Quantum Walking in Curved Spacetime

Pablo Arrighi

Joint work with S. Facchini, M. Forets.

[PA, S. Facchini, M. Forets, *“Quantum Walking in Curved Spacetime”*, QINP, arXiv:1505.07023]
[PA, S. Facchini, *“Quantum Walking in Curved Spacetime: 3+1-dimensions, and beyond”*, QIC, arXiv:1609.00305]

What's in the tin?

A stable numerical scheme for PDEs of the form

$$i\partial_0\psi = H\psi$$

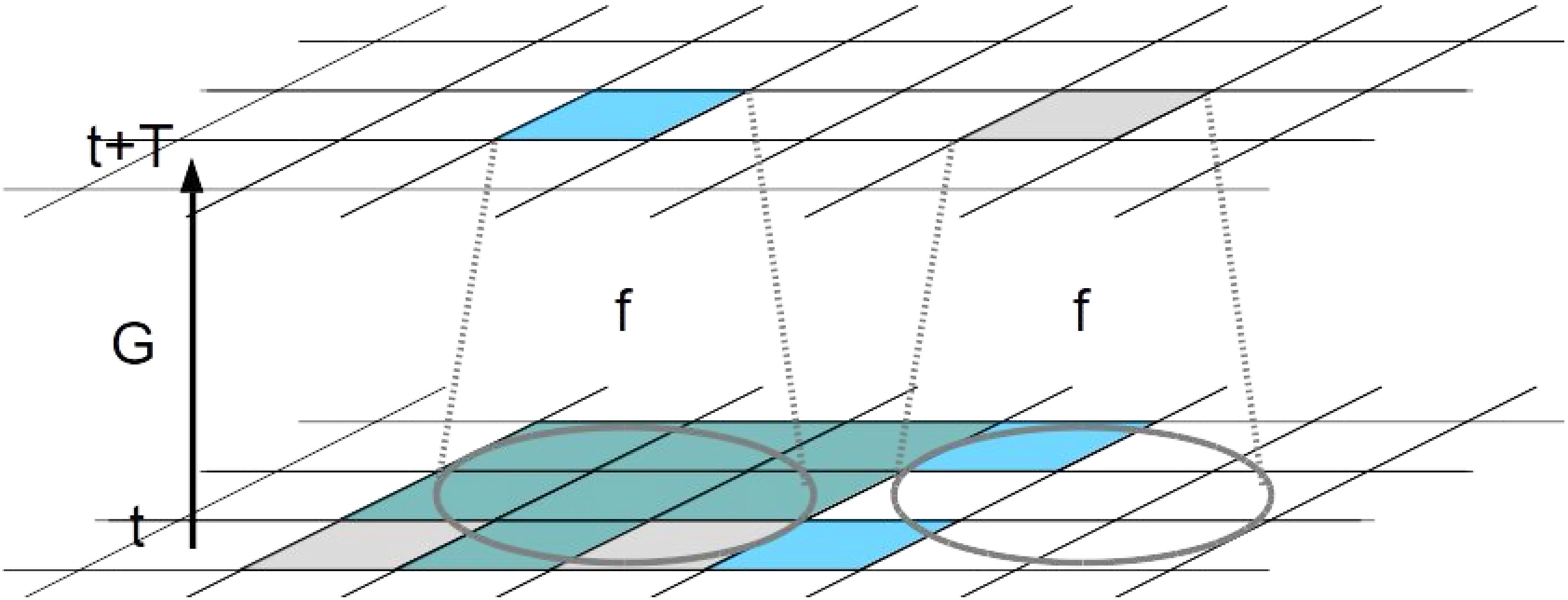
$$H = i\sum_{k=1\ldots d}\left(B_k\partial_k + \frac{1}{2}\partial_k B_k\right) - C$$

$$(\text{with } B_k,\, C \in \text{Herm}(\mathbb{C}) \text{ and } |B_k| \leq 1)$$

implementable by applying unitary matrices locally

i.e. by future quantum simulation devices.

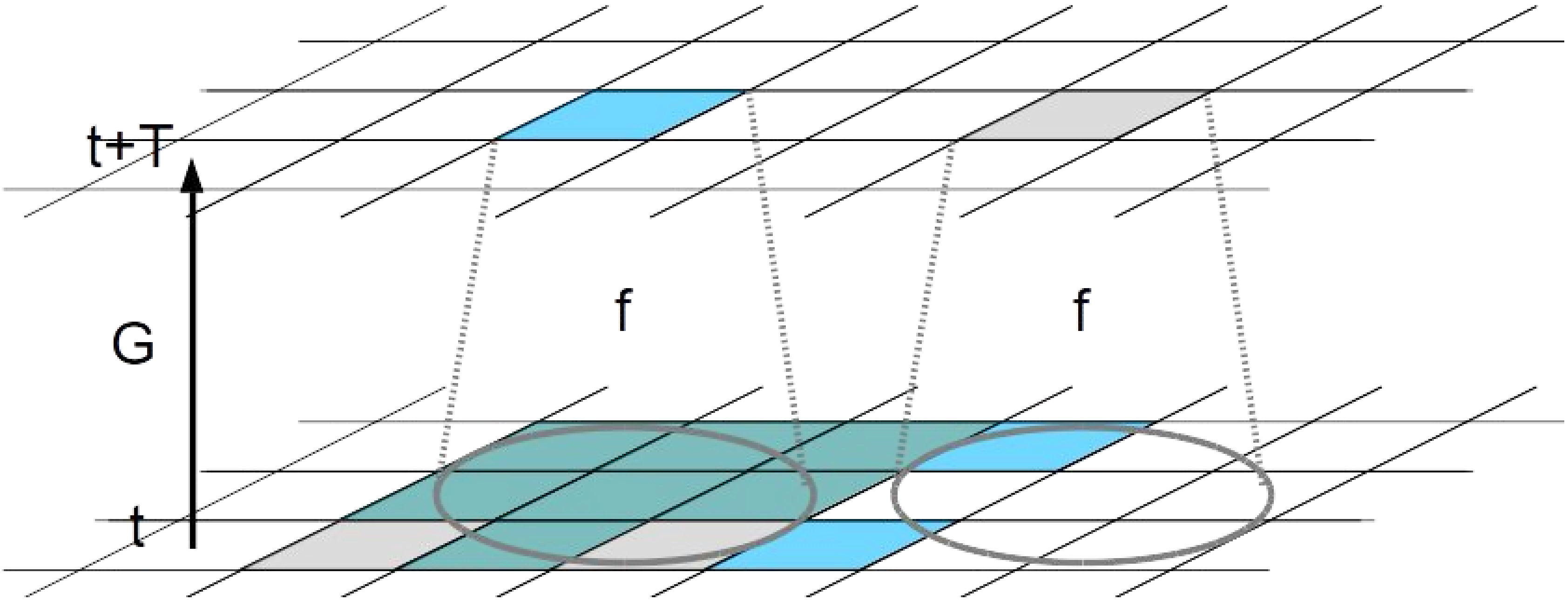
Discretize physics?



Cellular Automata

An old CompSci dream : to capture physics in this formalism.

Discretize physics?



… as Cellular Automata / Quantum Walks

Theorems about : the extent in which physics particles can be captured in this formalism.

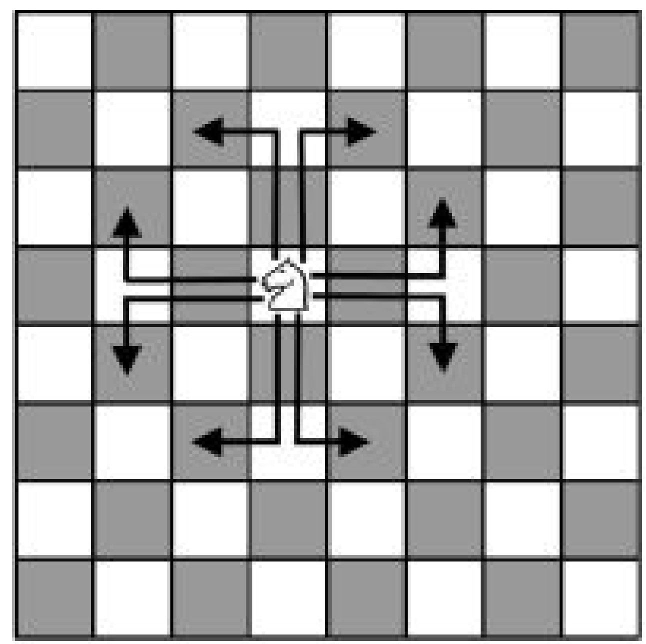
Discretize particules

Dirac equation

$$i\partial_0 \psi = D\psi, \quad \text{with} \quad D = m\alpha^0 - i\sum_j \alpha^j \partial_j$$

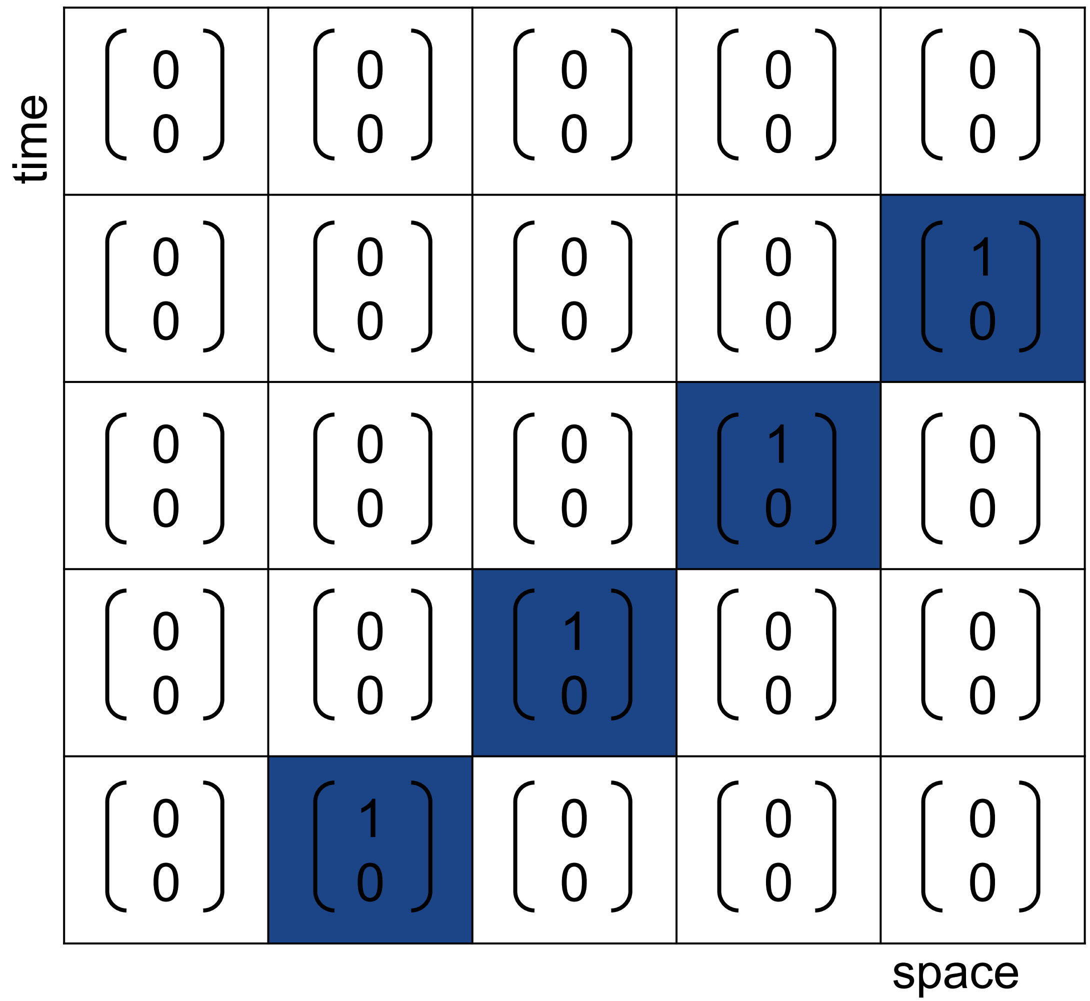
vs

Chess game



Chess game : neutrino

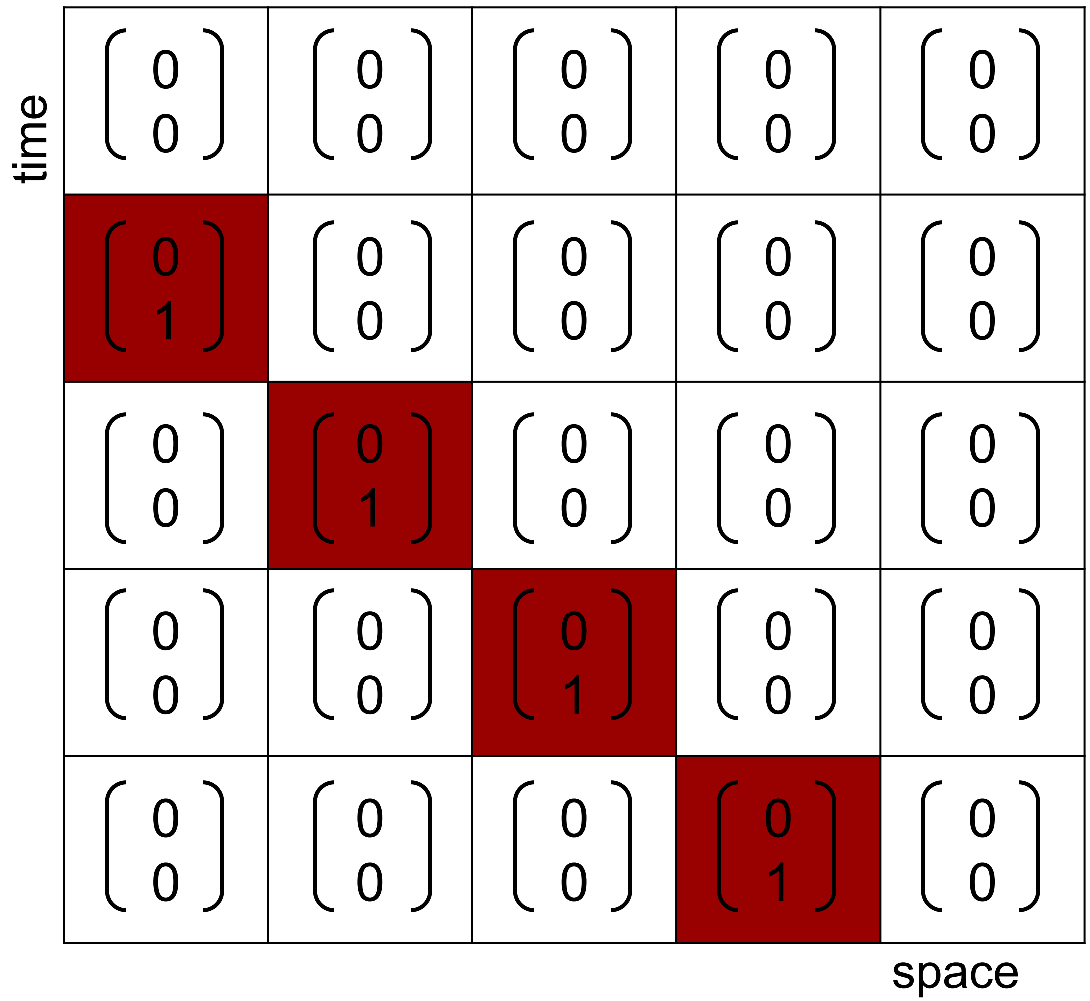
To the right



Chess game : neutrino

To the right

To the left



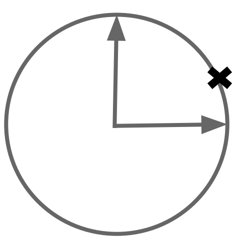
Chess game : neutrino

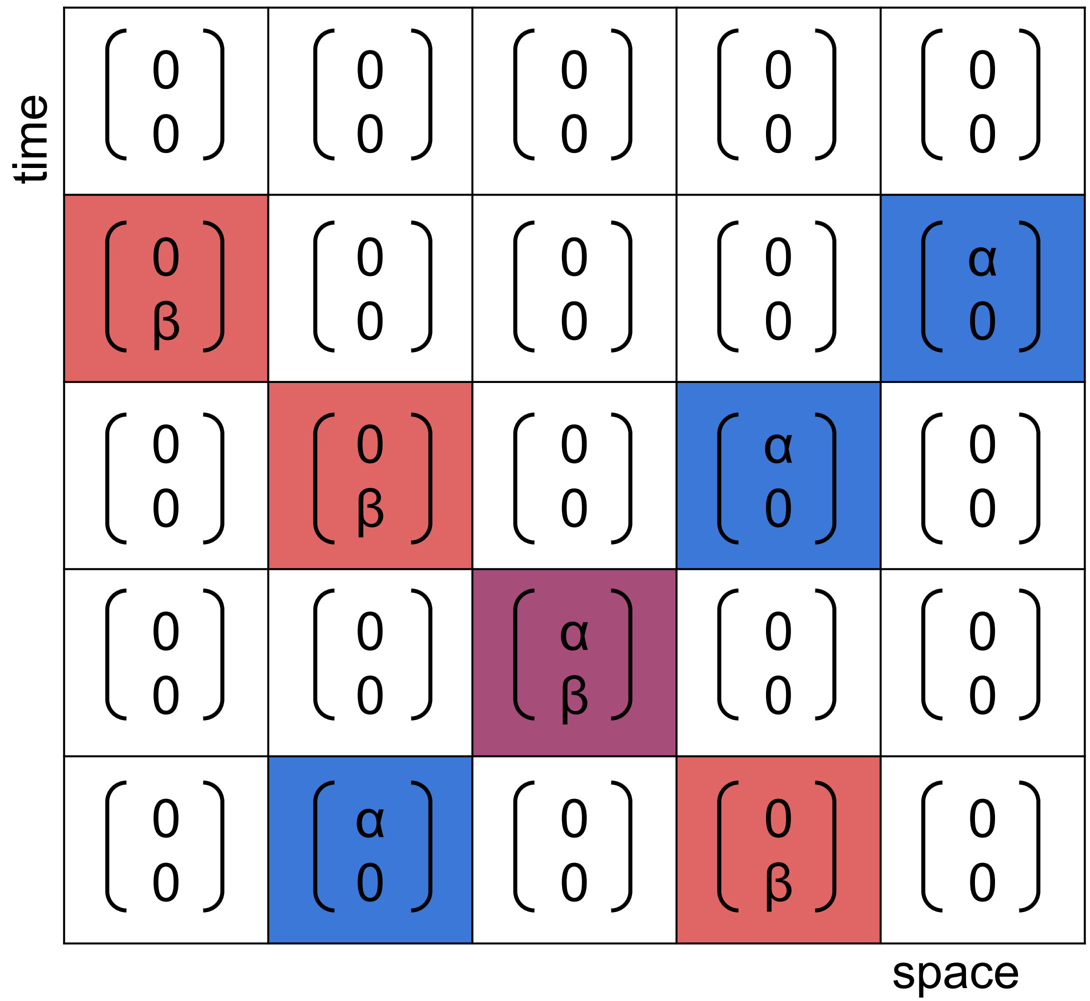
To the right

To the left

Amplitudes

$|\alpha|^2+|\beta|^2=1$





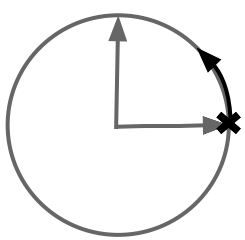
Chess game : electron

Rotations

$$C = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$c = \cos(\theta)$

$s = \sin(\theta)$



$\theta = m.\varepsilon$

m = mass

ε = step

time

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \dots \\ \dots \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} \dots \\ \dots \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} -cs^2 \\ c^2s \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -2cs^2 \\ -s^3+c^2s \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} c^3 \\ c^2s \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -s^2 \\ cs \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} c^2 \\ cs \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

space

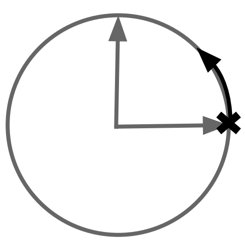
Chess game : electron

Rotations

$$C = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$c = \cos(\theta)$

$s = \sin(\theta)$



$\theta = m.\varepsilon$

m = mass

ε = step

Theorem : In the continuum limit, this Quantum Walk converges to the Dirac Equation.

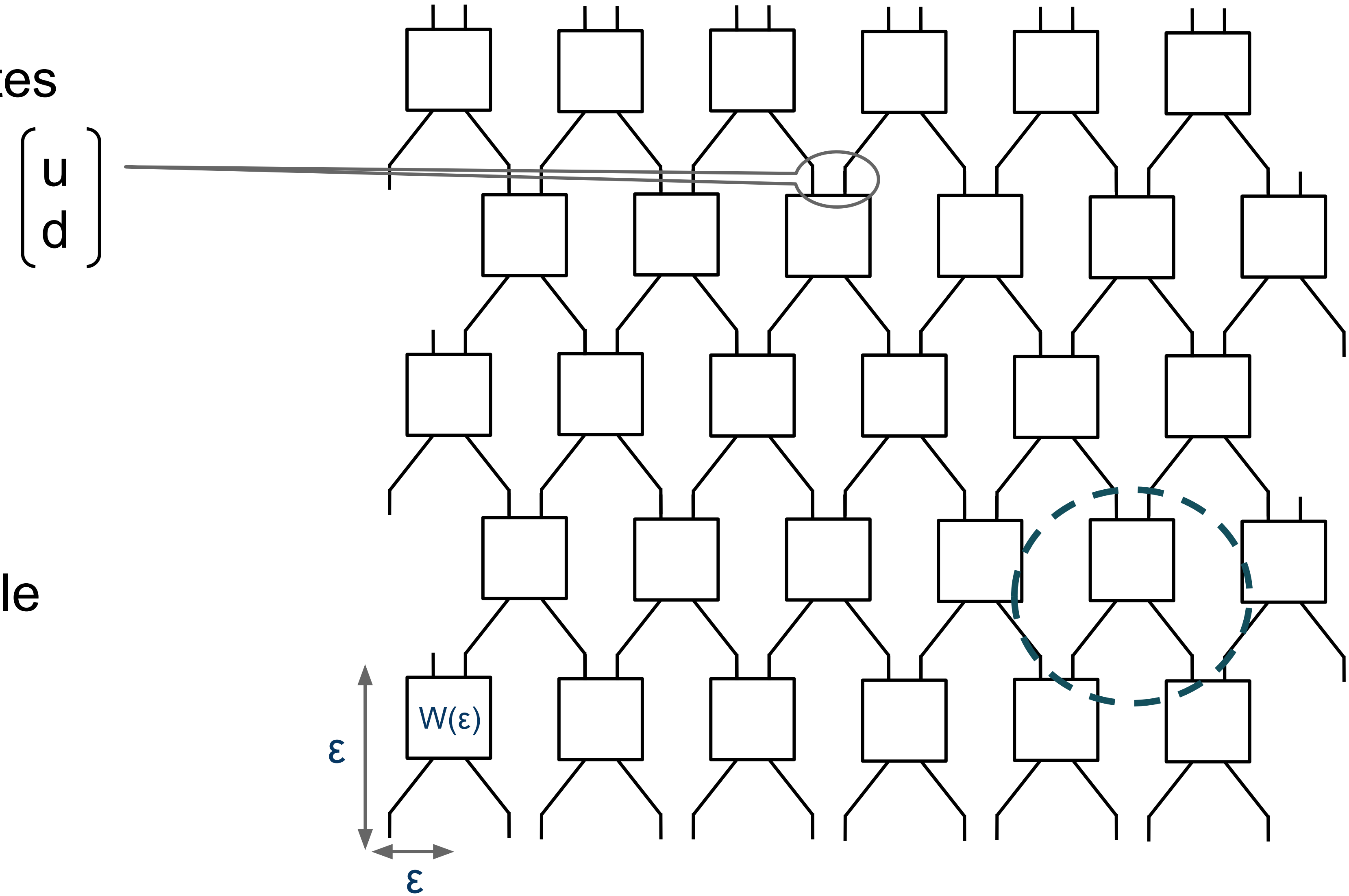
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -s^2 \\ cs \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} c^2 \\ cs \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

space

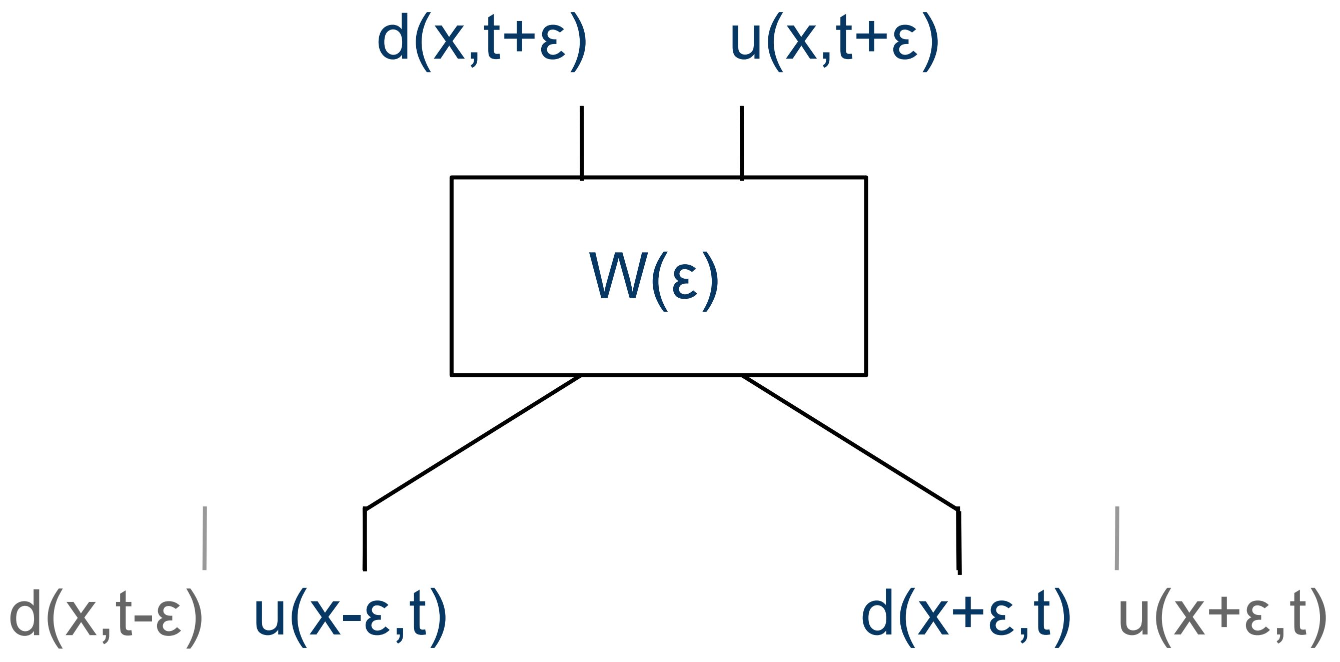
Chess game : electron

States

Scale

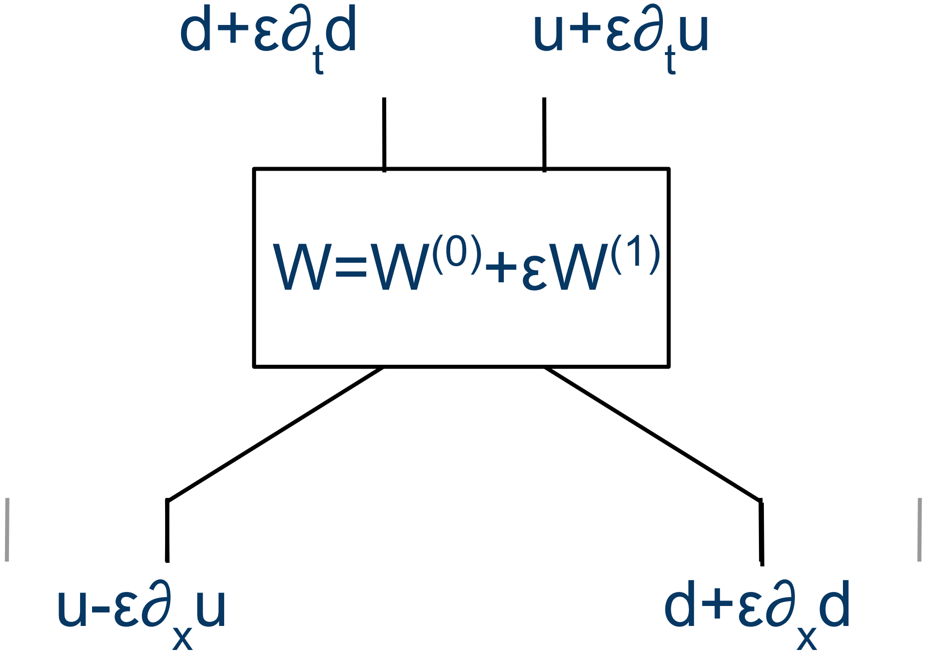


Chess game : electron



Chess game : electron

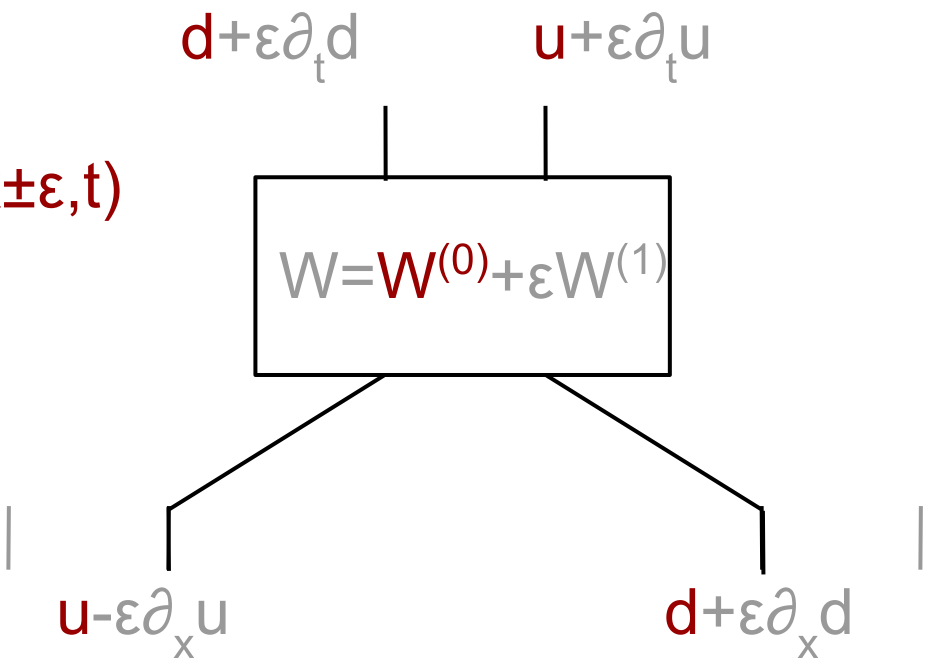
Expand



Chess game : electron

Order 0

$u(x,t+\varepsilon) \simeq u(x \pm \varepsilon,t)$



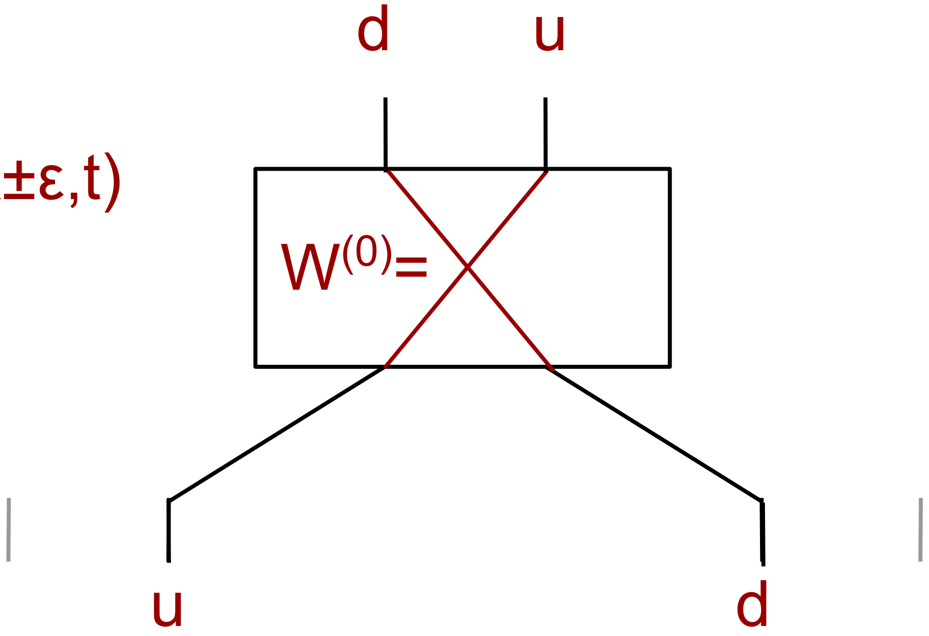
Chess game : electron

Order 0

$u(x,t+\varepsilon) \simeq u(x\pm\varepsilon,t)$

$\Rightarrow$

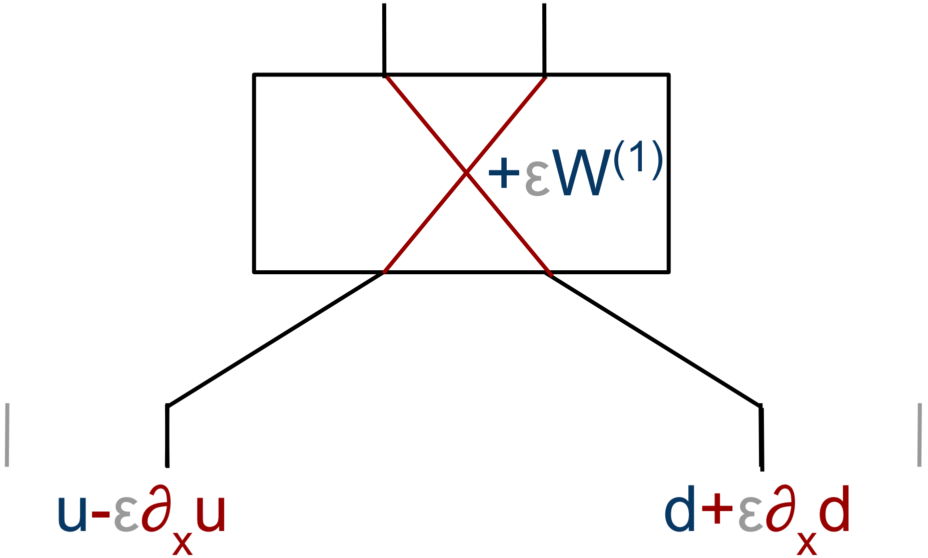
$W^{(0)} = X$



Chess game : electron

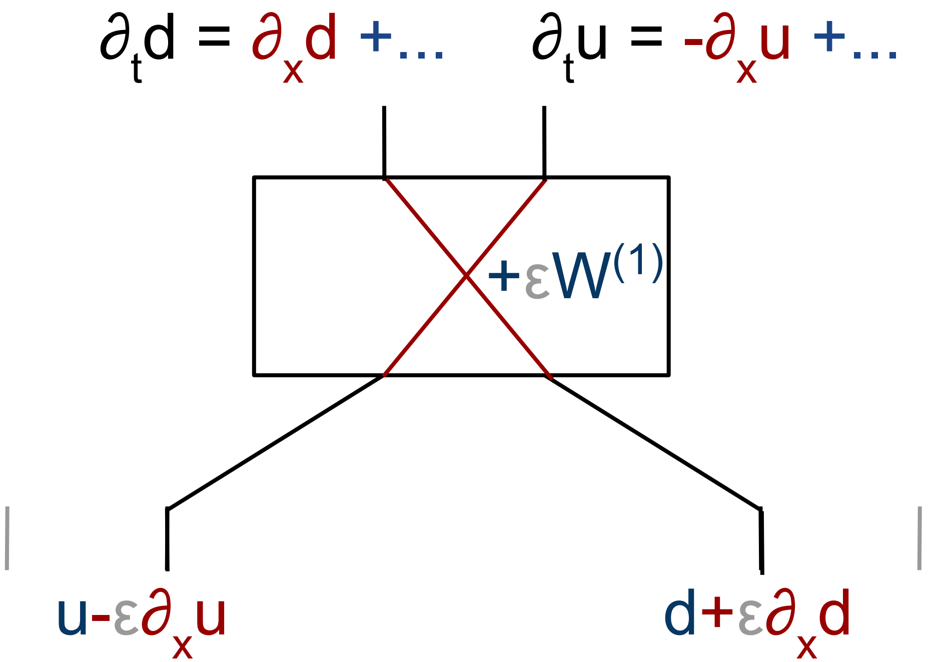
Order 1

$$d+\varepsilon\partial_t d = d + \varepsilon\partial_x d + \ldots \qquad u+\varepsilon\partial_t u = u - \varepsilon\partial_x u + \ldots$$



Chess game : electron

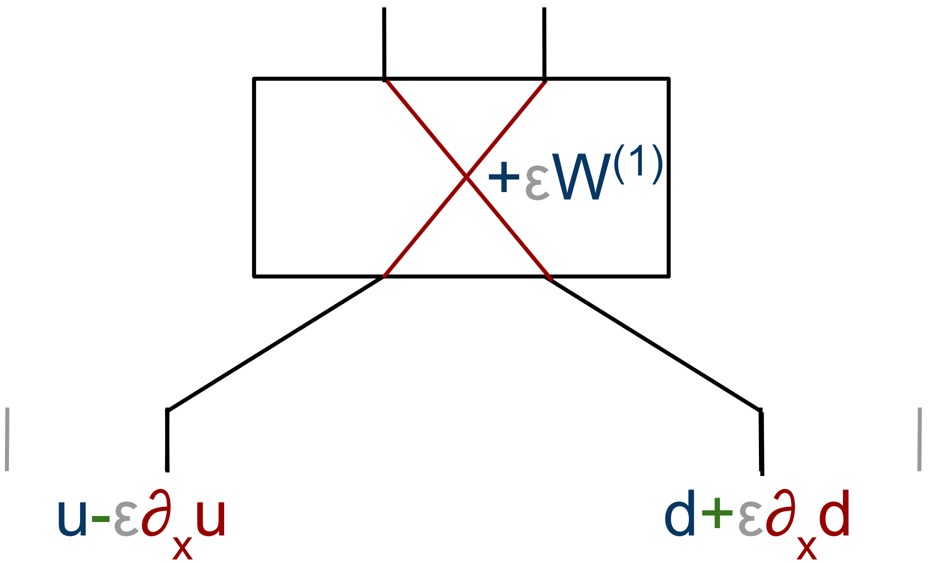
Order 1



Chess game : electron

Order 1

$$\partial_t \begin{pmatrix} u \\ d \end{pmatrix} = -\sigma_z \partial_x \begin{pmatrix} u \\ d \end{pmatrix} + W^{(1)} \begin{pmatrix} u \\ d \end{pmatrix}$$



Chess game : electron

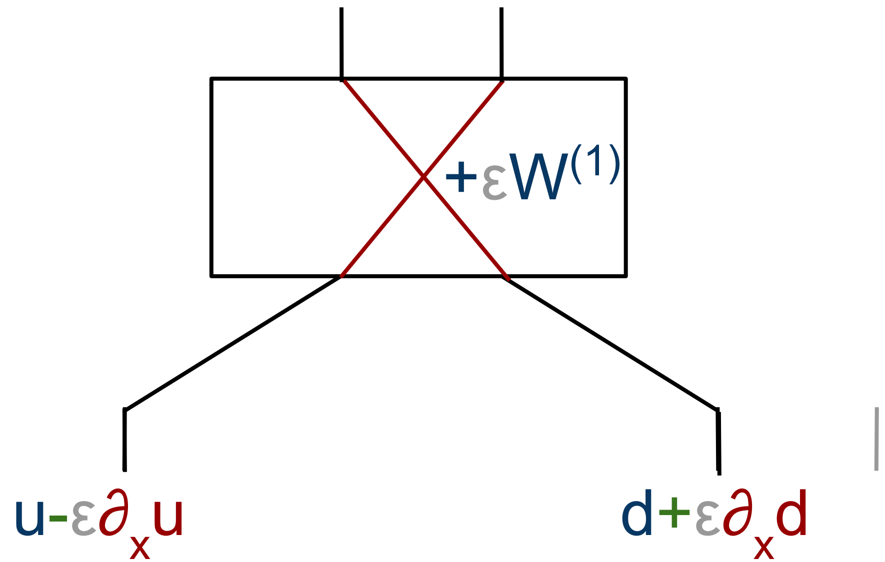
Order 1

$$C = \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \qquad \partial_t \begin{pmatrix} u \\ d \end{pmatrix} = -\sigma_z \partial_x \begin{pmatrix} u \\ d \end{pmatrix} + W^{(1)} \begin{pmatrix} u \\ d \end{pmatrix}$$

$c = \cos(\theta)$

$s = \sin(\theta)$

$\theta = m.\varepsilon$



$C^{(1)} = -im\sigma_y$

$W = CX$

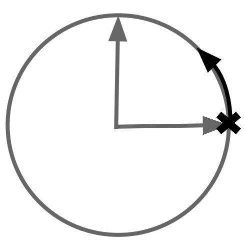
Chess game : electron

Rotations

$$C = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$c = \cos(\theta)$

$s = \sin(\theta)$



$\theta = m.\varepsilon$

m = mass

ε = step

Theorem : In the continuum limit, this Quantum Walk converges to the Dirac Equation.

$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -s^2 \\ cs \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} c^2 \\ cs \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

space

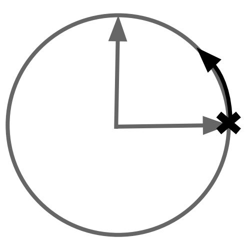
Chess game : electron

Rotations

$$C = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}$$

$c = \cos(\theta)$

$s = \sin(\theta)$



$\theta = m.\varepsilon$

m = mass

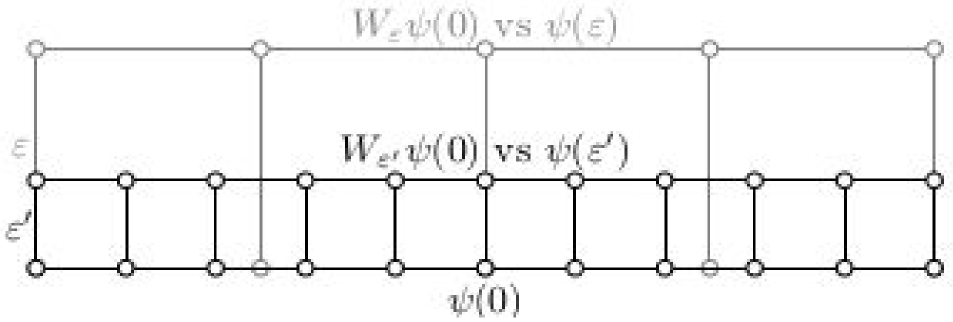
ε = step

Theorem : In the continuum limit, this Quantum Walk *converges* to the Dirac Equation.

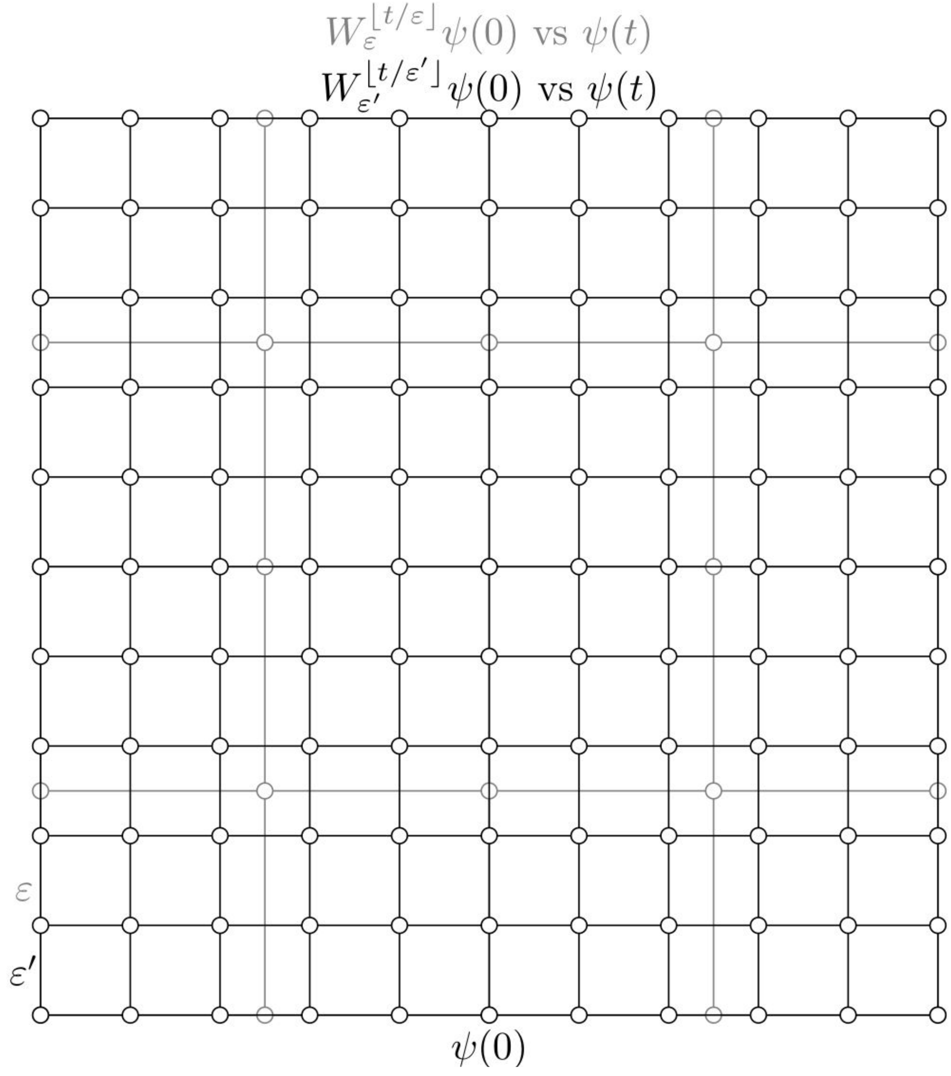
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -s^2 \\ cs \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} c^2 \\ cs \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

space

Consistency *vs* convergence

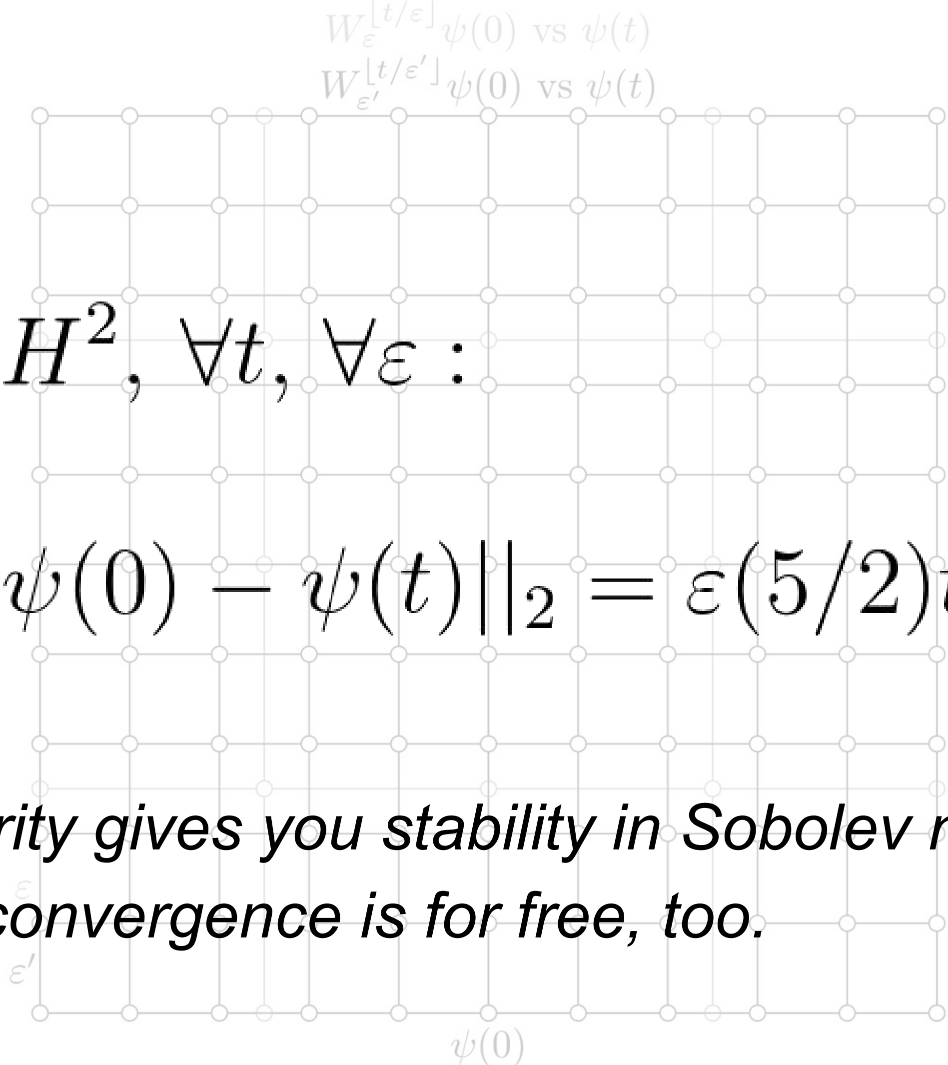


Consistency *vs* convergence



Consistency *vs* convergence

Theorem

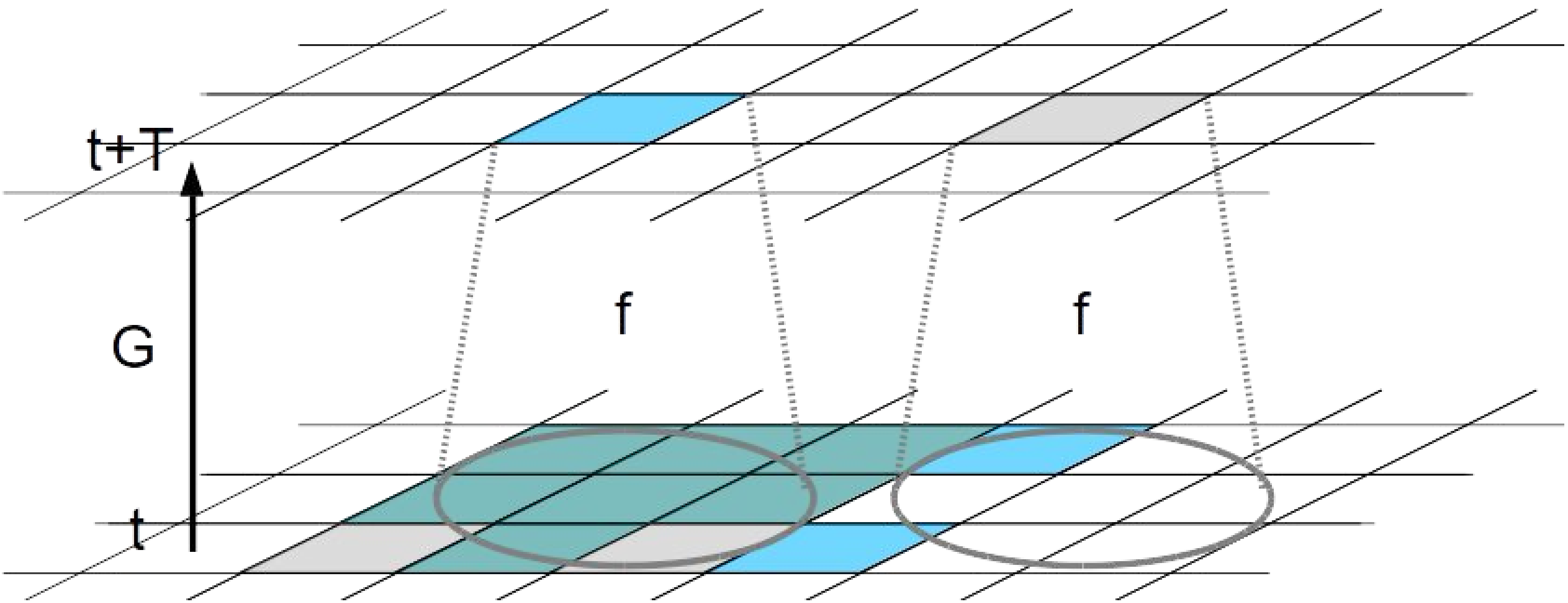


$$\forall \psi(0) \in H^2, \forall t, \forall \varepsilon :$$

$$||W_\varepsilon^{\lfloor t/\varepsilon \rfloor}\psi(0) - \psi(t)||_2 = \varepsilon(5/2)t||\psi(0)||_{H^2}.$$

Morale: unitarity gives you stability in Sobolev norm, for free, and so convergence is for free, too.

Discretize physics?

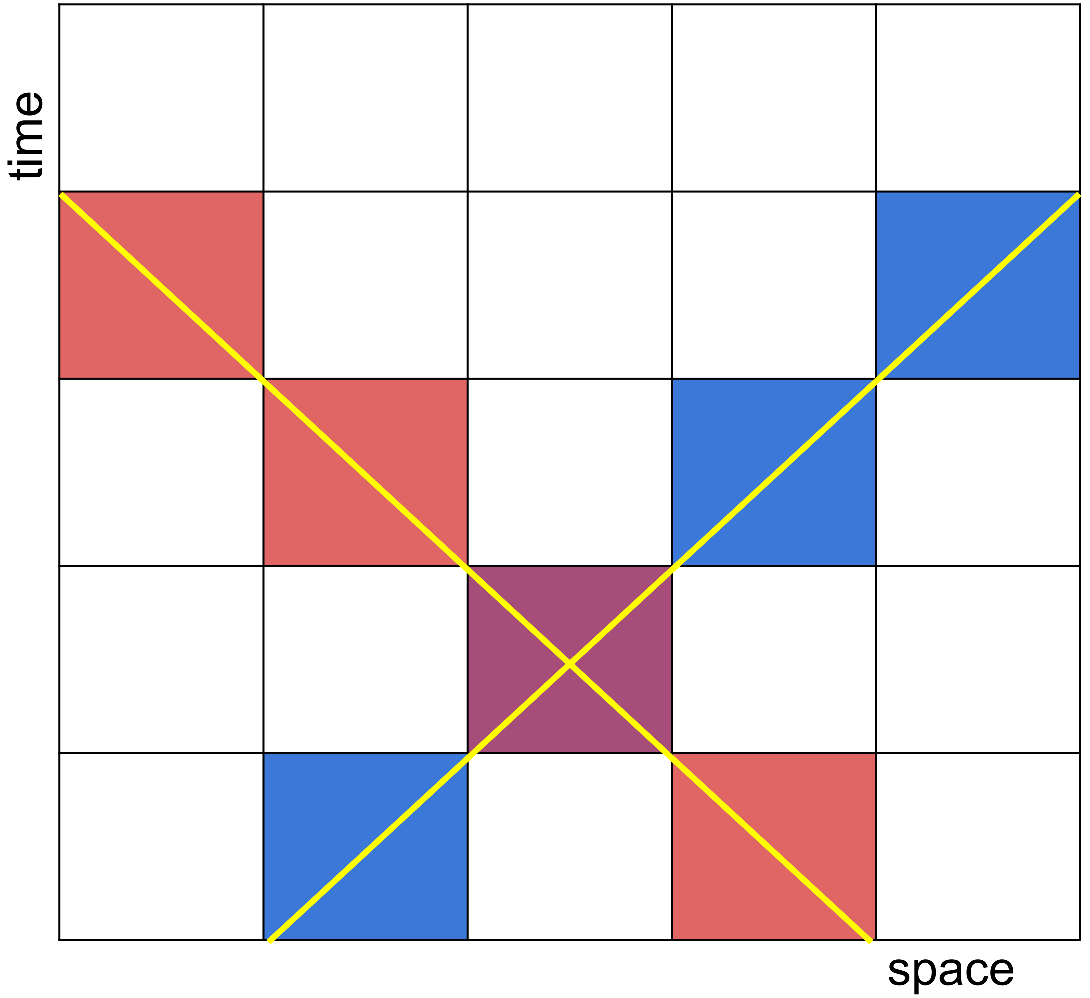


Cellular Automata / Quantum Walks

Theorems about : the extent in which Curved Spacetime can be captured in this formalism.

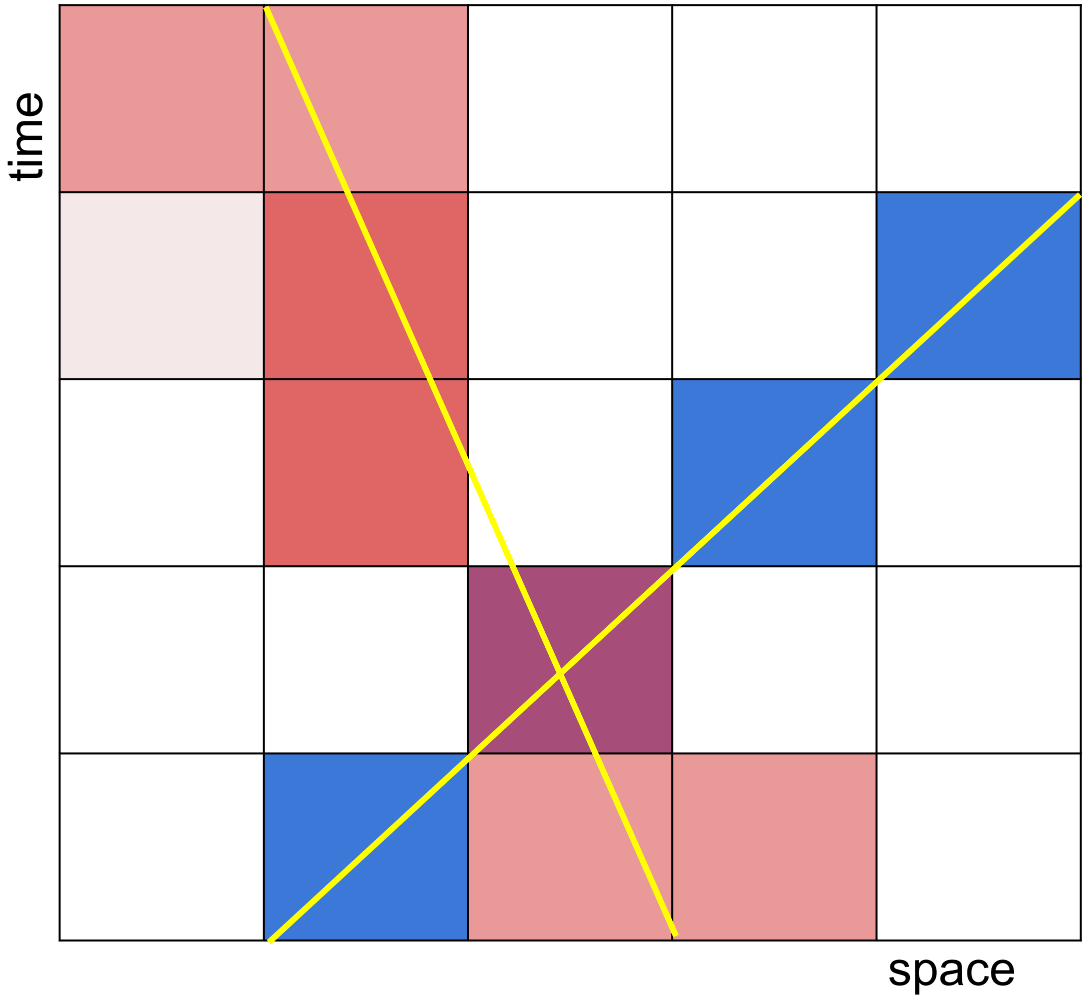
Curved space : problem 1

From

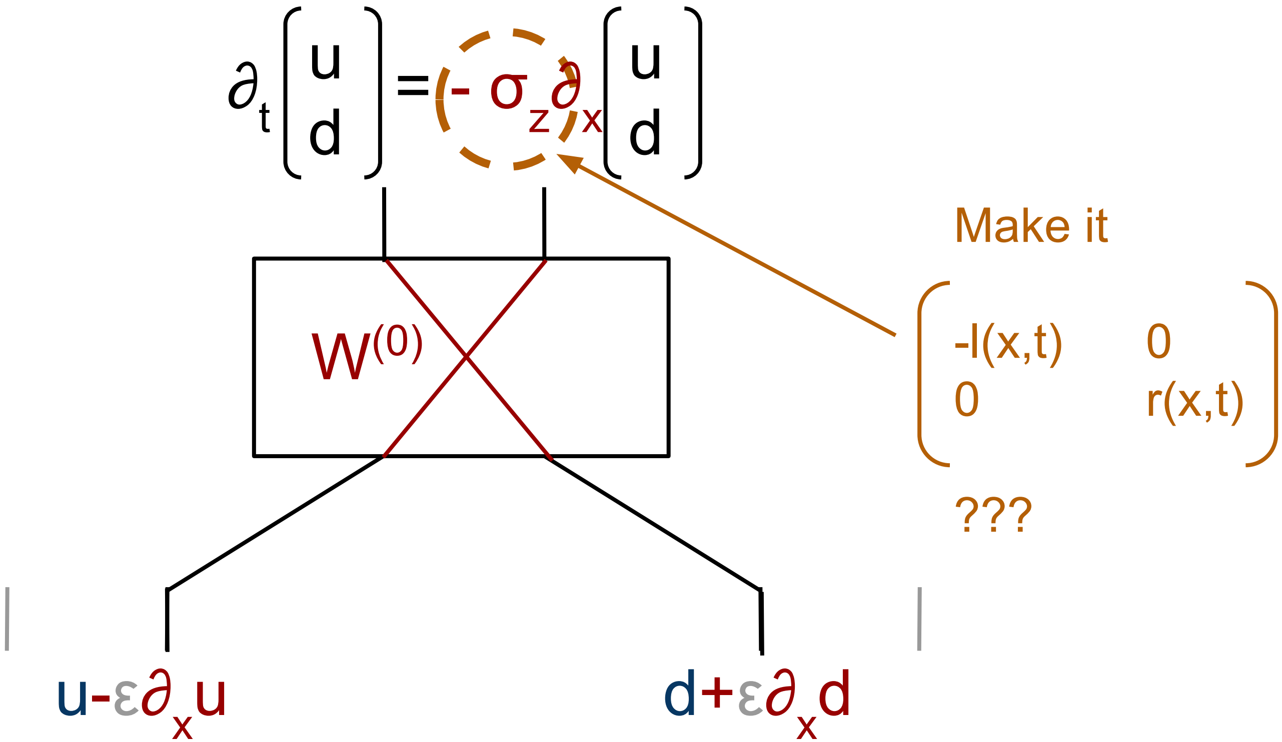


Curved space : problem 1

To



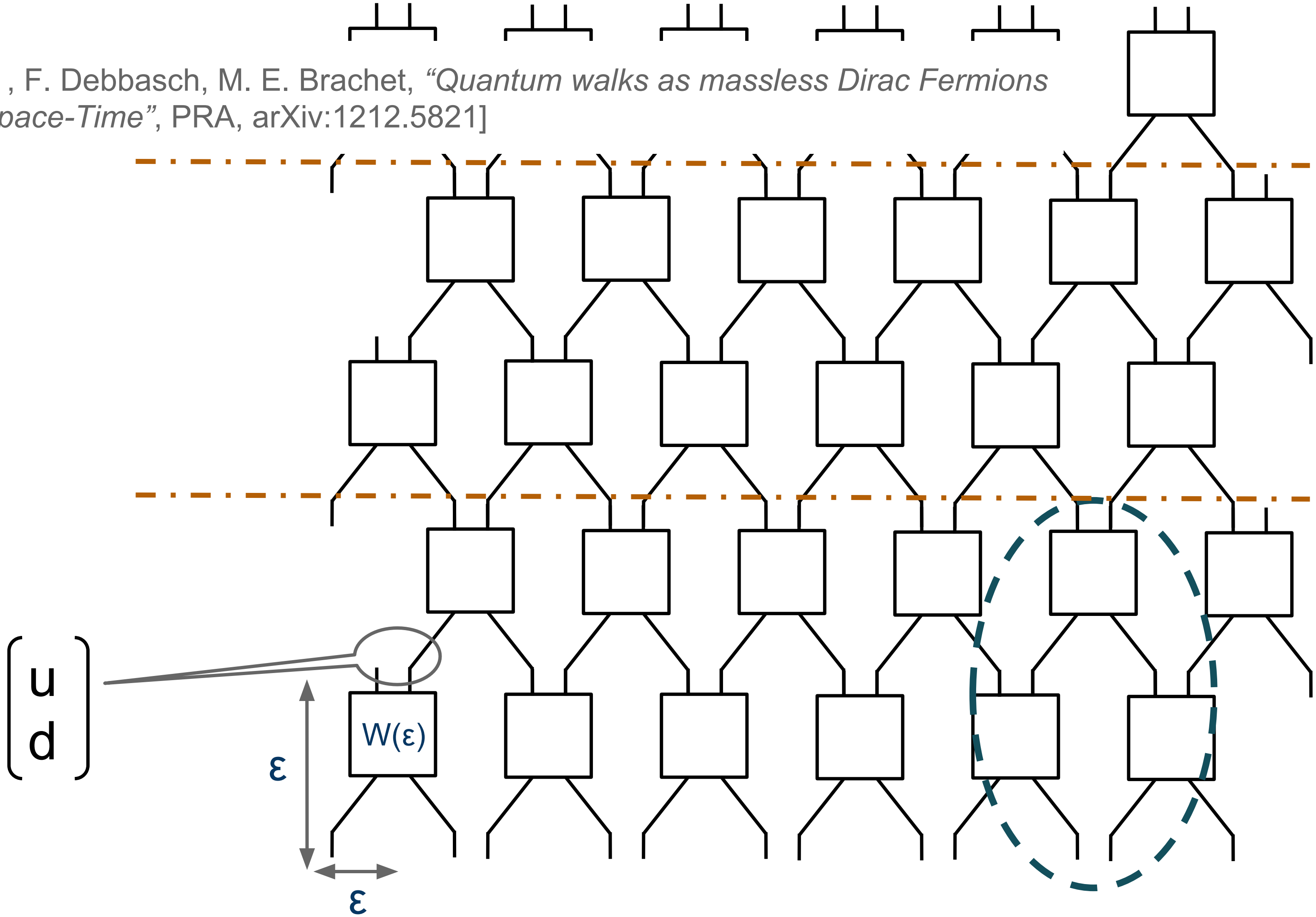
Curved space : problem 1



Transport term is fixed by 0^{th} order & grid :-((

Curved space : idea 0

[Di Molfetta , F. Debbasch, M. E. Brachet, *“Quantum walks as massless Dirac Fermions in curved Space-Time”*, PRA, arXiv:1212.5821]



Curved space : idea 0

[Di Molfetta , F. Debbasch, M. E. Brachet, *“Quantum walks as massless Dirac Fermions in curved Space-Time”*, PRA, arXiv:1212.5821]

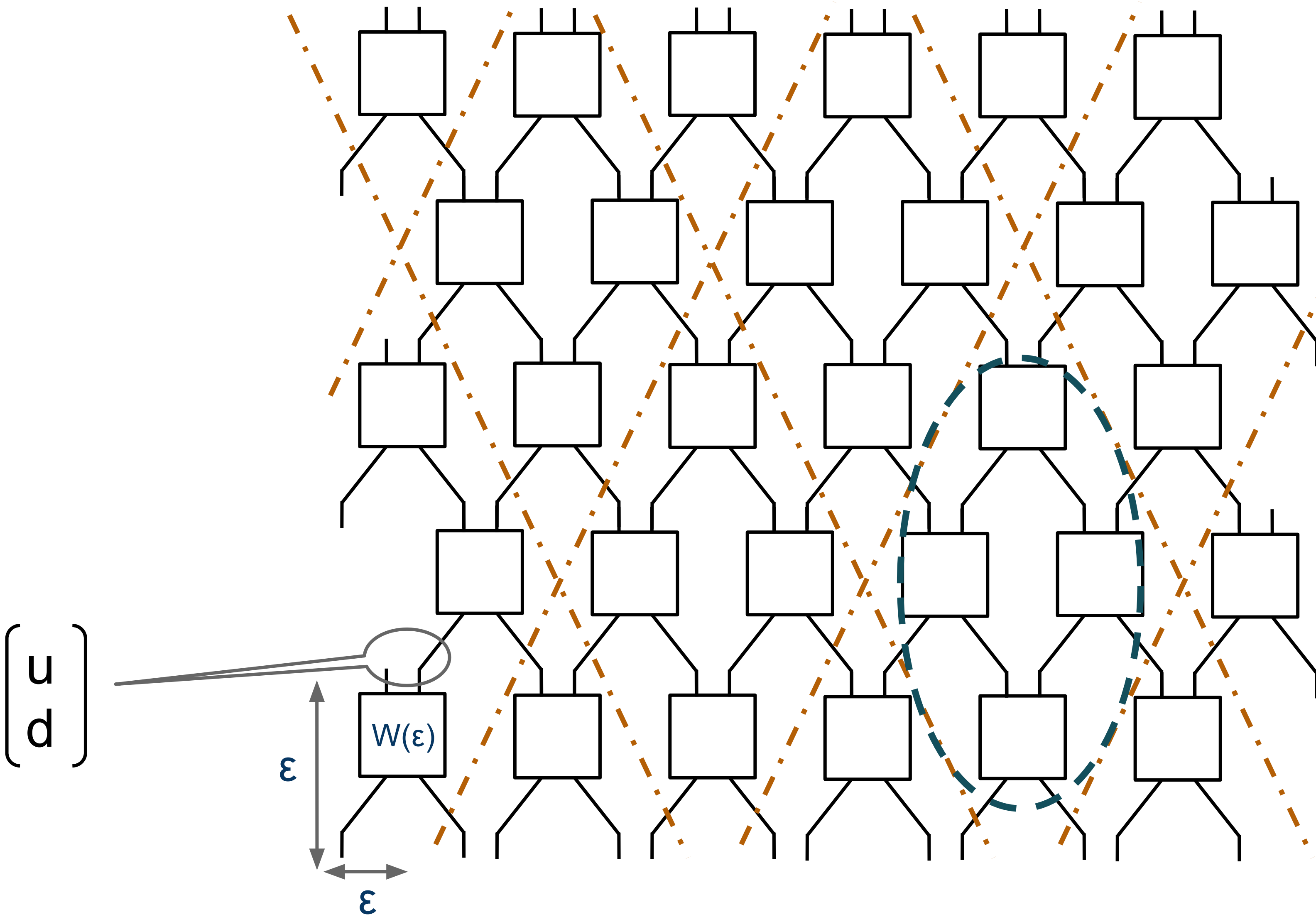
$$\partial_t \begin{pmatrix} u \\ d \end{pmatrix} = \left(- \sigma_z \partial_x\right) \begin{pmatrix} u \\ d \end{pmatrix}$$

- l = r = c
- massless
- 1+1

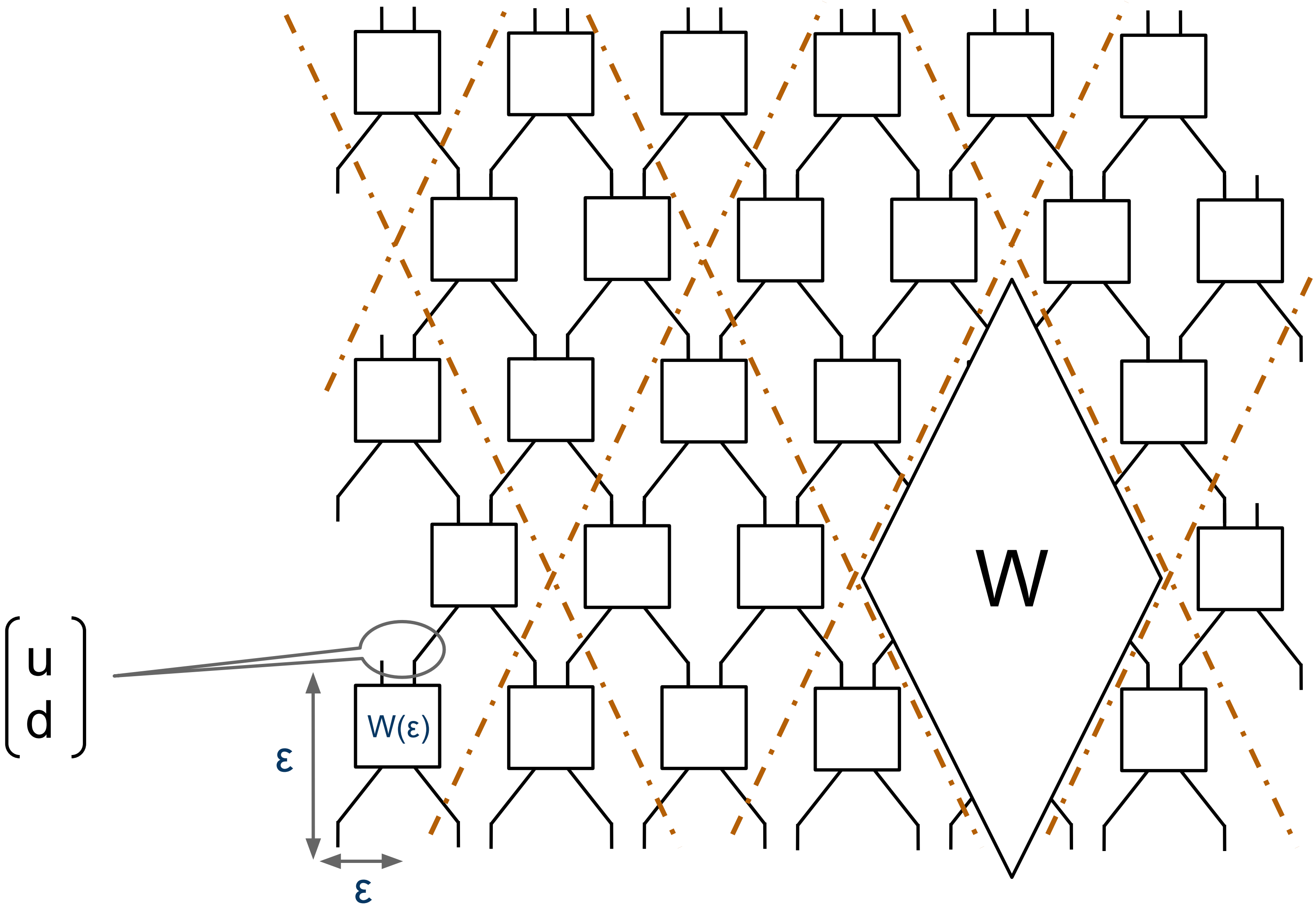
Made it

$$\begin{pmatrix} -c(x,t) & 0 \\ 0 & c(x,t) \end{pmatrix}$$

Curved space : idea 1

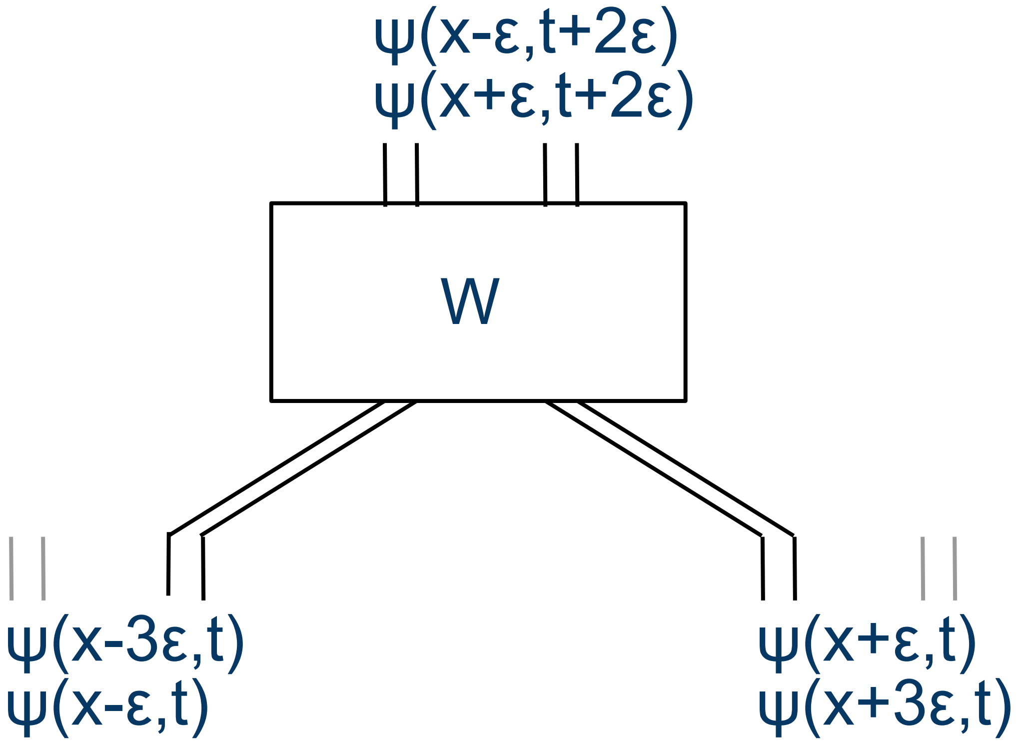


Curved space : idea 1



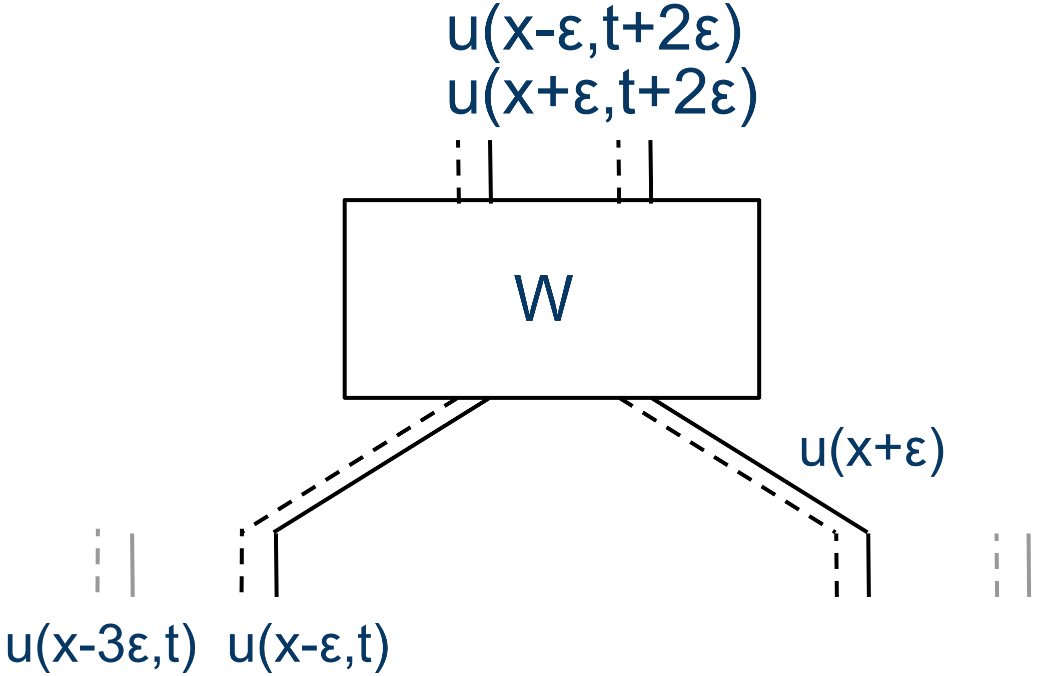
Curved space : idea 1

States



Curved space : idea 1

States

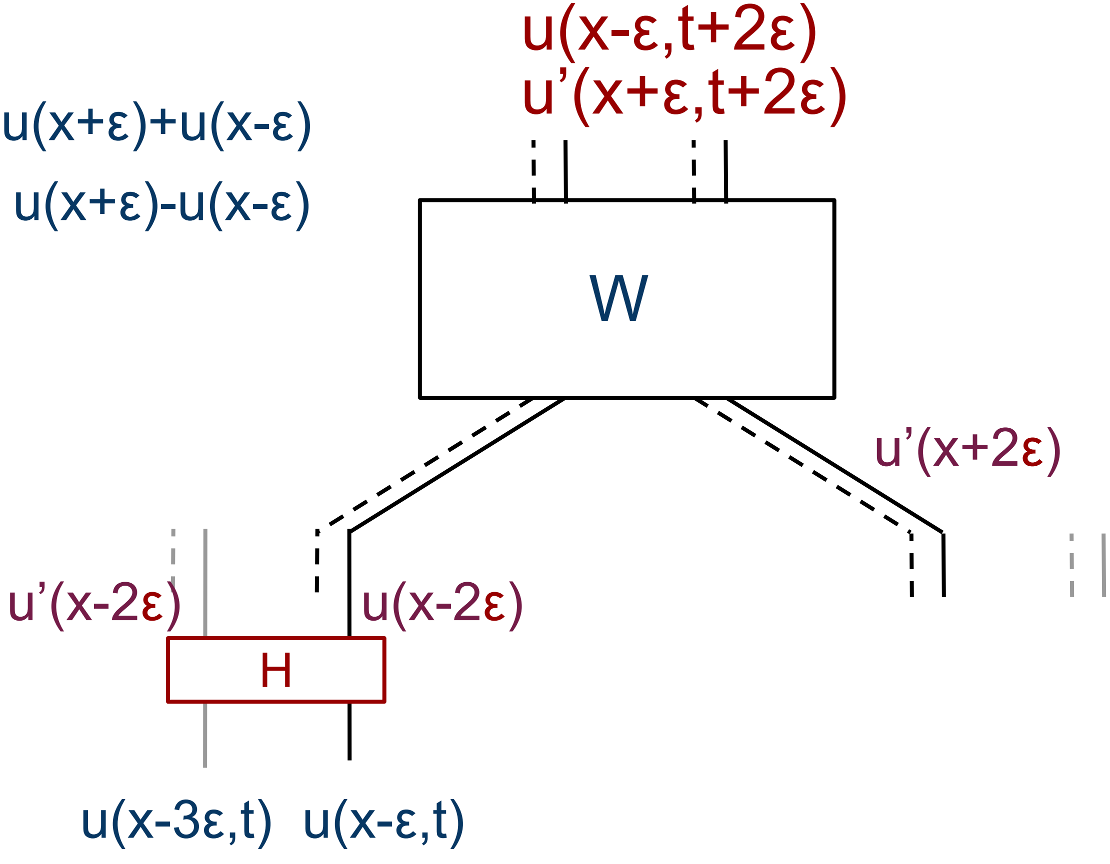


Curved space : idea 1

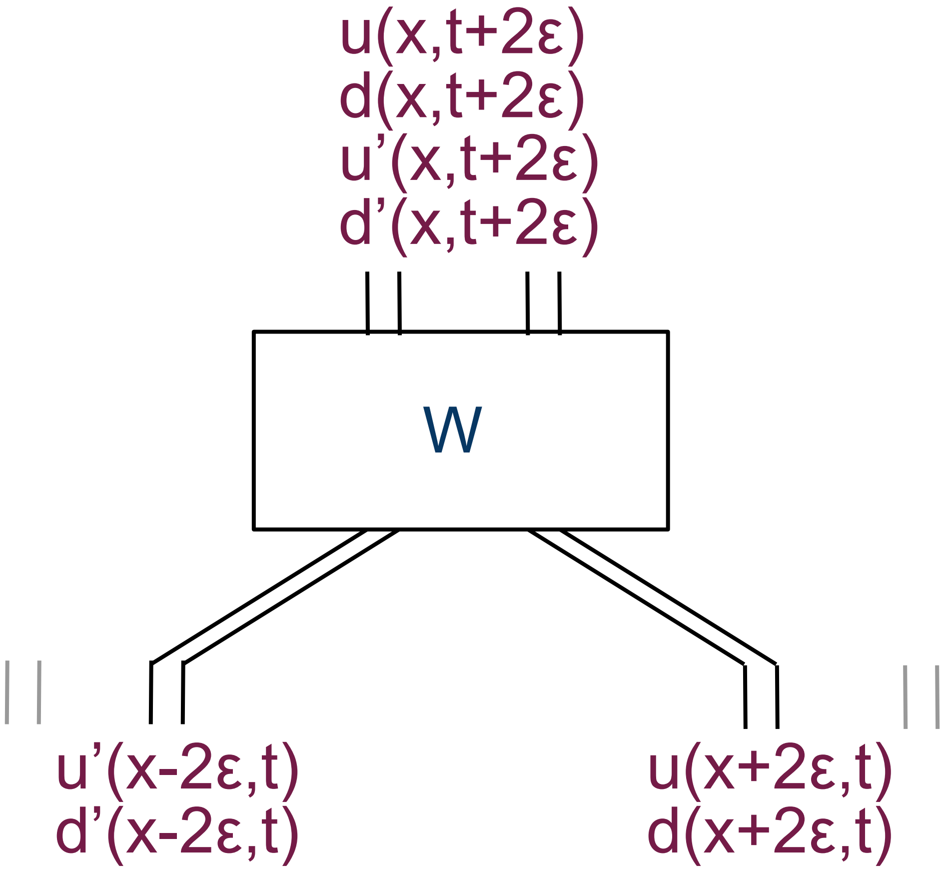
States

$u(x) := u(x+\varepsilon)+u(x-\varepsilon)$

$u'(x) := u(x+\varepsilon)-u(x-\varepsilon)$



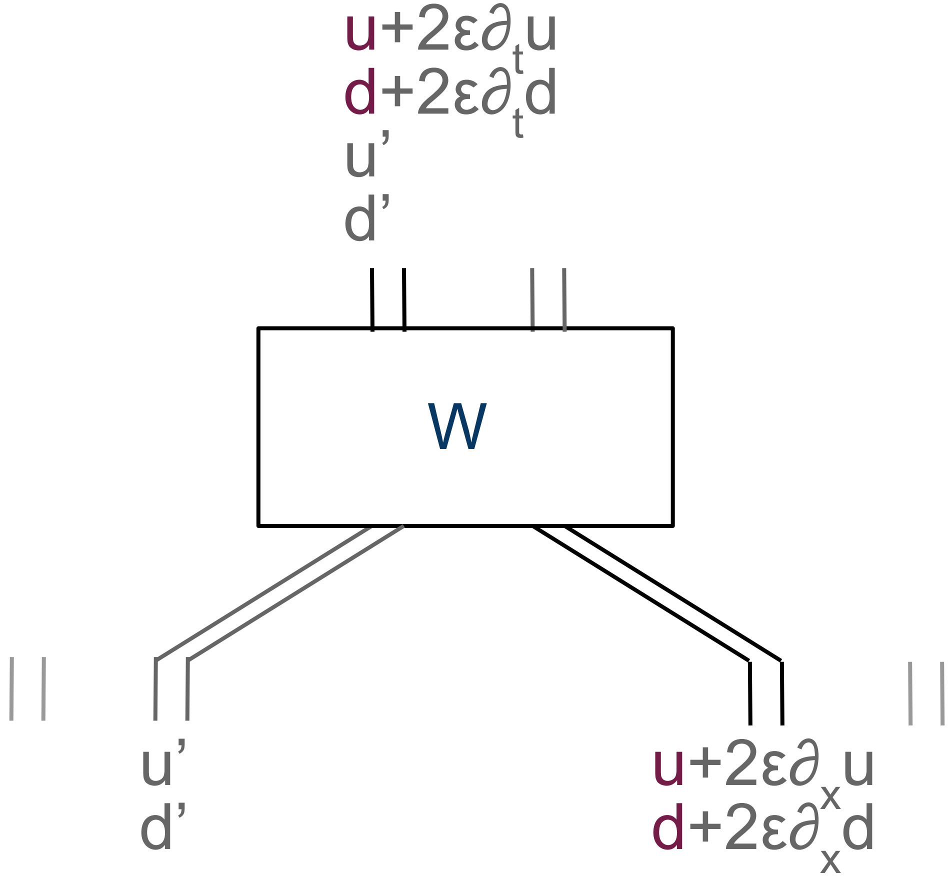
Curved space : idea 1



Curved space : idea 1

Expand

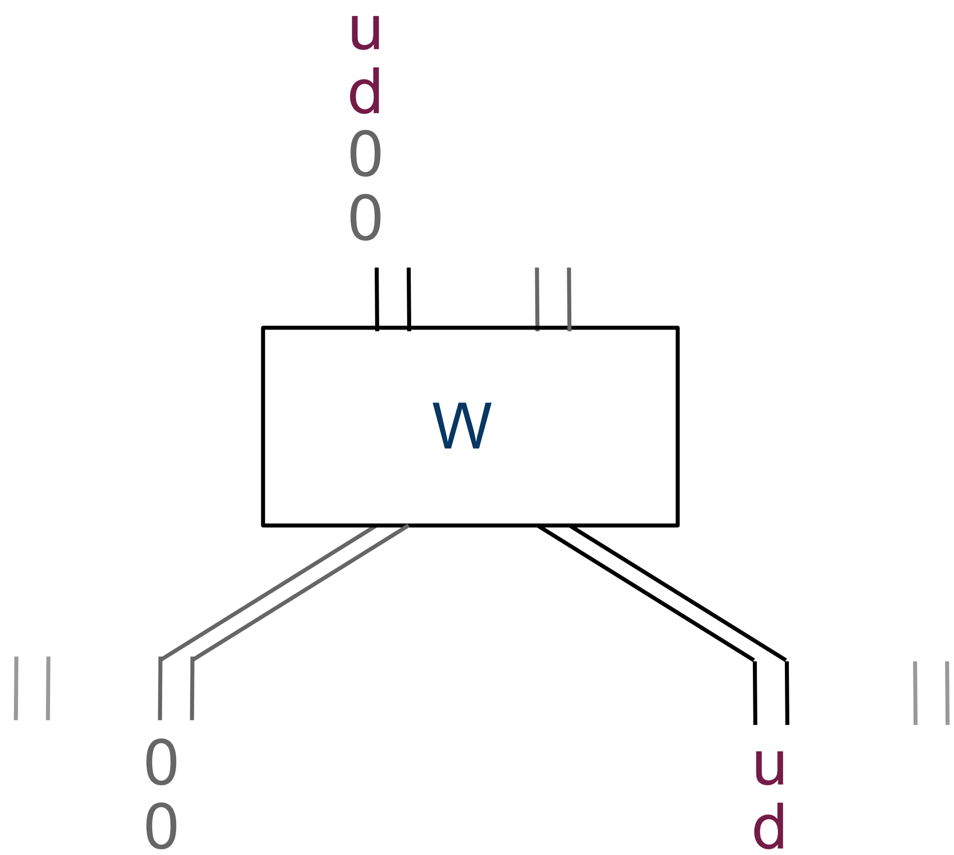
-



Curved space : idea 1

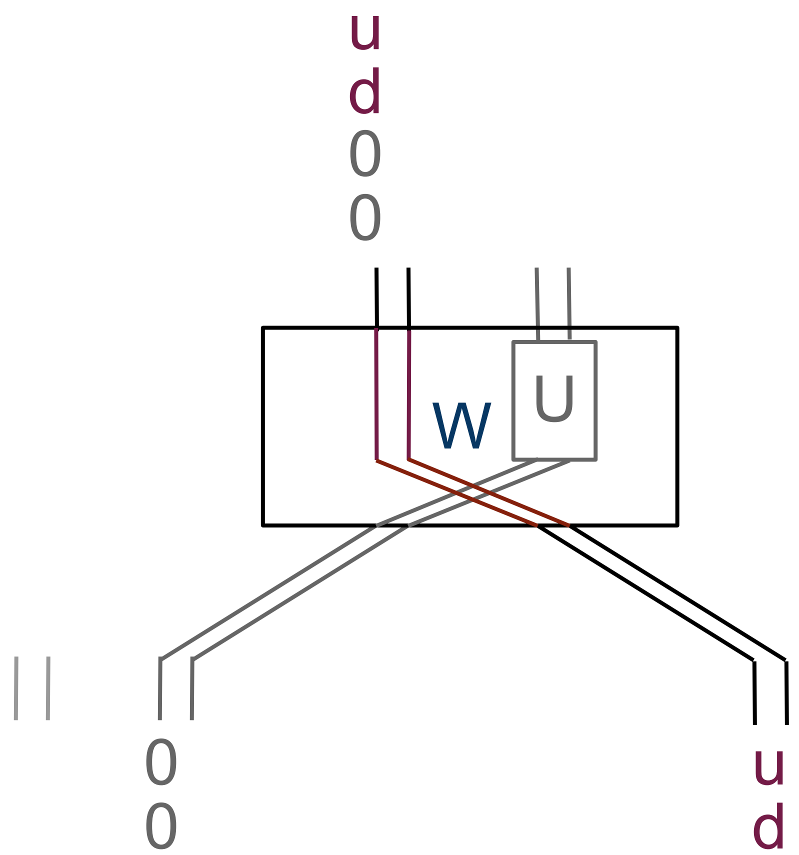
0th order

-



Curved space : idea 1

0th order

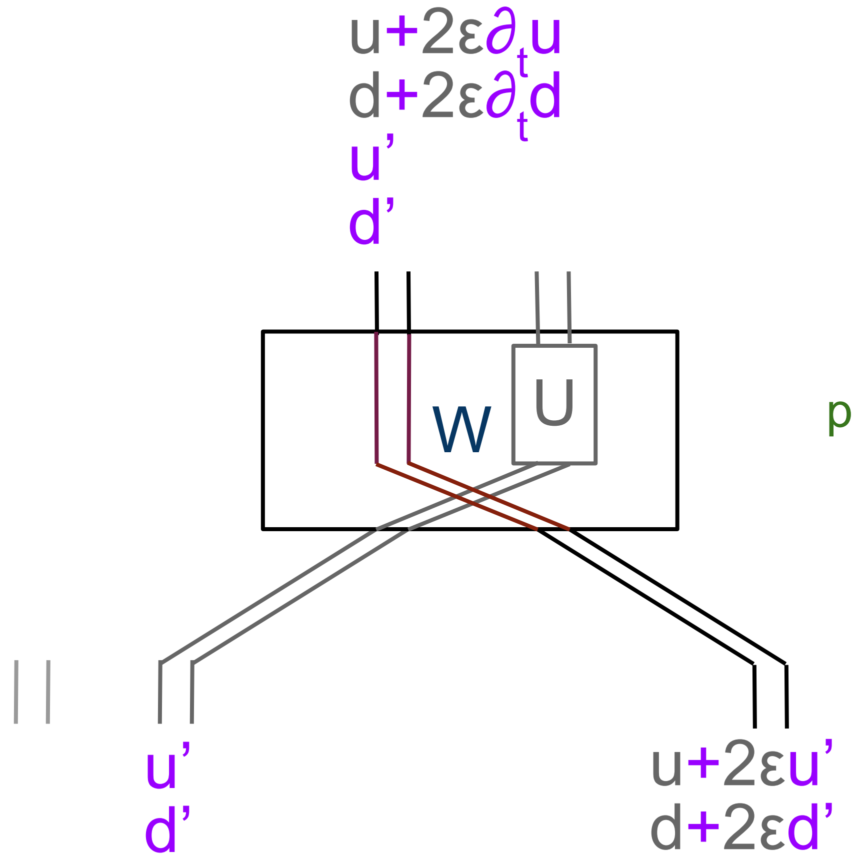


$W^{(0)}$, which governs the speed of propagation, is no longer trivial!

Primes and non-primes evolve separately: an inconsistency?

Curved space : problem 2

1st order

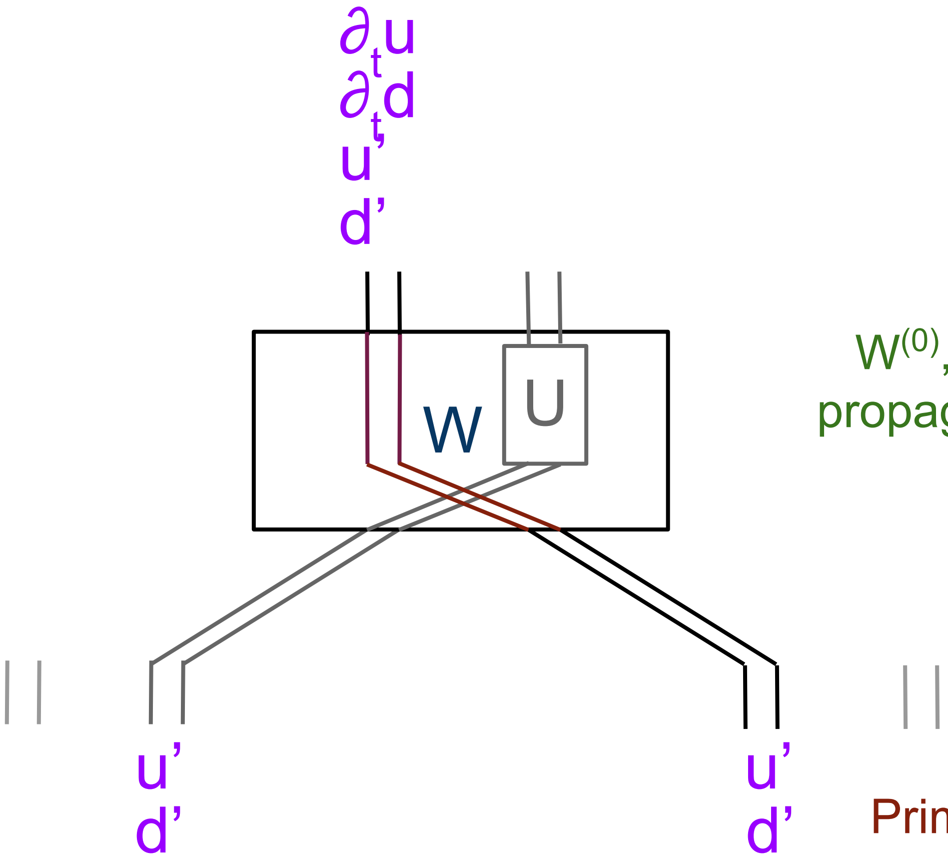


$W^{(0)}$, that which governs propagation, is non-trivial, and this still has a continuous limit.

Primes and non-primes evolve separately: an inconsistency?

Curved space : problem 2

1st order

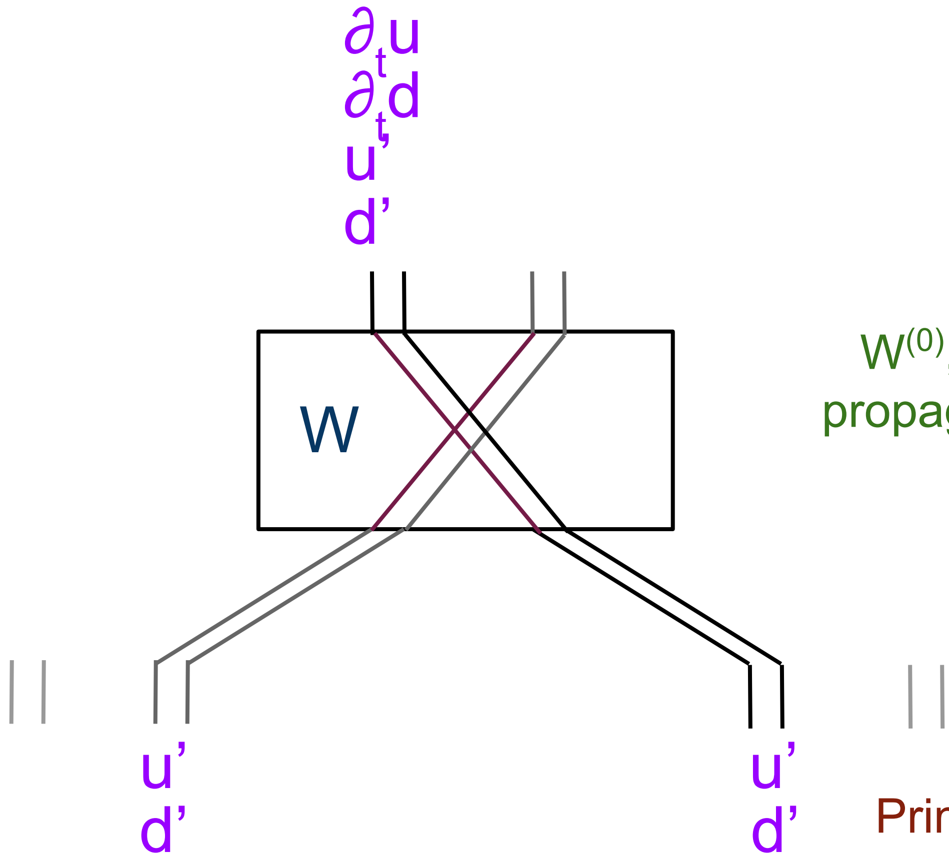


$W^{(0)}$, that which governs propagation, is non-trivial, and this still has a continuous limit.

Primes and non-primes evolve separately: an inconsistency?

Curved space : problem 2

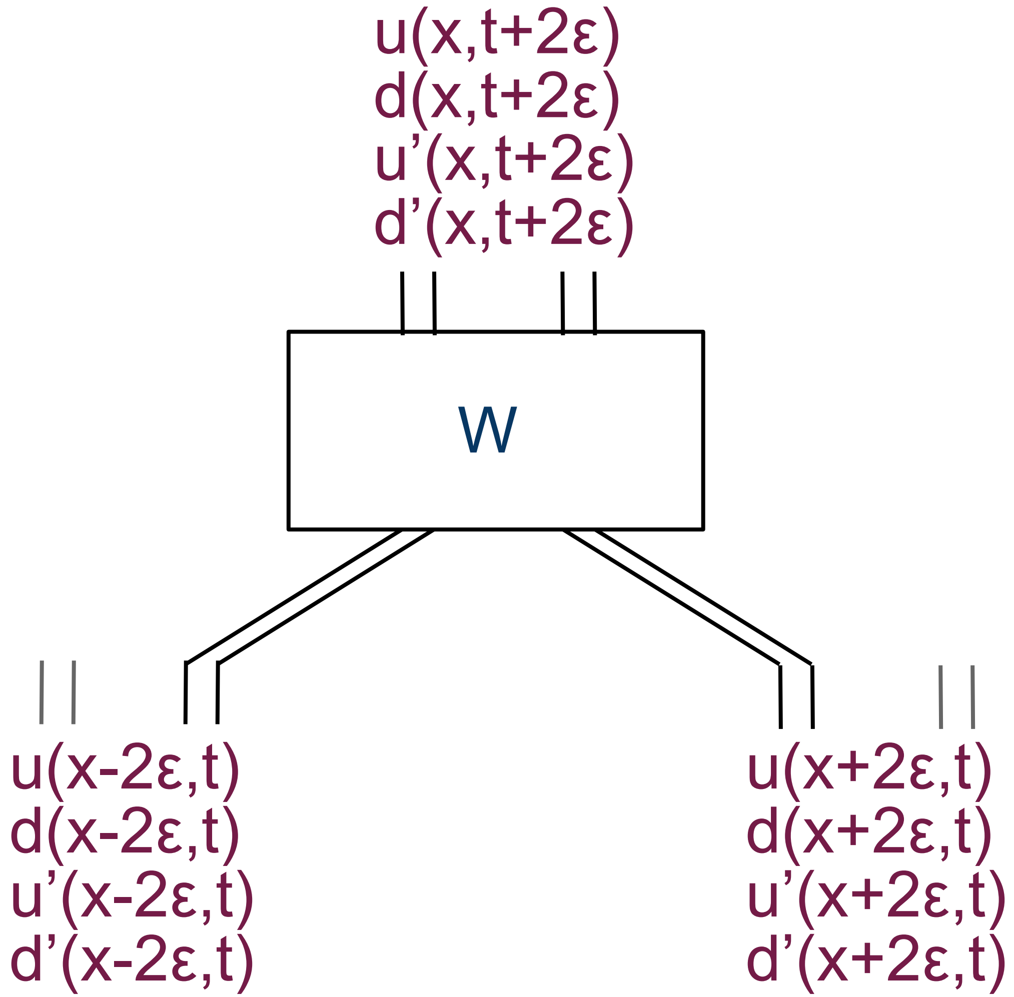
1st order



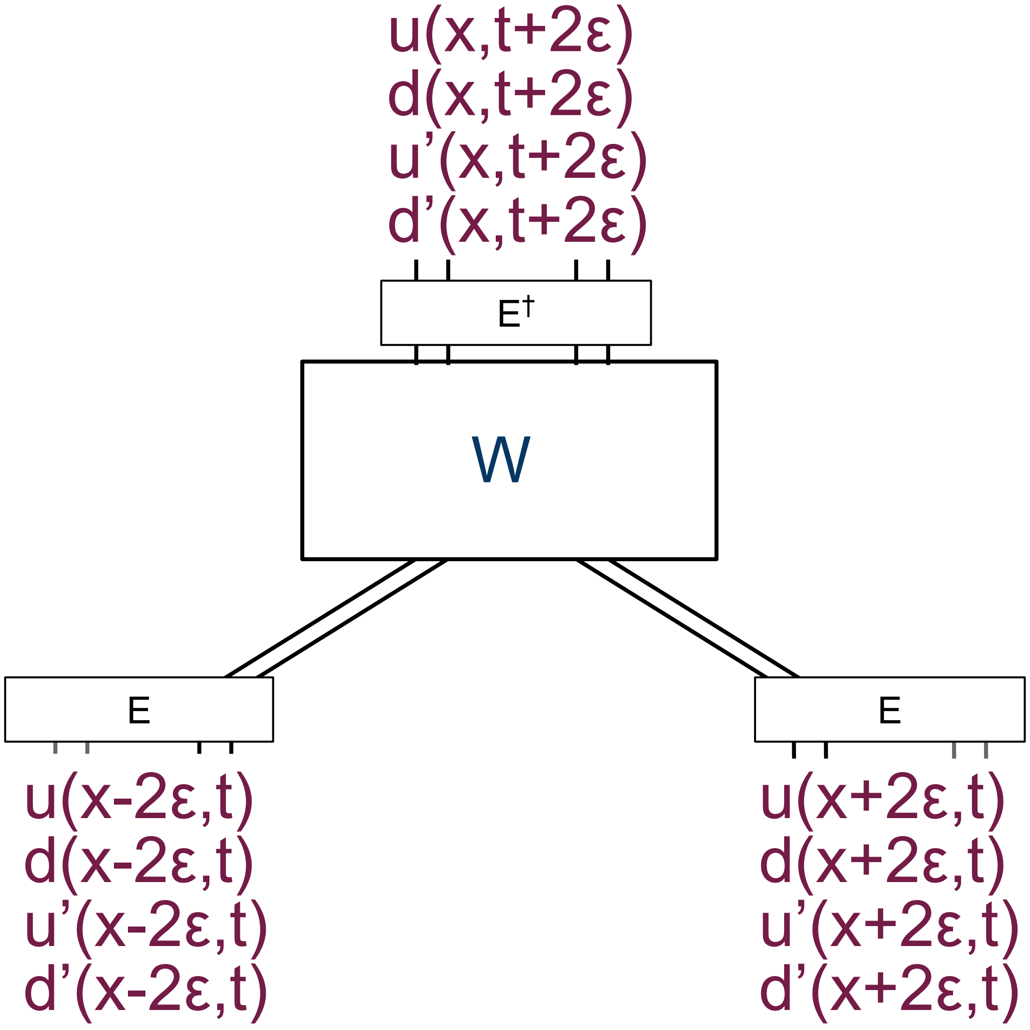
$W^{(0)}$, that which governs propagation, is non-trivial, and this still has a continuous limit.

Primes and non-primes evolve separately: an inconsistency?

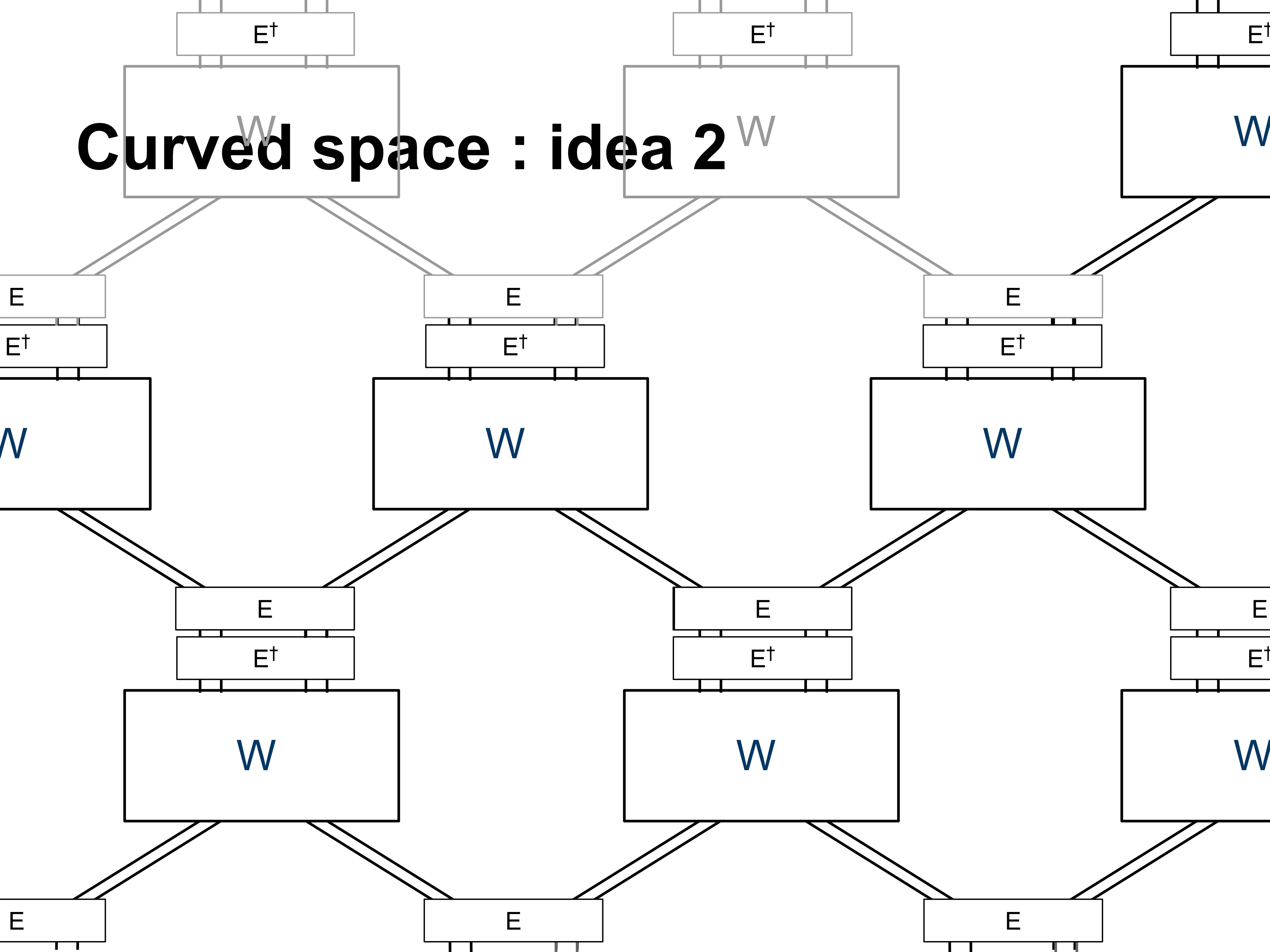
Curved space : idea 2



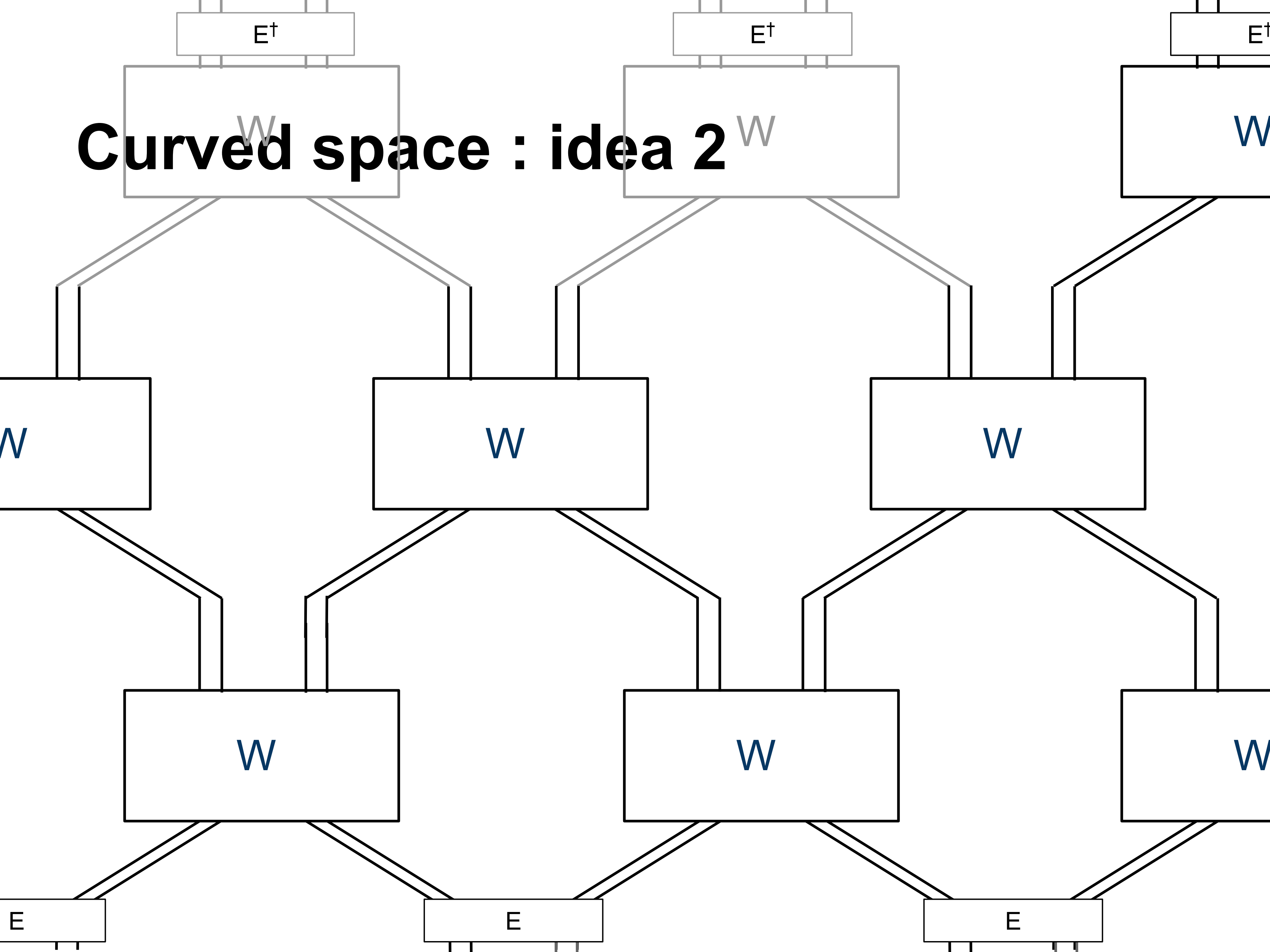
Curved space : idea 2



Curved space : idea 2



Curved space : idea 2



Tin content

Theorem

A stable numerical scheme for PDEs of the form

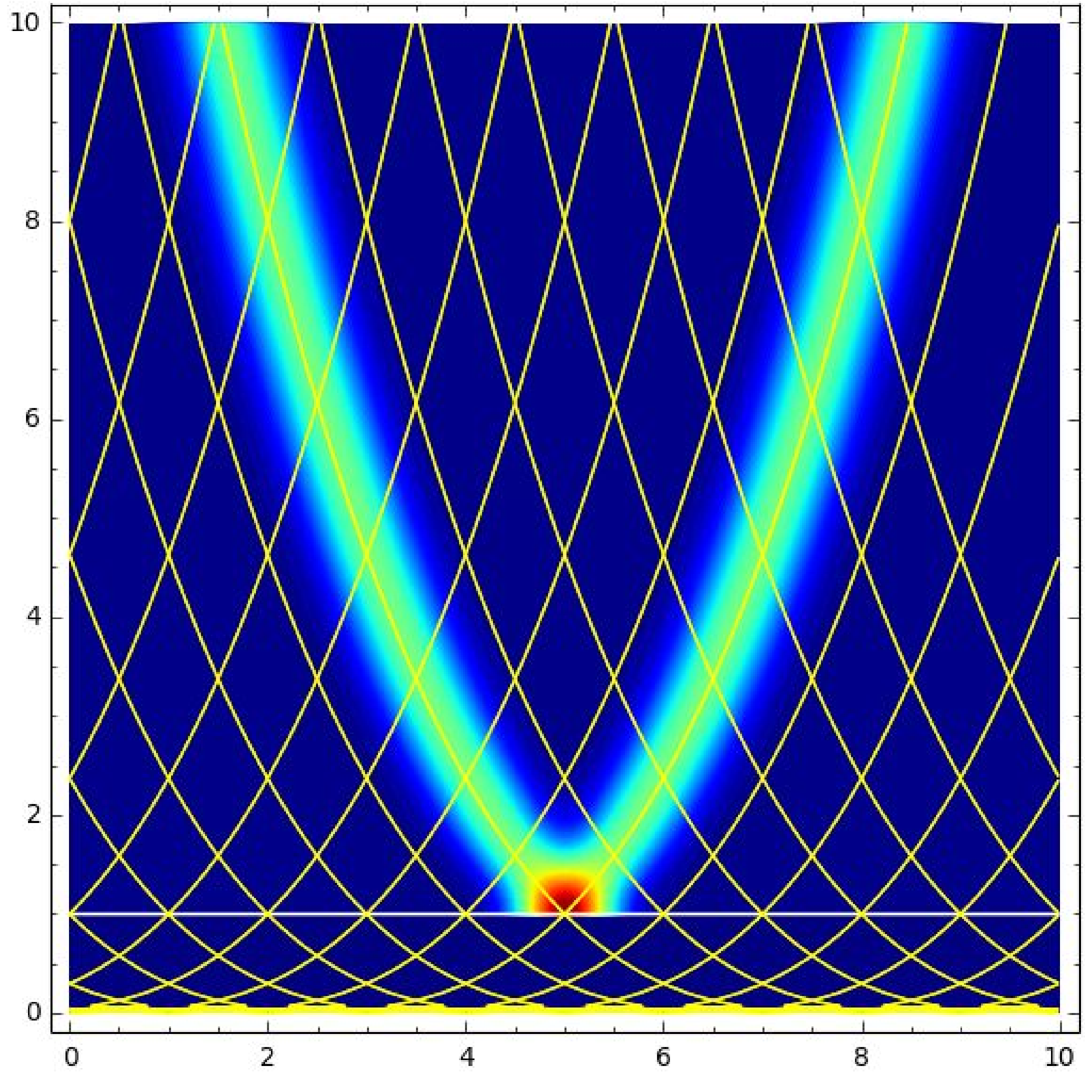
$$i\partial_0\psi = H\psi$$

$$H = i\sum_{k=1\ldots d}\left(B_k\partial_k + \frac{1}{2}\partial_k B_k\right) - C$$

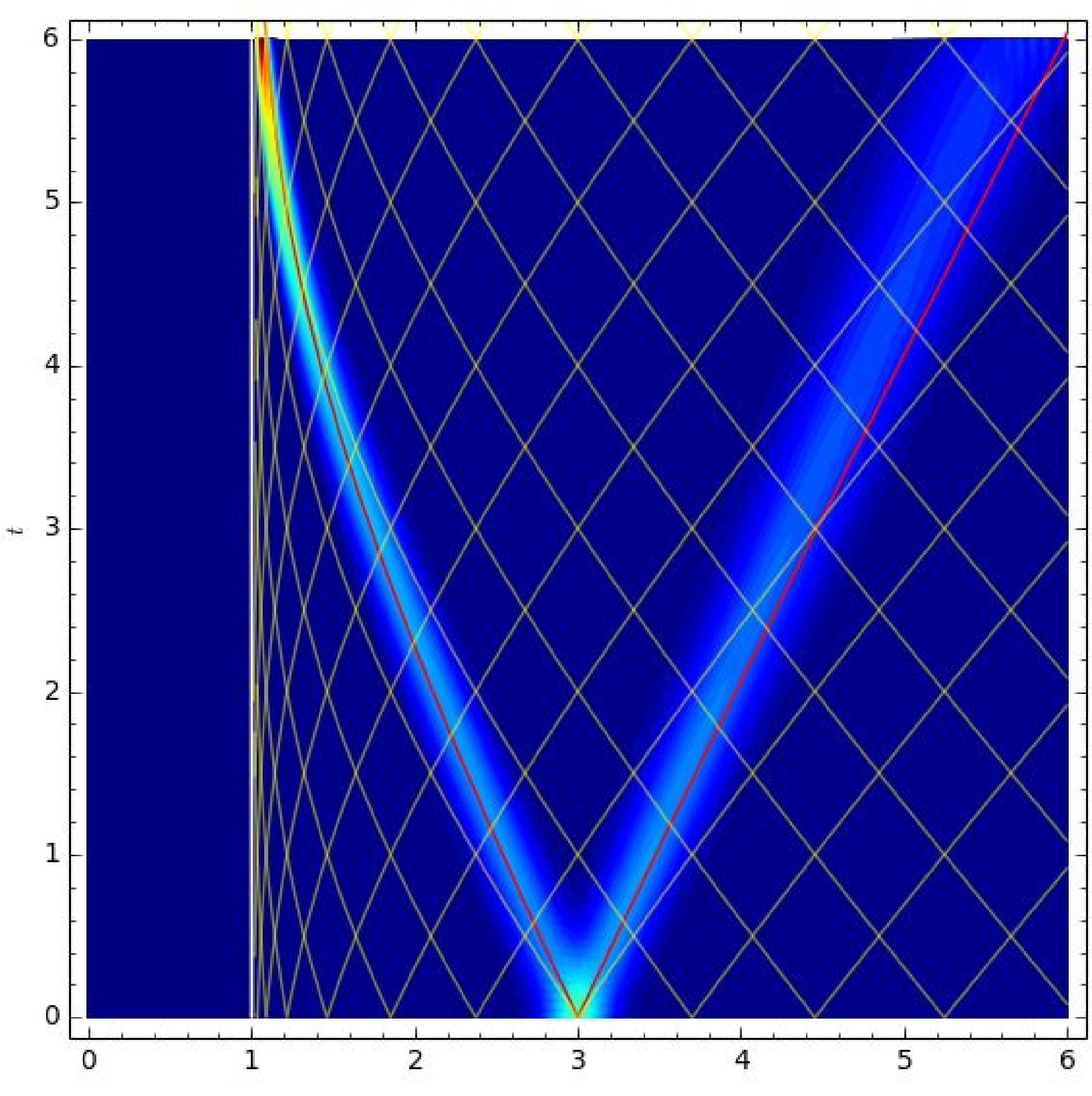
$$(\text{with } B_k,\ C \in \text{Herm}(\mathbb{C}) \text{ and } |B_k| \leq 1)$$

implementable by applying unitary matrices locally.

Curved space simulations : RW



Curved space simulations : BH



Conclusion

Non-interacting physics particles in curved space-time

...as a Quantum Walk.

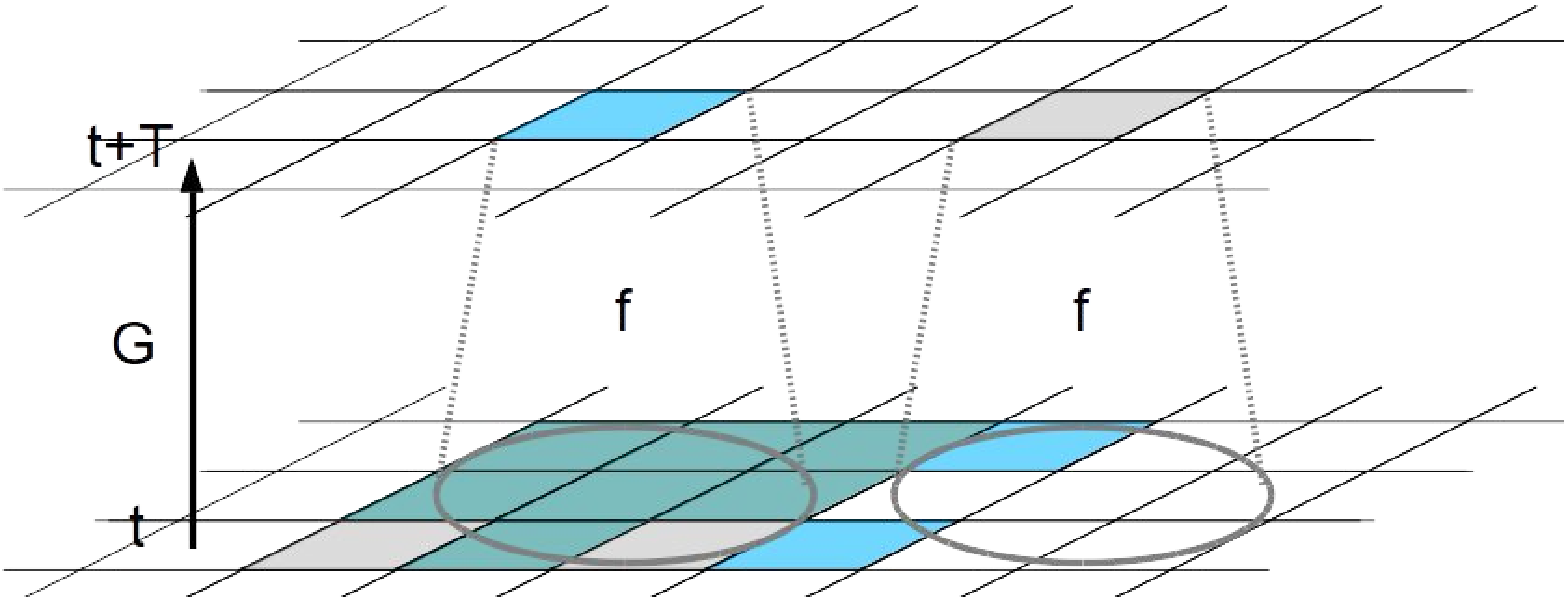
The point?

- stable numerical scheme
- quantum simulation device compatible
- to simplify, understand, offer toy models.

OK, but what about symmetries?

Extra 1

Discretize physics?

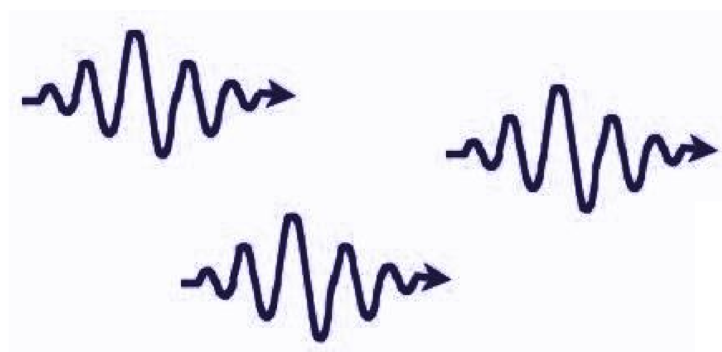


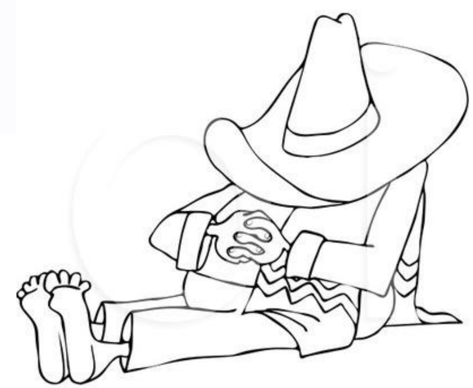
Cellular Automatas / Quantum Walks

Theorems about : the extent in which the SR notion of time can be captured in this formalism.

Time in SR

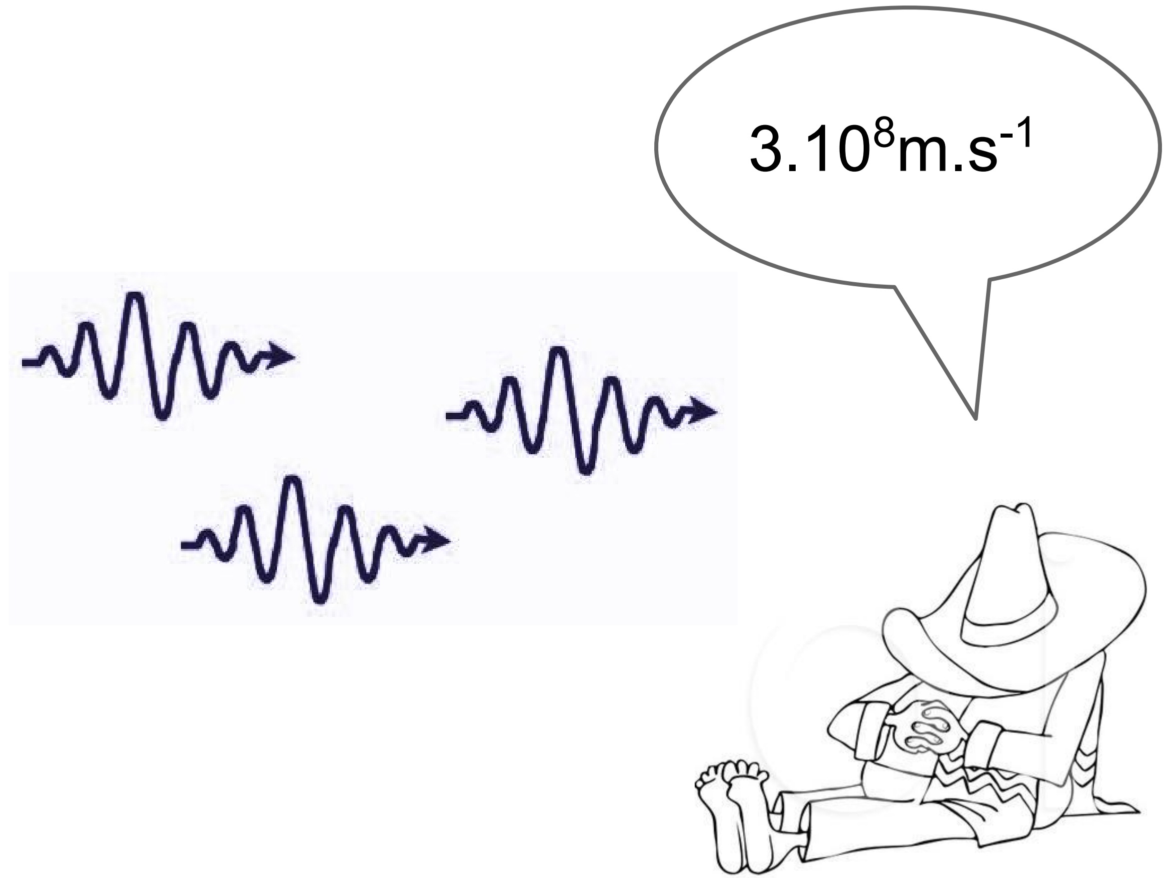
Observer at rest





Time in SR

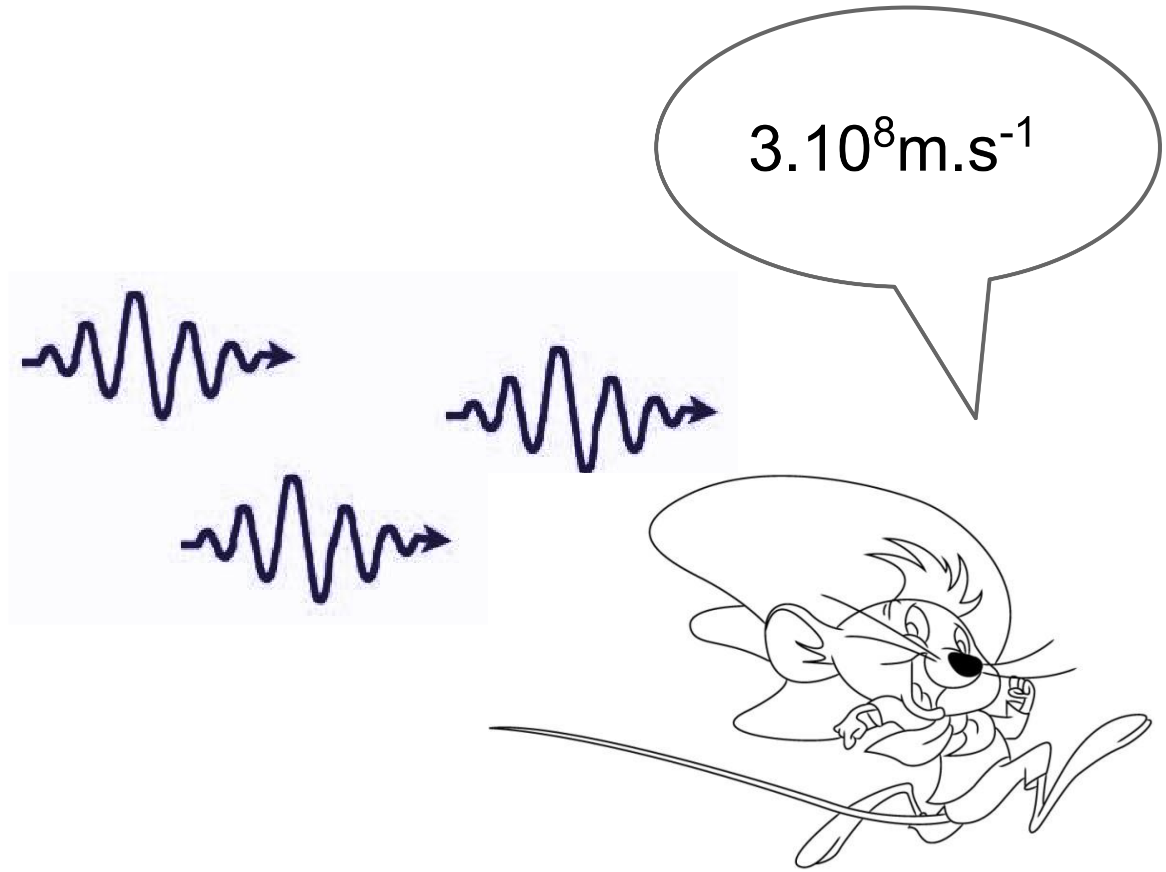
Observer at rest



Time in SR

Observer at rest

Uniform observer



Time in SR

Observer at rest

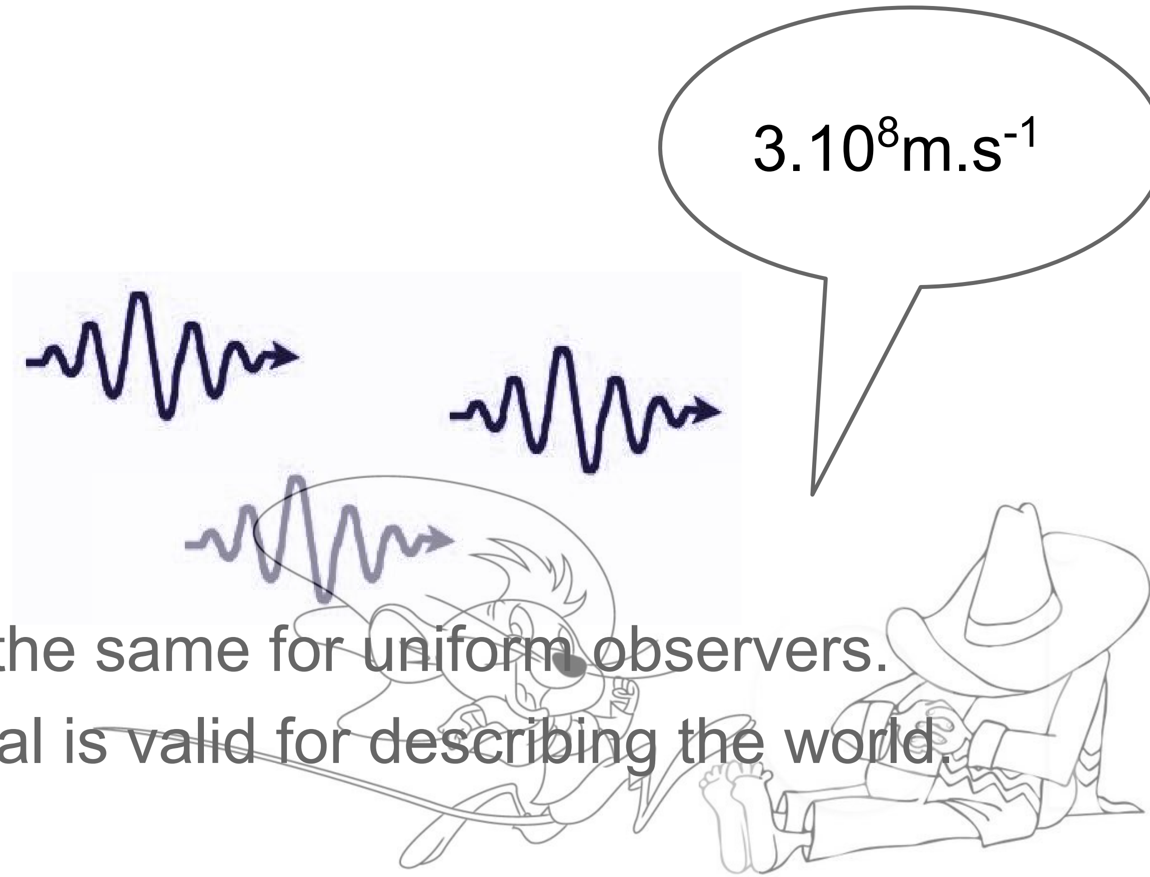
Uniform observer

Relativity

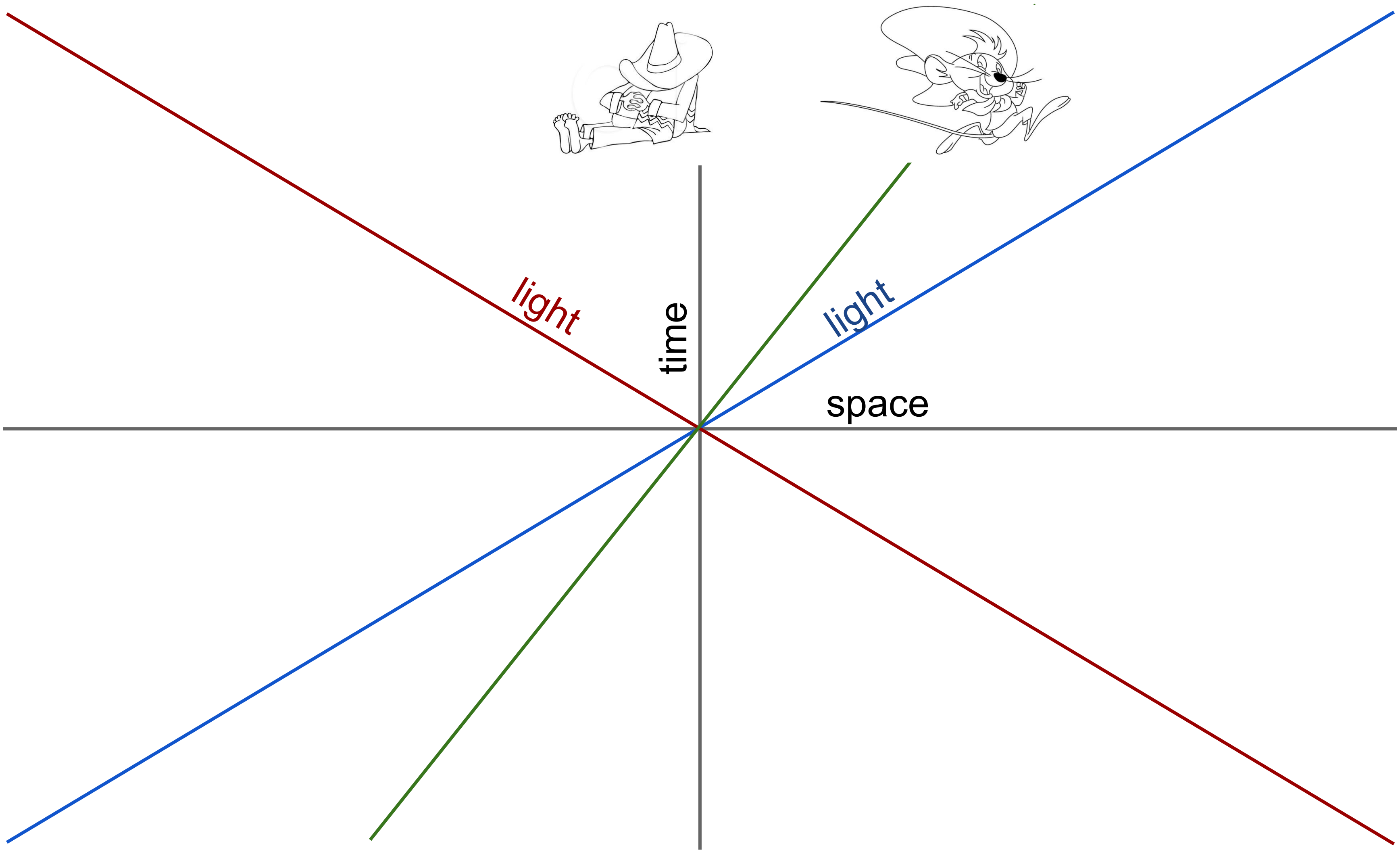
Both are right.

Laws of Physics are the same for uniform observers.

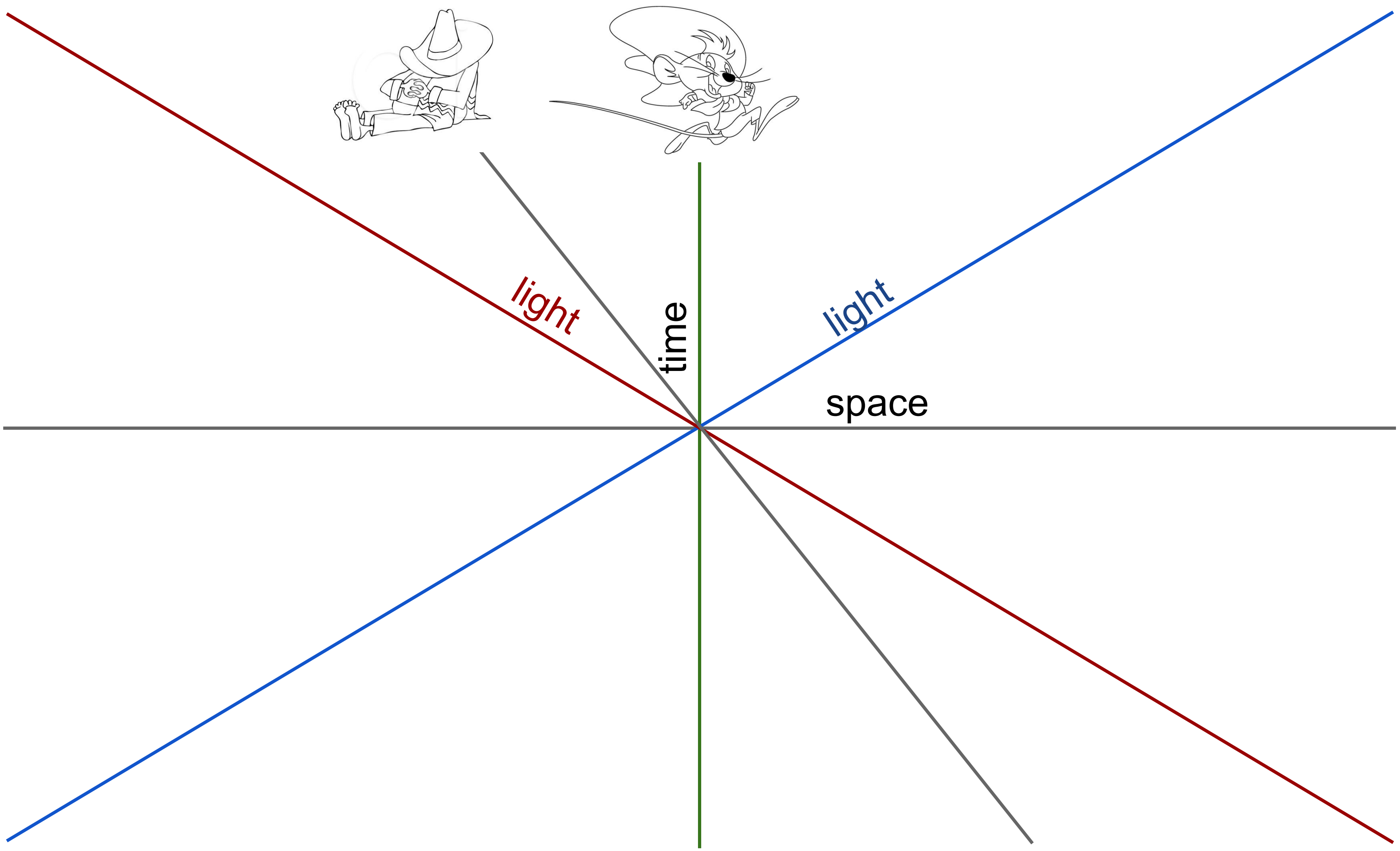
Any uniform referential is valid for describing the world.



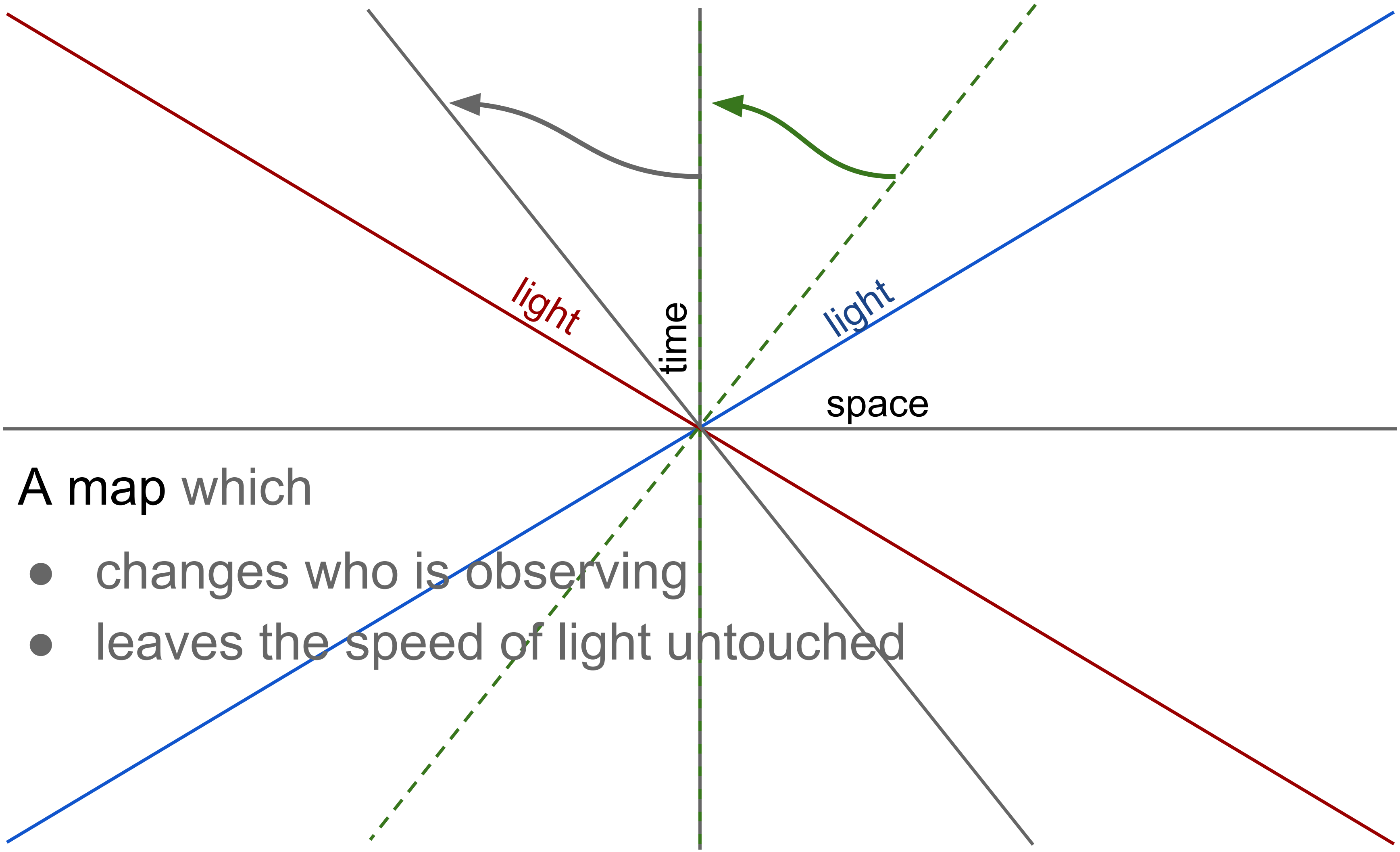
Time in SR



Time in SR



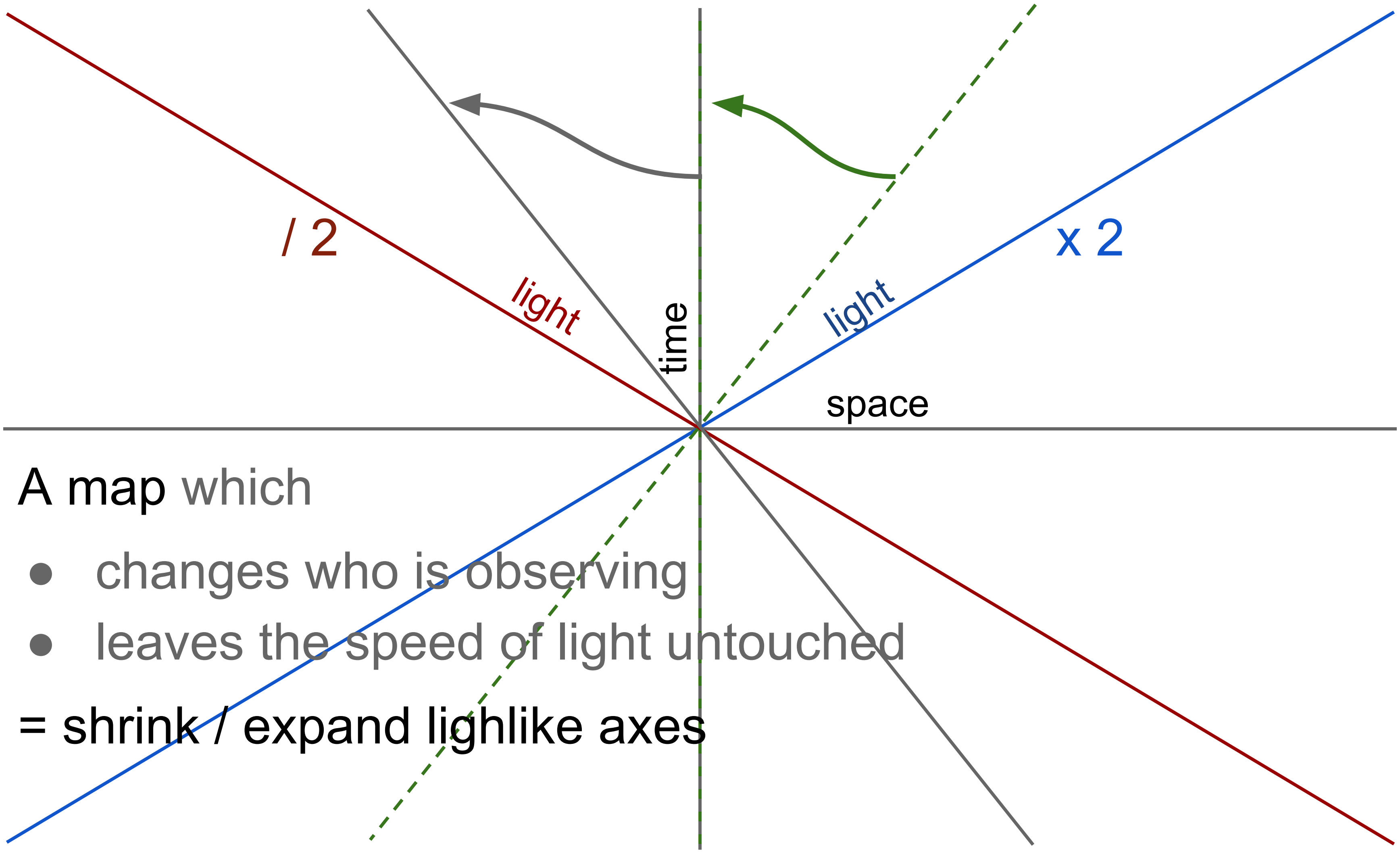
Lorentz transform



A map which

- changes who is observing
- leaves the speed of light untouched

Lorentz transform

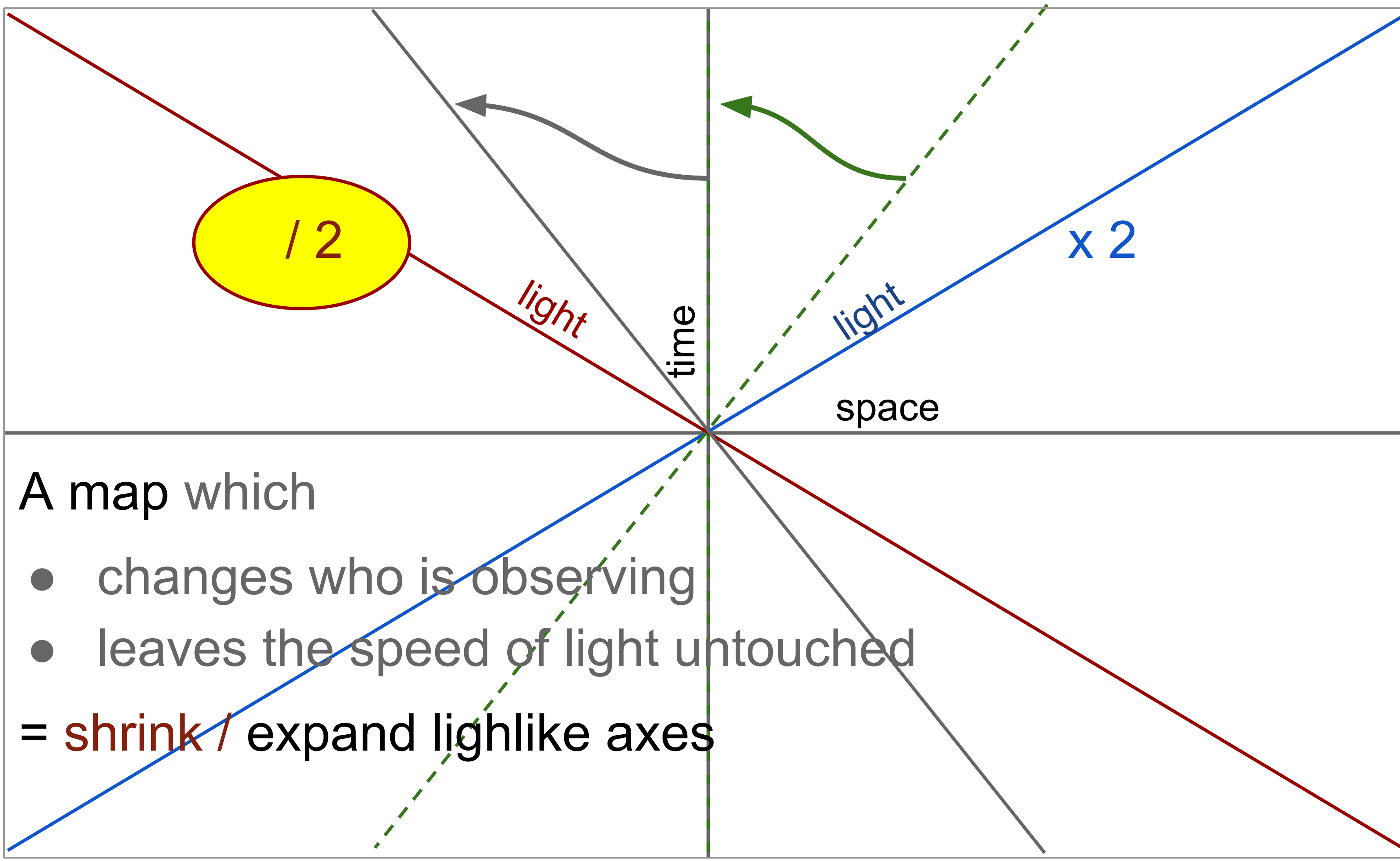


A map which

- changes who is observing
- leaves the speed of light untouched

= shrink / expand lighlike axes

Lorentz transform

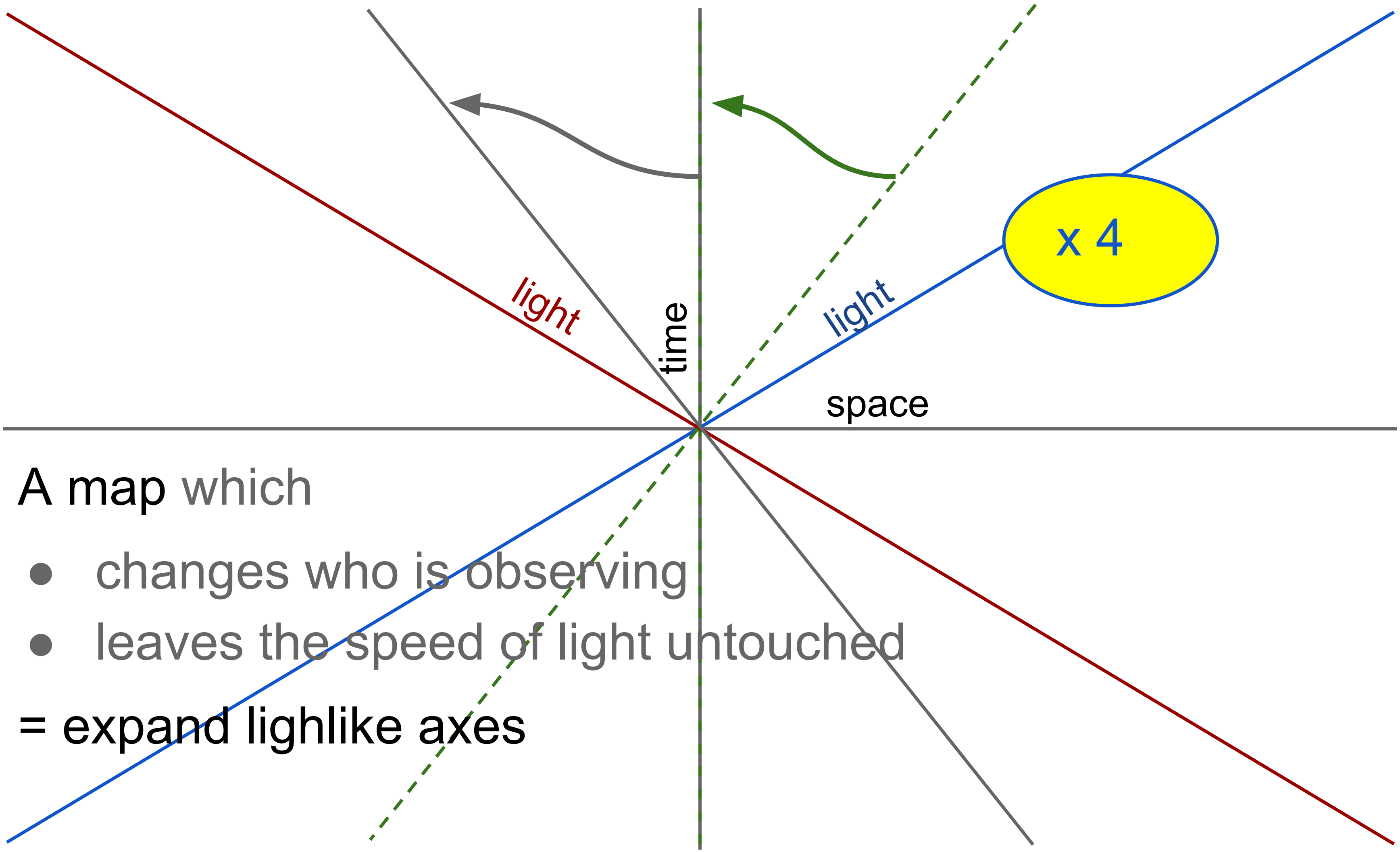


A map which

- changes who is observing
- leaves the speed of light untouched

= shrink / expand lighlike axes

Discrete Lorentz transform

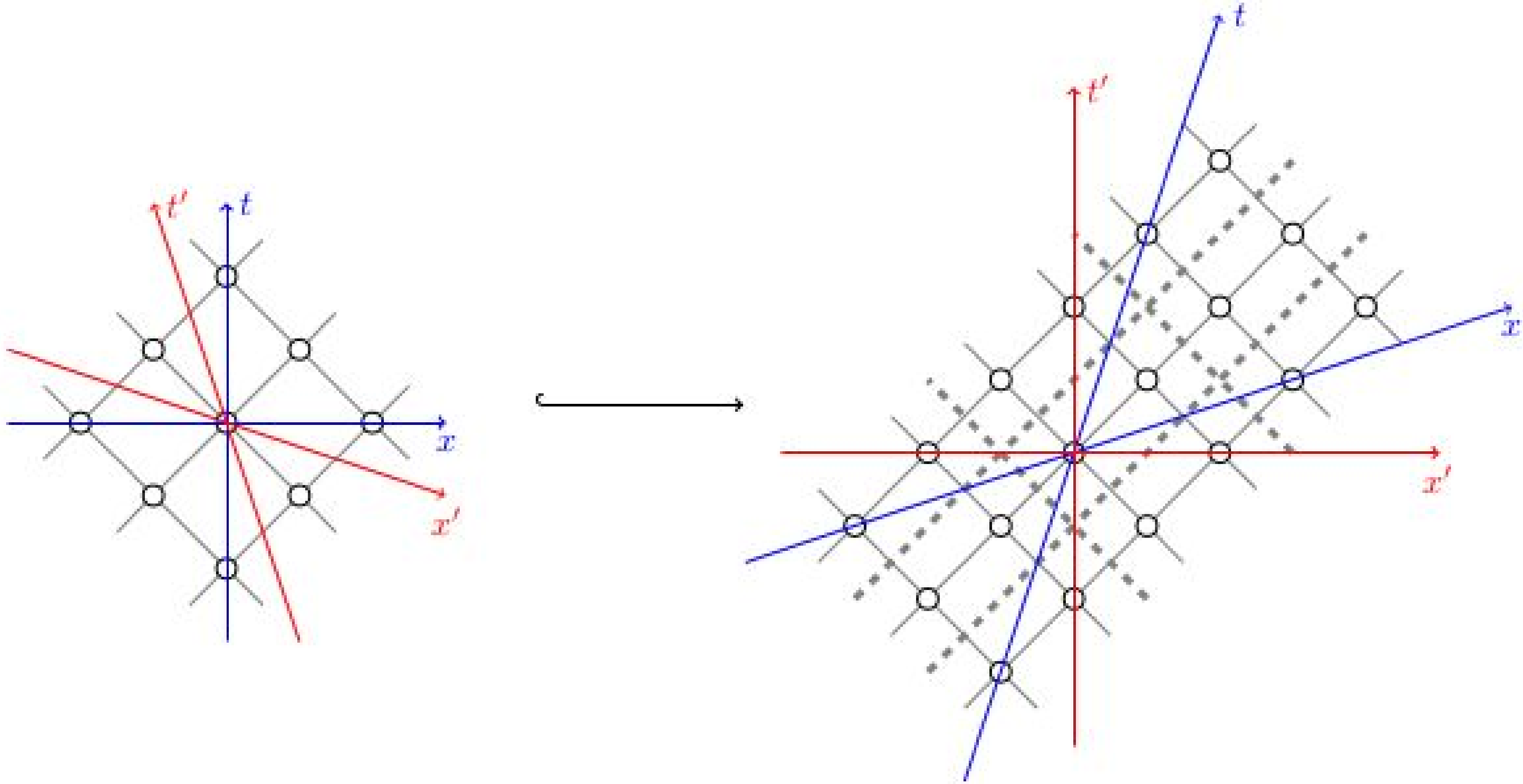


A map which

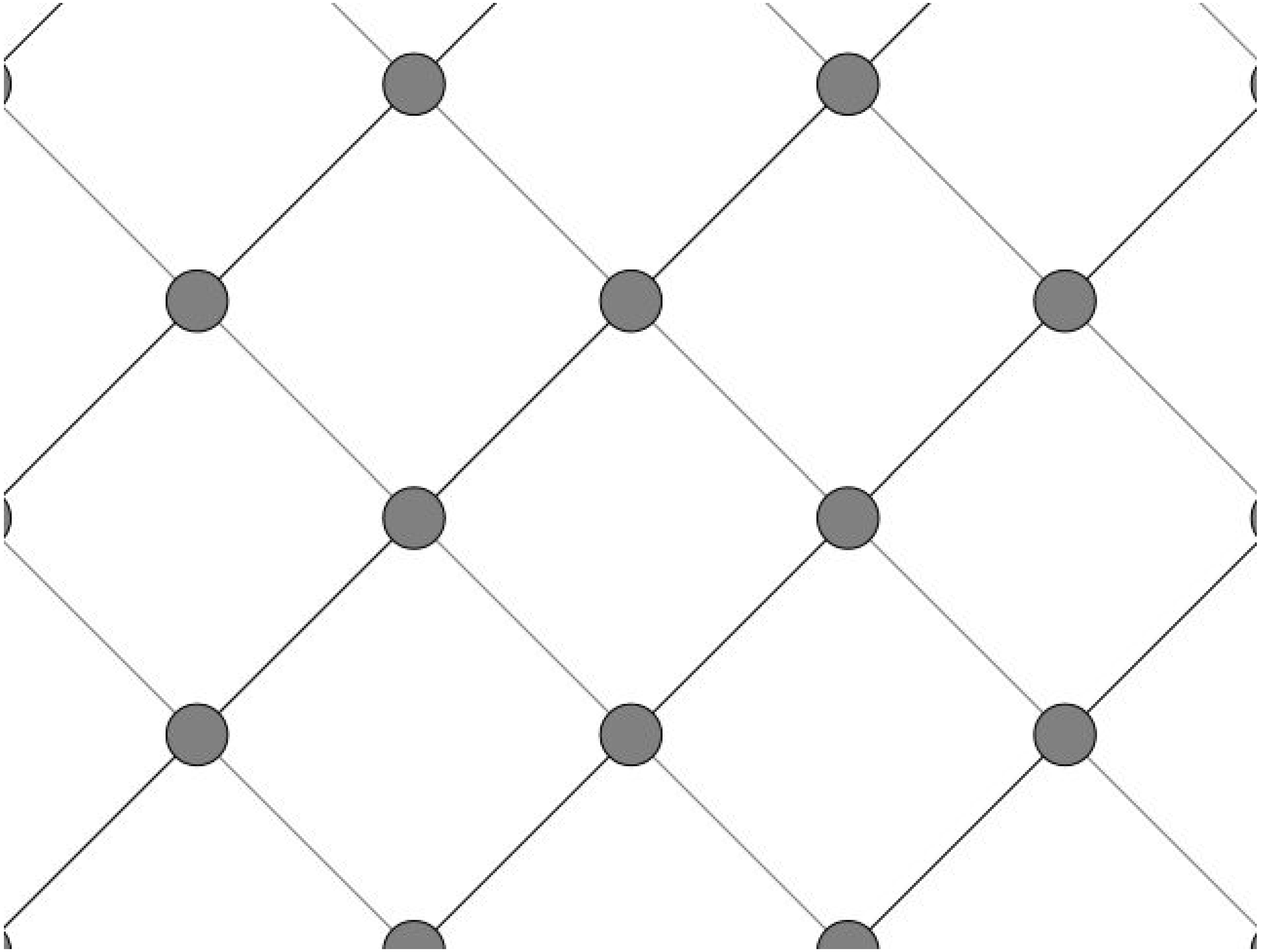
- changes who is observing
- leaves the speed of light untouched

= expand lighlike axes

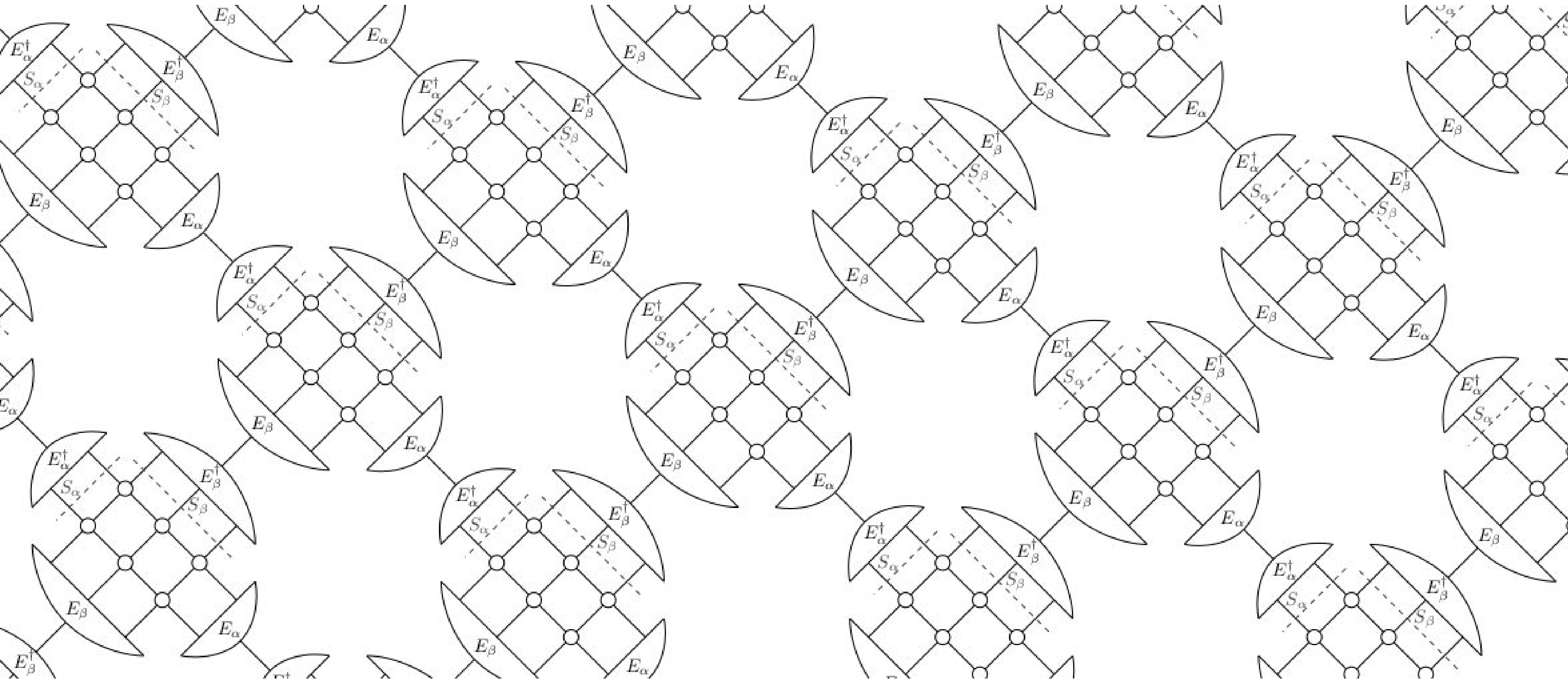
Discrete Lorentz transform



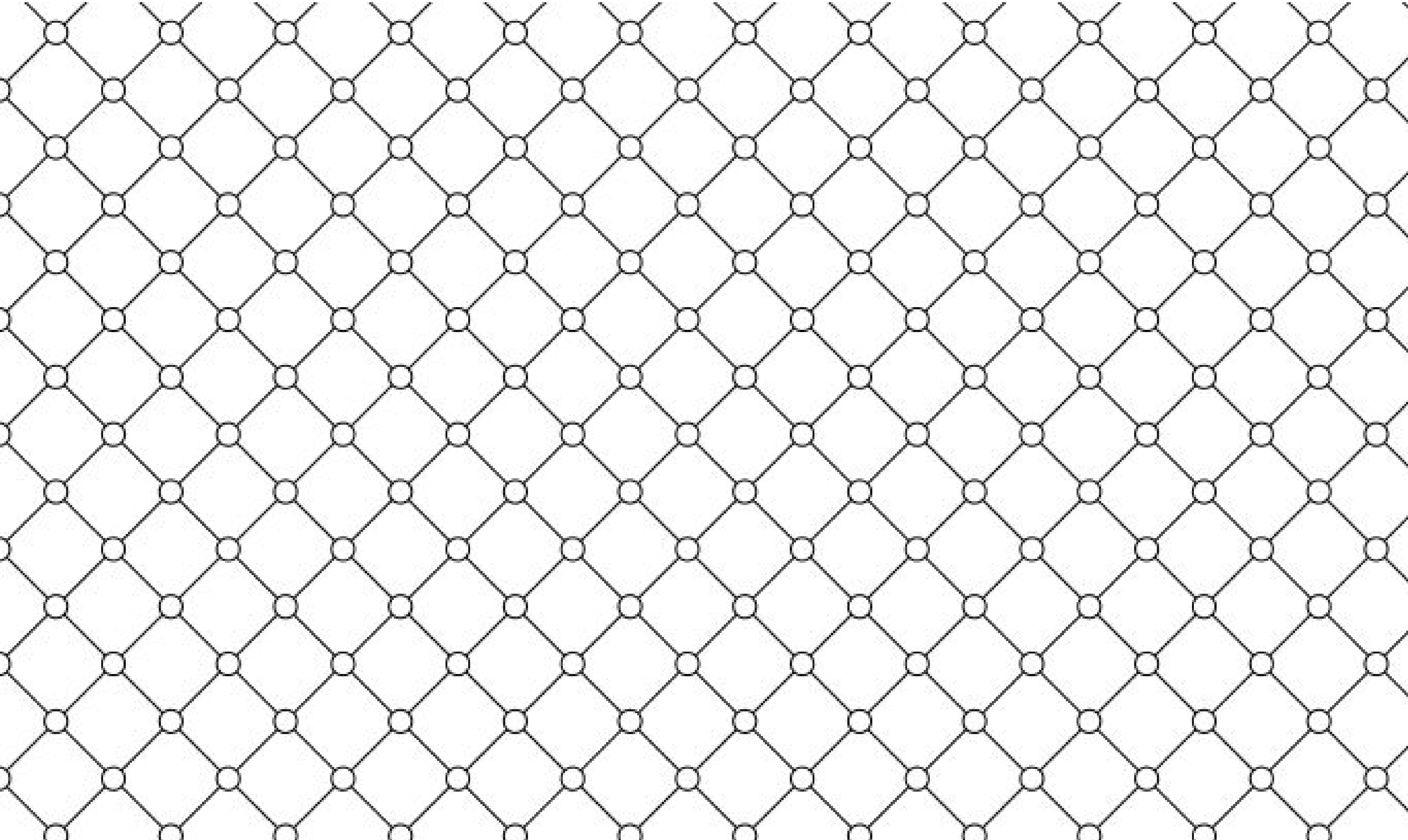
Discrete Lorentz transform



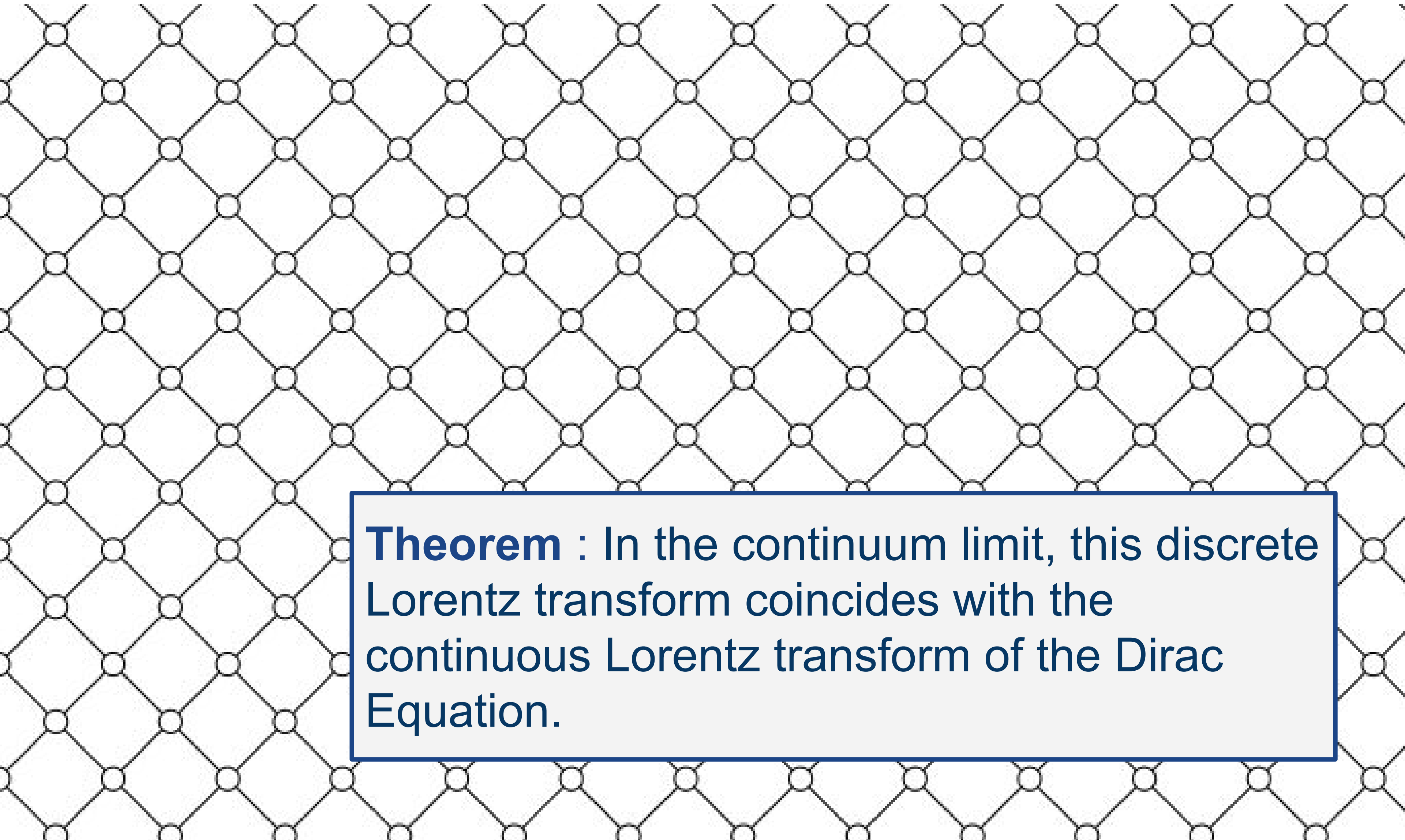
Discrete Lorentz transform



Discrete Lorentz transform



Discrete Lorentz transform



Theorem : In the continuum limit, this discrete Lorentz transform coincides with the continuous Lorentz transform of the Dirac Equation.

Covariance

Relativity

Laws of Physics are the same for uniform observers.

Any uniform referential is valid for describing the world.

Transform(Dirac Equation) = Dirac Equation

Transform(Physics Law) = Physics Law

A fundamental symmetry of physics.

Can it be discretized?

Covariance

Relativity

Laws of Physics are the same for uniform observers.

Any uniform referential is valid for describing the world.

Transform(Dirac Equation) = Dirac Equation

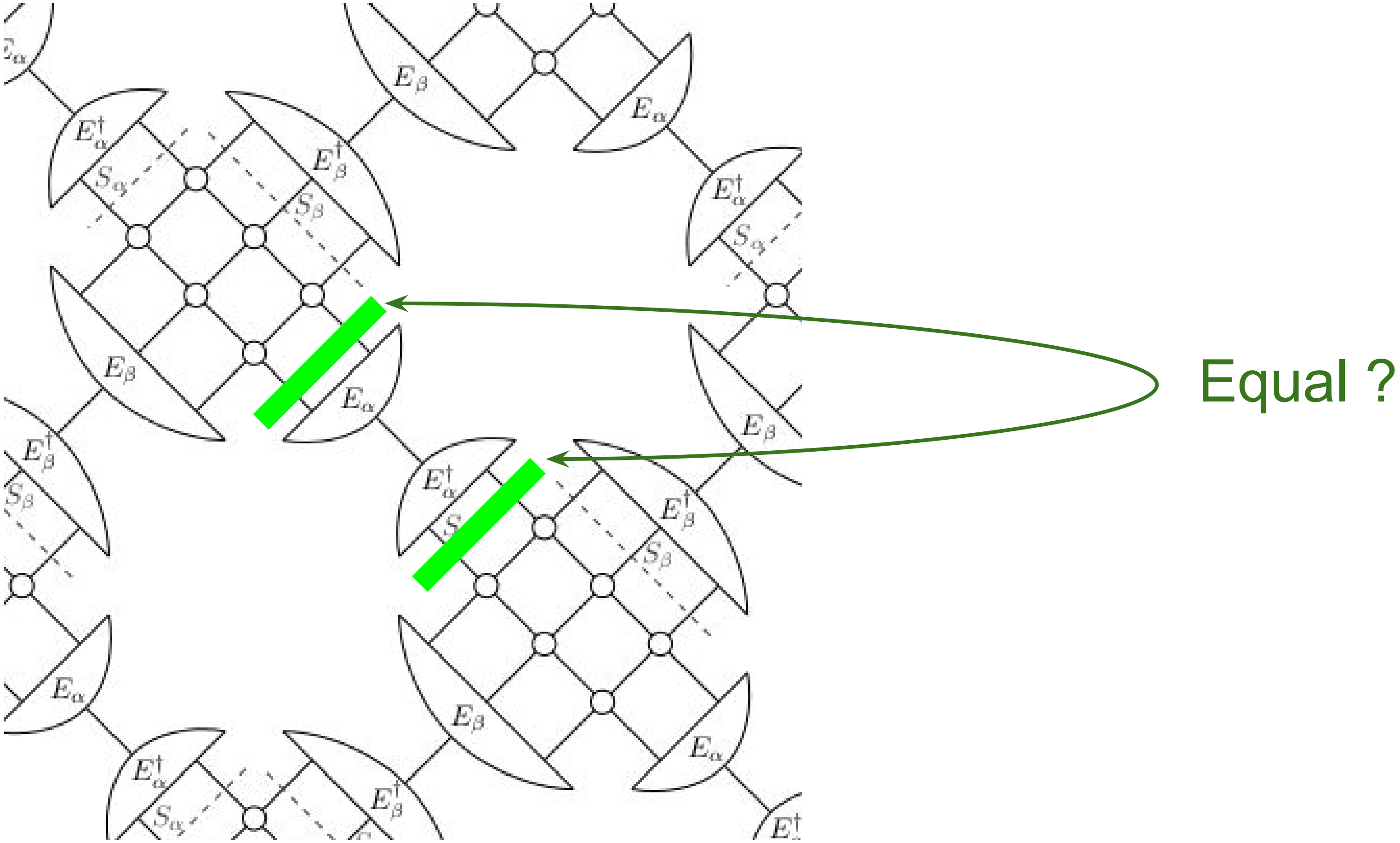
Transform(Quantum Walk) = Quantum Walk?

A fundamental symmetry of physics.

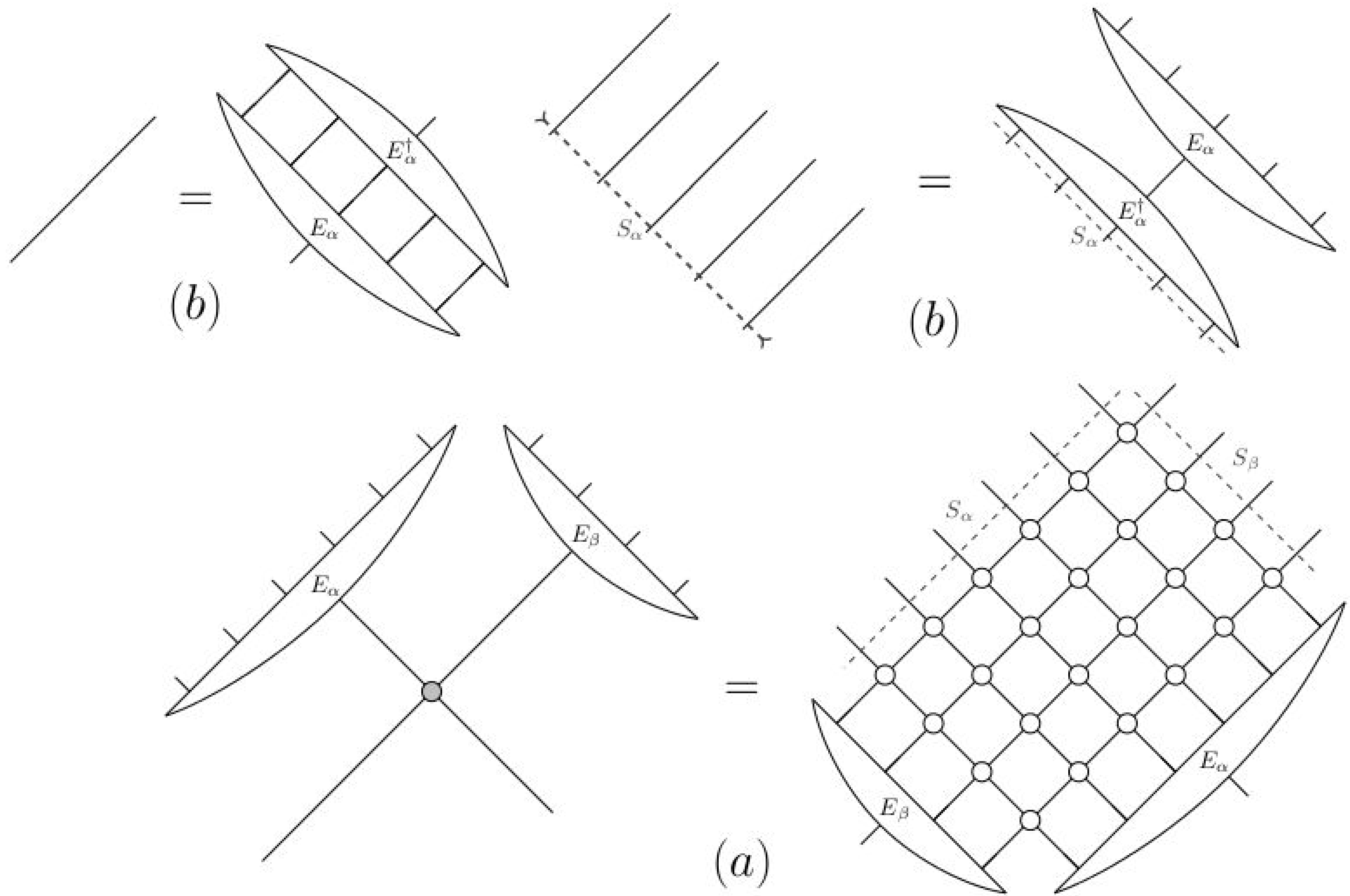
Can it be discretized?

Covariance

Transform(Quantum Walk) = Quantum Walk?

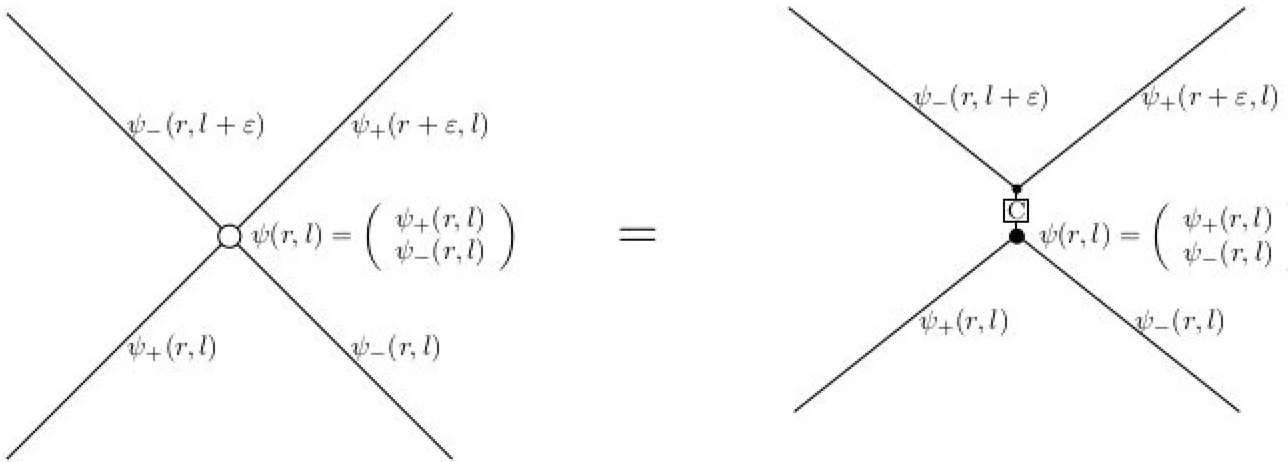


Discrete covariance

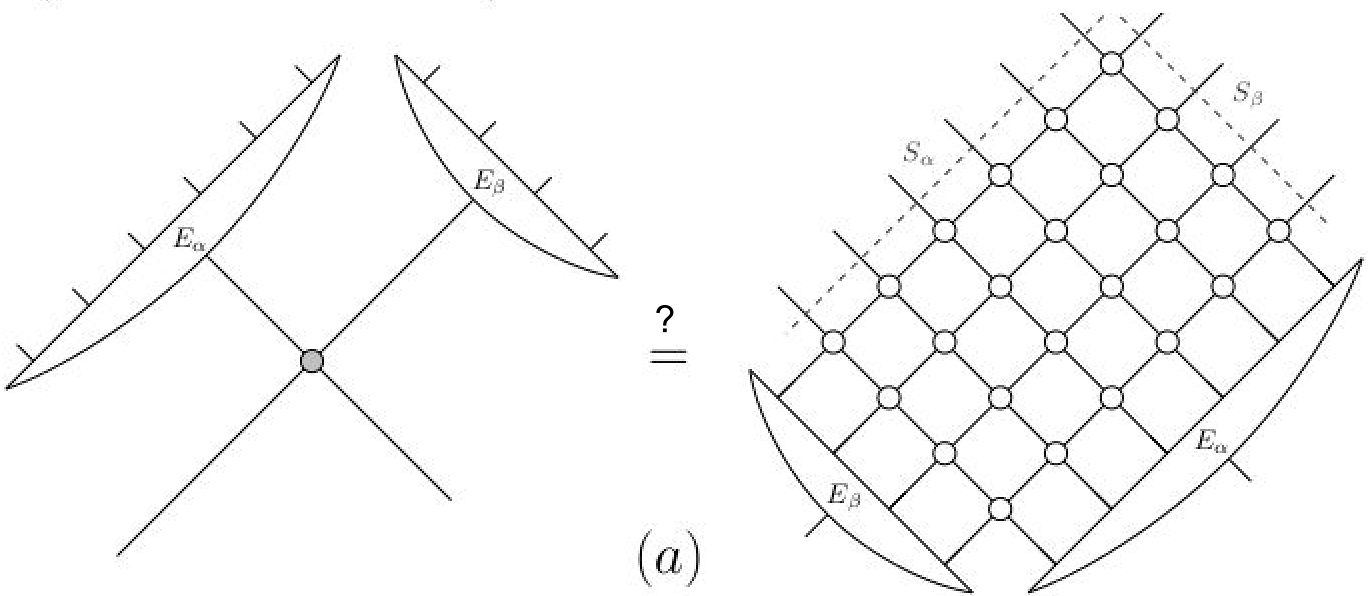


Discrete covariance

If

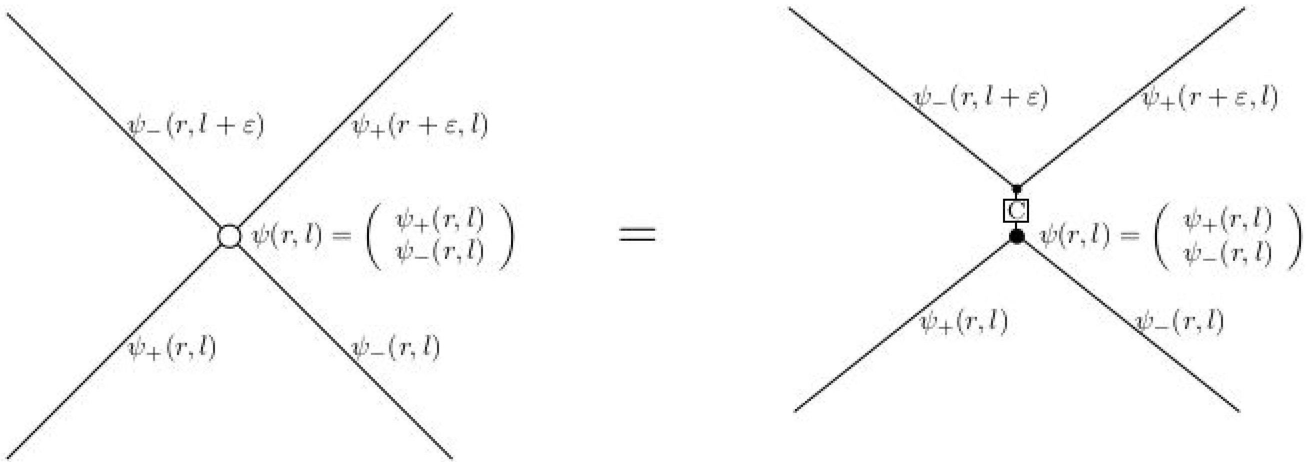


Then

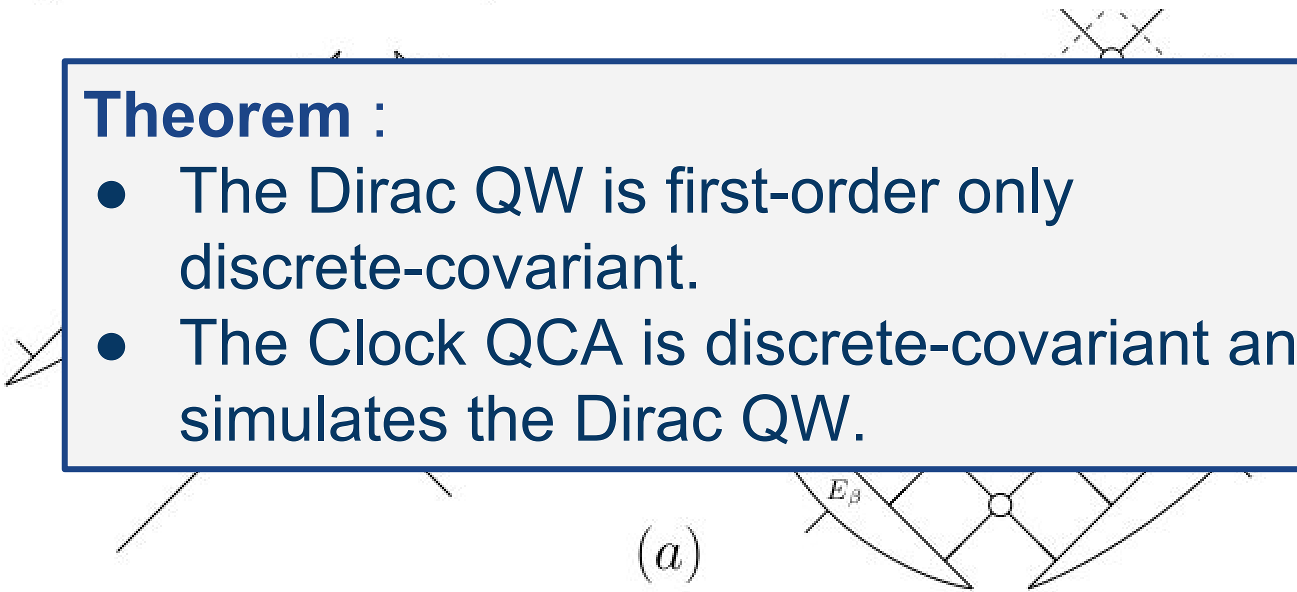


Discrete covariance

If



Then



Theorem :

- The Dirac QW is first-order only discrete-covariant.
- The Clock QCA is discrete-covariant and simulates the Dirac QW.

Indulging into reductionism

Indulging into reductionism

We might leave in a “great quantum circuit”.

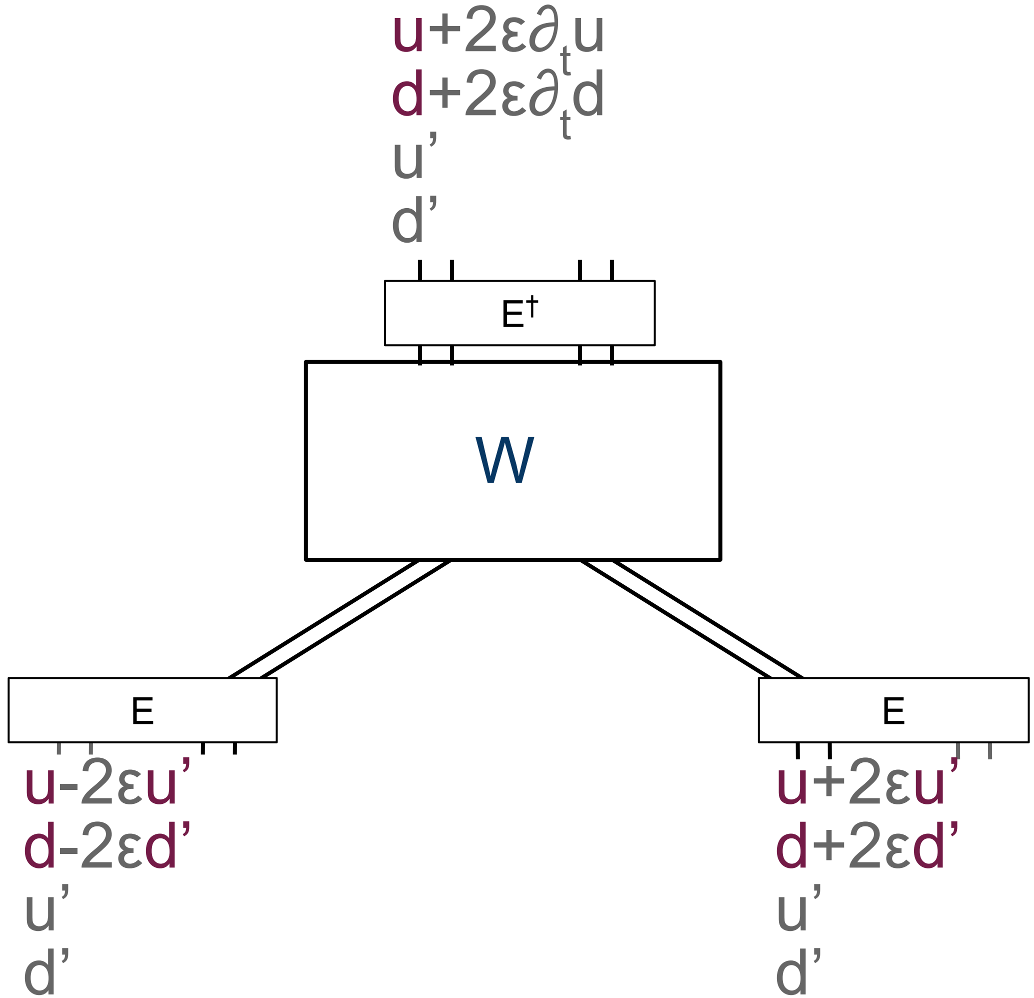
This great quantum circuit would be equivalent to some others… each of which would be a valid representation of our world.

The notion of time would then be relative to this choice of representation, just like in SR.

Extra 2

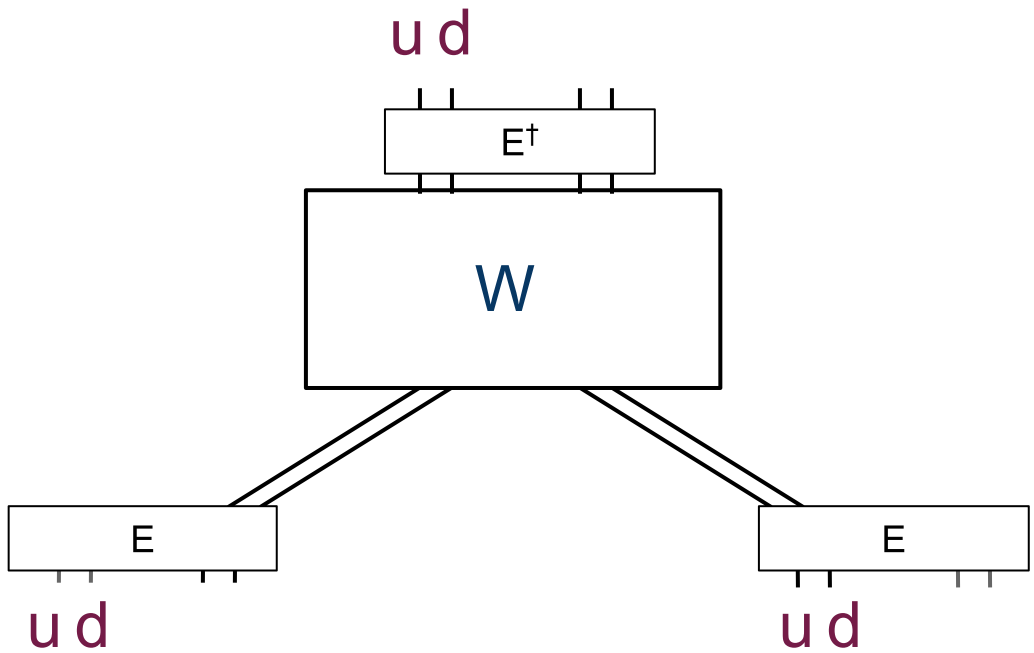
Curved space : idea 2

0^{th} order



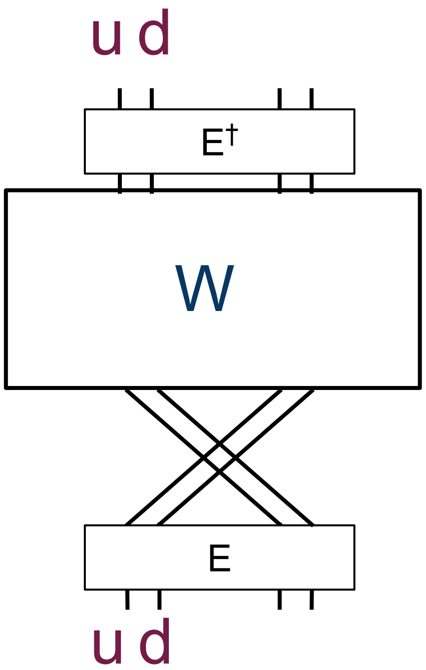
Curved space : idea 2

0th order



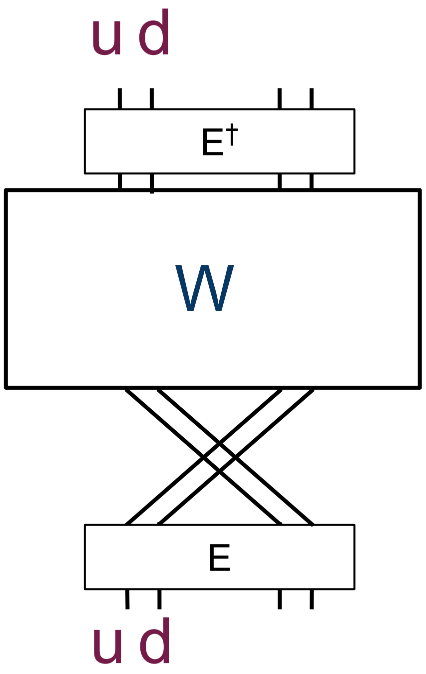
Curved space : idea 2

0^{th} order



Curved space : idea 2

0^{th} order

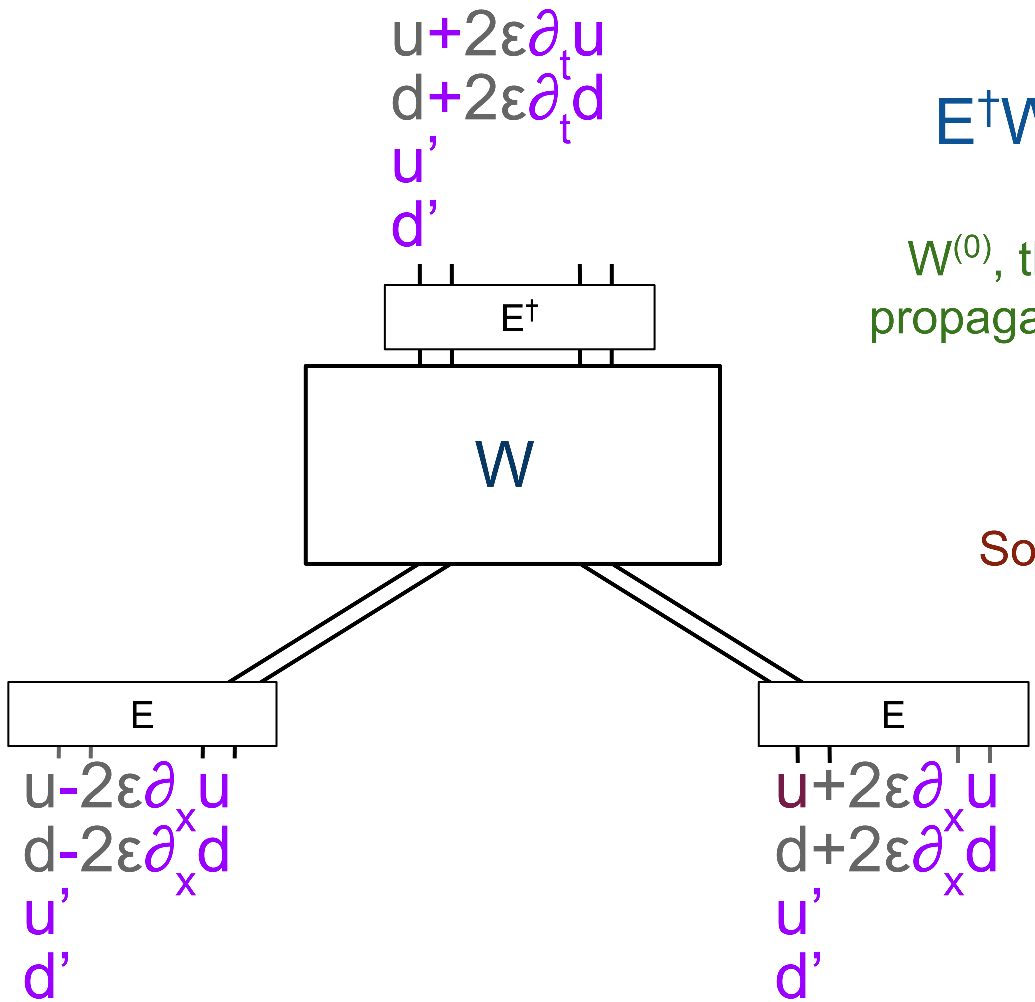


$E^{\dagger}W^{(0)}XE = I \oplus U$

$W^{(0)}$, that which governs propagation, is non-trivial, and this still has a continuous limit.

Curved space : idea 2

1^{st} order



$E^{\dagger}W^{(0)}XE = I \oplus U$

$W^{(0)}$, that which governs propagation, is non-trivial, and this still has a continuous limit.

Some constraints for consistency?

Curved space : idea 2

1^{st} order

$2\partial_t u$
$2\partial_t d$
u'
d'

$E^\dagger W^{(0)} X E = I \oplus U$

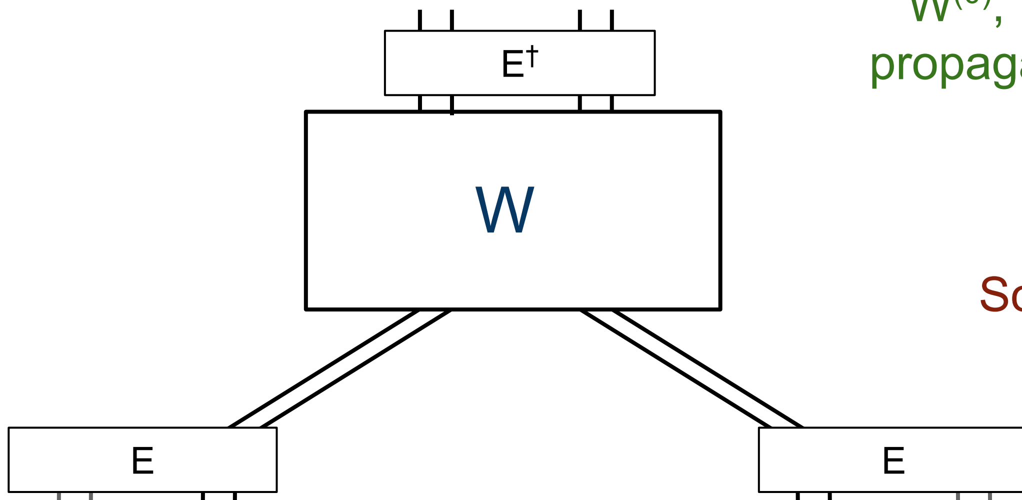
$W^{(0)}$, that which governs propagation, is non-trivial, and this still has a continuous limit.

$u = \psi^+$

$d = \psi^-$

$u' = 2\varepsilon\partial_x\psi^-$

$d' = 2\varepsilon\partial_x\psi^-$



Some constraints for consistency?

-2u'
-2d'
u'
d'

2u'
2d'
u'
d'

Curved space : idea 2

1^{st} order

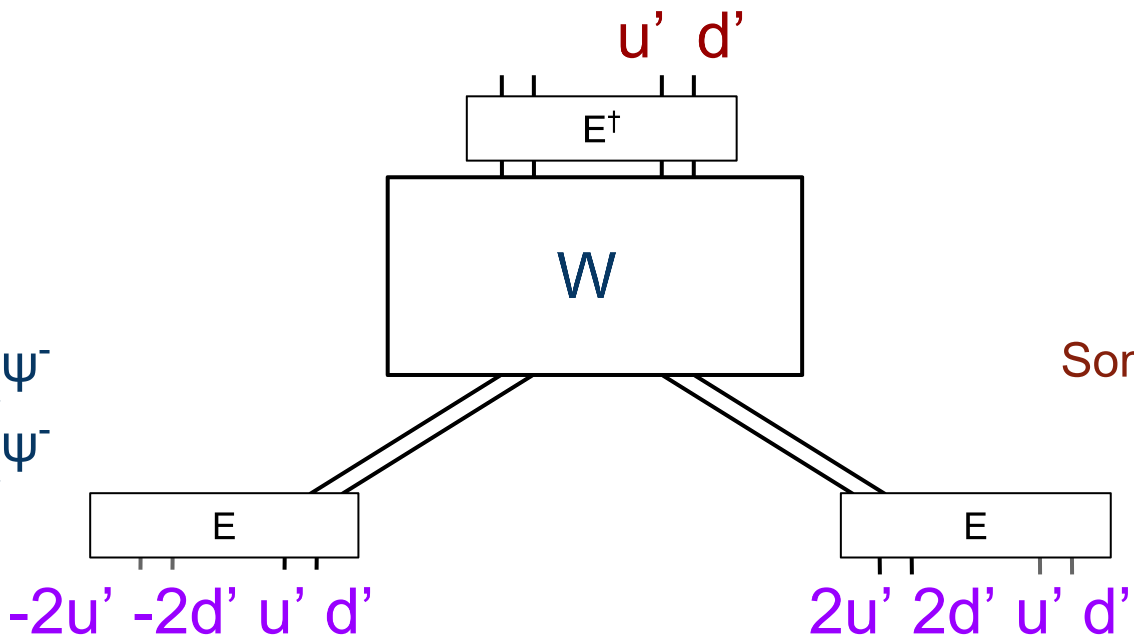
$E^{\dagger}W^{(0)}XE = I \oplus U$

$u = \psi^+$

$d = \psi^-$

$u' = 2\varepsilon\partial_x\psi^-$

$d' = 2\varepsilon\partial_x\psi^-$



Some constraints for consistency?

Yes, but with non-trivial solutions.

Curved space : idea 2

1^{st} order

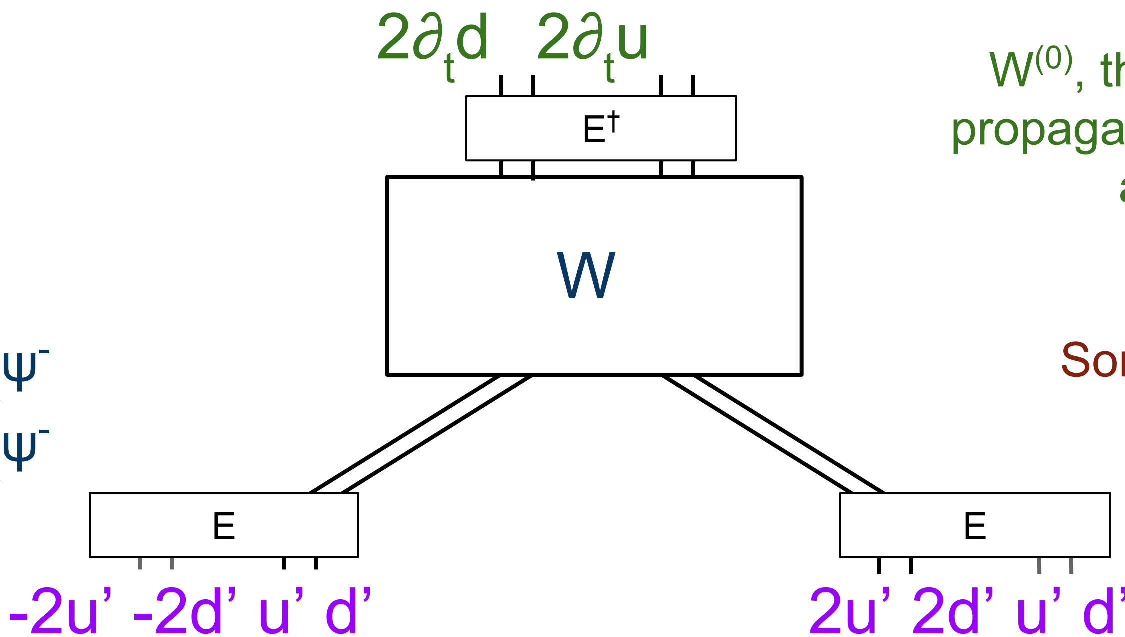
$E^{\dagger}W^{(0)}XE = I \oplus U$

$u = \psi^{+}$

$d = \psi^{-}$

$u' = 2\varepsilon\partial_x\psi^{-}$

$d' = 2\varepsilon\partial_x\psi^{-}$

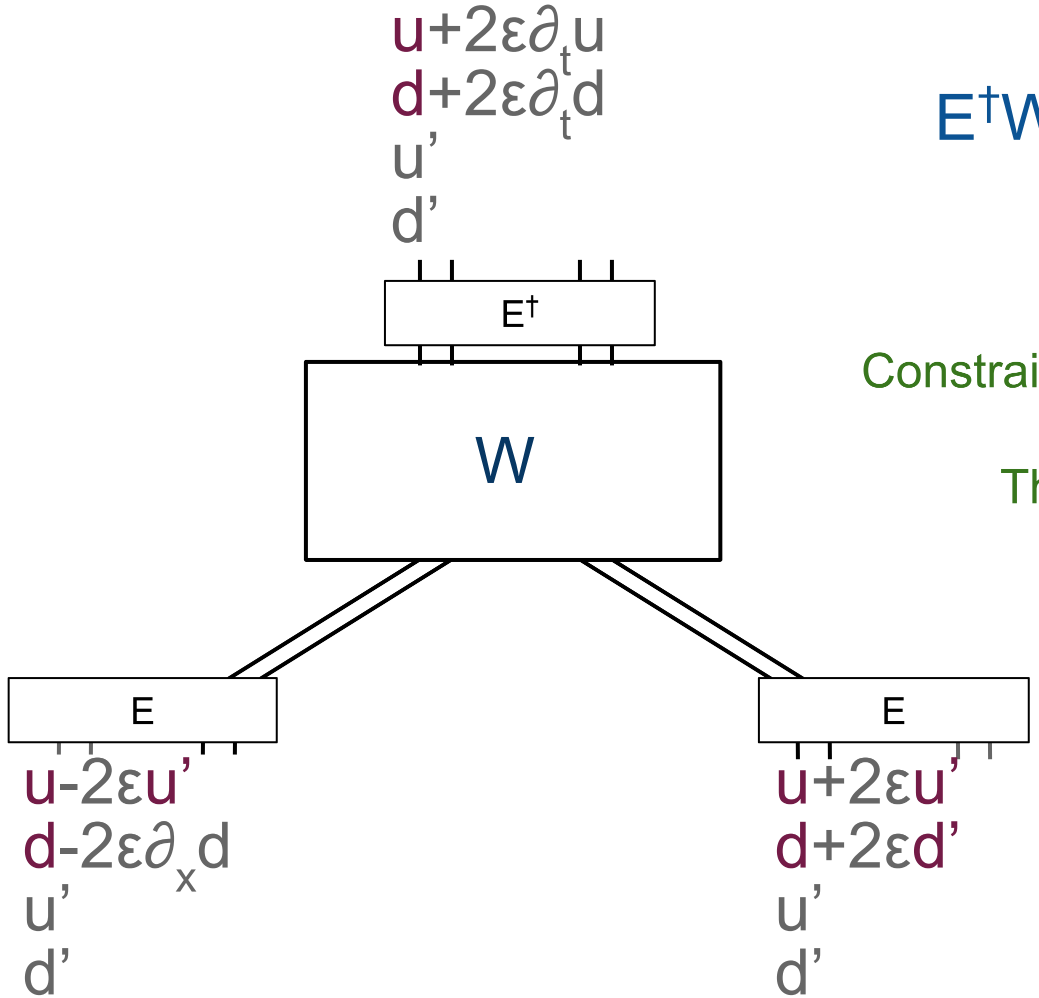


$W^{(0)}$, that which governs propagation, is non-trivial, and this still has a continuous limit.

Some constraints for consistency?

Yes, but with non-trivial solutions.

Curved space : Dirac Eq.



$E^{\dagger}W^{(0)}XE = I \oplus U$

A QW.

Constrained to have a lim.

Those limits are the Curved Dirac Eq.