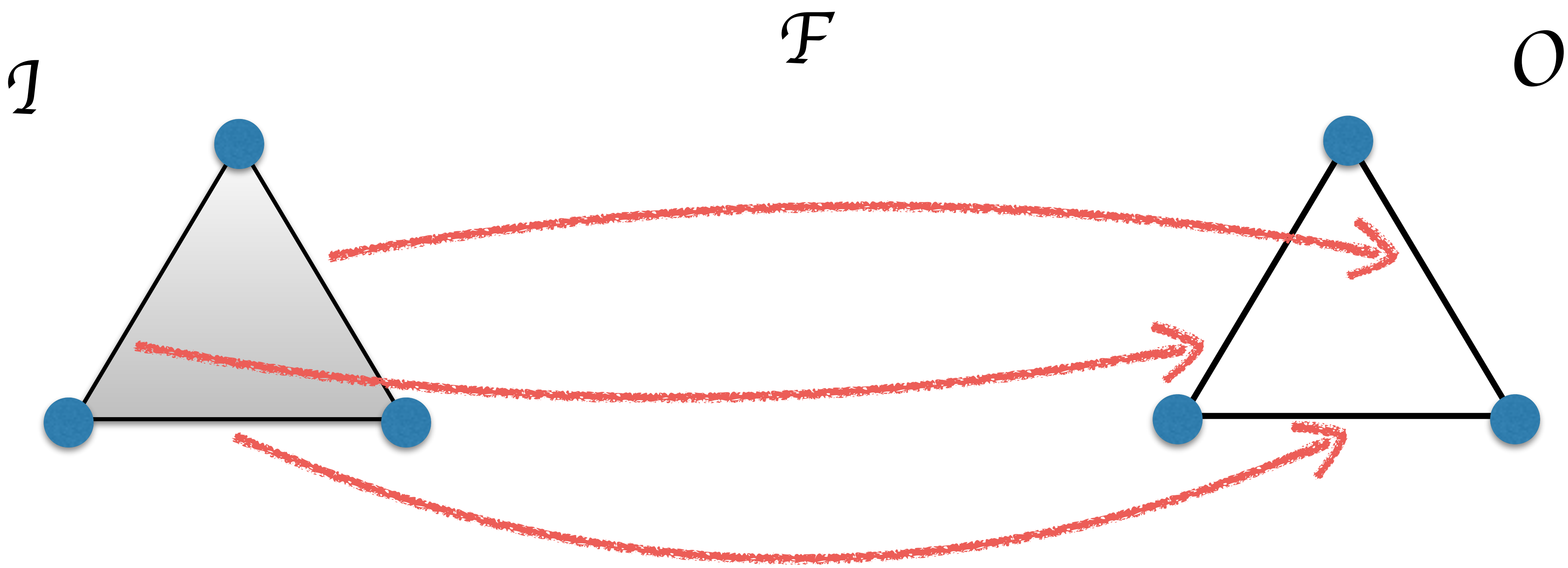
1. The case n= 3 is enough.
2. Prove the triangle is impossible.
3. Solve the triangle from any cyclic graph.

The Triangle is Impossible

The Triangle is Impossible



The Triangle is Impossible



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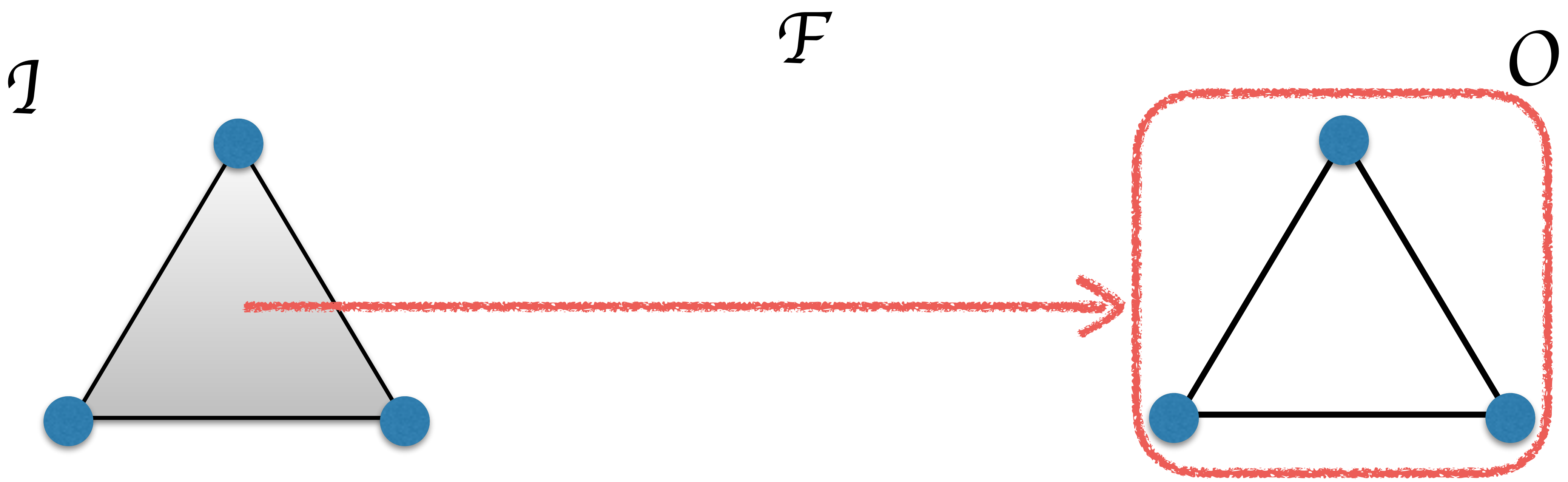

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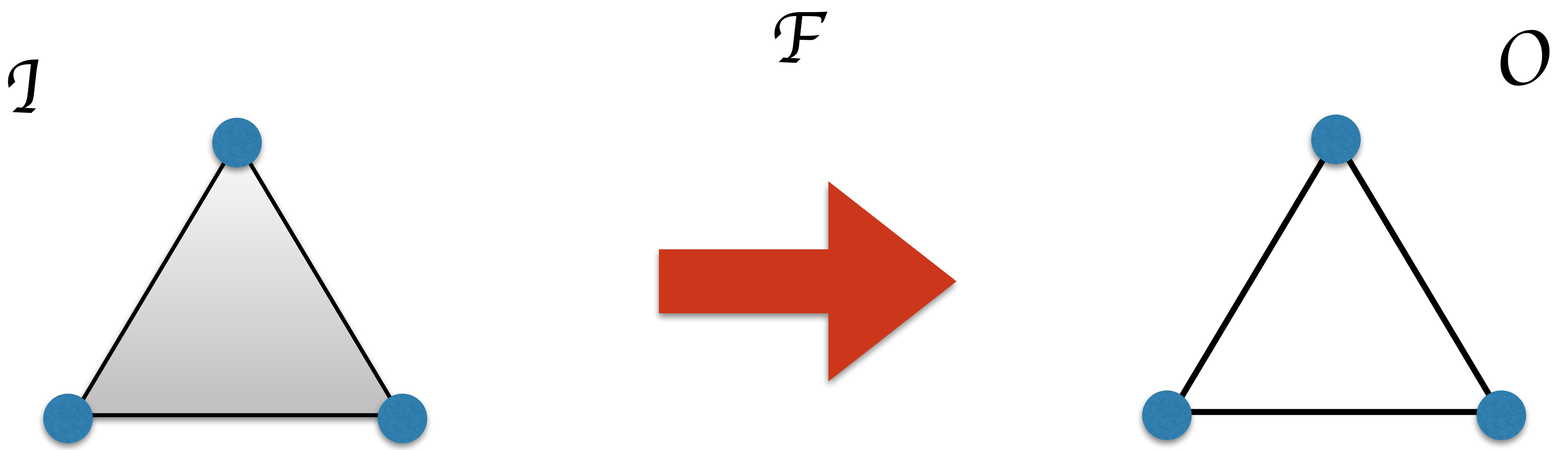

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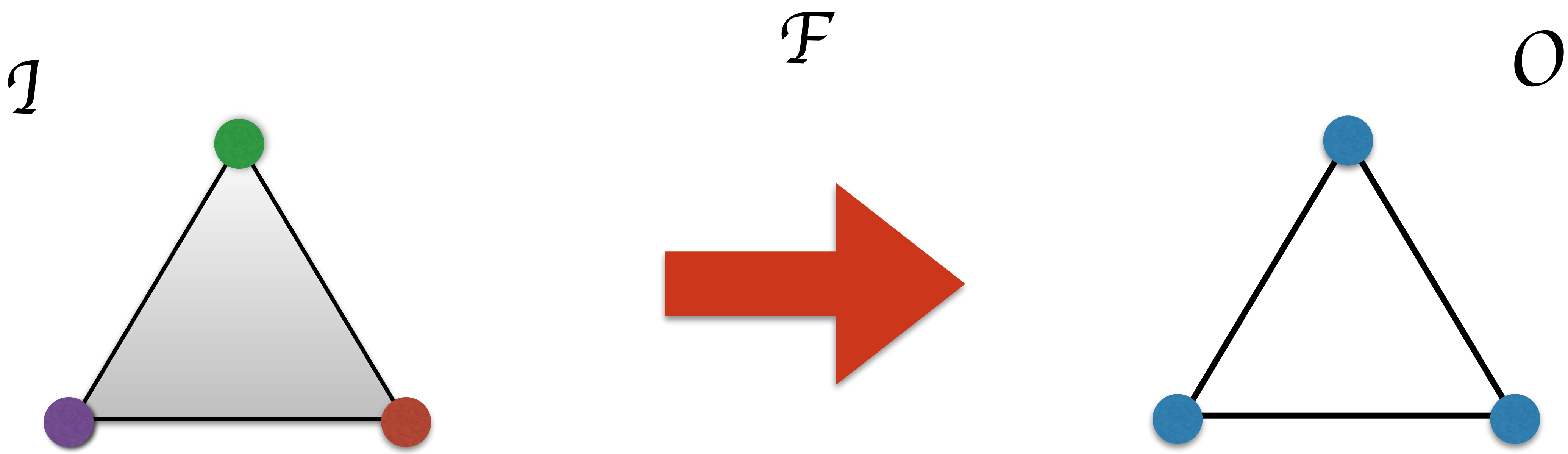

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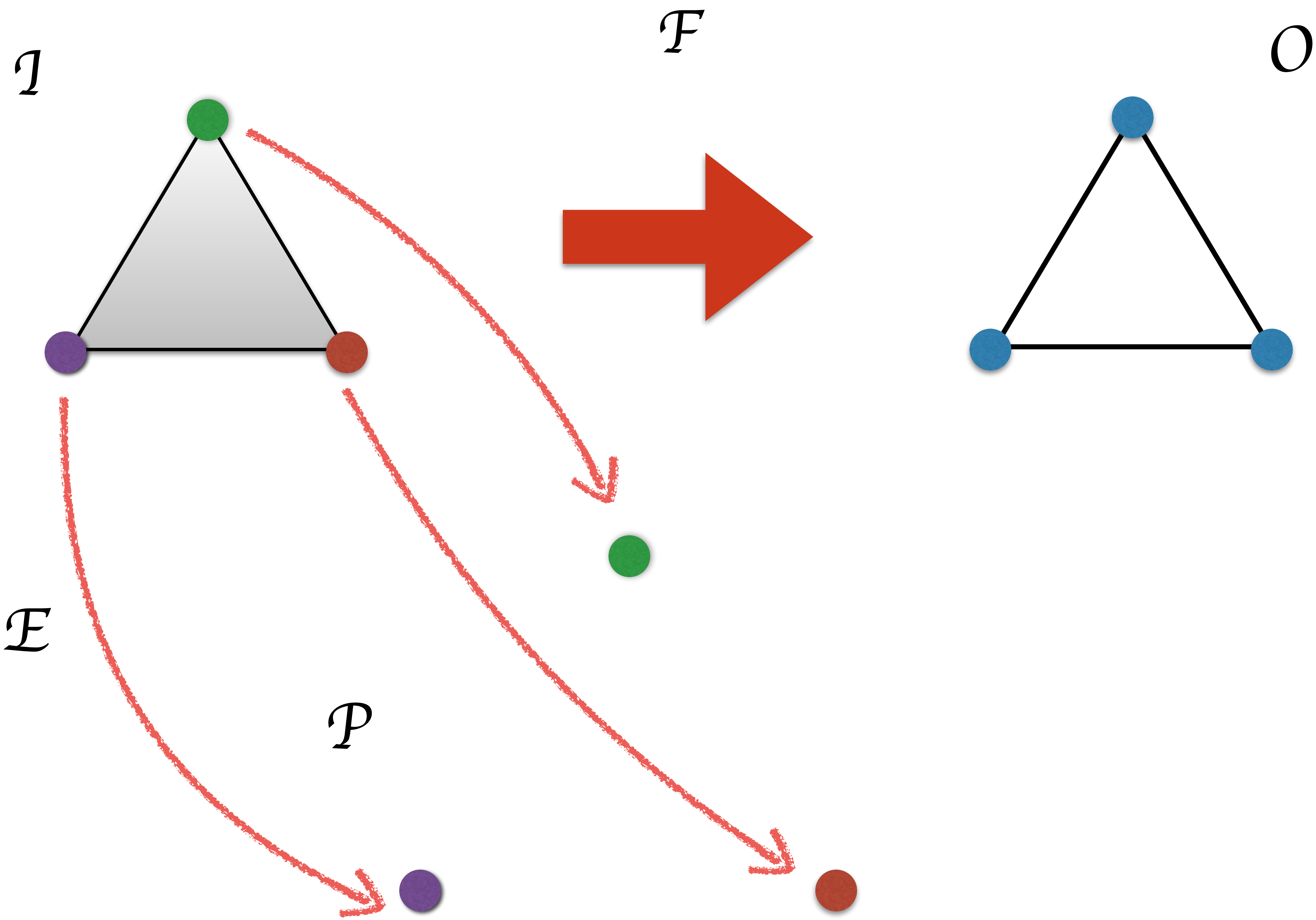
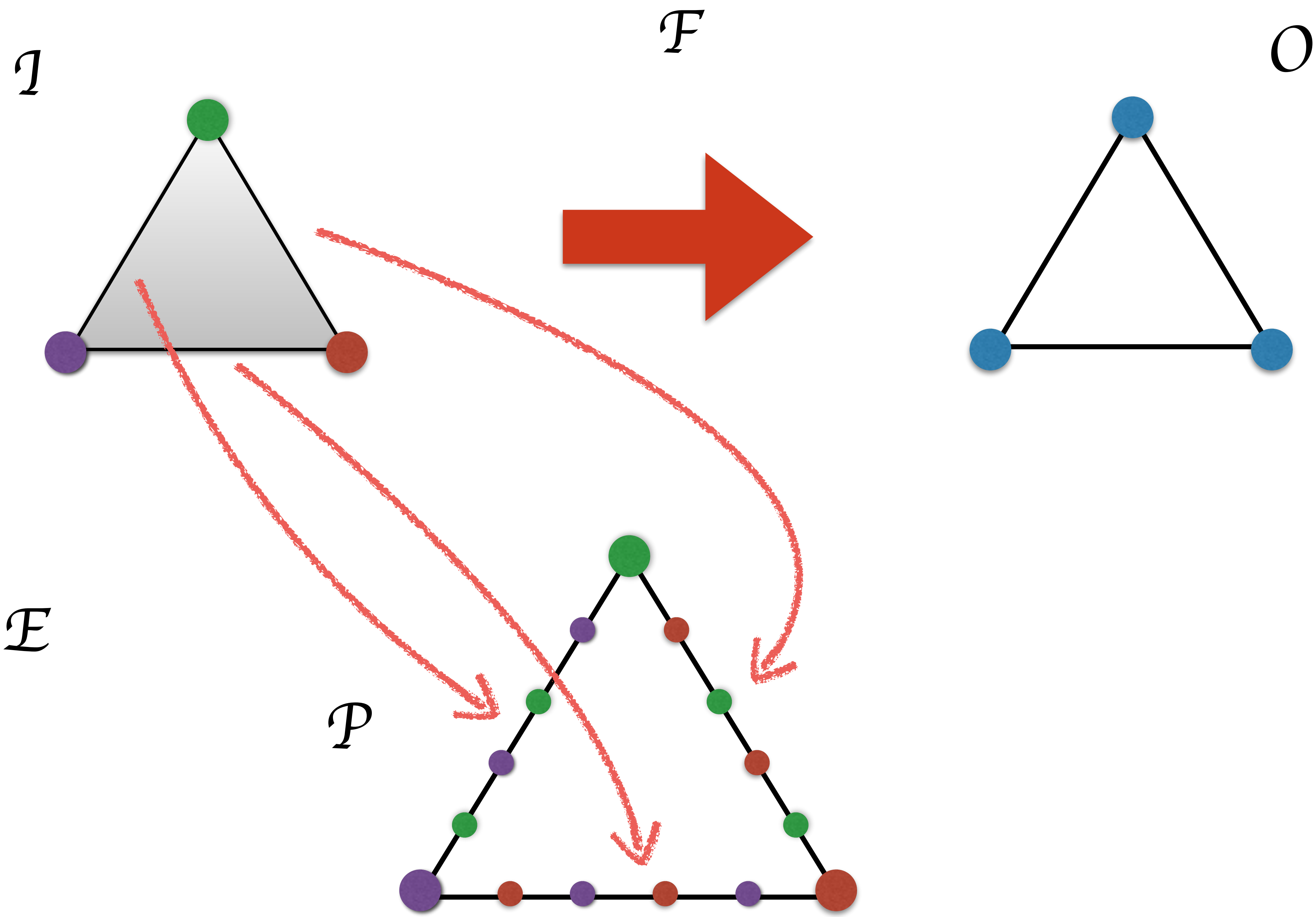
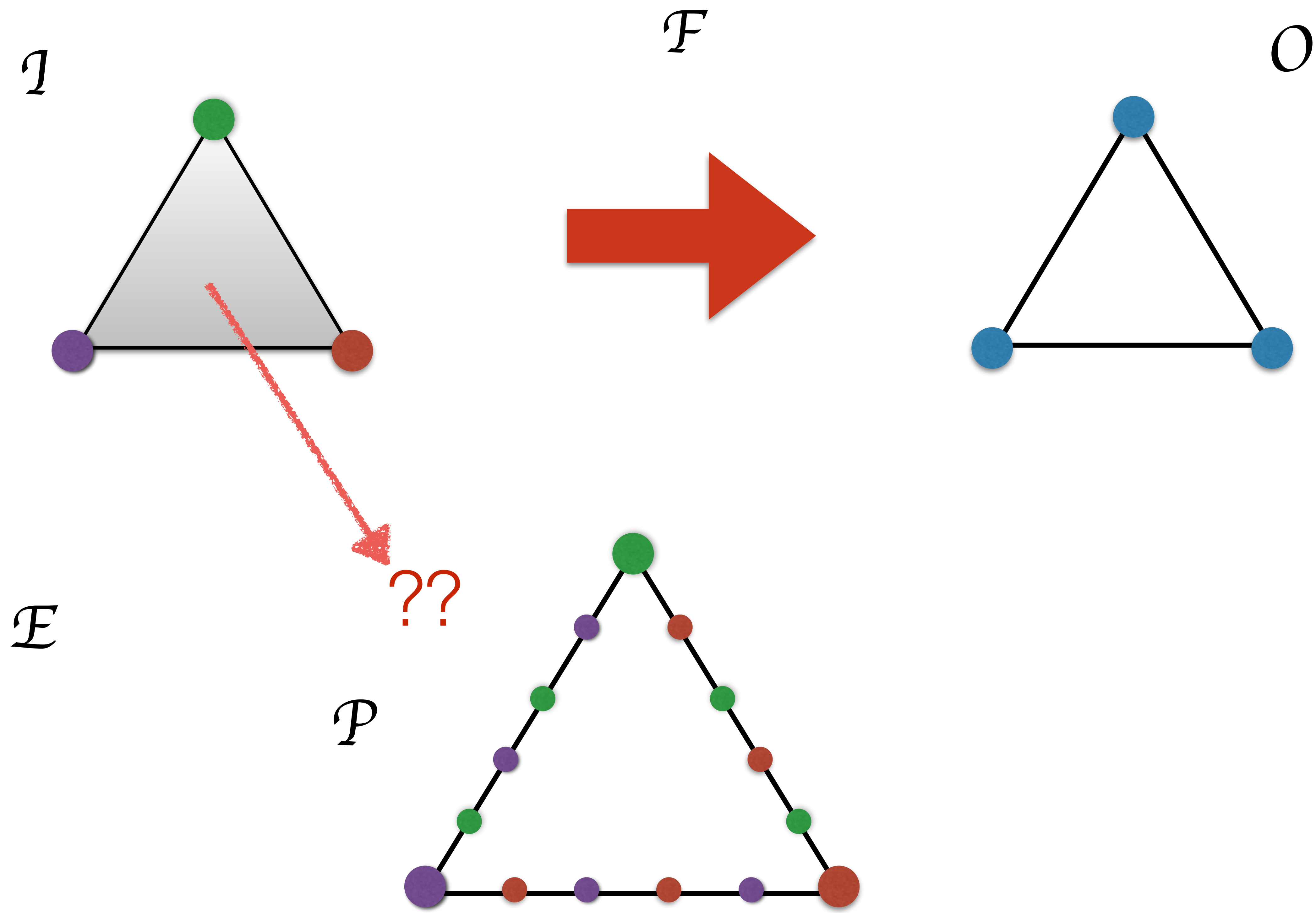


The Triangle is Impossible



The Triangle is Impossible



Immediate Snapshot Executions (ISE)

- Subset of nice structured executions.
- Robots proceed in a sequence of concurrency classes:

$$\{A,B,C\}\ \{B\}\ \{A,C\}\ \{B\}\ \{B\}\ \{A,C\}\ \dots$$

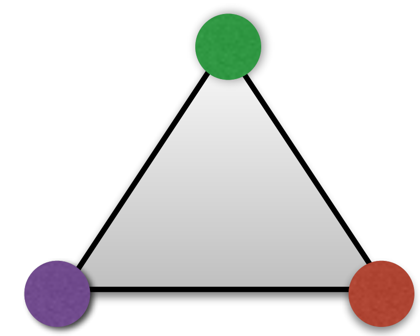
- Concurrency class: concurrent move, then concurrent look.

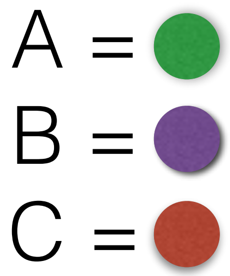
A =

B =

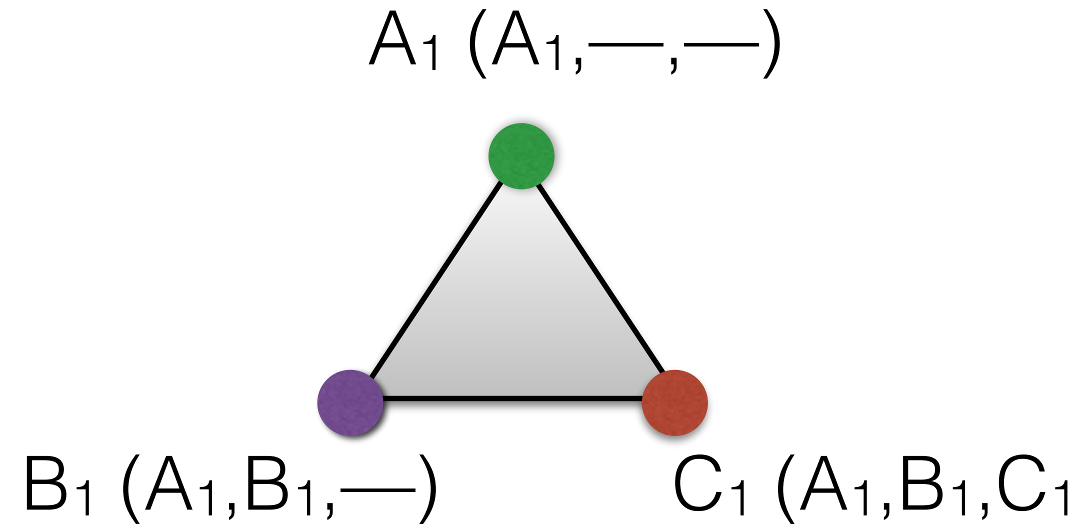
C =

{A}{B}{C}

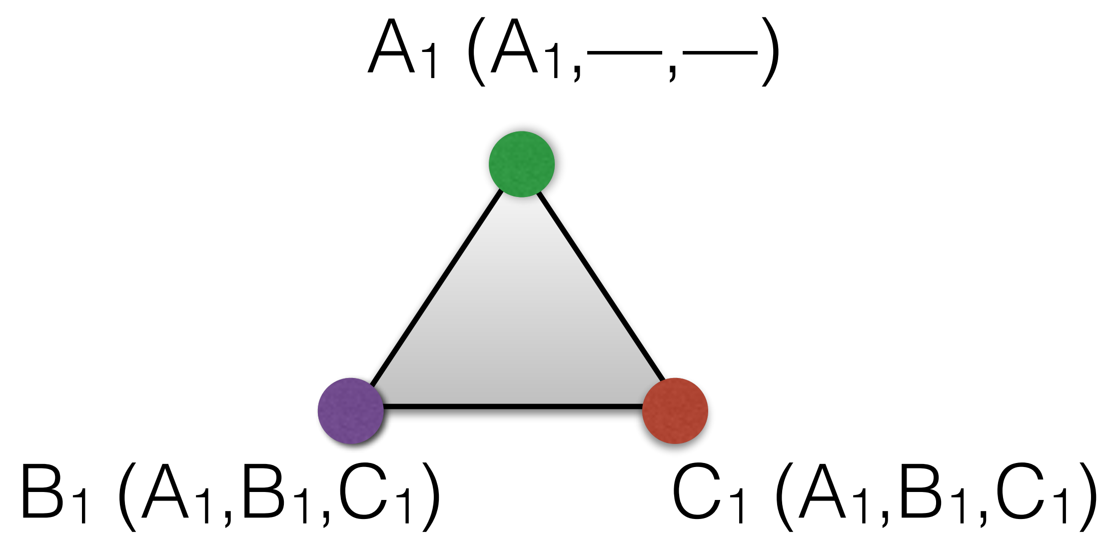


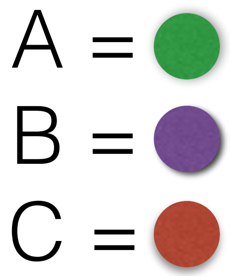


{A}{B}{C}

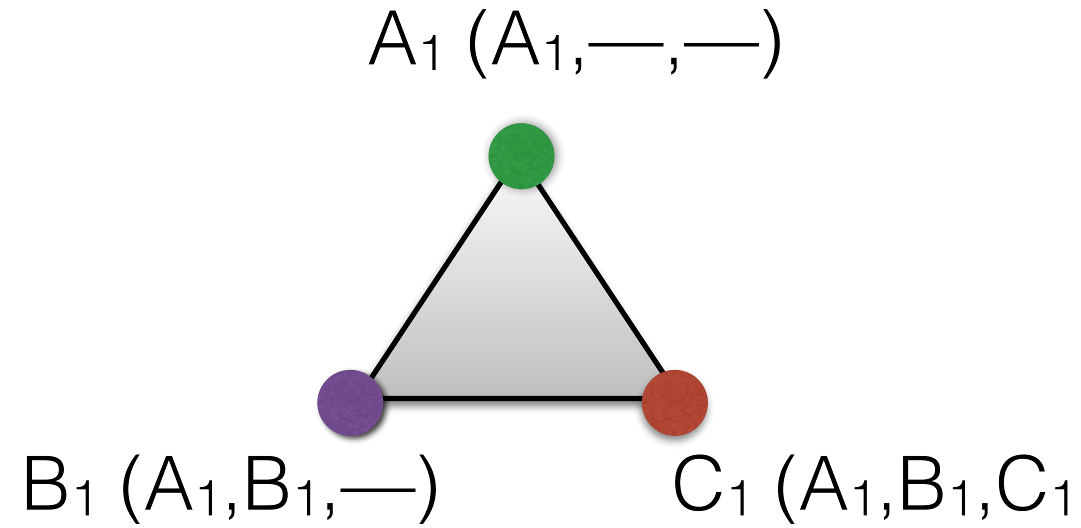


{A}{B,C}

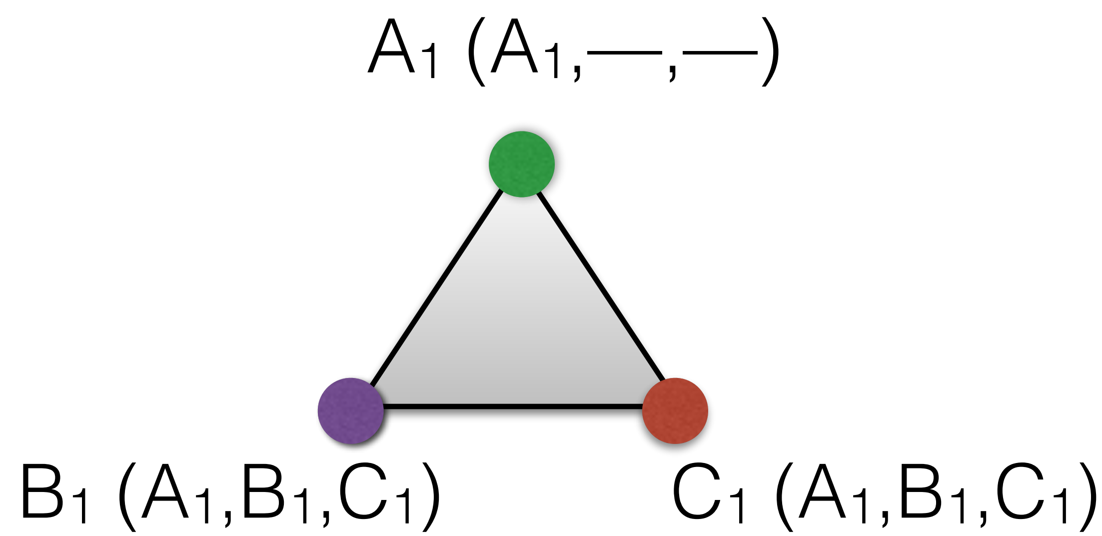



A =
B =
C =

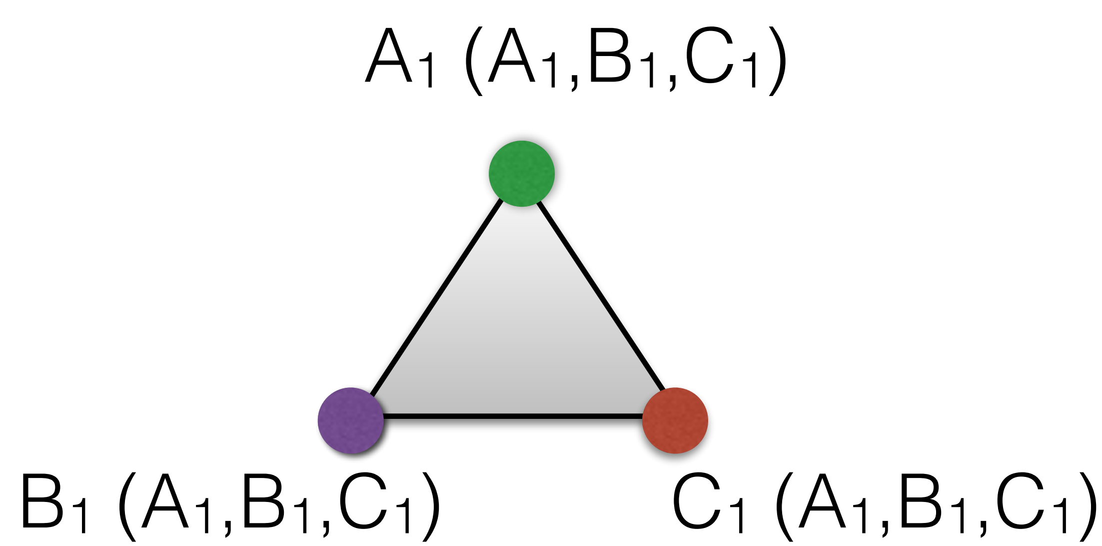
{A}{B}{C}

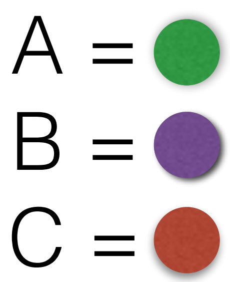

A_1 (A_1,—,—)
B_1 (A_1,B_1,—)
C_1 (A_1,B_1,C_1)

{A}{B,C}

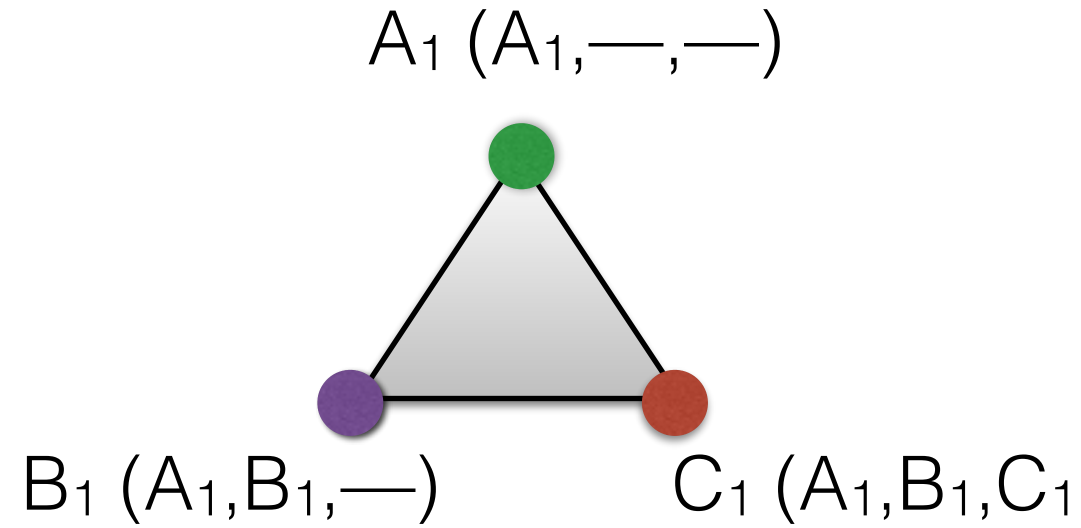

A_1 (A_1,—,—)
B_1 (A_1,B_1,C_1)
C_1 (A_1,B_1,C_1)

{A,B,C}


A_1 (A_1,B_1,C_1)
B_1 (A_1,B_1,C_1)
C_1 (A_1,B_1,C_1)

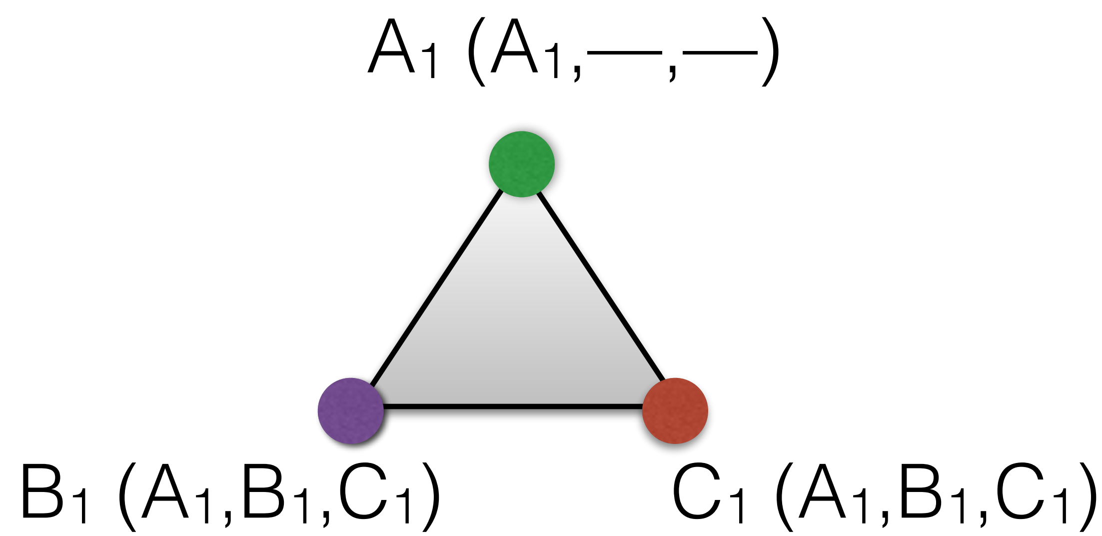


$\{A\}\{B\}\{C\}$

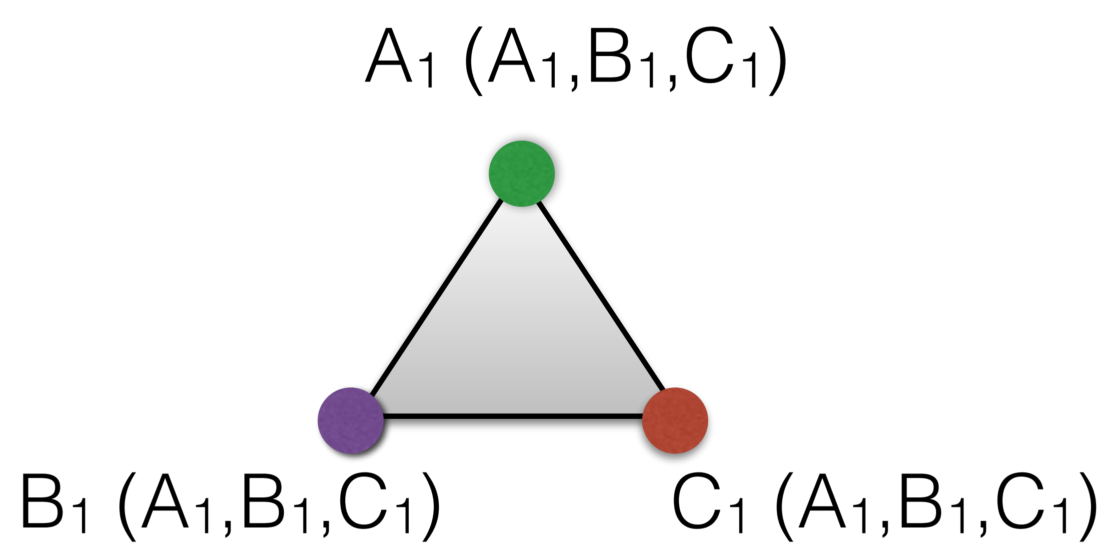


And so on …

$\{A\}\{B,C\}$

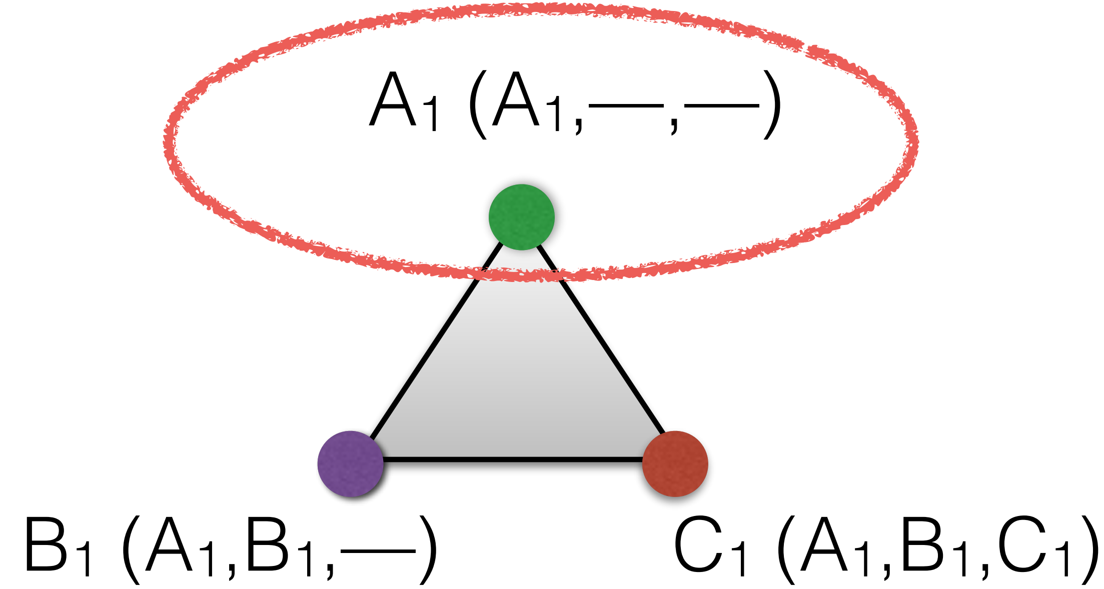


$\{A,B,C\}$



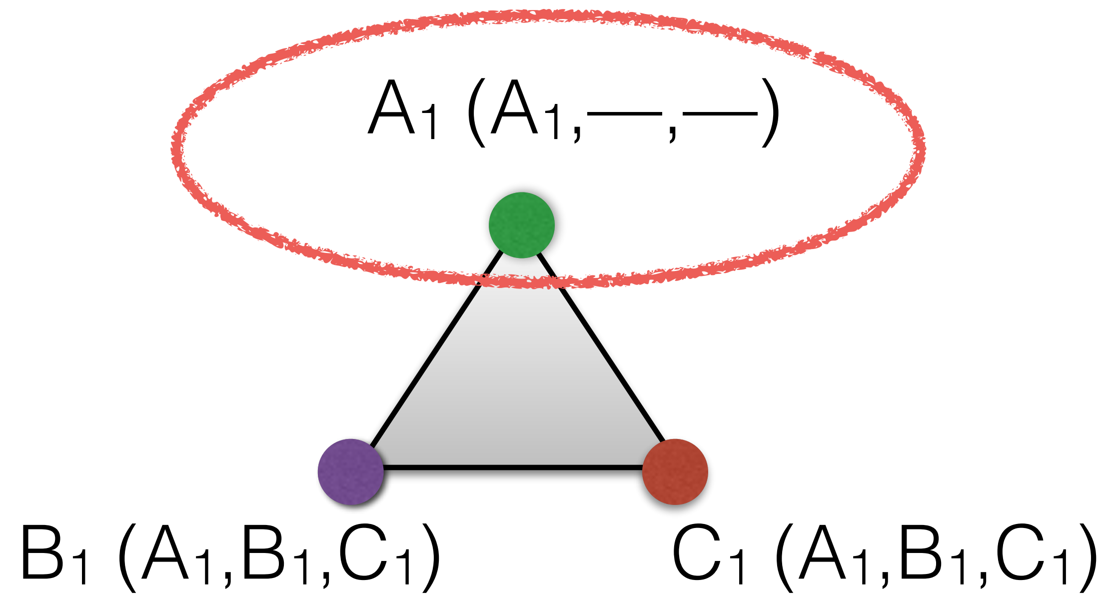
A =
B =
C =

{A}{B}{C}

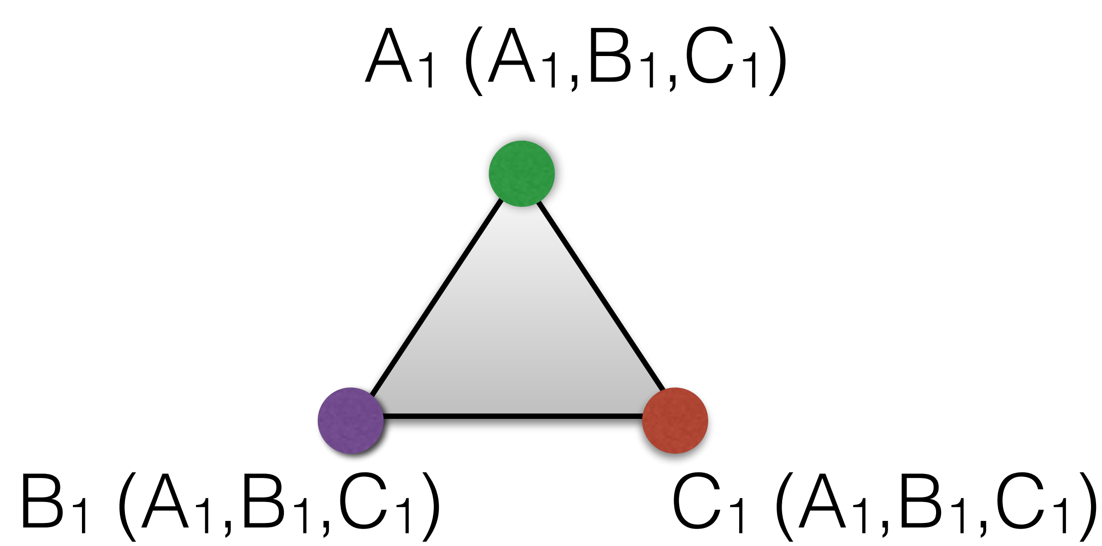


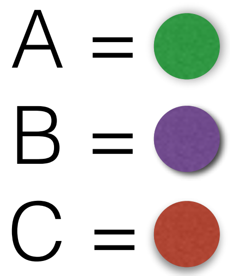
And so on …

{A}{B,C}

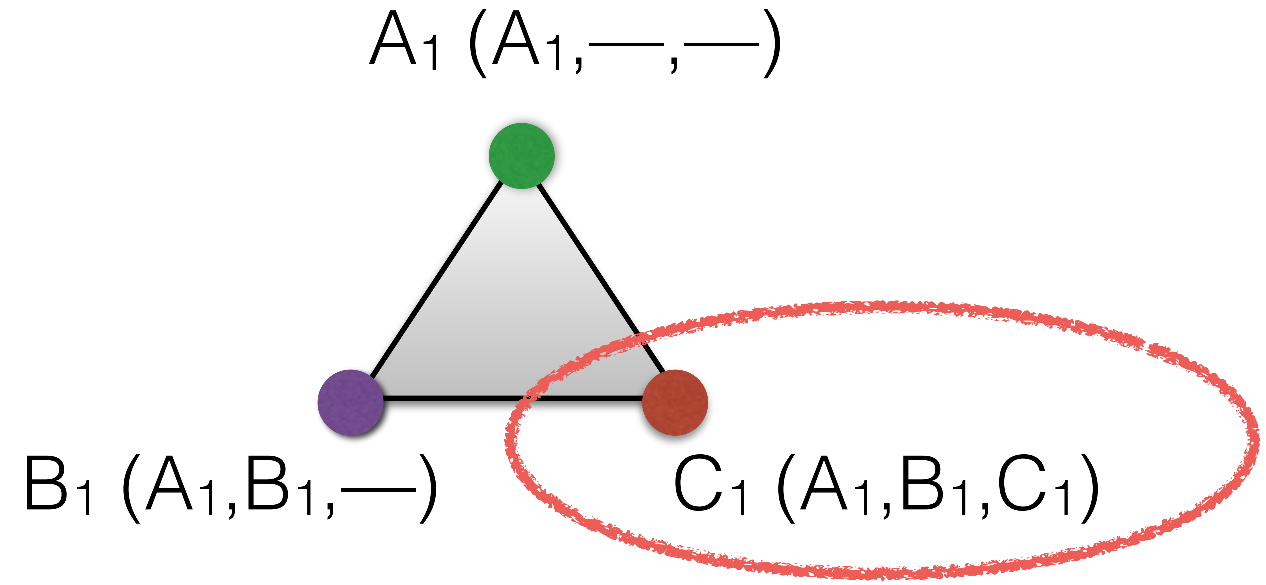


{A,B,C}



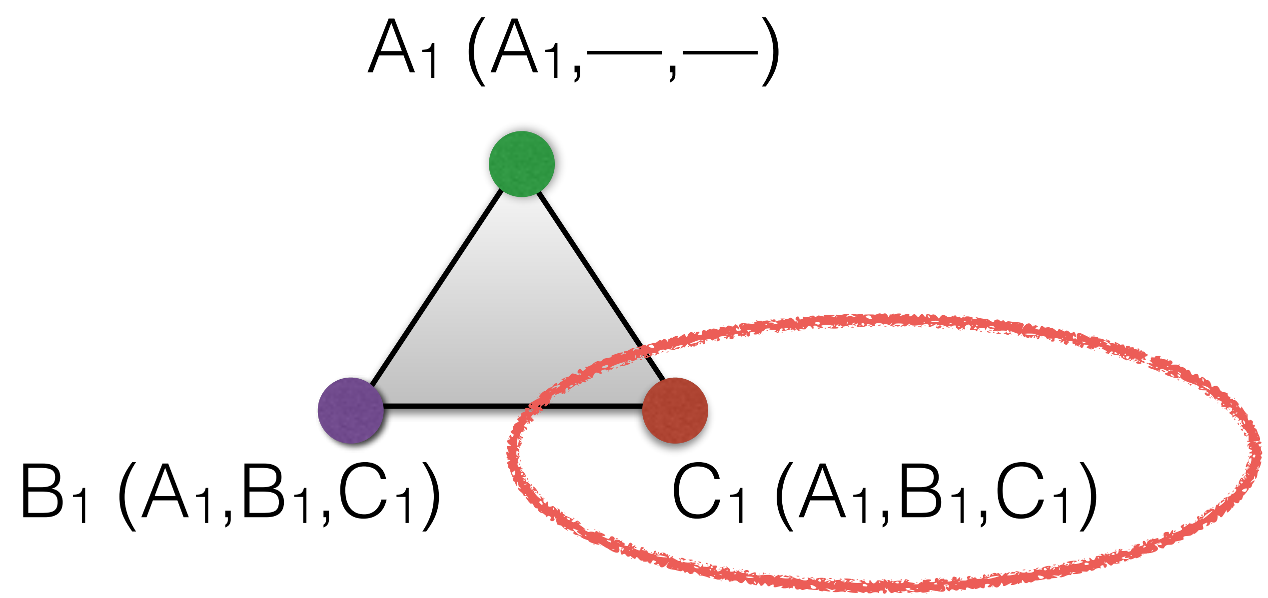


{A}{B}{C}

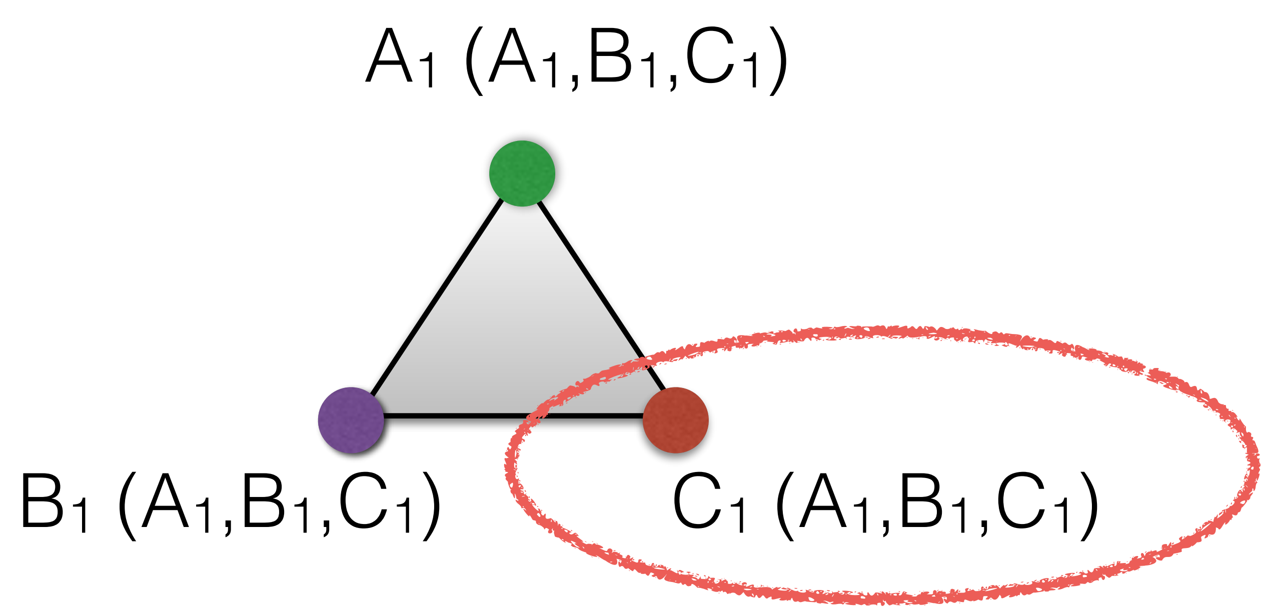


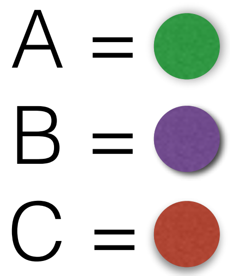
And so on …

{A}{B,C}

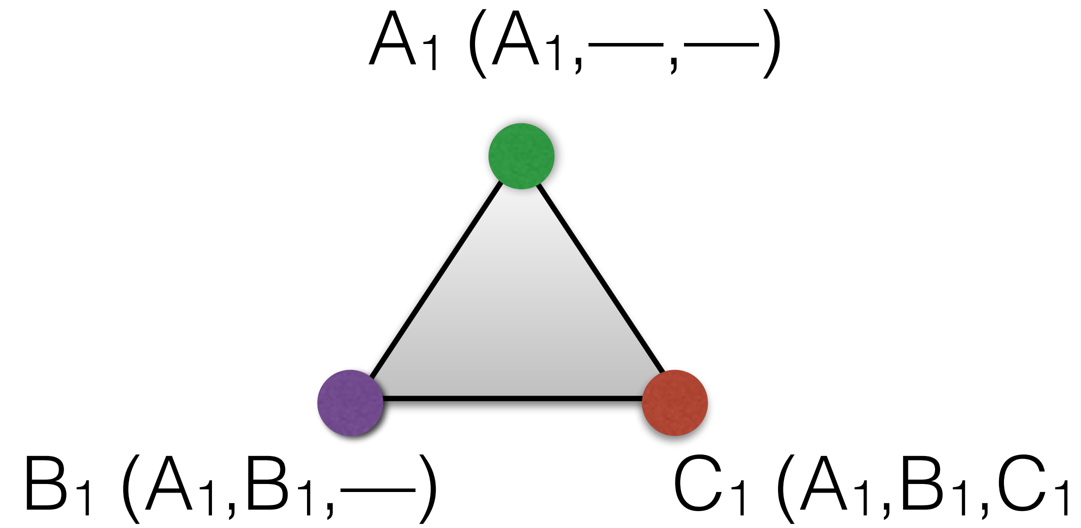


{A,B,C}



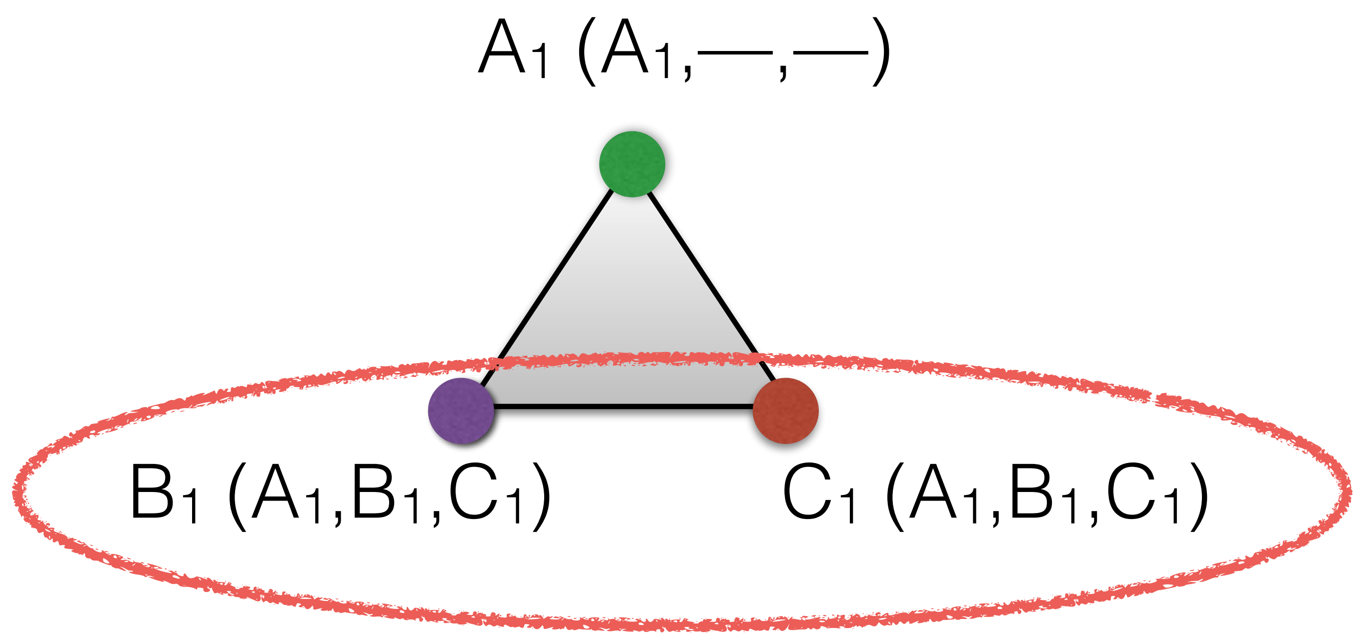


{A}{B}{C}

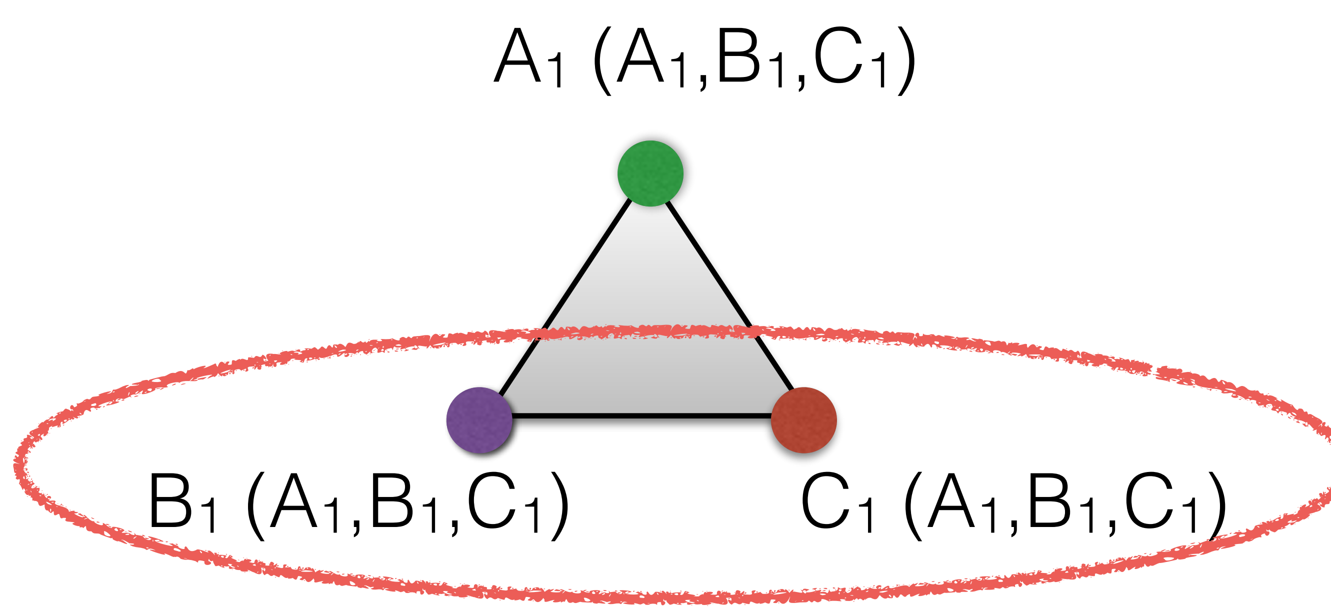


And so on …

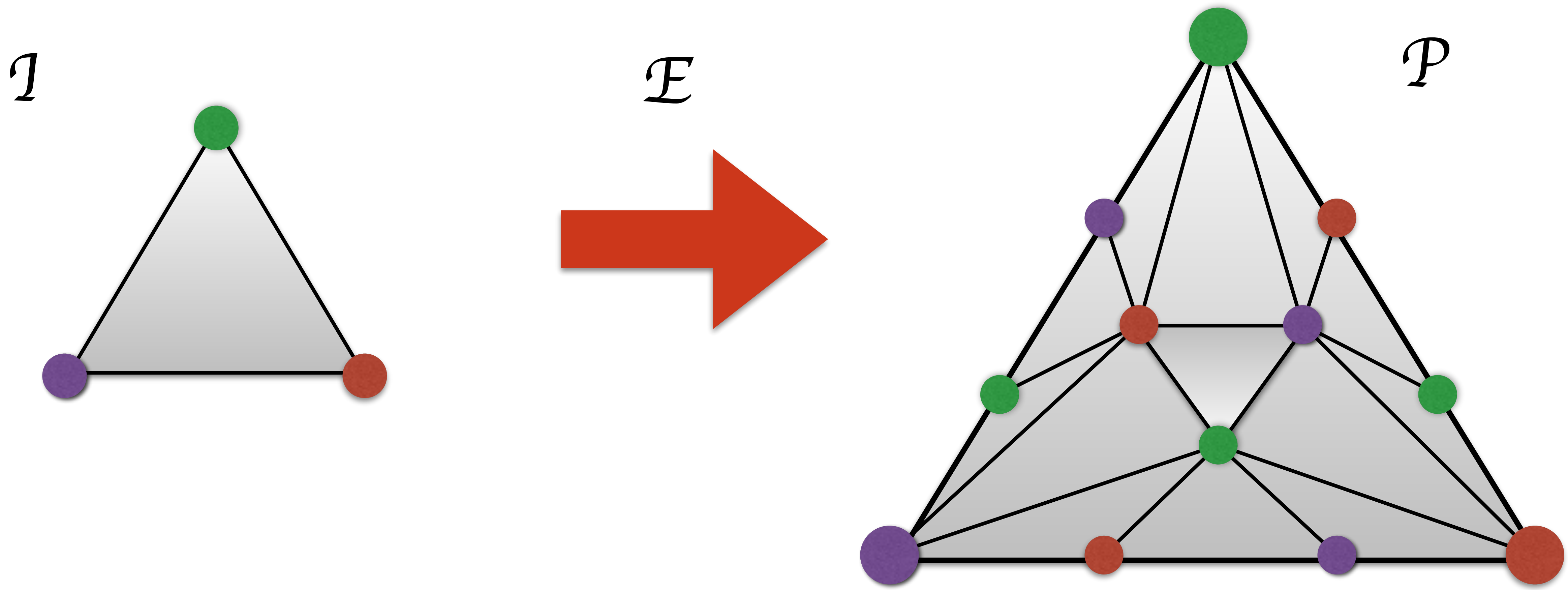
{A}{B,C}



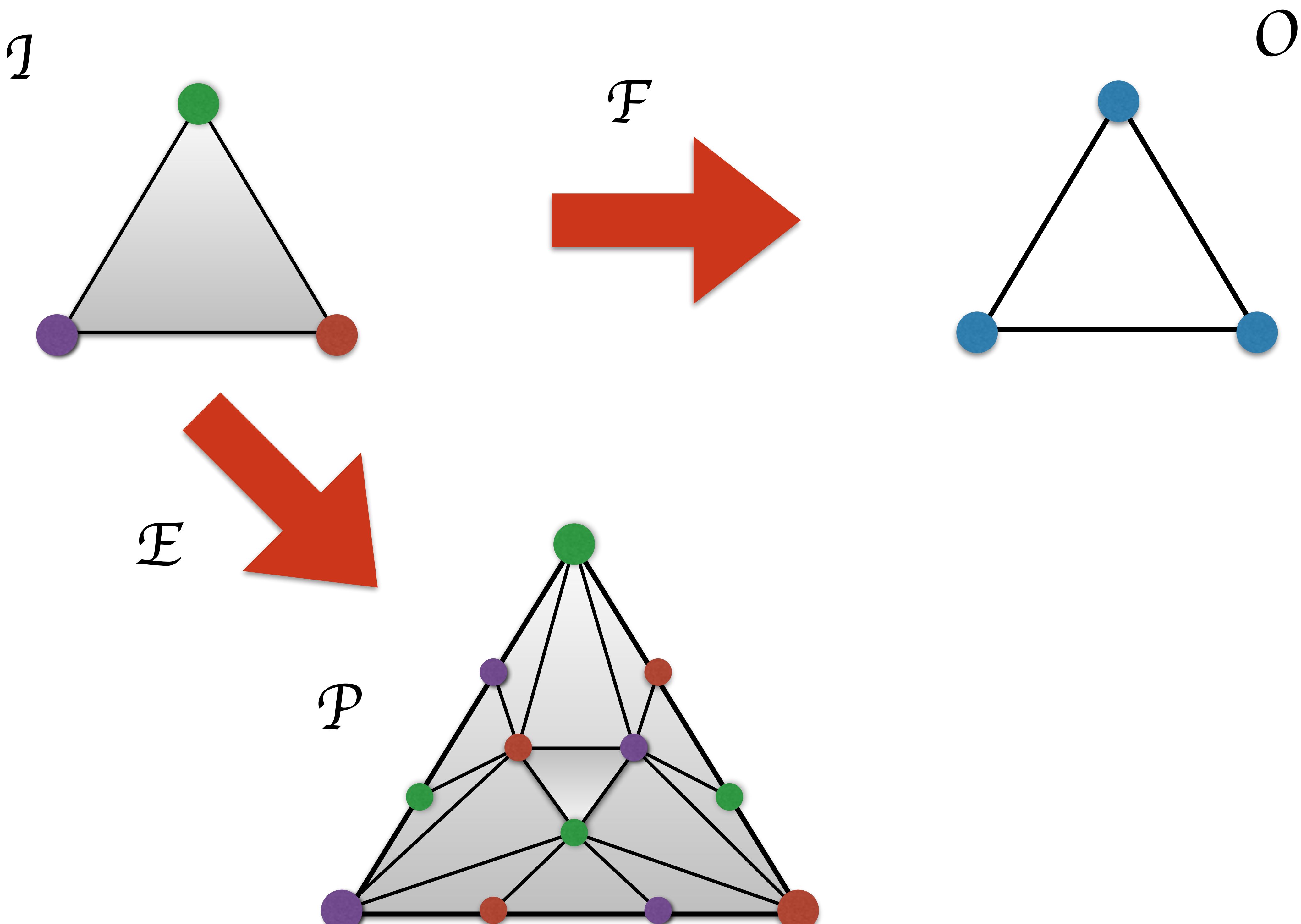
{A,B,C}



1-Round ISE Complex



The Triangle is Impossible



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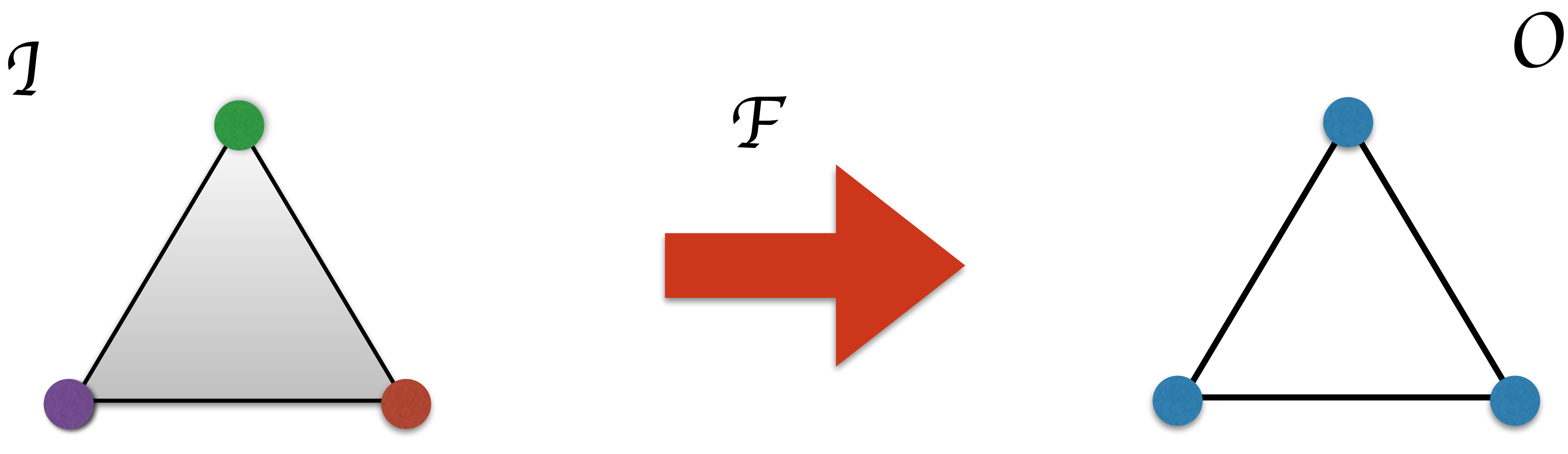
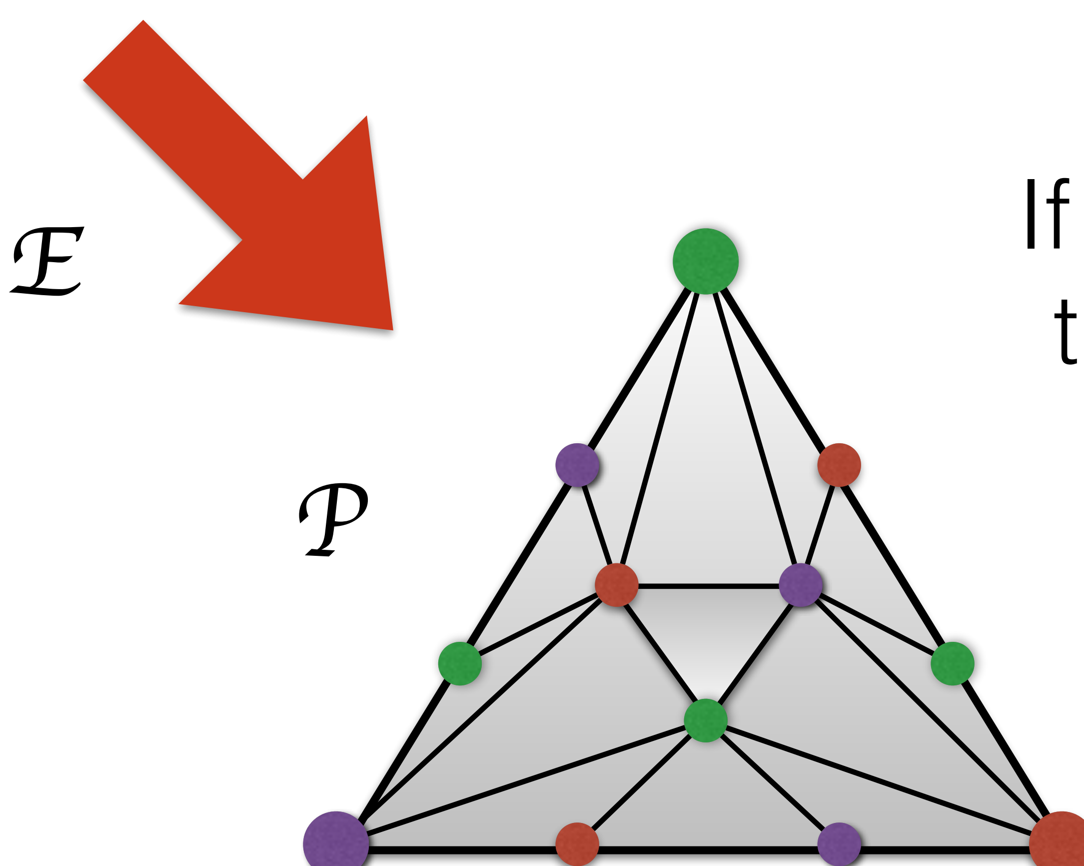
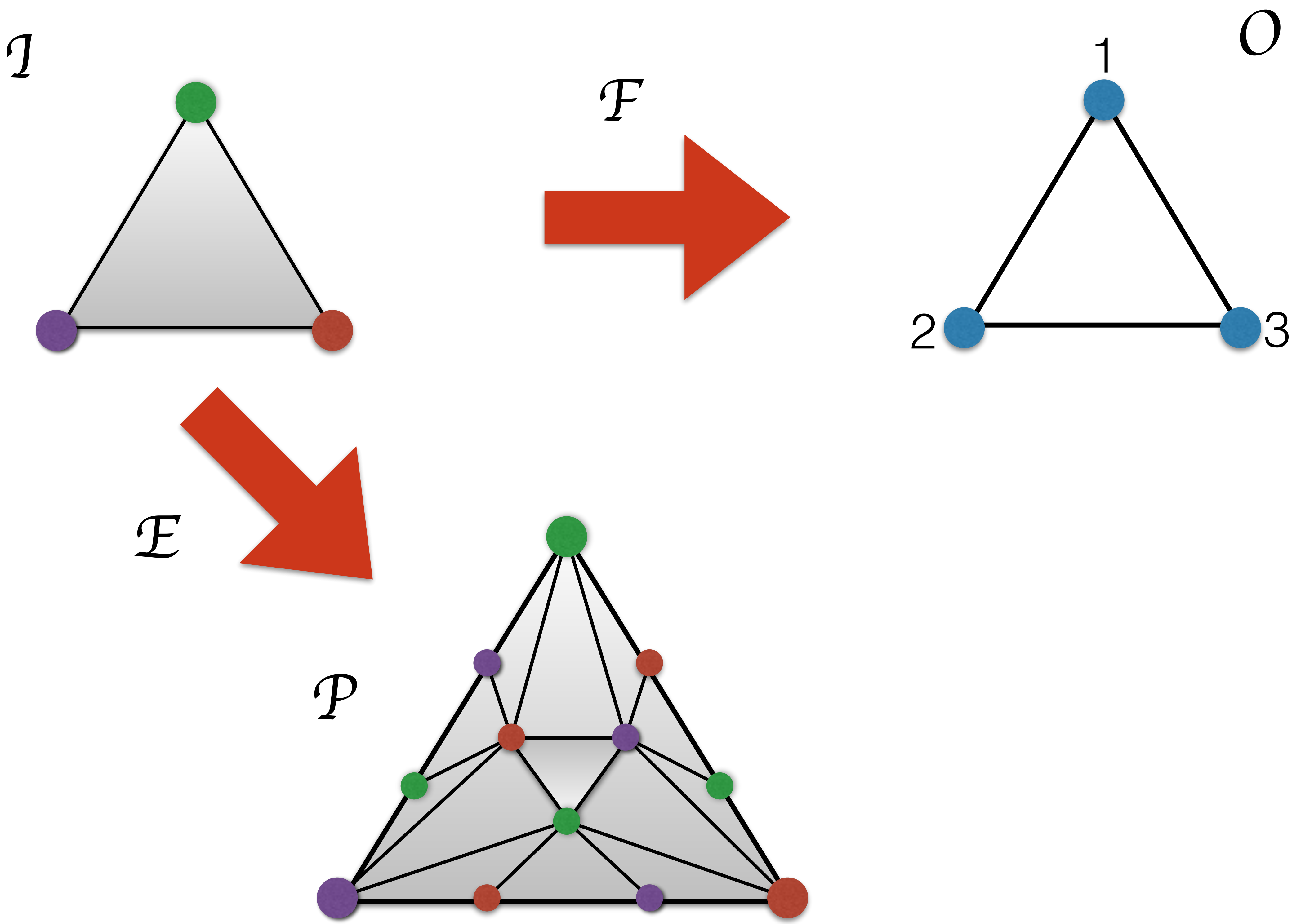




If there is an algorithm =>
there is a simplicial map
d: $\mathcal{V}(\mathcal{P}) \longrightarrow \mathcal{V}(\mathcal{O})$
respecting $\mathcal{F}$

The Triangle is Impossible



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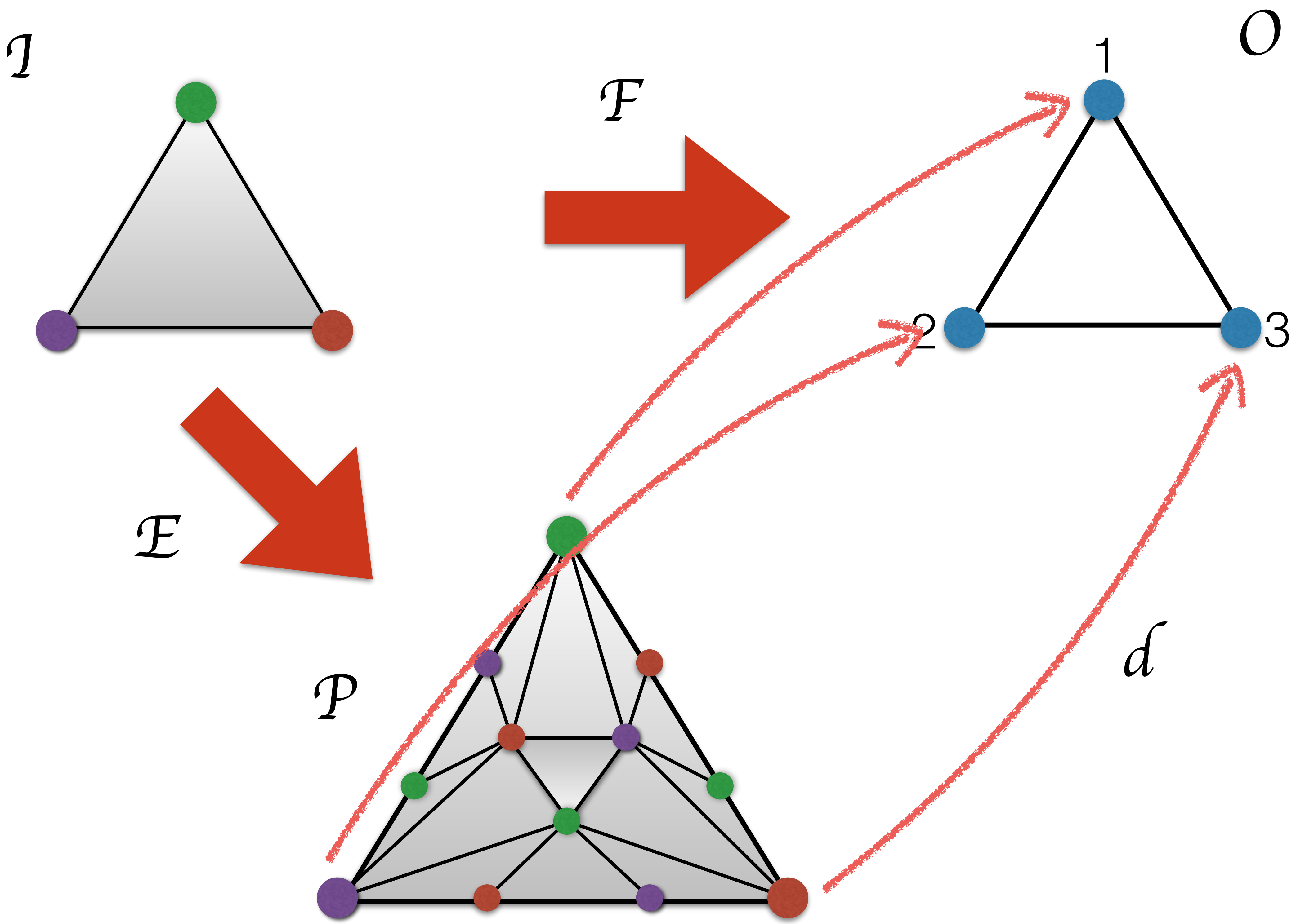

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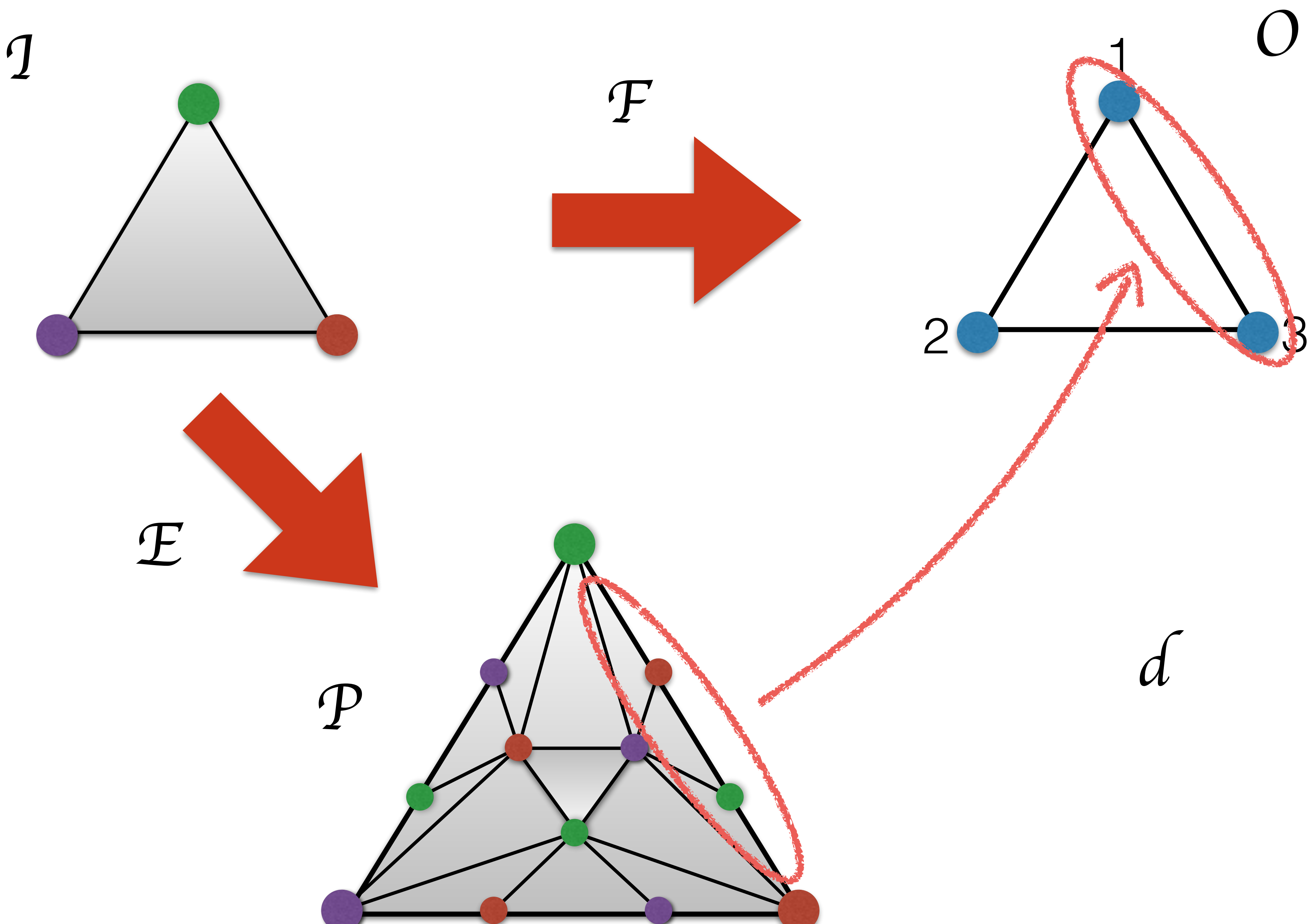

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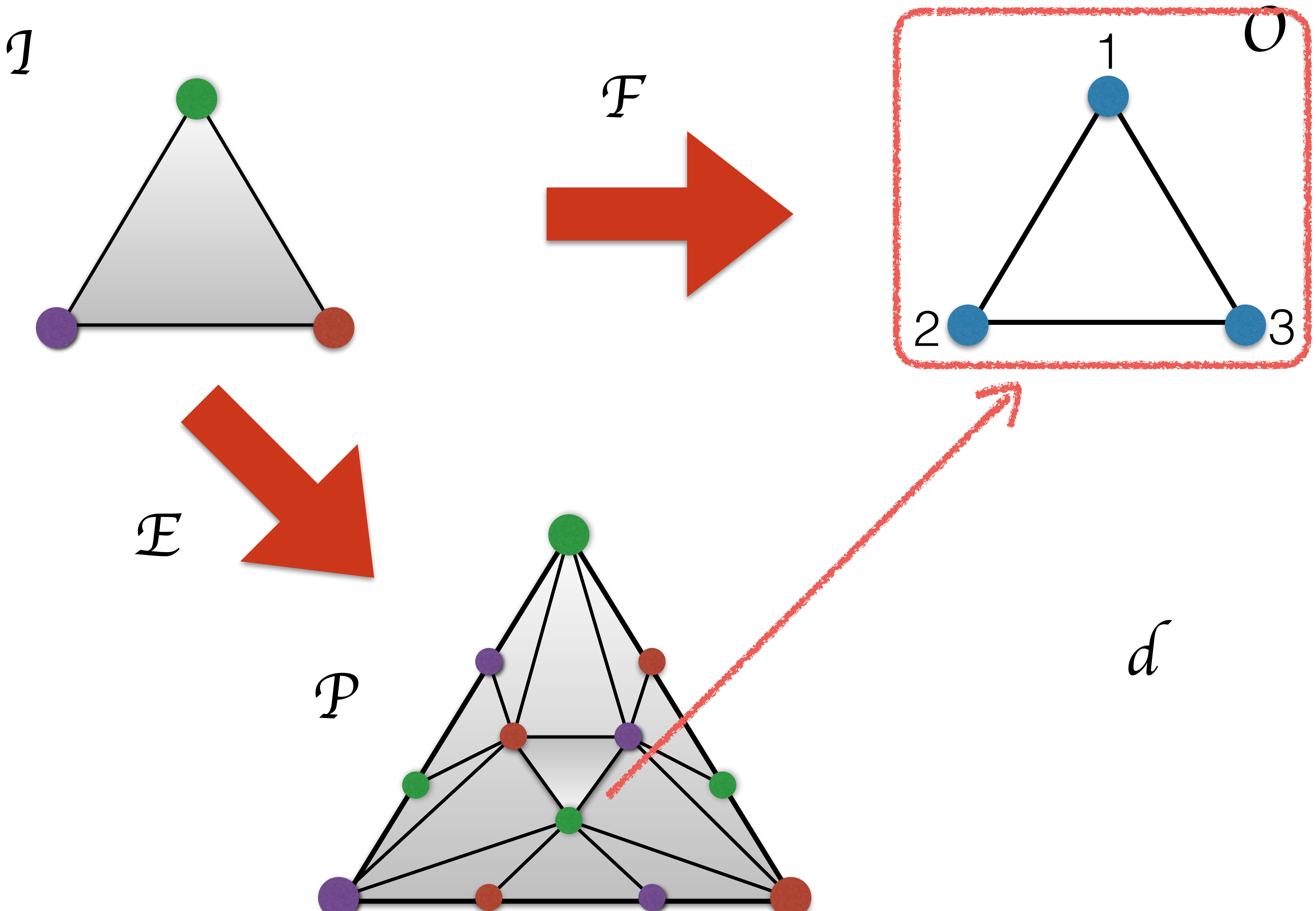

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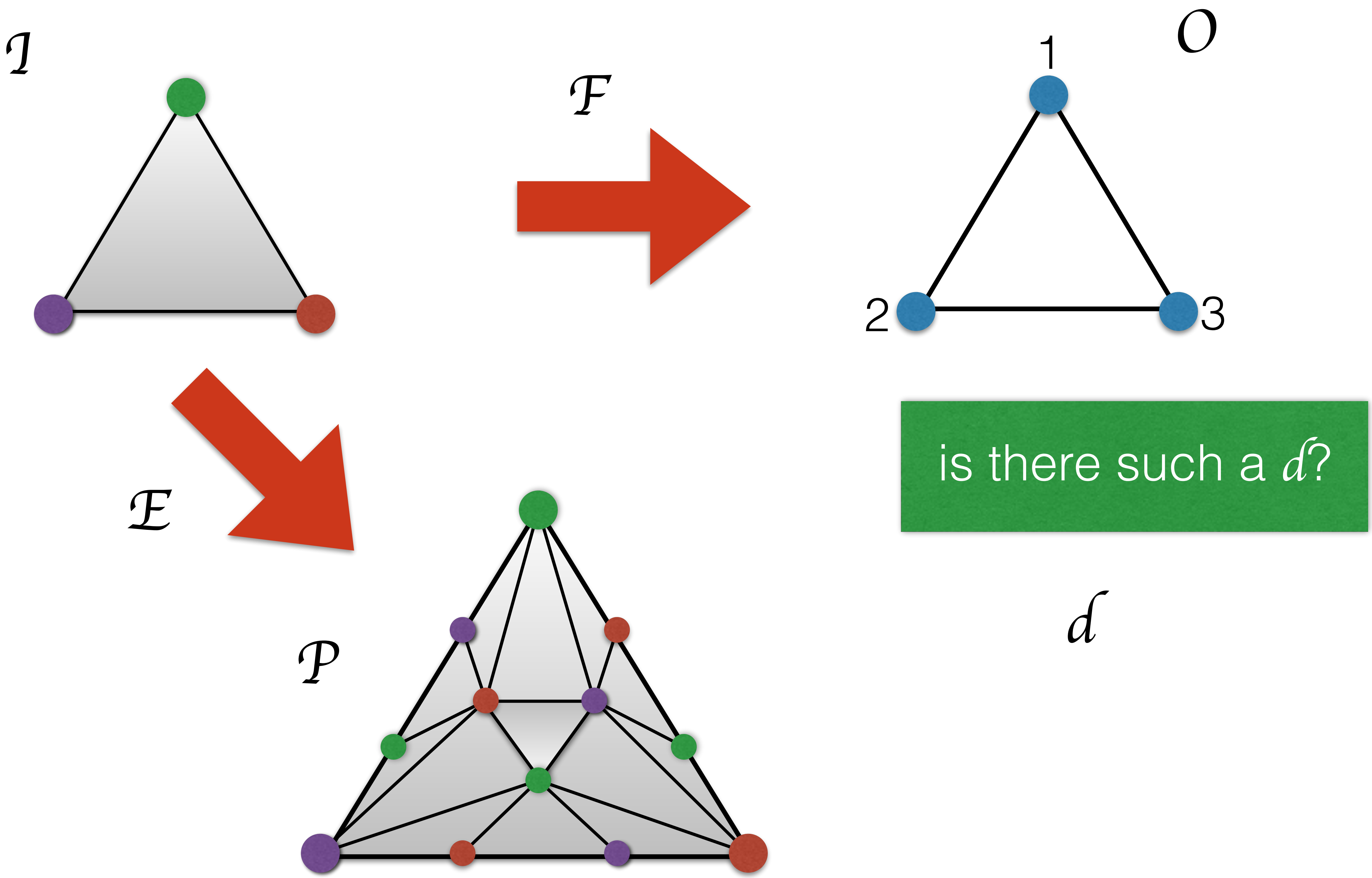
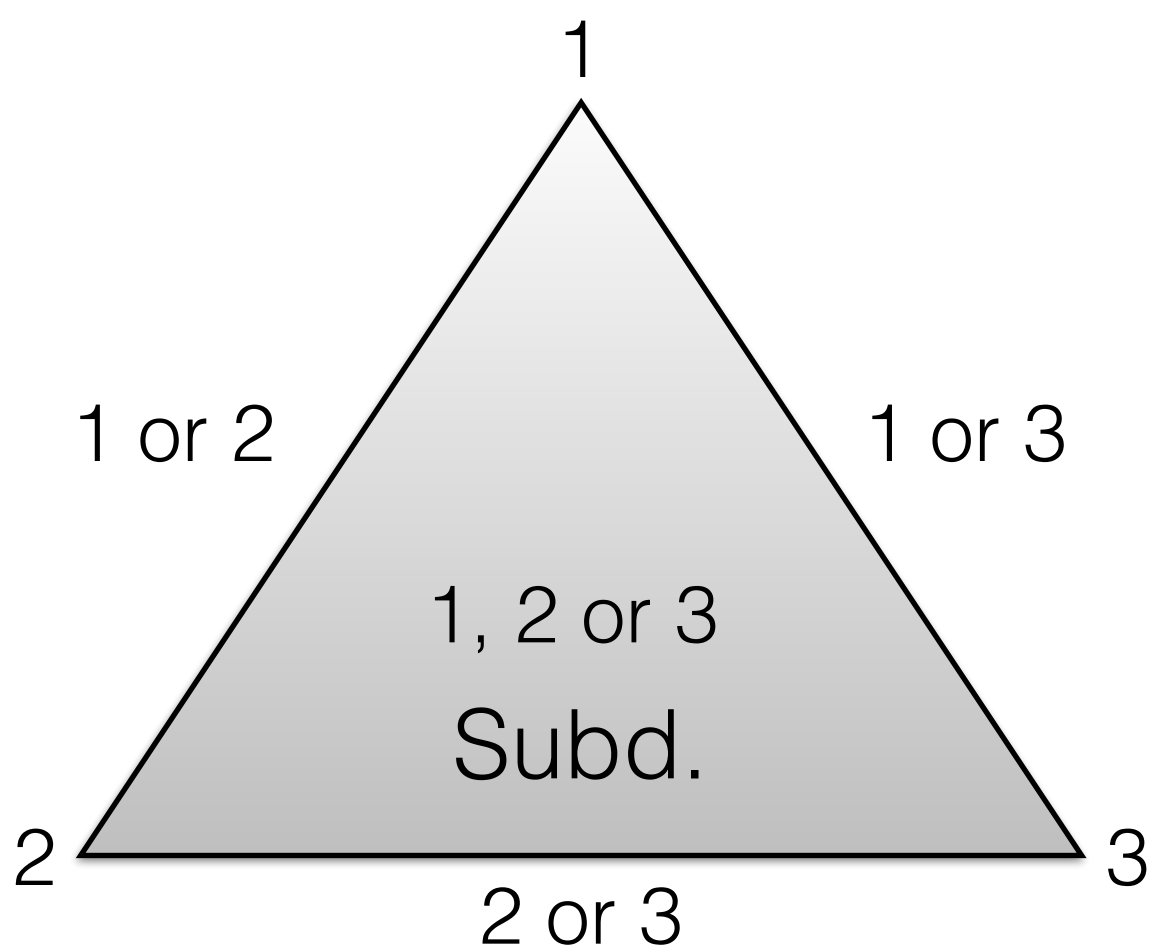


2-Dim Sperner's Lemma

Every subdivision of a triangle with a Sperner coloring has an odd number of 3-chromatic triangles



2-Dim Sperner's Lemma

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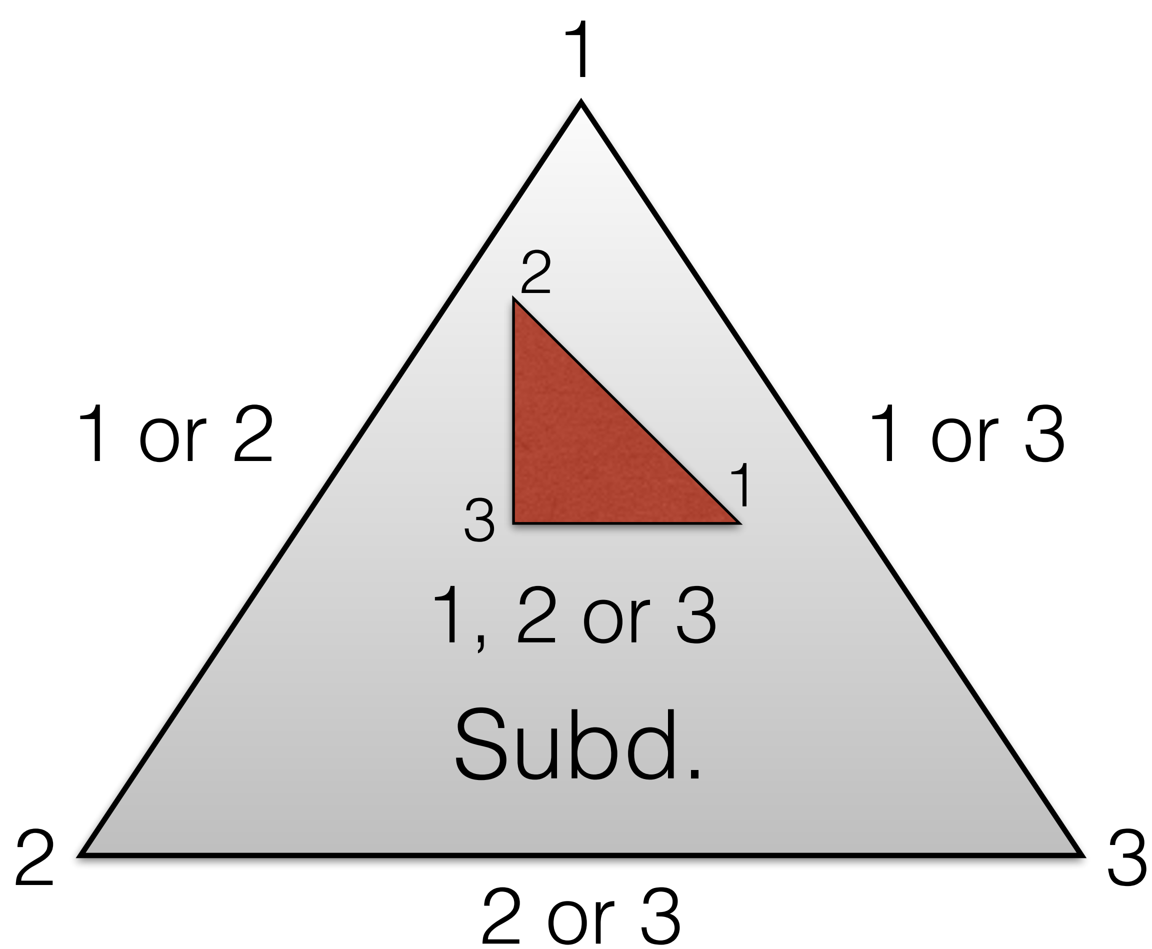
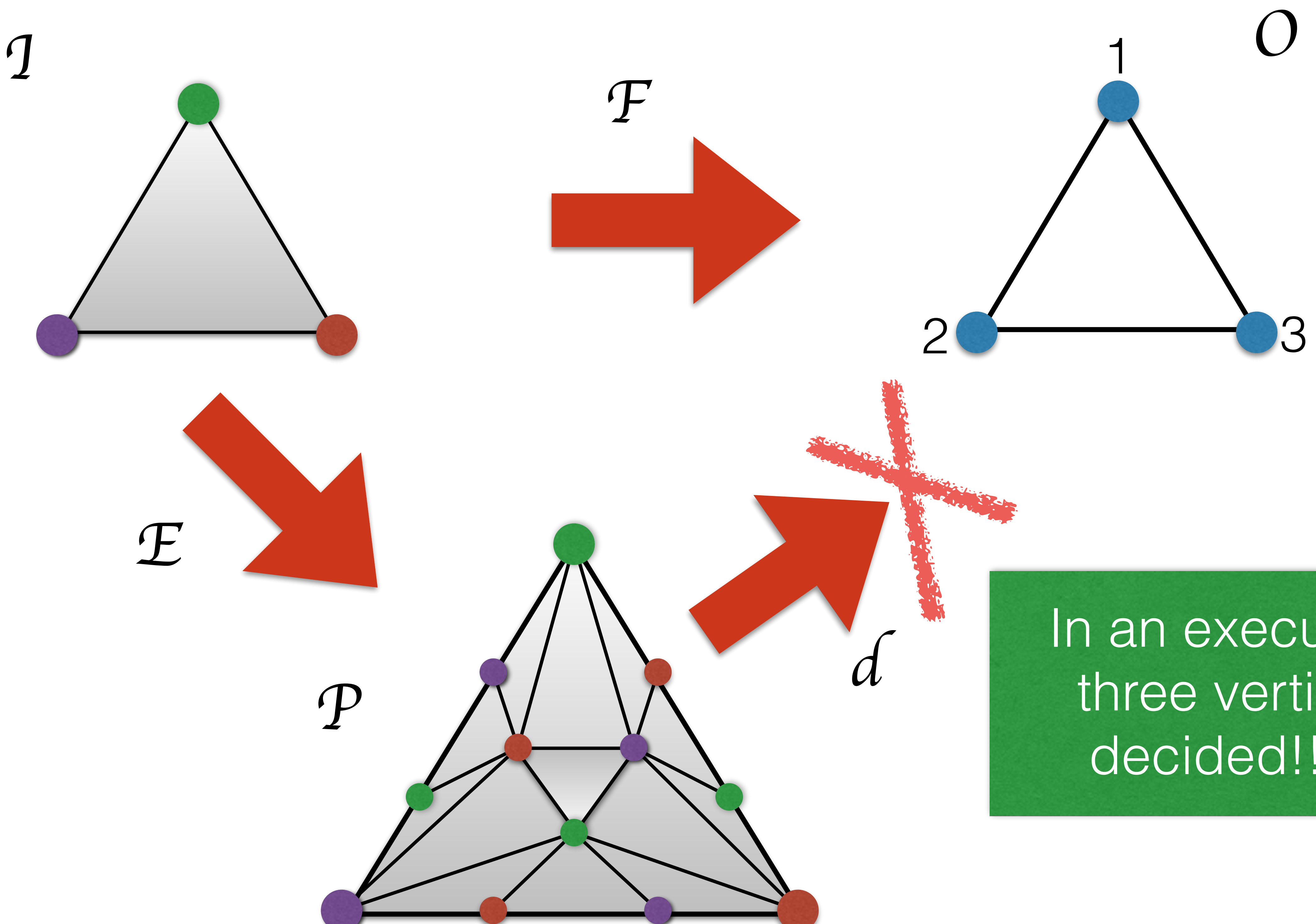


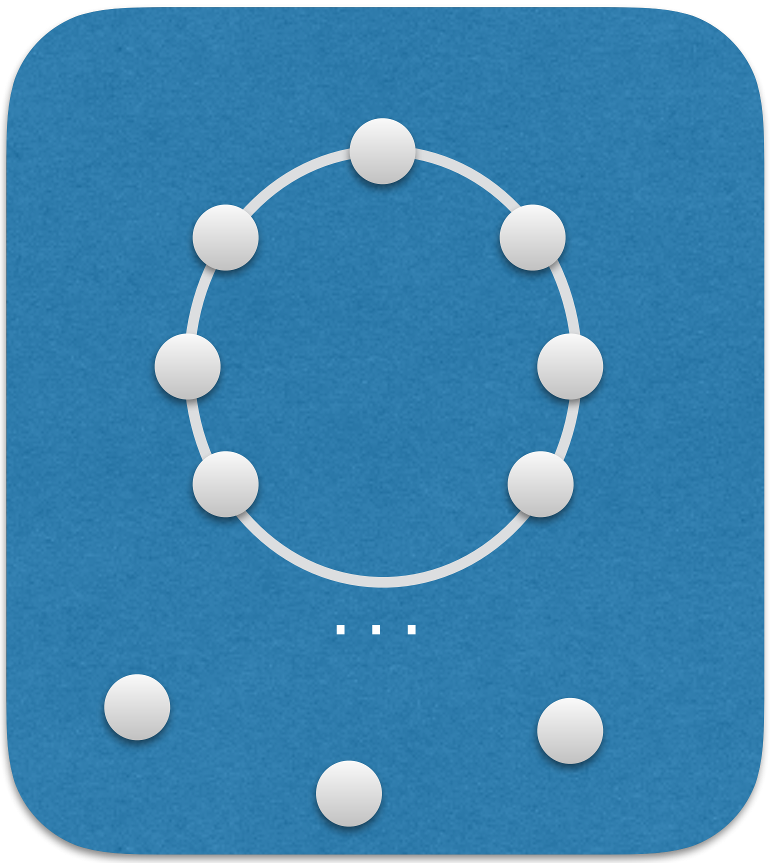
The Triangle is Impossible

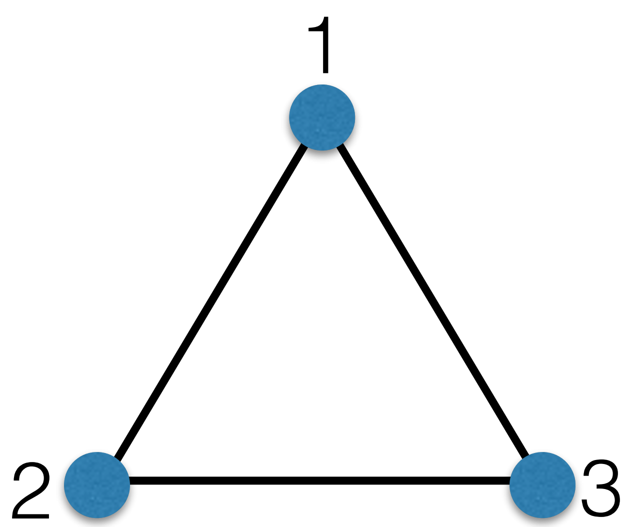


In an execution, three vertices decided!! :-(

Solving Triangle from Cyclic Graphs

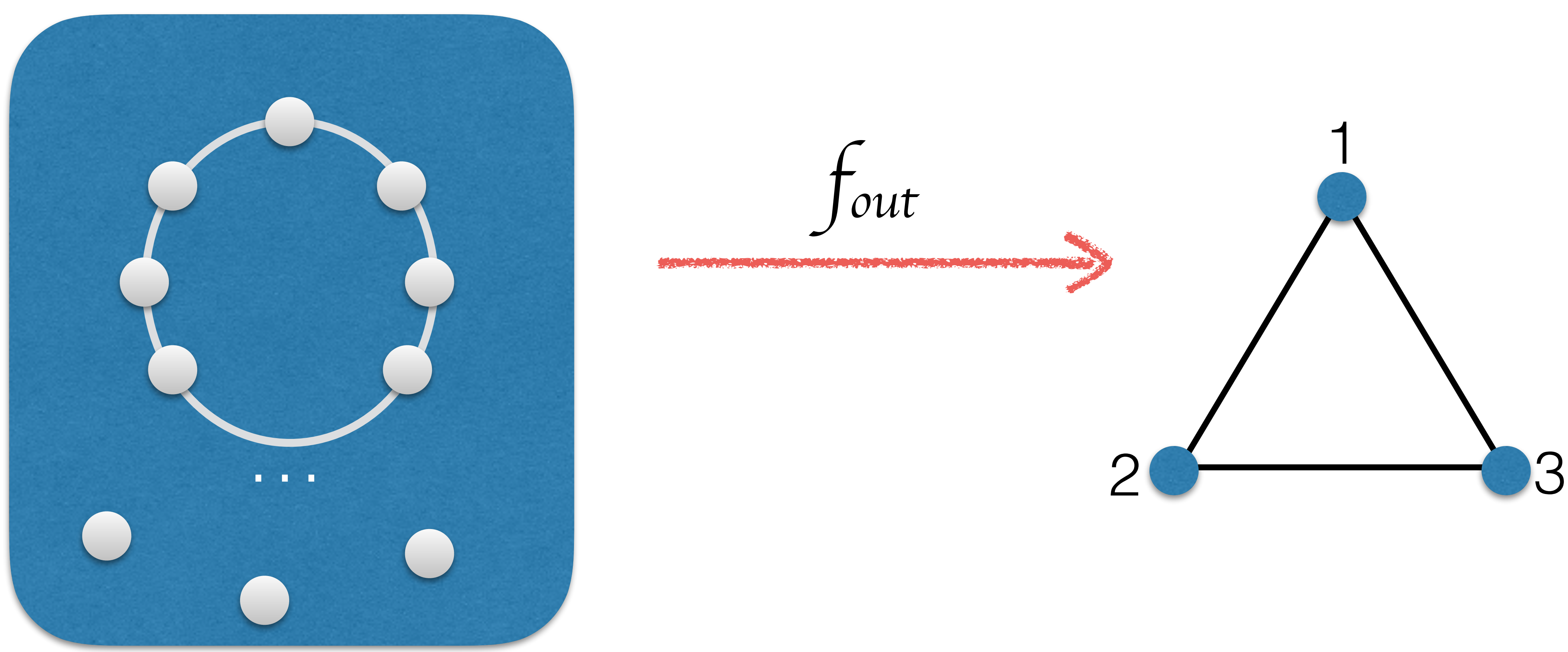
$\mathcal{G}$



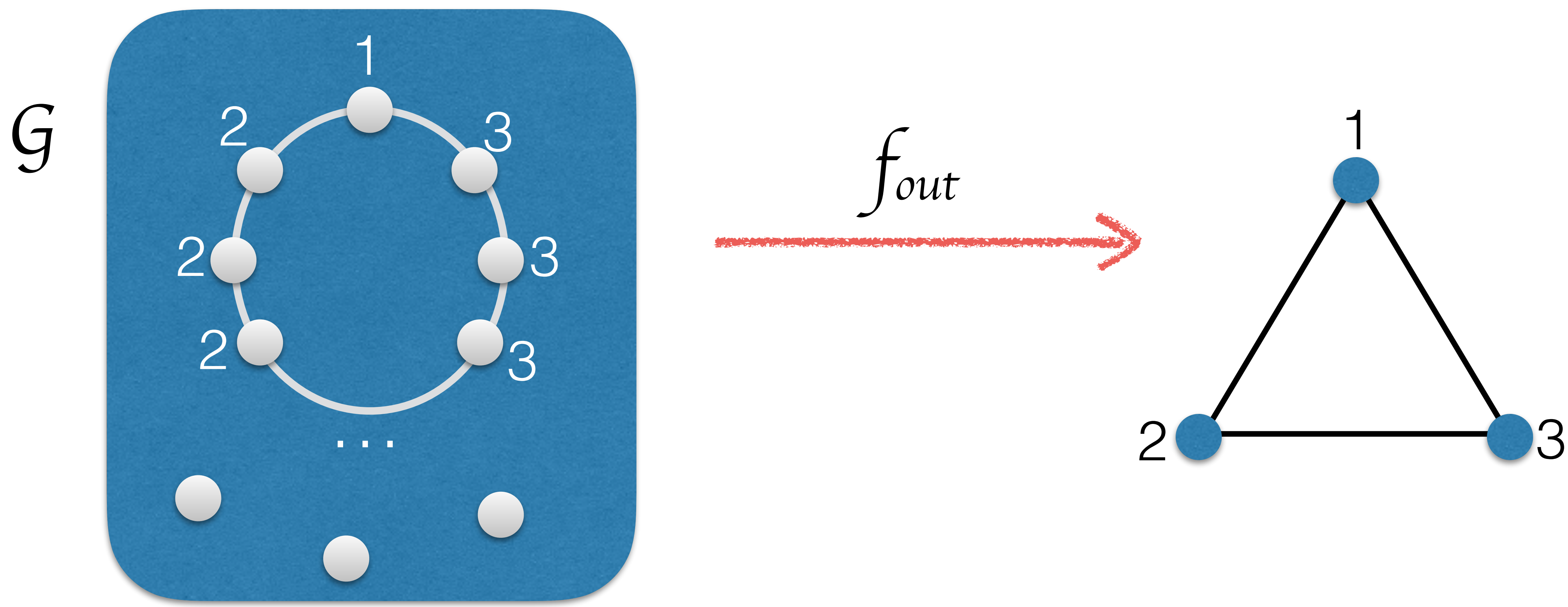


Solving Triangle from Cyclic Graphs

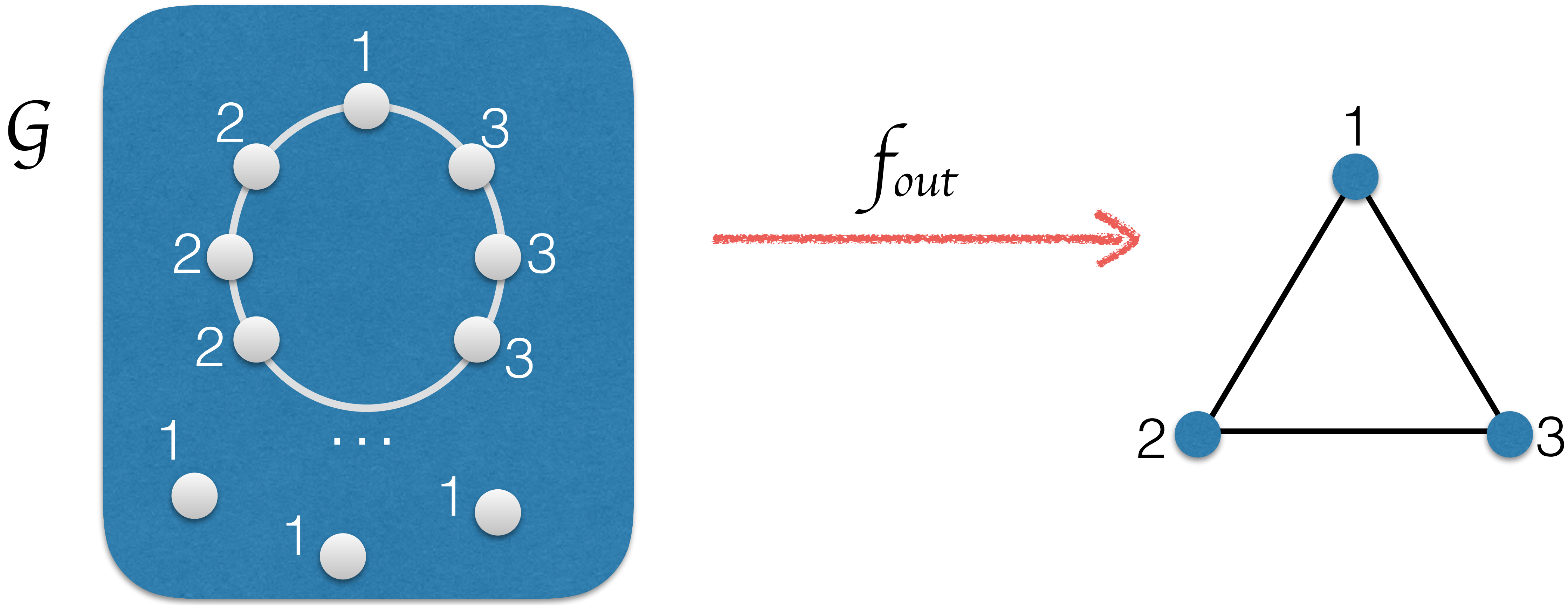
$\mathcal{G}$



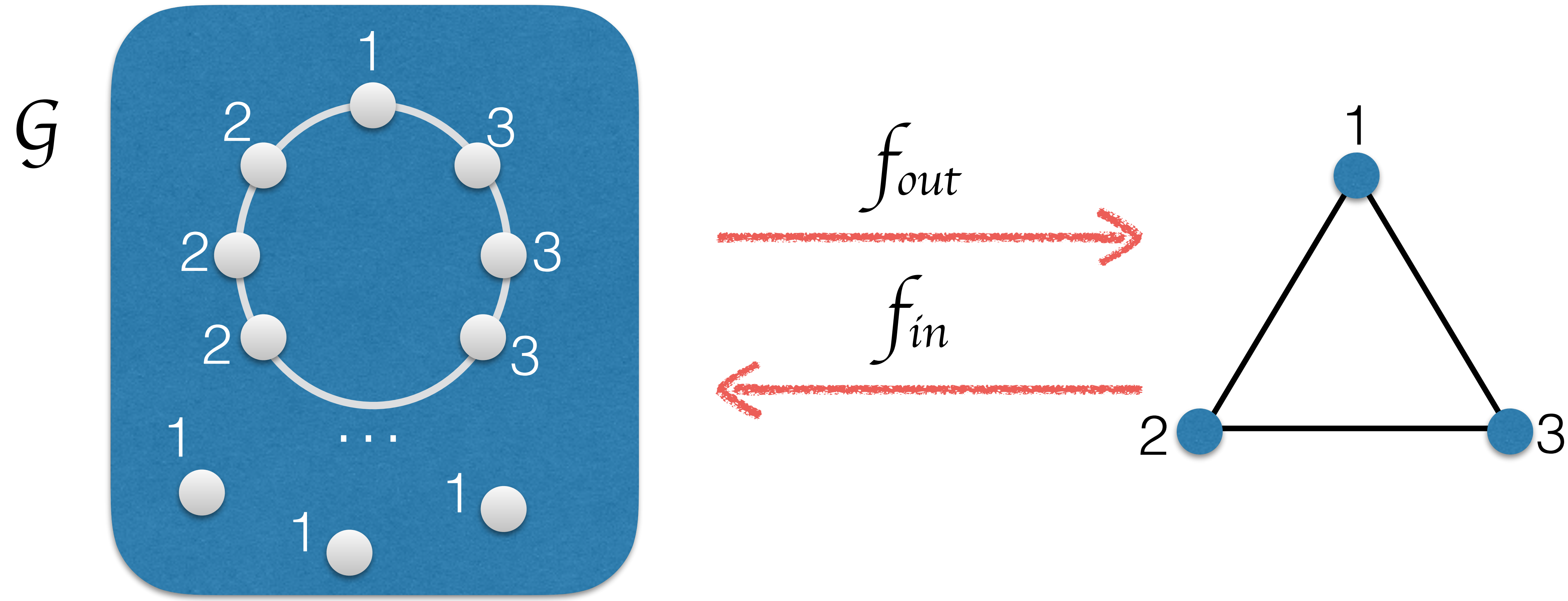
Solving Triangle from Cyclic Graphs



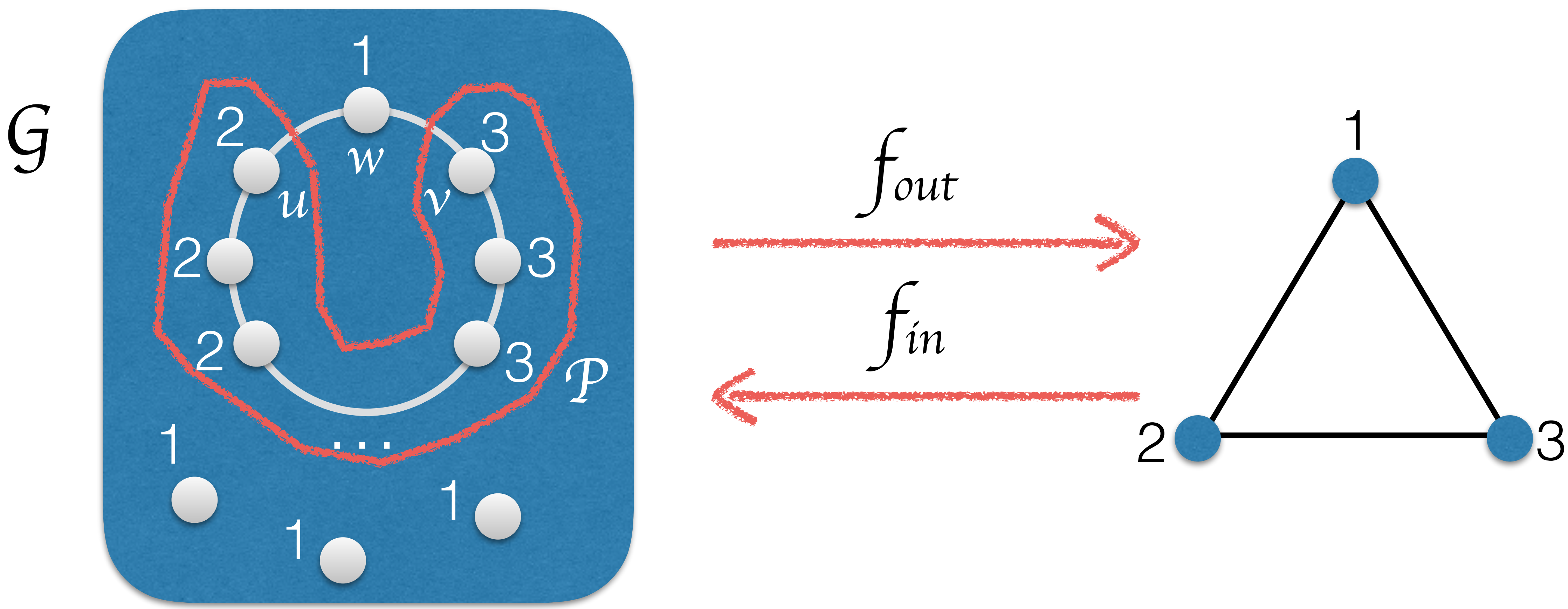
Solving Triangle from Cyclic Graphs



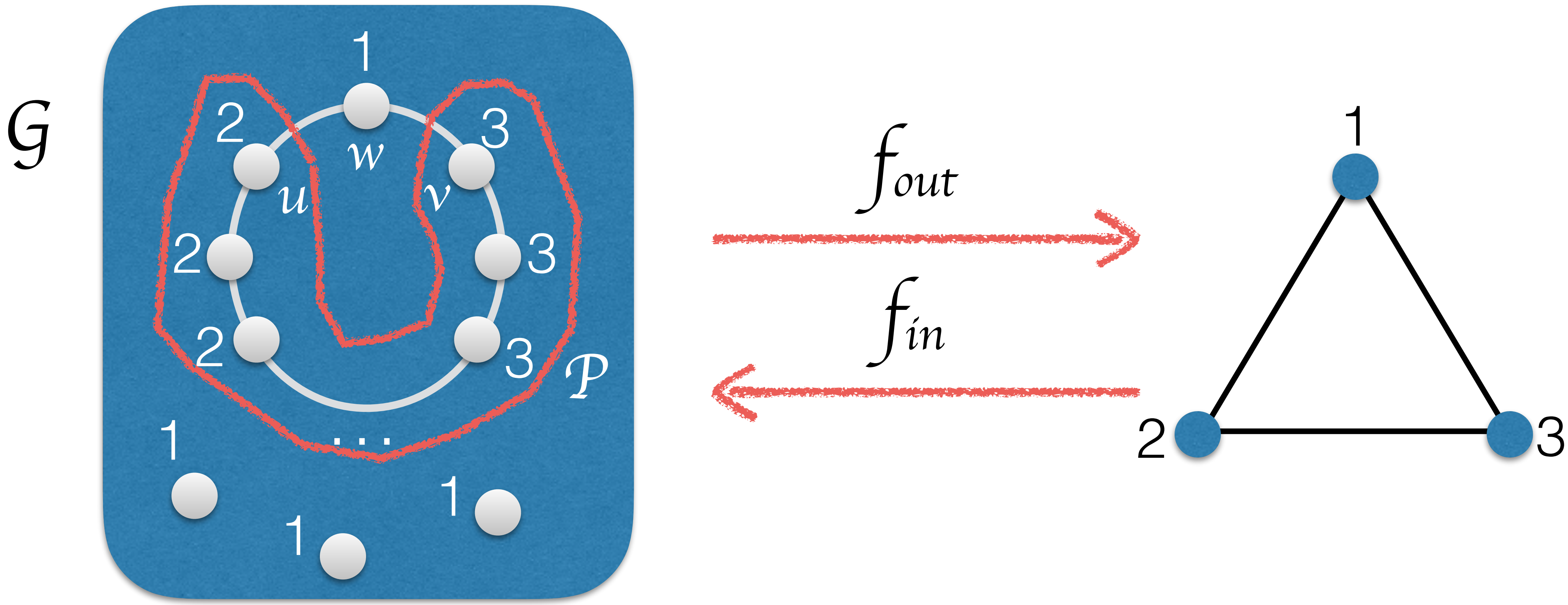
Solving Triangle from Cyclic Graphs



Solving Triangle from Cyclic Graphs



Solving Triangle from Cyclic Graphs



f$_{\text{in}}$(x):

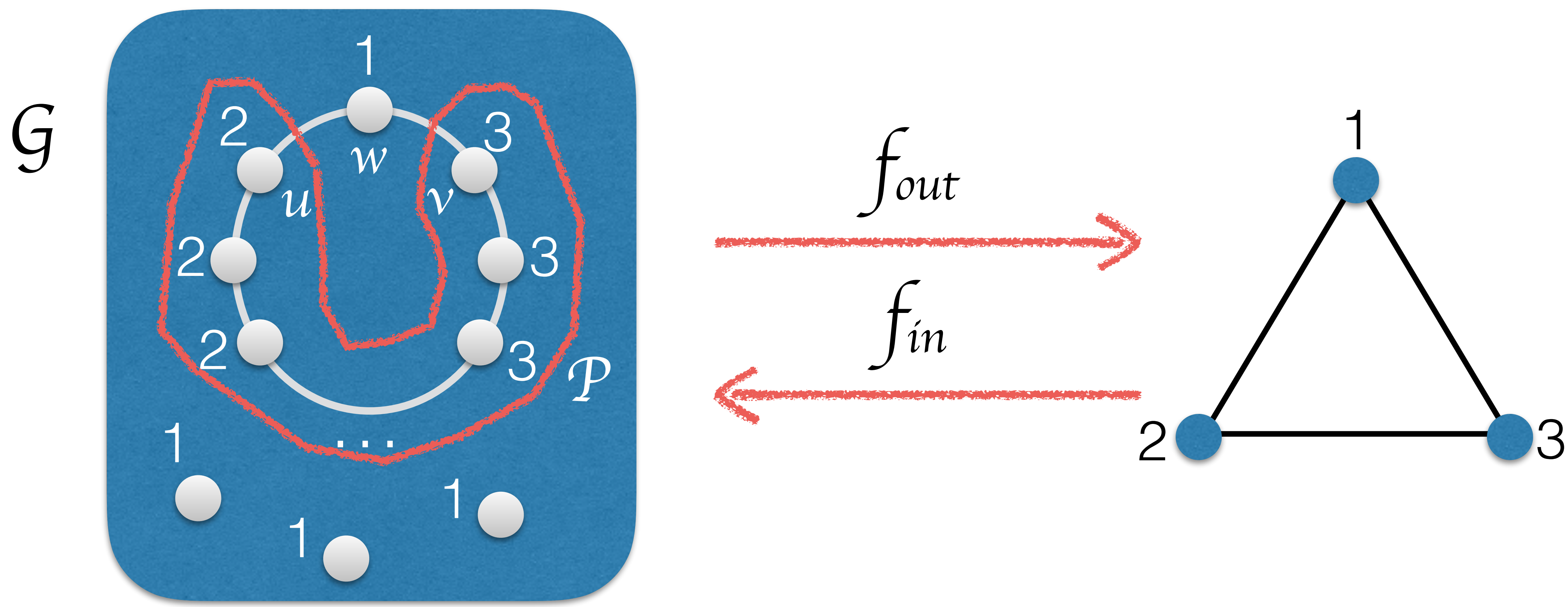
$\mathcal{A}$ = Edge gath. alg. on $\mathcal{P}$

if $x == 1$ **then return** w

elseif $x == 2$ then **return** $\mathcal{A}$.decide(u)

elseif $x == 3$ then **return** $\mathcal{A}$.decide(v)

Solving Triangle from Cyclic Graphs



$\mathrm{f_{in}}(x)$:

$\mathcal{A}$ = Edge gath. alg. on $\mathcal{P}$

if x == 1 **then return** w

elseif x == 2 then **return** $\mathcal{A}$.decide(u)

elseif x == 3 then **return** $\mathcal{A}$.decide(v)

EdgeGathTriangle(x):

$\mathcal{B}$ = Edge gath. alg. on $\mathcal{G}$

return f_{out}($\mathcal{B}$.decide(f_{in}(x)))

Let’s Do More

- **Edge Covering**:
  - **Termination.** Correct robots decide a vertex.
  - **Validity.** If participating robots start on the same vertex, they stay there. If start on an edge, decide vertices of the edge.
  - **Edge Covering.** If more than one decided vertex, decisions cover an edge.

Solvability of Edge Covering

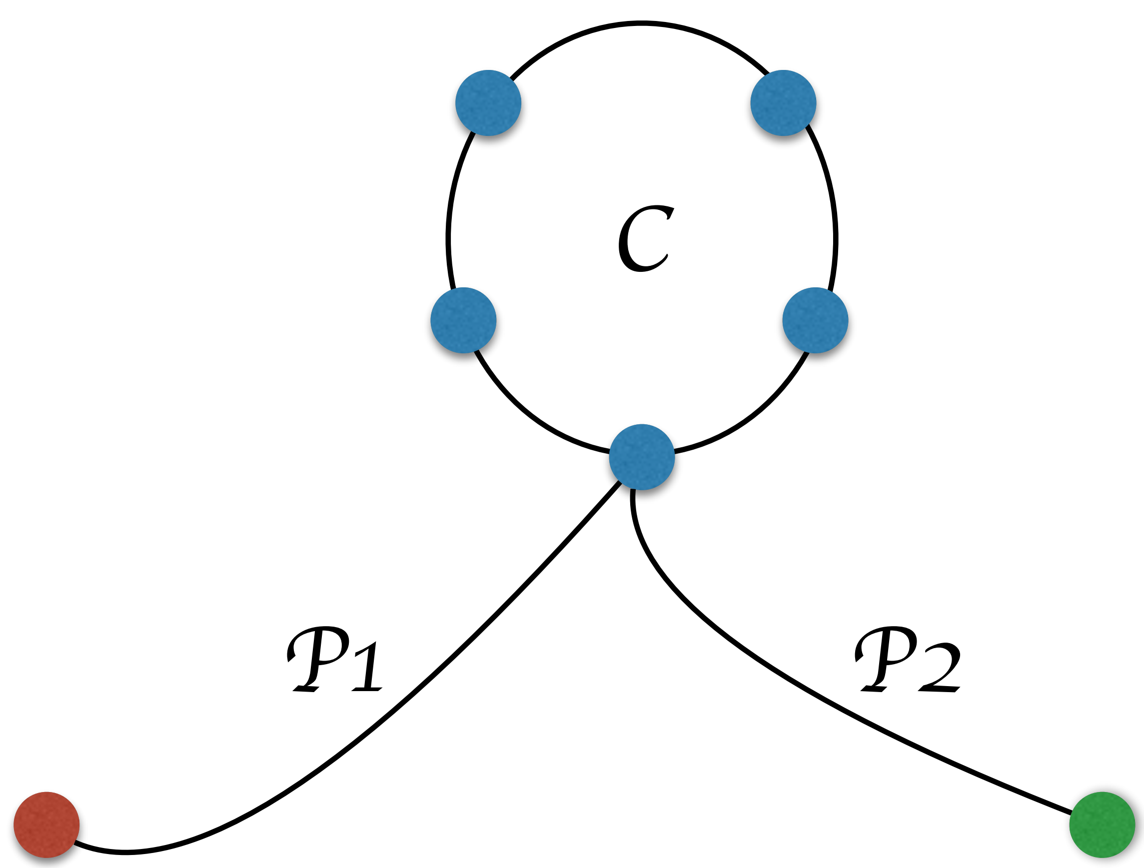
For n = 2, edge covering is solvable if and only if the base graph $\mathcal{G}$ is not bipartite

For n > 2, edge covering is imposible on every base graph $\mathcal{G}$

2-Robot Edge Covering Algorithm

For n = 2, if the base graph $\mathcal{G}$ is is not bipartite then edge covering is possible

There is an odd length path or cycle between any pair of nodes.

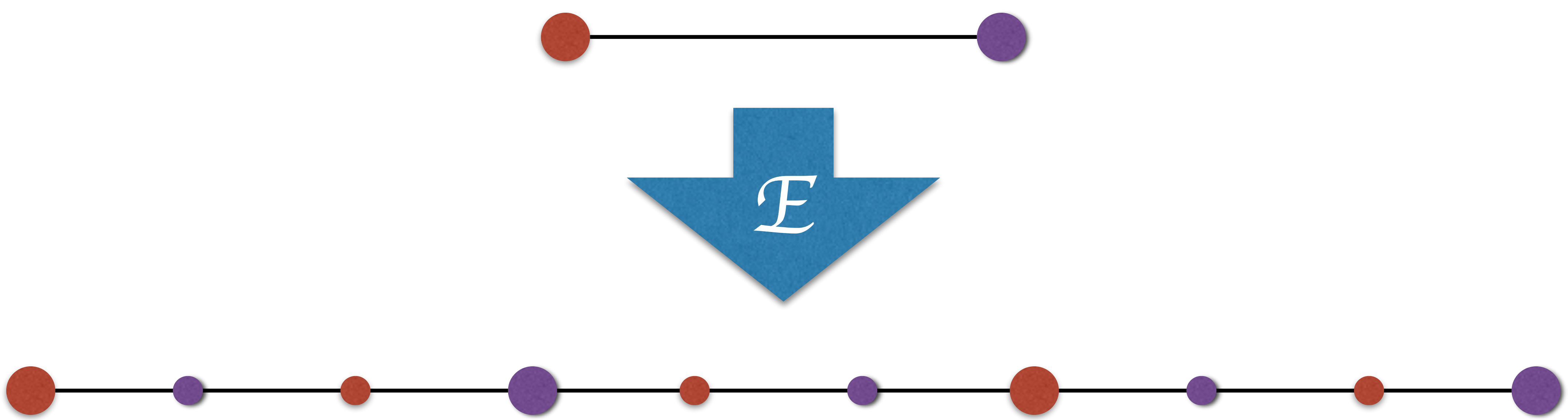


1) $|\mathcal{P}_1 — \mathcal{P}_2|$ is odd.
   Done

2) $|\mathcal{P}_1 — \mathcal{P}_2|$ is even.
   Take $\mathcal{P}_1 — \mathcal{C} — \mathcal{P}_2$.

2-Robot Edge Covering Algorithm

Protocol complex (path) can be mapped to those paths.
Why? Length of the complex (path) is odd.



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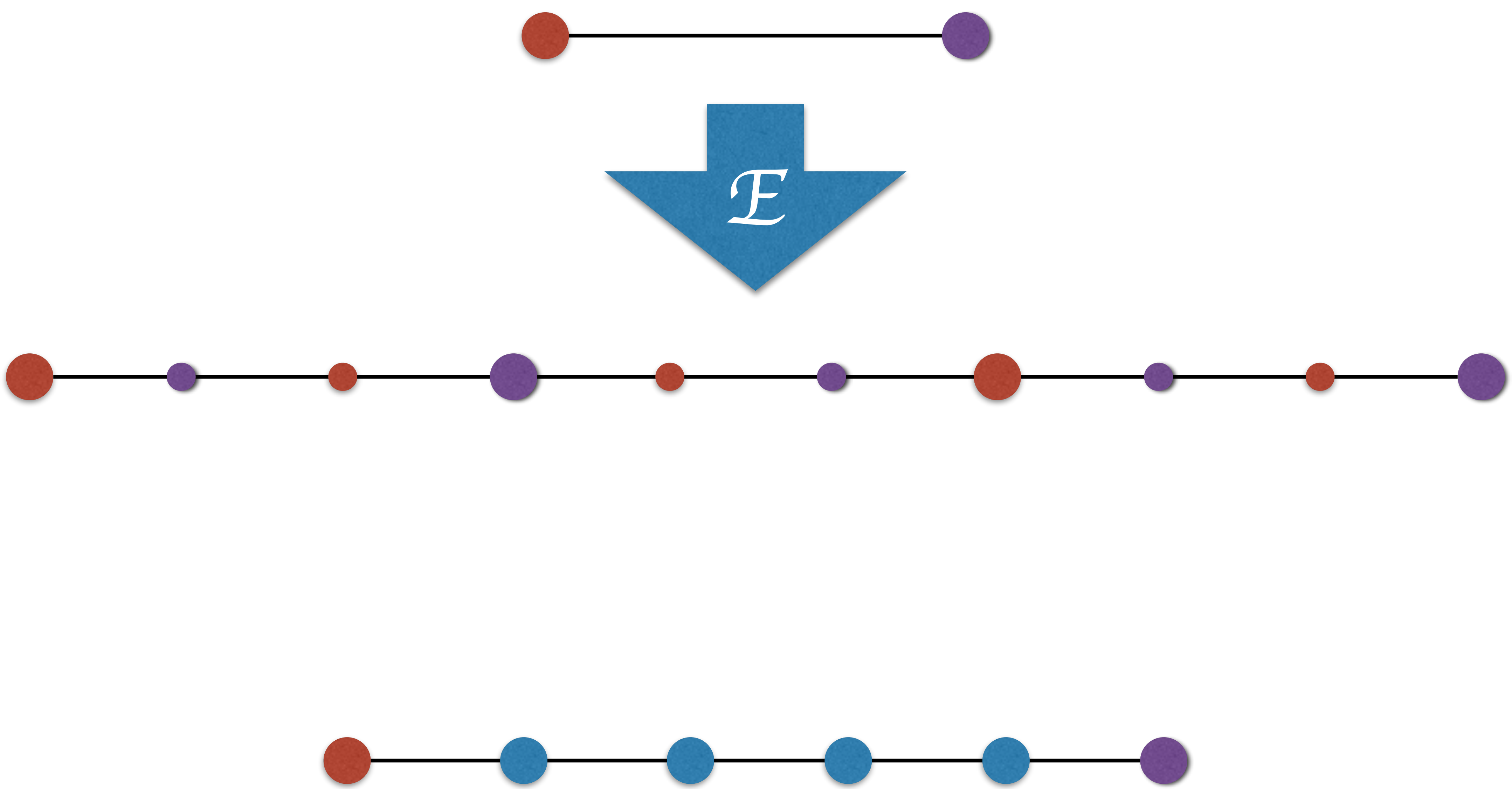

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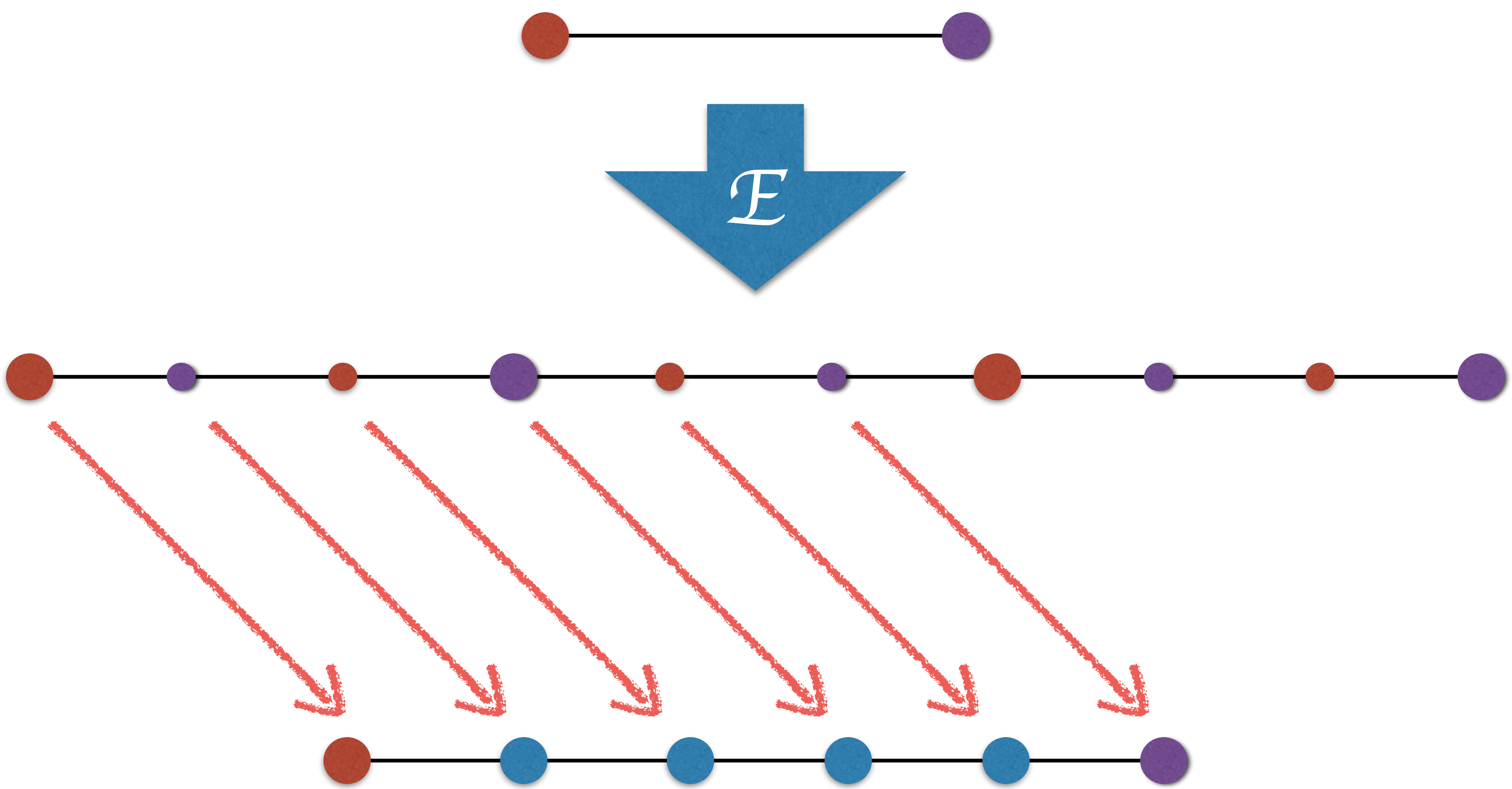

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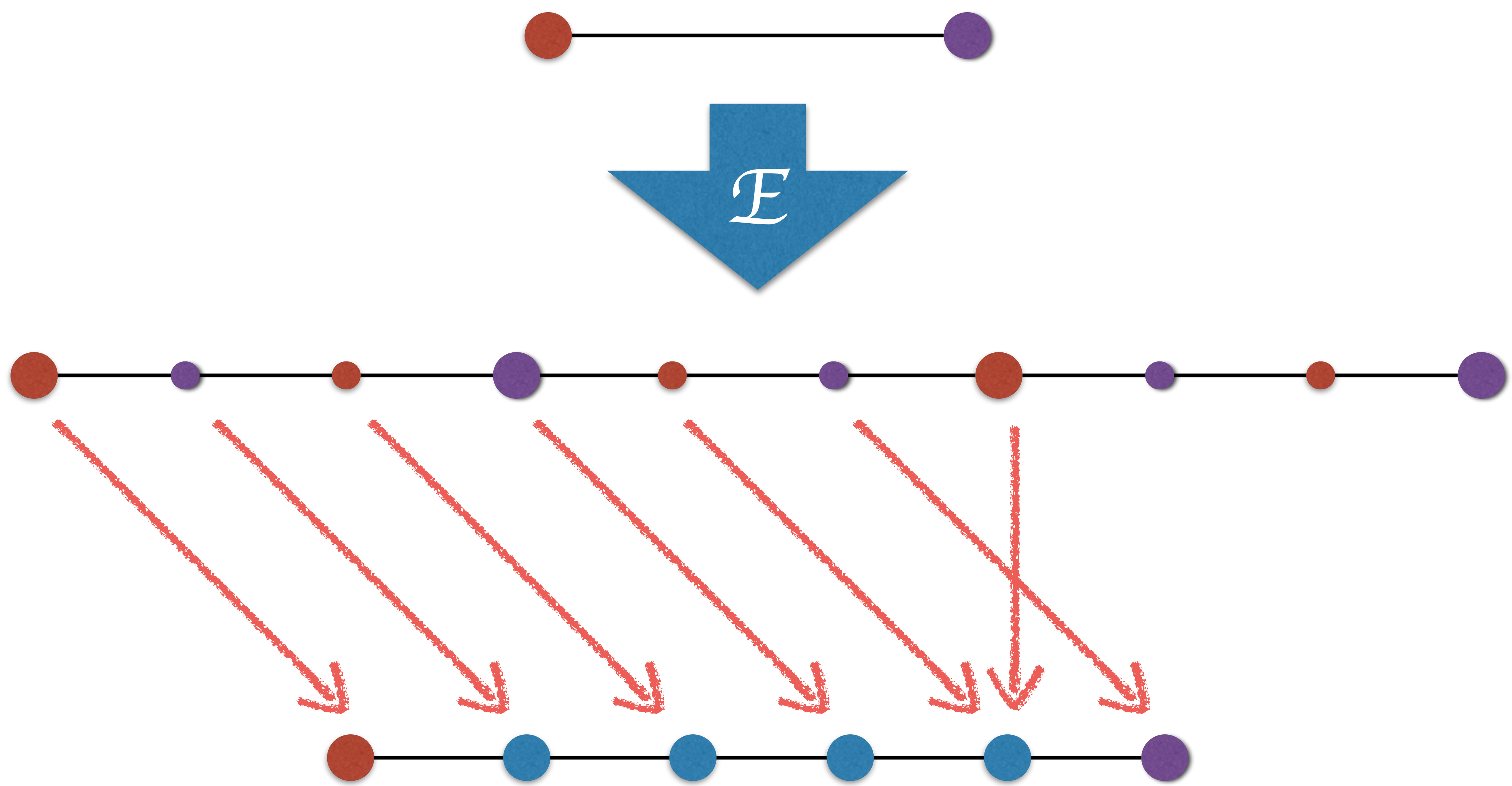

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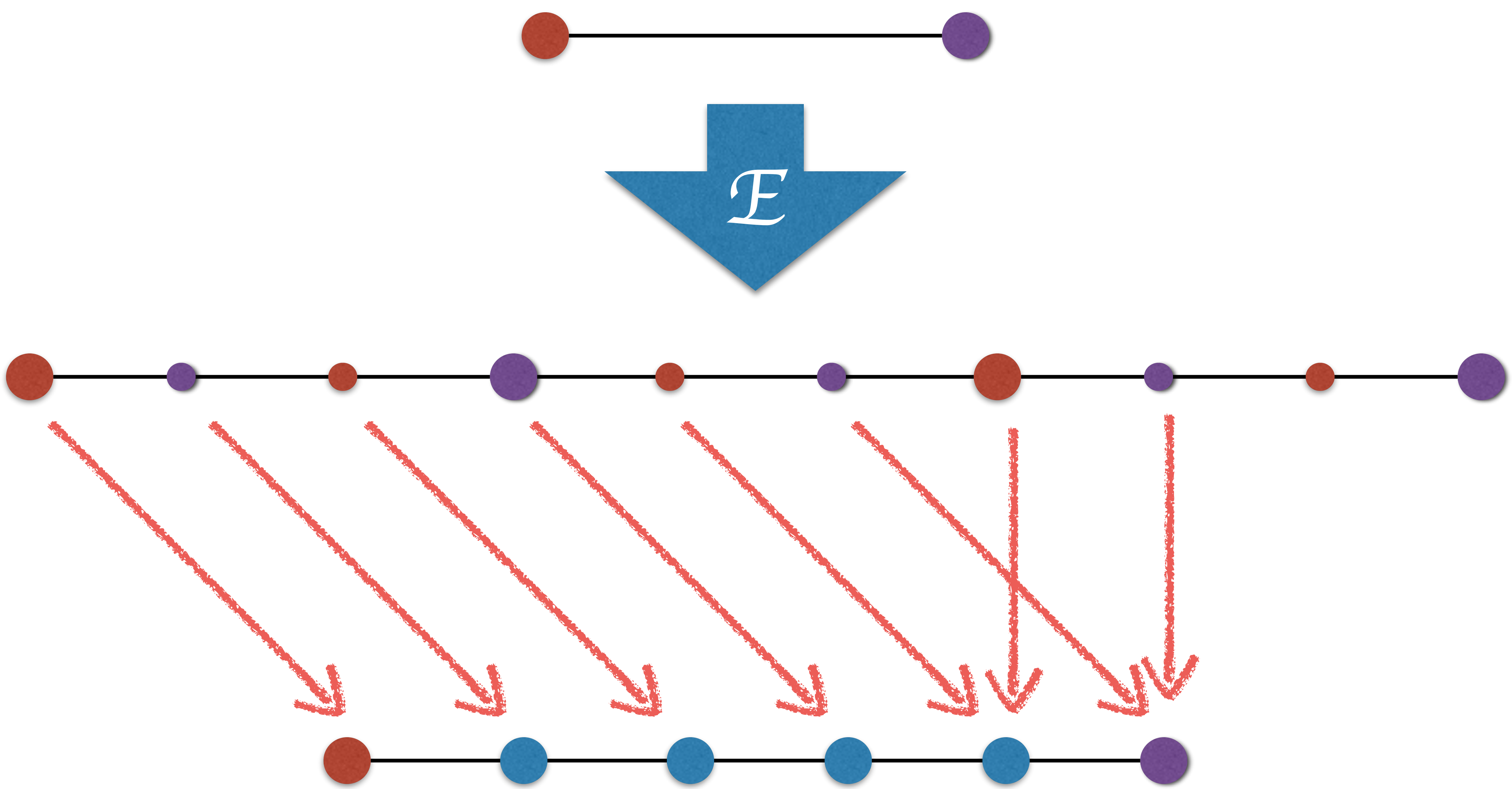

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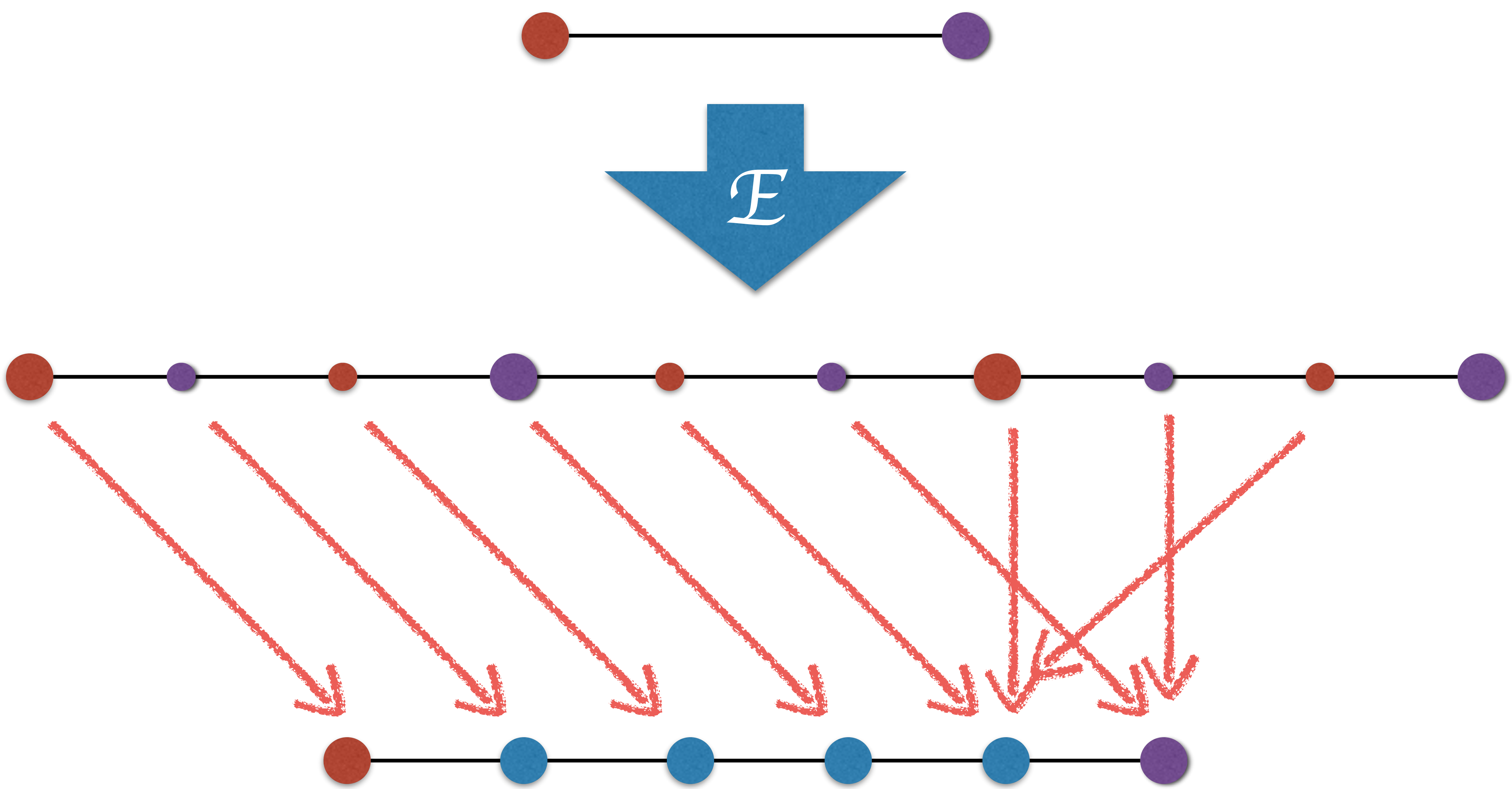

2-Robot Edge Covering Algorithm

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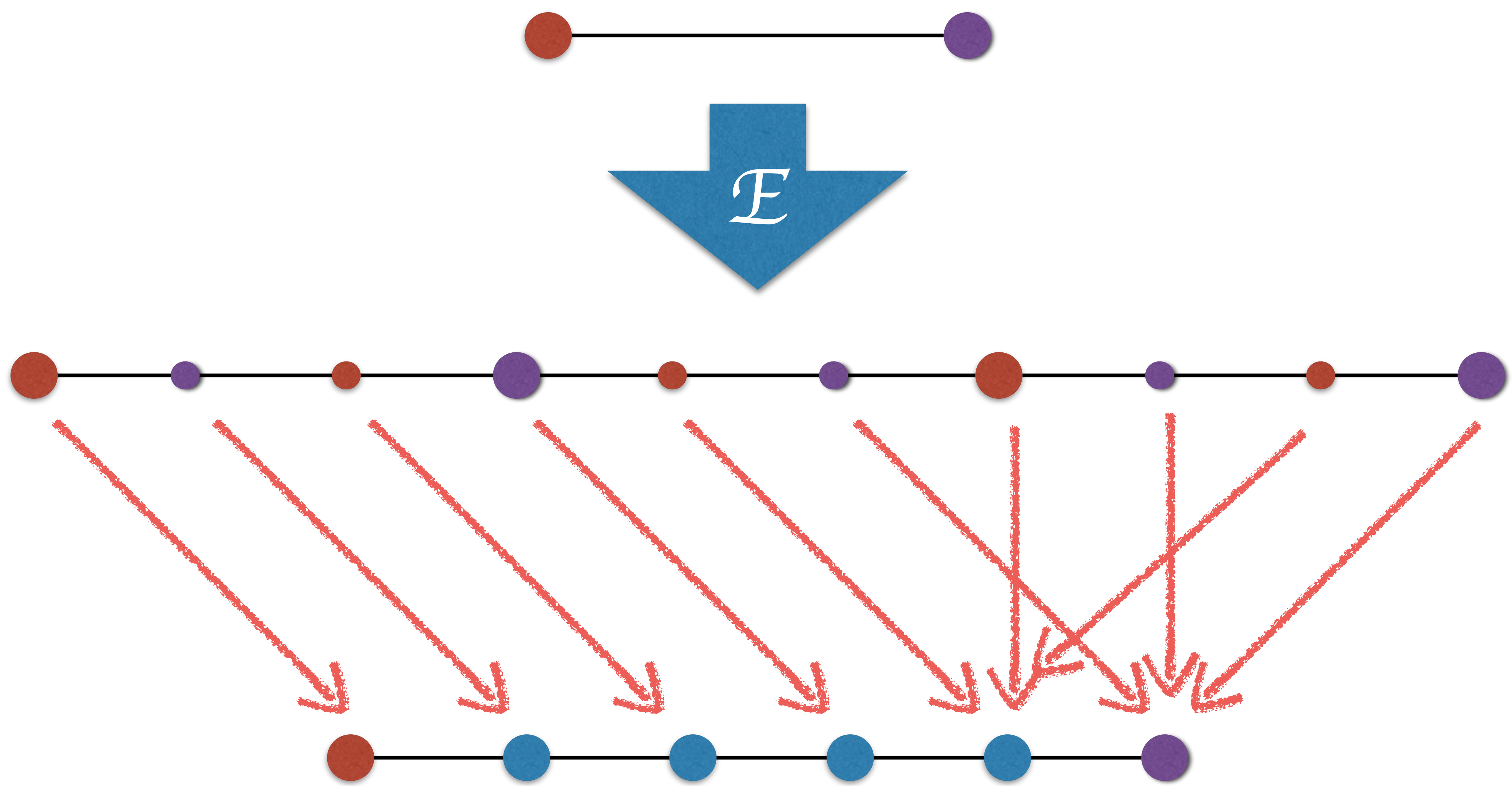


2-Robot Edge Covering Impossibility

For n = 2, if the base graph $\mathcal{G}$ is bipartite, then edge covering is impossible

- Bipartite => for some pair, there is no odd length path or cycle
- Protocol complex cannot be mapped to even length paths: Endpoints to endpoints and edges to edges.

2-Robot Edge Covering Impossibility

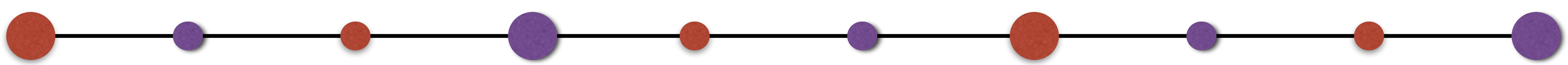
For n = 2, if the base graph $\mathcal{G}$ is bipartite, then edge covering is impossible

Proof:

1. Prove the edge is impossible.
2. Solve the edge from any bipartite graph.

Impossibility of Edge Covering on Edge

What if the two robots start on the same vertex?



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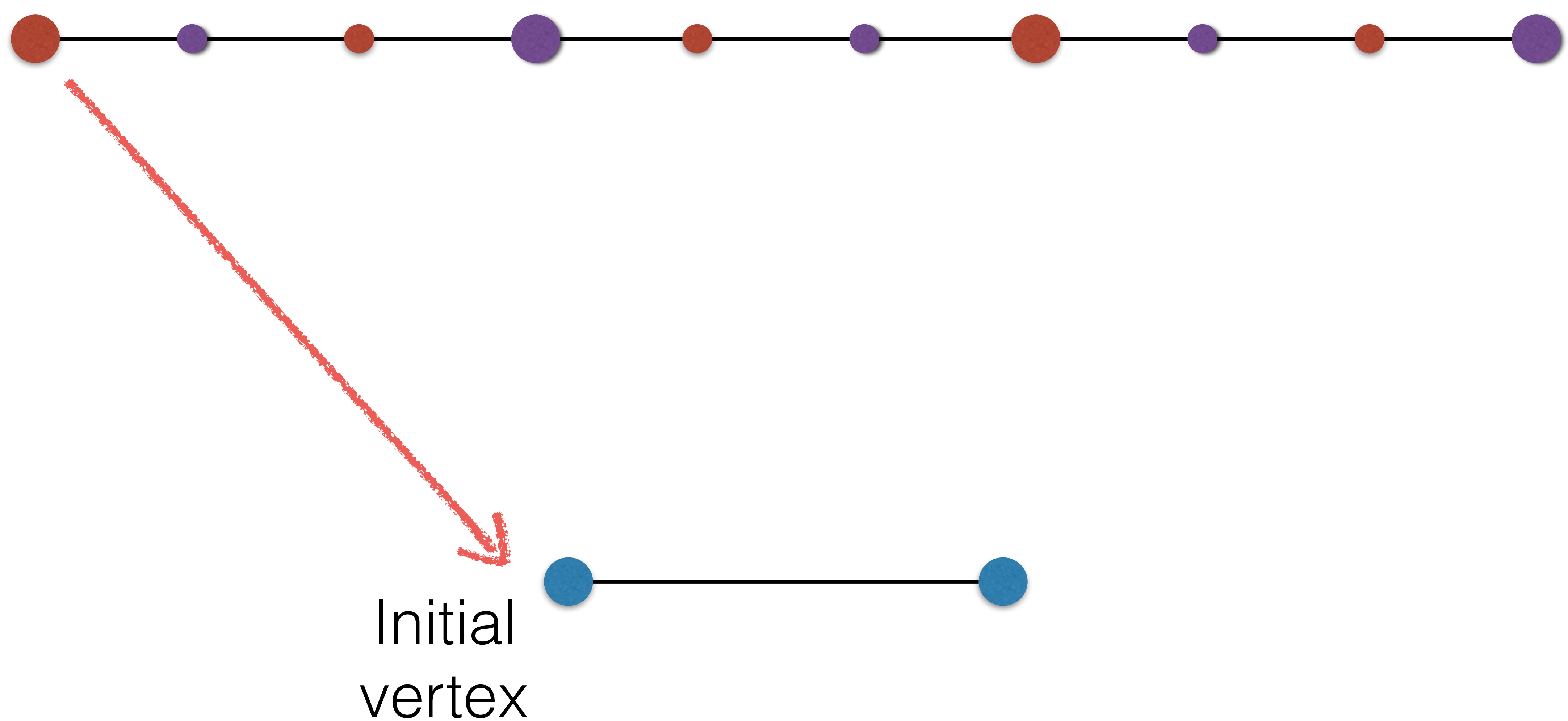

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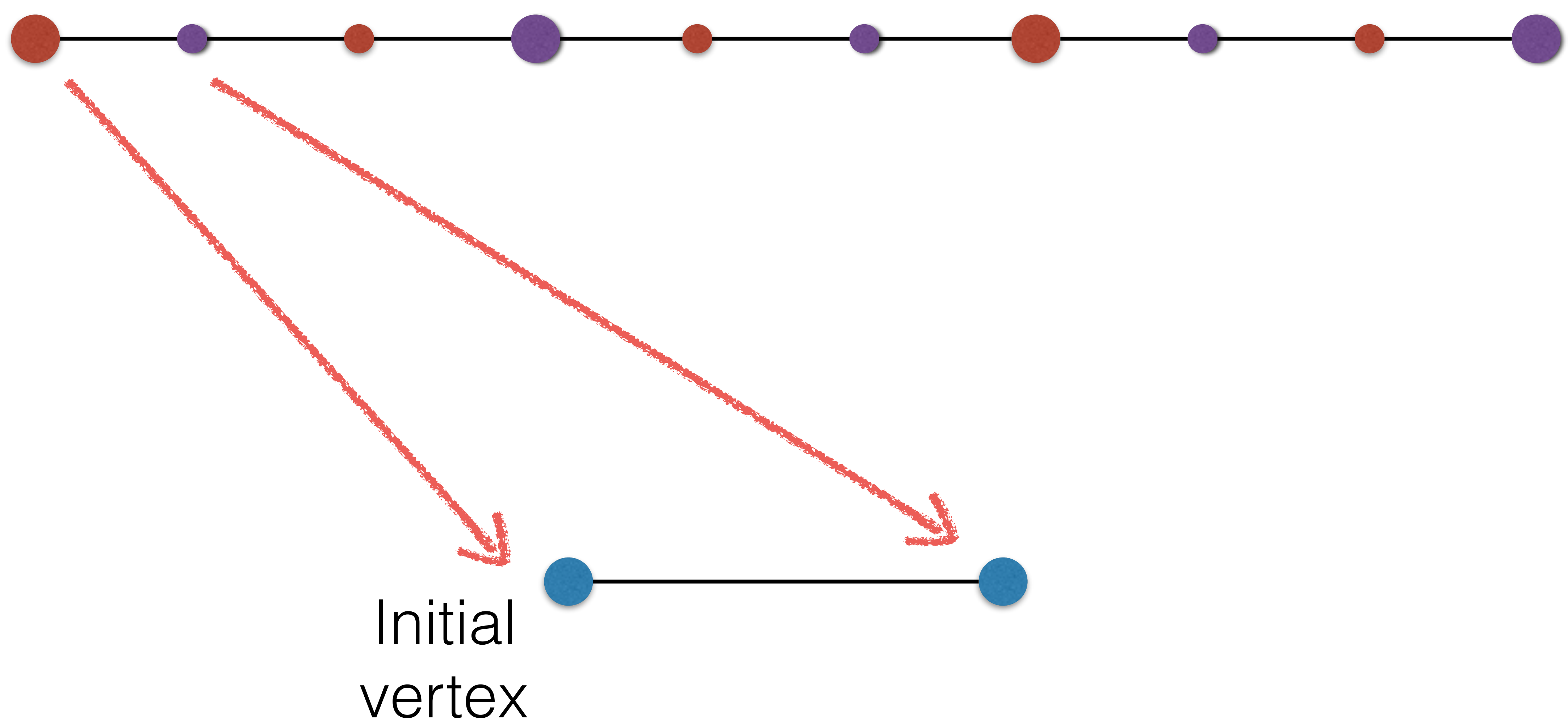

Impossibility of Edge Covering on Edge

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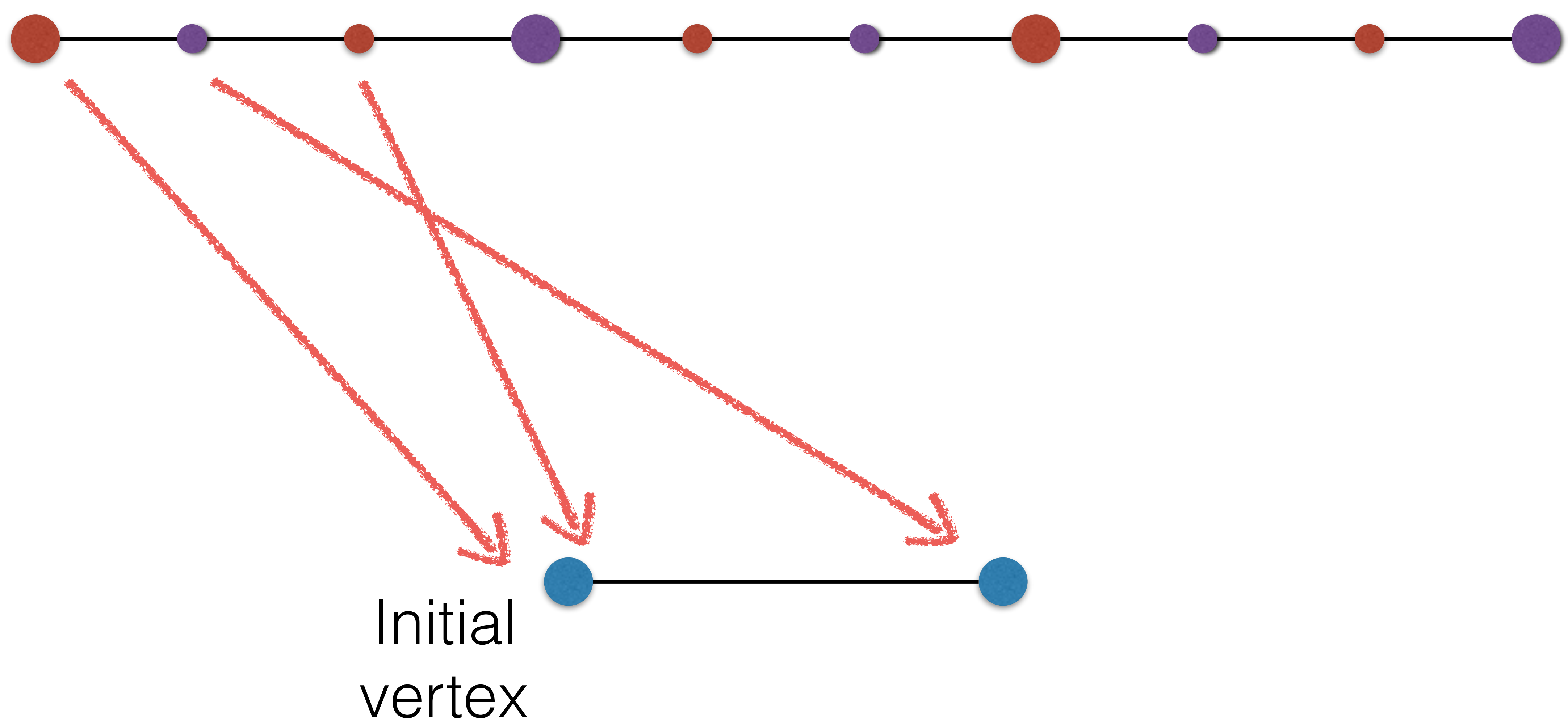

Impossibility of Edge Covering on Edge

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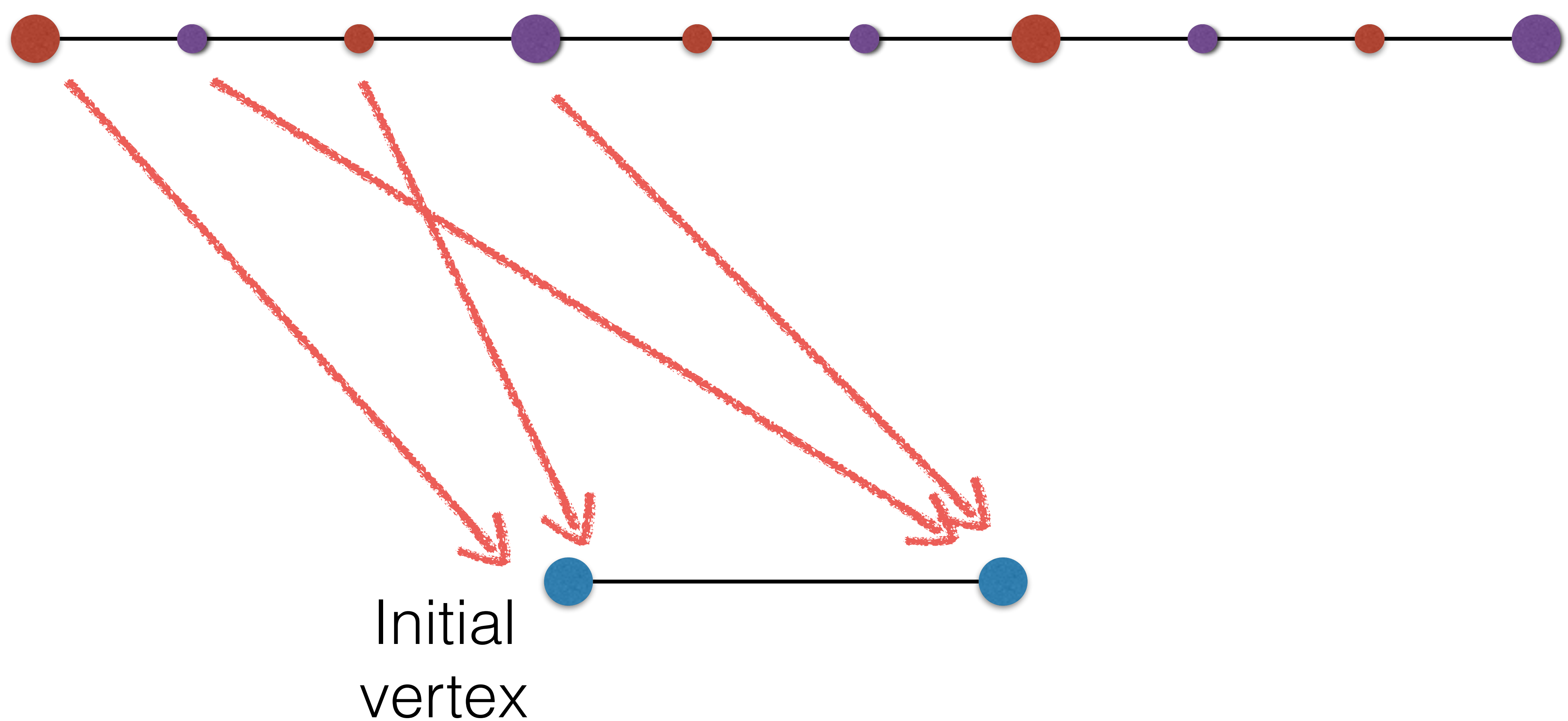

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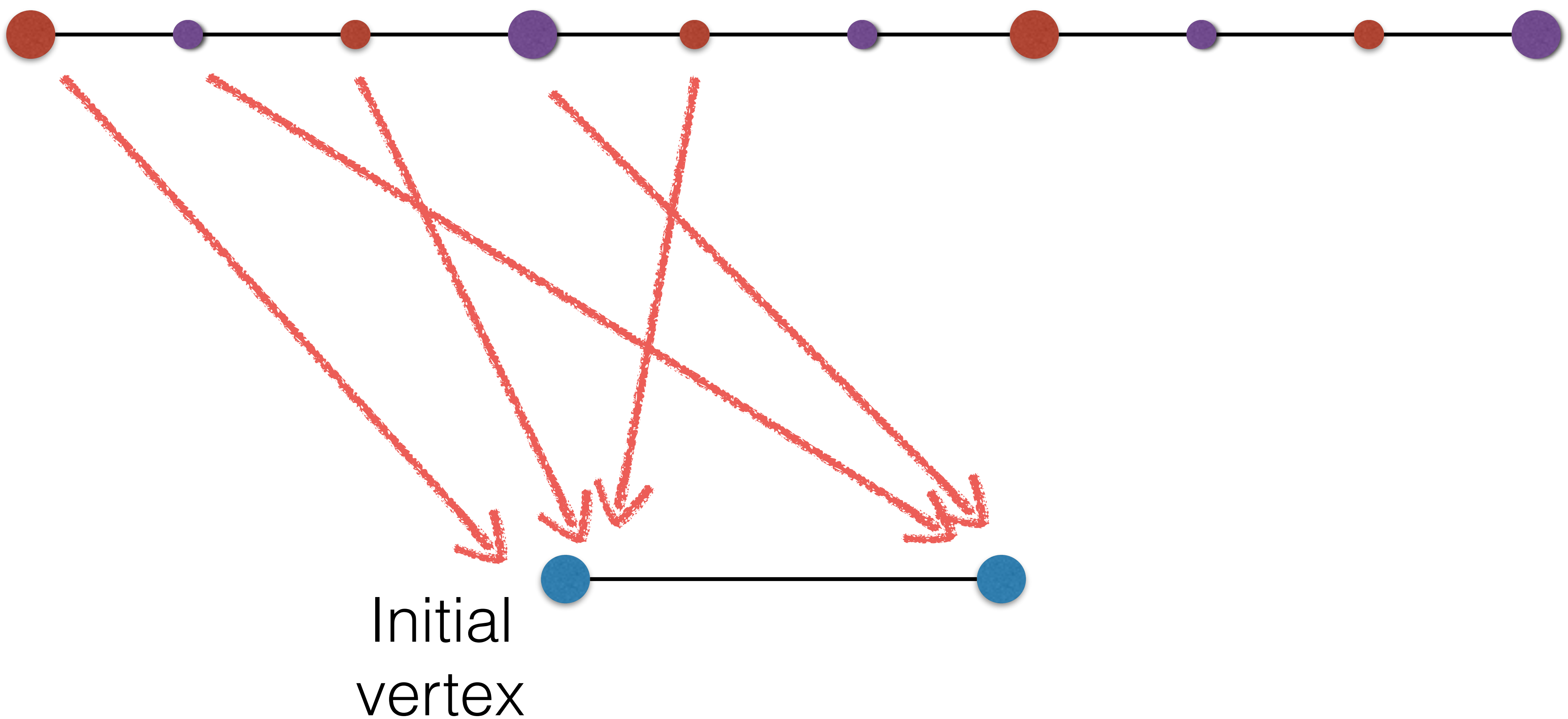

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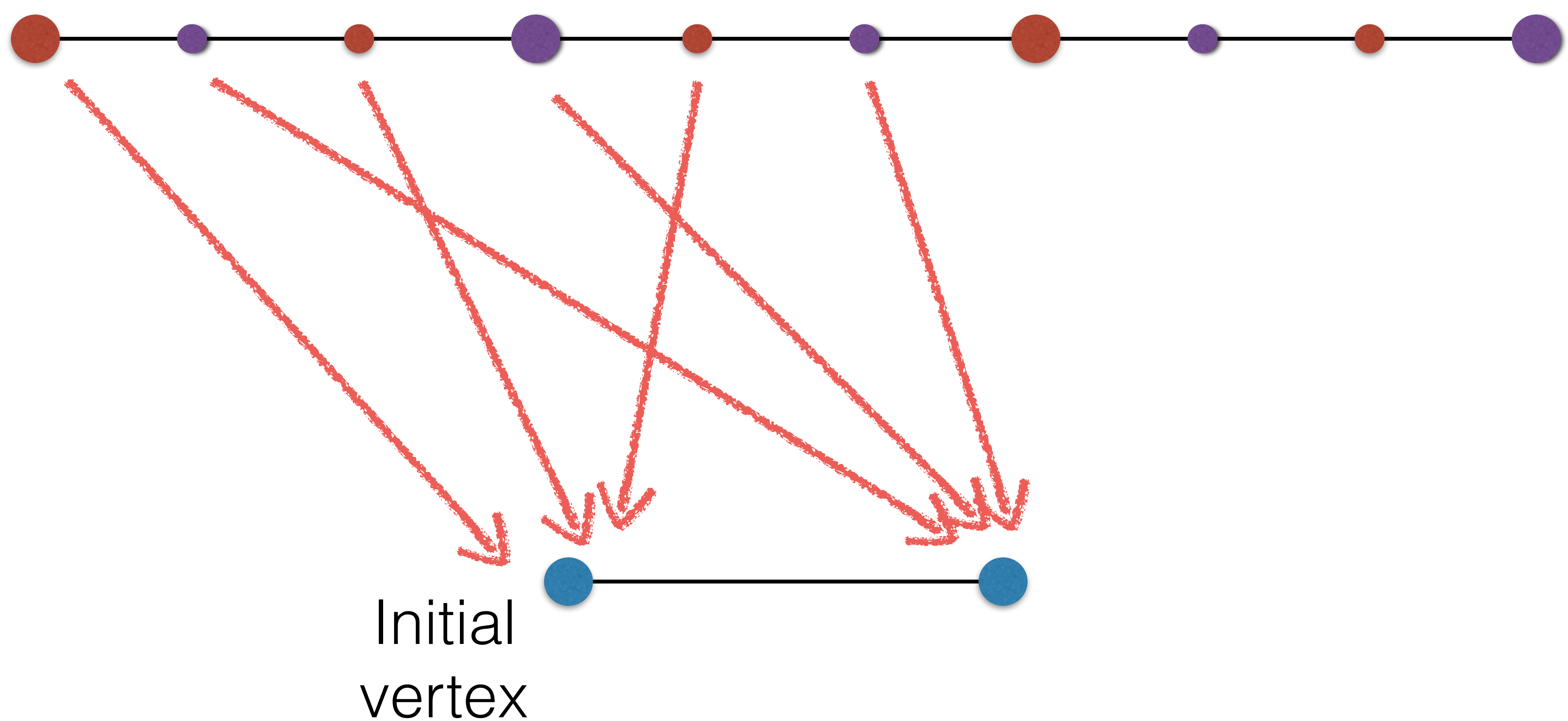

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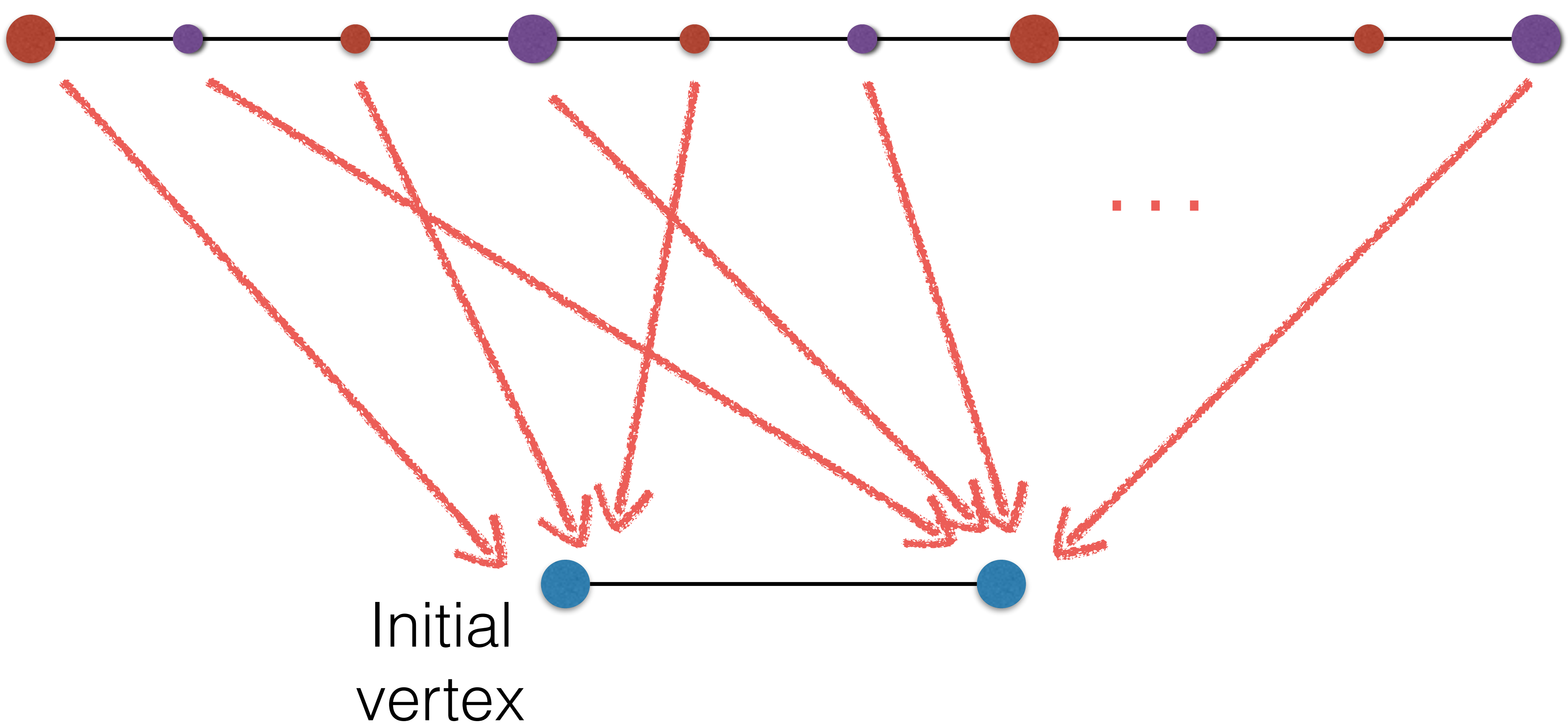

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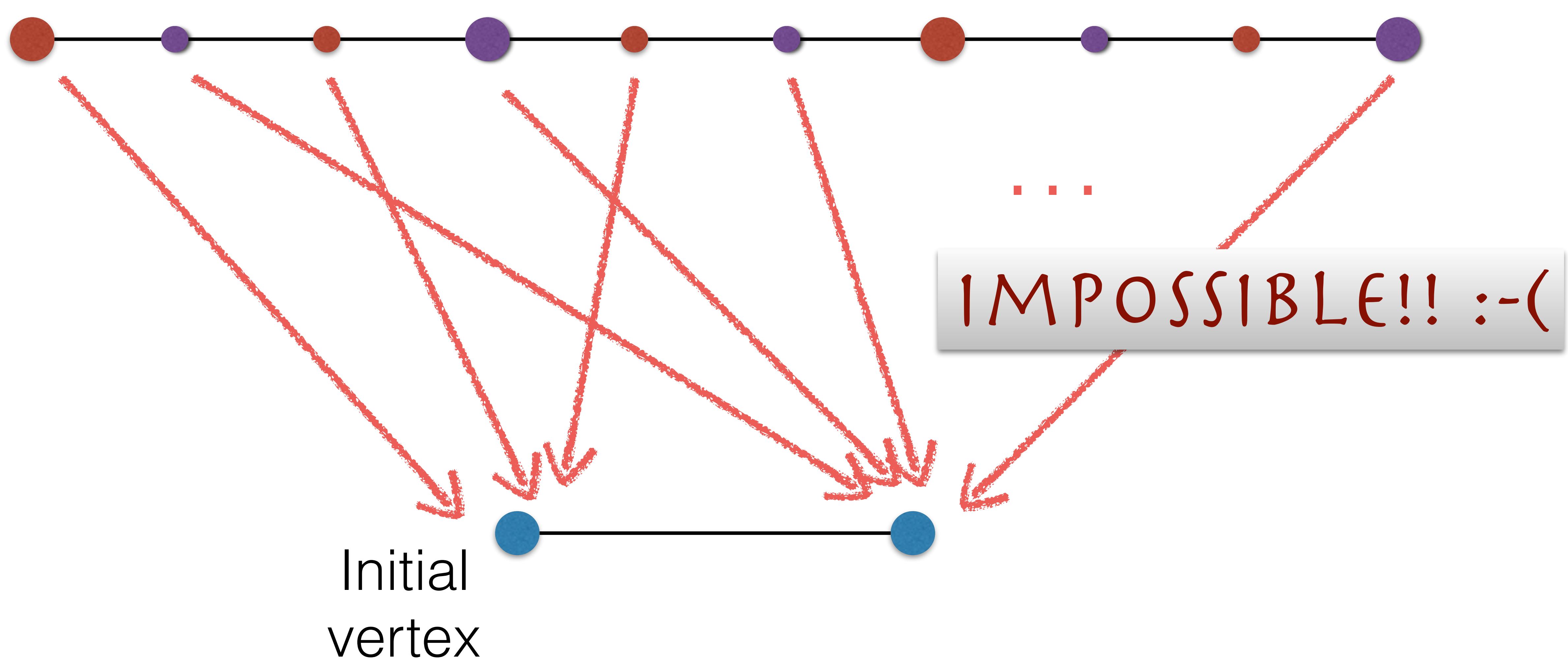
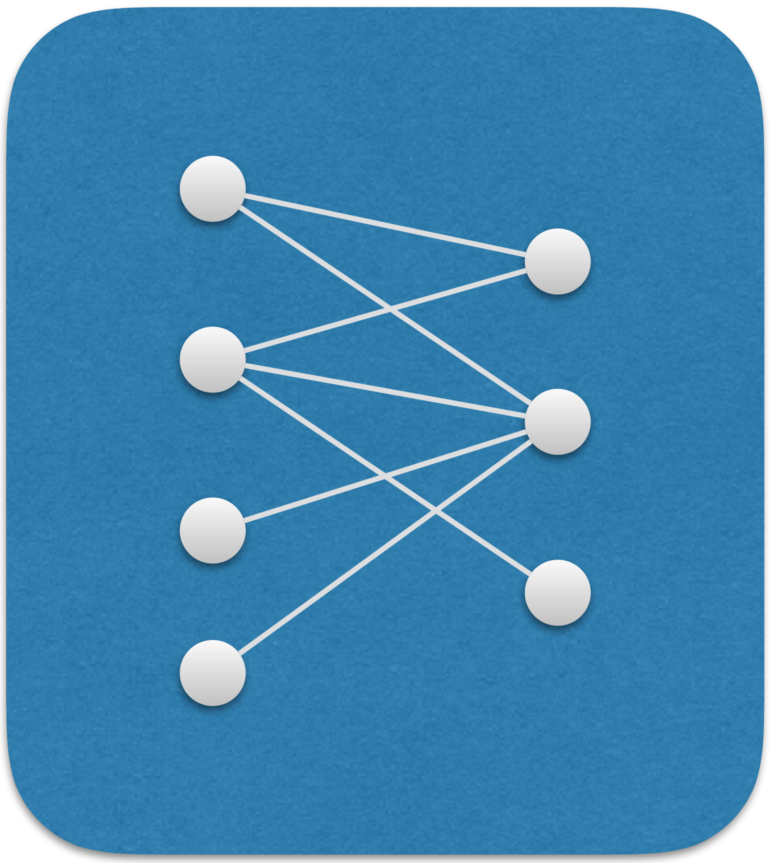
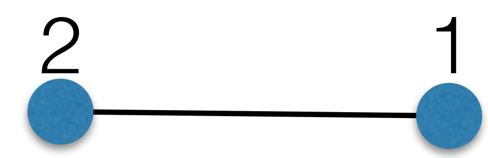


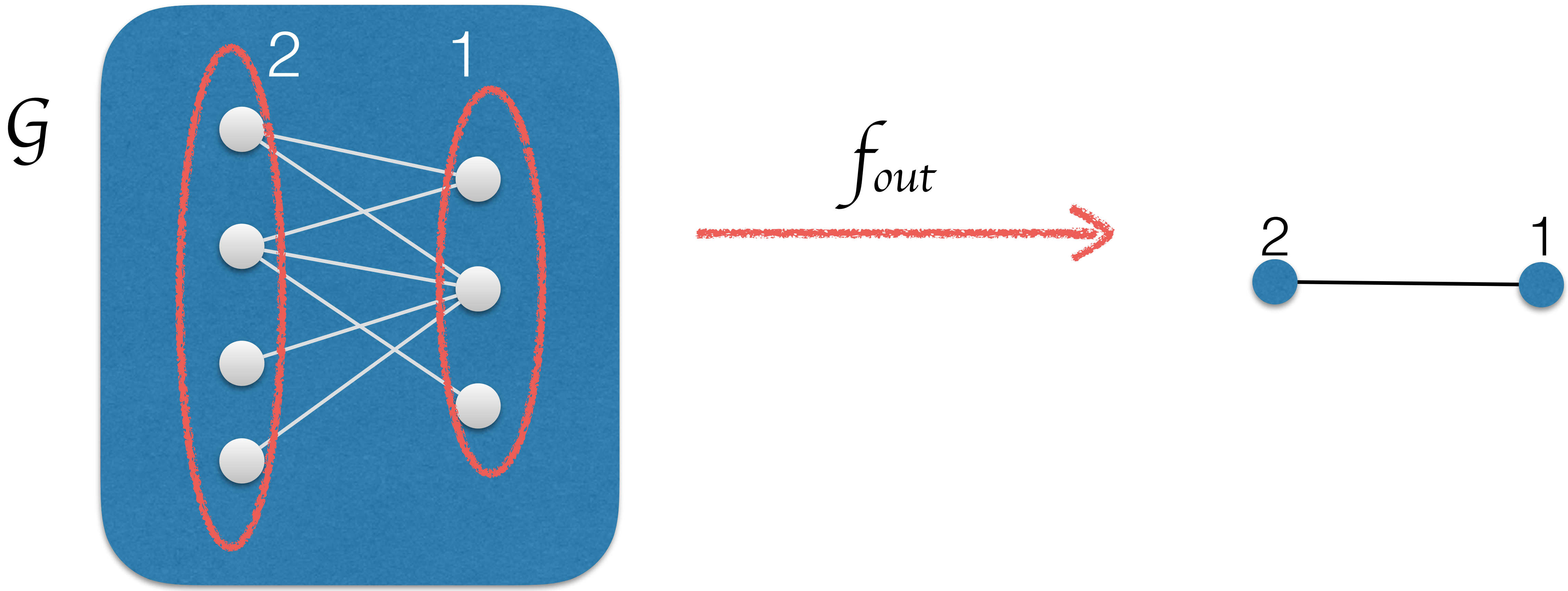
Solving Edge from Bipartite Graphs

$\mathcal{G}$





Solving Edge from Bipartite Graphs



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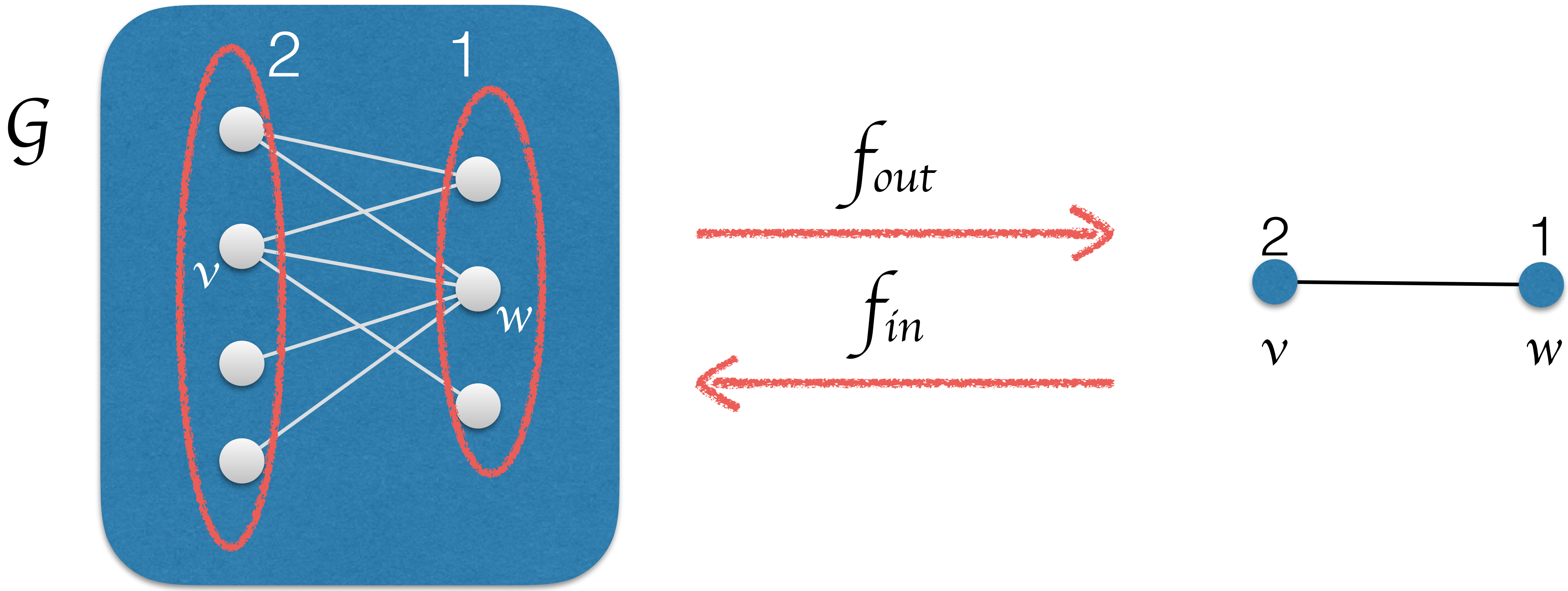
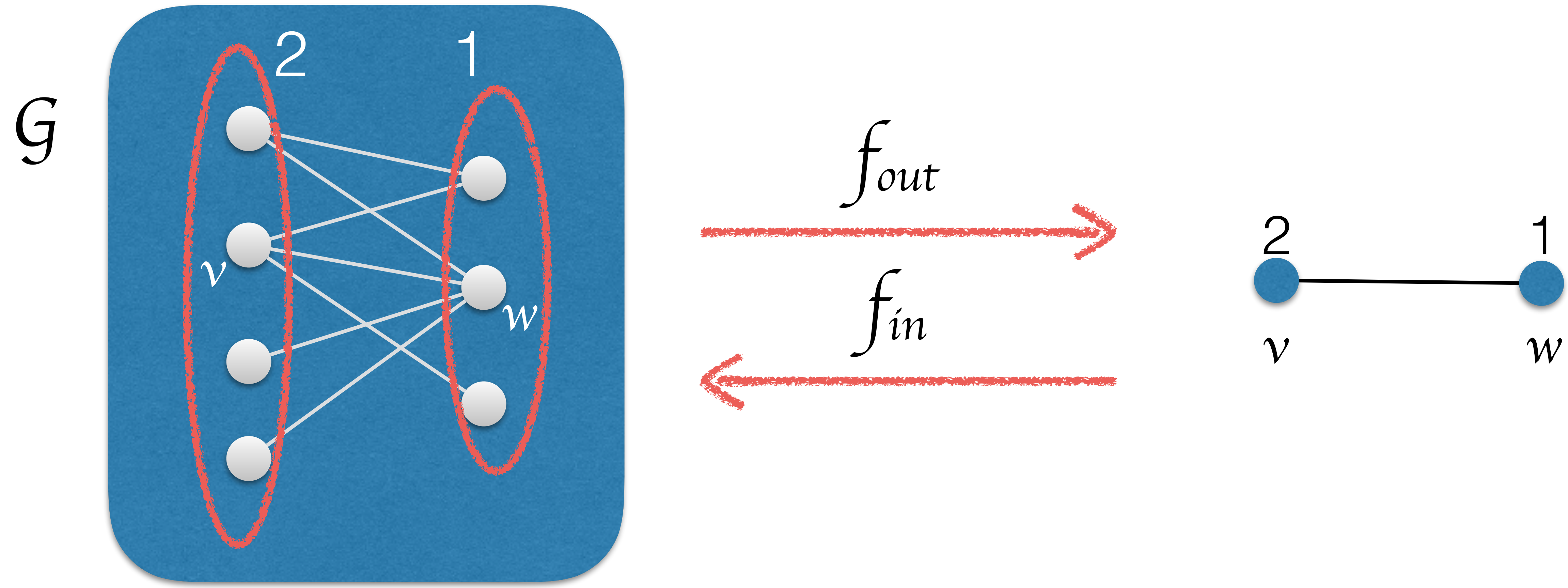


Solving Edge from Bipartite Graphs



EdgeCoveringEdge(x):

$\mathcal{B}$ = Edge covering alg. on $\mathcal{G}$

return $f_{out}(\mathcal{B}.\text{decide}(f_{in}(\text{x})))$

Edge Covering Impossibility

For n > 2, edge covering is imposible on every base graph $\mathcal{G}$

Proof:

1. Suppose there is an edge covering algorithm $\mathcal{A}$ on $\mathcal{G}$.
2. $\mathcal{A}$ solves 2-robot edge covering on $\mathcal{G}$.
3. $\mathcal{G}$ is not bipartite => $\mathcal{G}$ has cycles.
4. $\mathcal{A}$ solves edge gathering on $\mathcal{G}$ for n > 2 robots. Contradiction!!

Summary

1. Gathering. Impossible
2. Edge Gathering:
    - For $n=2$, solvable on any graph.
    - For $n>2$, solvable if an only if acyclic.
3. Edge Covering:
    - For $n=2$, solvable if an only if not bipartite.
    - For $n>2$, impossible.

ALR = R/W Wait-Free

A task (maybe non-colorless) is solvable in ALR if and only if it is solvable in Async. R/W Wait-Free

Reduction based proofs:

1. Same connectivity properties.
2. Gathering => Consensus
3. Edge Gathering => 2-Set Consensus
4. Edge Covering => WSB