Numerical modeling of tracer tests in single injection/extraction wells

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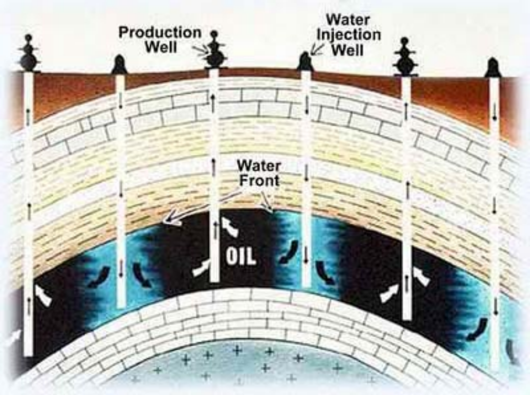
Contents

Introduction

Primary recovery uses only natural reservoir pressure to transport oil or gas toward of the production wells

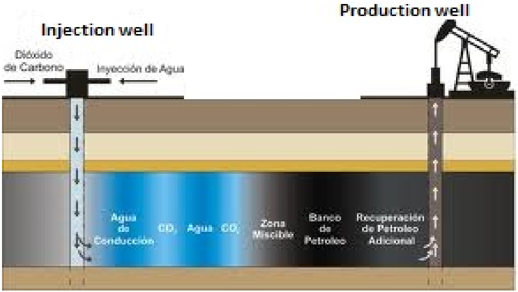
Secondary recovery

In **secondary recovery** the reservoir pressure is increased or maintained by the injection of water or other fluid into the porous formation



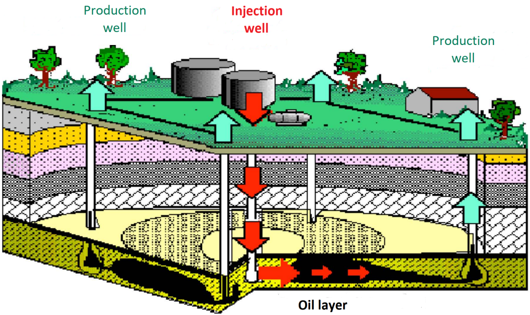
Enhanced Oil recovery (EOR)

These methods consider changes in properties of rock (wettability) or fluid (viscosity or interfacial tension). They are extremely expensive and are only used when the characteristics of the rock are fully known.



Tracer tests

A tracer test consists of the injection of a radioactive or chemical substance dissolved in the injection fluid that is monitored until arriving at the neighboring production wells.

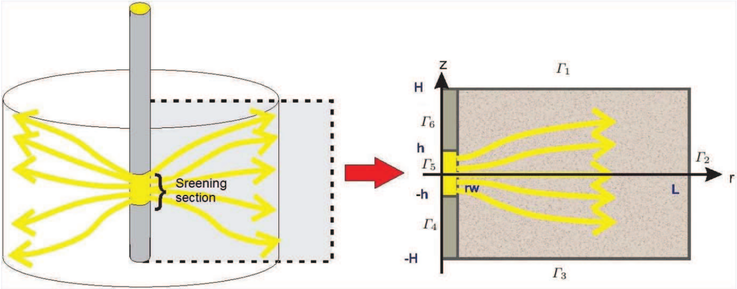


Tracer tests

Tracer tests are employed for secondary and enhanced oil recovery

They are used for characterize reservoirs and determine:

1. communication channels in the reservoirs and
2. properties such as porosity, absorption delay coefficient and thickness of the oil layer.

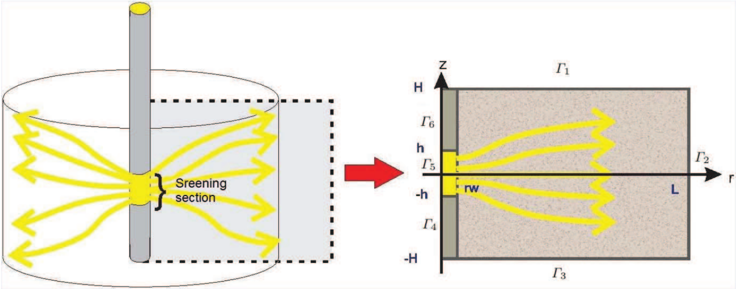


Our goal is to analyze the tracer behavior in partially penetrating injection wells when the vertical component of velocity is taken into account

Flow with axial symmetry, it depends on r and z

3D region of interest

2D system domain, $\overline{\Omega}$



$\overline{\Omega} = [r_w, L] \times [-H, H]$, cylindrical coordinates with fixed θ

Zemel, 1995; Kass, 1998; Ptak et al., 2004, Cosler, 2004; Chen et al., 2010

Assumptions. Porous media is homogeneous, the solutions are continuous and the fluid mass is much greater than the tracer mass

- **Fluid velocity**

  1. ***Find the pressure*** p
  2. ***Calculate the velocity*** $\overrightarrow{U}$

- **Tracer transport**

Fluid velocity

Darcy equation:

$$\overrightarrow{U} = -\frac{\kappa}{\mu}\nabla p$$

where κ is the permeability (constant), μ the fluid viscosity (constant) and p the pressure.

Mass conservation equation with constant porosity

$$\frac{\partial(\Phi\rho)}{\partial t} + \nabla \cdot (\rho\vec{U}) = 0.$$

for a non-compressible fluid in stationary-state is written as

$$\nabla \cdot \left(\frac{\kappa}{\mu}\nabla p\right) = 0$$

Dimensionless model of pressure

1. Find p such that

$$\nabla \cdot (\nabla p) = 0$$

Subject to the boundary conditions

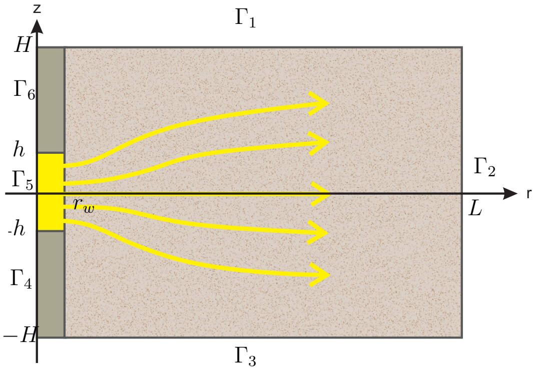
$$\begin{aligned}
\frac{\partial p}{\partial z}(r,z)\Big|_{z=-1} &= 0, \quad r \in \left[\frac{r_w}{H}, \frac{L}{H}\right], \\
\frac{\partial p}{\partial r}(r,z)\Big|_{r=\frac{L}{H}} &= -\frac{\mu Q}{4\pi\, p_0\, L\, \kappa}, \quad z \in [-1, 1], \\
\frac{\partial p}{\partial z}(r,z)\Big|_{z=1} &= 0, \quad r \in \left[\frac{r_w}{H}, \frac{L}{H}\right], \\
\frac{\partial p}{\partial r}(r,z)\Big|_{r=\frac{r_w}{H}} &= 0, \quad z \in \left[\frac{h}{H}, 1\right], \\
\frac{\partial p}{\partial r}(r,z)\Big|_{r=\frac{r_w}{H}} &= \frac{\mu Q H}{8\, p_0\, h\, r_w\, \kappa} \cos\left(\frac{\pi H}{2h} z\right), \quad z \in \left[-\frac{h}{H}, \frac{h}{H}\right], \\
\frac{\partial p}{\partial r}(r,z)\Big|_{r=\frac{r_w}{H}} &= 0, \quad z \in \left[-1, -\frac{h}{H}\right].
\end{aligned}$$

$Q = \int_{-h}^{h} \int_0^{2\pi} U_r r_w d\theta dz$, $U_r = A cos(\pi z/2h)$ is the fluid volume injection rate through the screening section.

Dimensionless model of velocity

2. Calculate the velocity

$$\overrightarrow{U} = -\frac{\kappa}{\mu}\nabla p$$



Variational formulation: Pressure

Find $p \in W$ such that

$$\int_{-1}^{1}\int_{r_w/H}^{L/H} \nabla w \cdot \nabla p \, r dr \, dz \quad = \quad \frac{L\delta_1}{H}\int_{-1}^{1} w \; dz - \frac{r_w \delta_2}{H}\int_{-h/H}^{h/H} w \cos\left(\frac{\pi H}{2h} z\right) \, dz, \quad \forall \; w \; \in \; W_0,$$

where

$$\begin{aligned} \delta_1 &= \frac{\mu \, Q}{4\pi \, p_0 \, L \, \kappa}, \\ \delta_2 &= \frac{\mu \, Q \, H}{8 \, p_0 \, h \, r_w \, \kappa}, \\ W &= \left\{ w \in H^1(\Omega) : w\left(\vec{n}_\gamma\right) = p_\gamma \right\} \quad \text{and} \\ W_0 &= \left\{ w \in H^1(\Omega) : w\left(\vec{n}_\gamma\right) = 0 \right\}. \end{aligned}$$

Girault and Raviart 1986,

Finite element method: Pressure

$$W_{\mathfrak{h}} = \left\{ w \in \mathfrak{C}^0(\Omega) : w|_{\Omega_e} \in \mathbb{Q}_n \ \forall\, \Omega_e \in \Omega_{\mathfrak{h}}, w(\vec{n}_\gamma) = p_\gamma \right\} \quad \text{con } n = 1, 2$$

$$W_{\mathfrak{h},0} = \left\{ w \in \mathfrak{C}^0(\Omega) : w|_{\Omega_e} \in \mathbb{Q}_n \ \forall\, \Omega_e \in \Omega_{\mathfrak{h}}, w(\vec{n}_\gamma) = 0 \right\}.$$

Linear system obtained is

$$K\,\vec{p}_{\mathfrak{h}} = \vec{F}$$

where $\vec{p}_{\mathfrak{h}}$ is the pressure vector and

$$K_{ij} = \sum_{e=1}^{ne} \int_{\Omega_e} \nabla\phi_i \cdot \nabla\phi_j \, r \, d\Omega$$

$$F_i = \sum_{e=1}^{ne} \frac{L\delta_1}{H} \int_{\Gamma_2} \phi_i \Big|_{\Gamma_2} d\Gamma - \sum_{e=1}^{ne} \frac{r_w \delta_2}{H} \int_{\Gamma_5} \phi_i \Big|_{\Gamma_5} \cos\left(\frac{\pi H}{2h} z\right) d\Gamma - \sum_{e=1}^{ne} p_\gamma \int_{\Omega_e} \nabla\phi_i \cdot \nabla\phi_\gamma \, r \, d\Omega.$$

Discrete fluid Velocity

Let be $\vec{U}_{\mathfrak{h}} = (U_{1,\mathfrak{h}}, U_{2,\mathfrak{h}})$ with

$$\vec{U}_{\mathfrak{h}} = -\frac{L}{H}\nabla p_{\mathfrak{h}}.$$

If $U_{1,\mathfrak{h}} = \sum_{i=1}^{nnt} U_{1,i}\,\phi_i(r,z)$ and $p_{\mathfrak{h}}(r,z) = \sum_{j=1}^{nnt} p_j \phi_j(r,z)$, then

$$U_{1,i} = -\frac{L \displaystyle\sum_{e=1}^{sop(\phi_i)} \int_{\Omega_e} \sum_{j=1}^{nnt} p_j \frac{\partial \phi_j}{\partial r} \phi_i \, d\Omega}{H \displaystyle\sum_{e=1}^{sop(\phi_i)} \int_{\Omega_e} \phi_i \, d\Omega}, \qquad \forall i = 1, 2, \ldots, nnt.$$

Glowinski 2003

Tracer transport

The advection-dispersion equation for the tracer concentration

$$\phi(r,z)\frac{\partial C}{\partial t} + \nabla \cdot \left[\vec{U}C - D\nabla C\right] = 0$$

where:

- $C(r,z,t)$ is the tracer concentration in the position (r,z) at time t,
- $\phi(r,z)$ is the porosity of the porous medium,
- D is a scalar dispersion coefficient.

Dimensionless model for the tracer concentration

Find $C(r, z, t)$ such that

$$\frac{\partial C}{\partial t} + \frac{1}{\Phi} \nabla \cdot \left[\vec{U} C - D \nabla C \right] = 0.$$

Subject to the boundary conditions

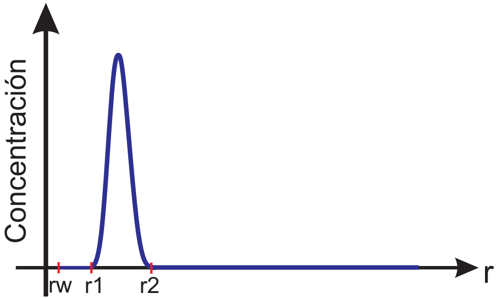
$$\begin{aligned}
C(r, z, t)\Big|_{r=\frac{L}{H}} &= 0, \quad z \in [-1, 1], \\
\left[\vec{U} C - D \nabla C \right] \cdot \vec{n}\Big|_{z=-1} &= 0, \quad r \in \left[\frac{r_w}{H}, \frac{L}{H} \right], \\
\left[\vec{U} C - D \nabla C \right] \cdot \vec{n}\Big|_{z=1} &= 0, \quad r \in \left[\frac{r_w}{H}, \frac{L}{H} \right], \\
\left[\vec{U} C - D \nabla C \right] \cdot \vec{n}\Big|_{r=\frac{r_w}{H}} &= 0, \quad z \in [-1, 1],
\end{aligned}$$

Initial condition

$$C^0 = \begin{cases} \mathcal{B}(r,z) & \text{si } r \in [r_1, r_2],\ z \in \left[-\frac{h}{H}, \frac{h}{H}\right] \\ & \\ 0 & \text{in other case} \end{cases}$$

with

$$\mathcal{B}(r,z) = \frac{1}{4}\left[\cos\left(\frac{\pi H}{h} z\right) + 1\right]\left[\cos\left(\frac{(2r - r_1 - r_2)\pi}{r_2 - r_1}\right) + 1\right].$$



Time discretization

Crank-Nicolson method:

$$C^{n+1} + \frac{\Delta t}{2}\mathfrak{L}C^{n+1} = C^n - \frac{\Delta t}{2}\mathfrak{L}C^n$$

where

$$\mathfrak{L}C^{n+1} = \frac{1}{\Phi}\left[\vec{U}\cdot\nabla C^{n+1} - \nabla\cdot(D\nabla C^{n+1})\right]$$

We define a spatial differential operator and the source term

$$\begin{aligned} L(C^{n+1}) &= \left(C^{n+1} + \frac{\Delta t}{2}\mathfrak{L}C^{n+1}\right) \\ f &= C^n - \frac{\Delta t}{2}\mathfrak{L}C^n \end{aligned}$$

Then

$$L(C^{n+1}) = f$$

Least square formulation

Sean $\mathcal{V} = \left\{ v \in H^1(\Omega) \;:\; v(r,z)\Big|_{\Gamma_5} = 0 \right\}$ y

$$J(C^{n+1}) = \frac{1}{2} \| L(C^{n+1}) - f \|^2$$

Least square problem:

$$J(C^{n+1}) \leq J(v) \qquad \forall v \in \mathcal{V}$$

Critical point:

$$\Big(L(v), L(C^{n+1}) \Big) - (L(v), f) = 0 \qquad \forall v \in \mathcal{V}$$

Find $C^{n+1} \in S_t$ such that

$$\left(v + \frac{\Delta t}{2} \mathfrak{L} v, C^{n+1} + \frac{\Delta t}{2} \mathfrak{L} C^{n+1} \right) = \left(v + \frac{\Delta t}{2} \mathfrak{L} v, C^{n} - \frac{\Delta t}{2} \mathfrak{L} C^{n} \right) \forall v \in \mathcal{V}$$

Finite element method: Tracer concentration

Crank-Nicolson method + Least Square formulation

Donea and Huerta, 2003

Linear system obtained

$$\left[M + \frac{\Delta t}{2\Phi}(G + DK + E_5) + \frac{\Delta t}{2\Phi}(G^T - DR) + \frac{\Delta t^2}{4\Phi^2}\left(\widehat{K} - D(S + S^T) + DT\right)\right]\vec{C}^{n+1} =$$

$$\left[M - \frac{\Delta t}{2\Phi}(G + DK + E_5) + \frac{\Delta t}{2\Phi}\left(G^T - DR\right) - \frac{\Delta t^2}{4\Phi^2}\left(\widehat{K} - D(S + S^T) + DT\right)\right]\vec{C}^{n}$$

Property:

- Matrices are constant in each time step
- Global matrix is not symmetric

Oil field data:

$$
\begin{aligned}
L &= \text{100 meters (a radial length)}\\
H &= \text{25 meters (thickness of porous layer)}\\
h &= \text{2.5 meters (height of screening section)}\\
r_w &= \text{10 centimeters (well radius)}\\
\mu &= \text{1 cp (viscosity)}\\
Q &= \text{2600 barrels/day (the injected flow rate)}\\
p_0 &= \text{3700 psi (pressure at z=0)}\\
\kappa_0 &= \text{12 miliDarcy (permeability)}\\
\Phi &= 10\,\% \text{ (porosity)}
\end{aligned}
$$

All algorithms are programmed and executed on **MATLAB** in a **PC** with Intel Core i5 processor, 3.2 GHz and 4 GB of RAM under Linux environment.

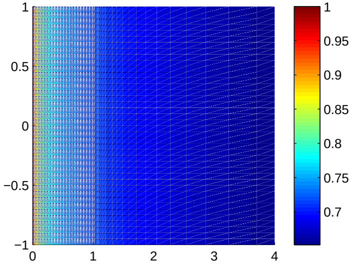
Fully penetrating wells: Code validation, $h = H$

Dimensionless analytical solution

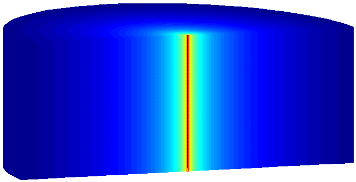
$$
\begin{aligned}
p_a(r) &= 1 - \frac{p_L}{p_0} \ln\left(\frac{H}{r_w} r\right), \quad \forall z \in [-1, 1] \\
\vec{U} &= \frac{L\, p_L}{H\, p_0\, r} \hat{r},
\end{aligned}
$$

where $p_0 = p(r_w)$ and $p_L = \frac{\mu Q}{4\pi H \kappa}$.

2D Pressure

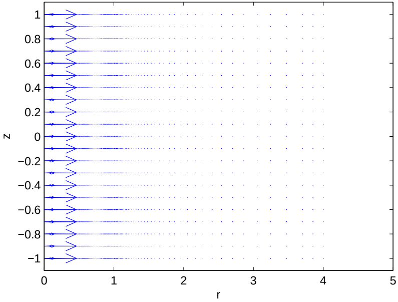


3D Pressure

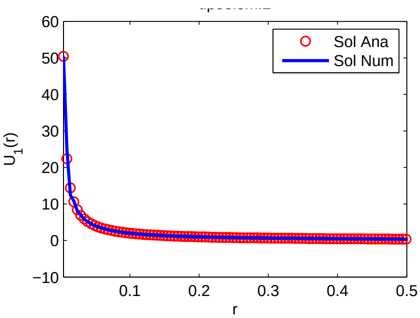


$h = H$ (validation problem)

Velocity field

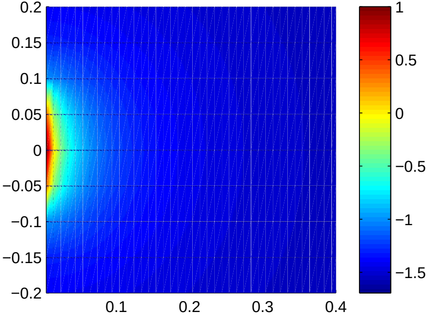


$z = 0$ (direction r)

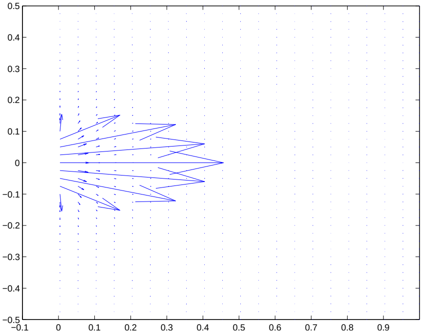


PARTIALLY PENETRATING WELLS: $h \neq H$

Normalized pressure



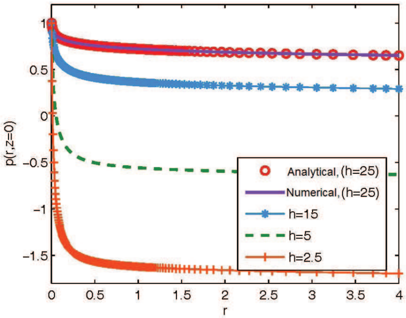
Injected fluid velocity field

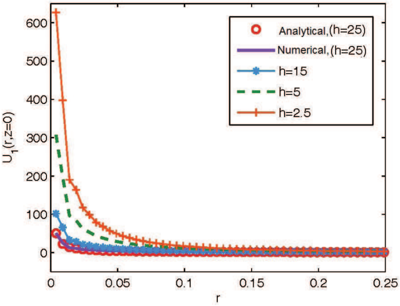


Partially penetrating wells: $h \neq H$

Pressure at $z = 0$

Radial velocity at $z = 0$

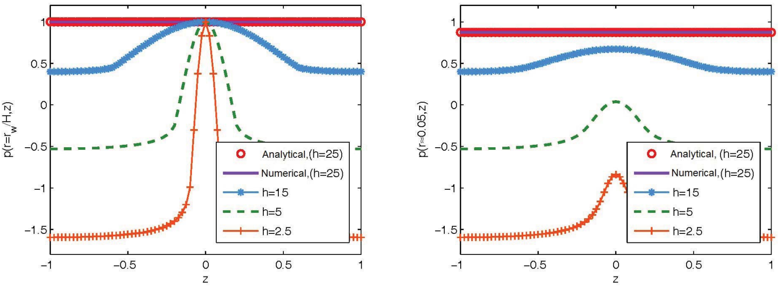




Pressure gradient and radial velocity near the wellbore increase as the screening zone thickness h decreases

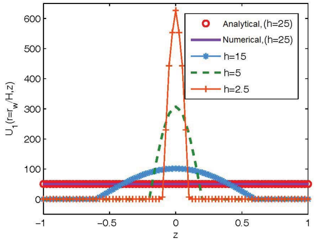
Vertical profiles of the pressure

r is fixed

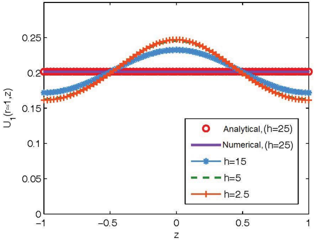


(a) $r = r_w/H = 0.004$ (b) $r = 0.05$

Radial velocity profiles

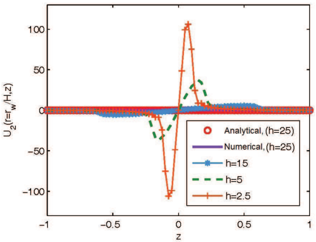


(c) $r = r_w/H = 0.004$

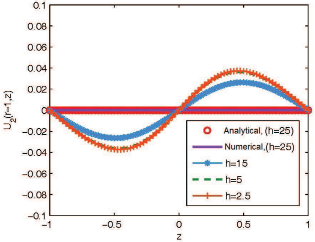


(d) $r = 0.05$

Vertical velocity profiles



(e) $r = 0.004$

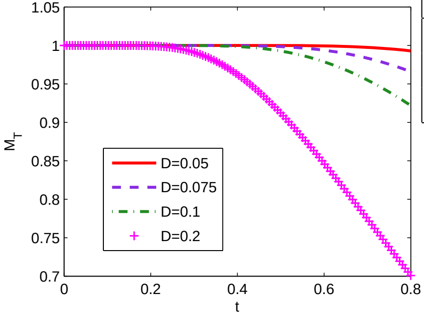


(f) $r = 1$

Tracer concentration

$$M_T = \int_V C(r, z, t)\; dV, \quad \forall t \in [0, T], \quad T = 0.8$$

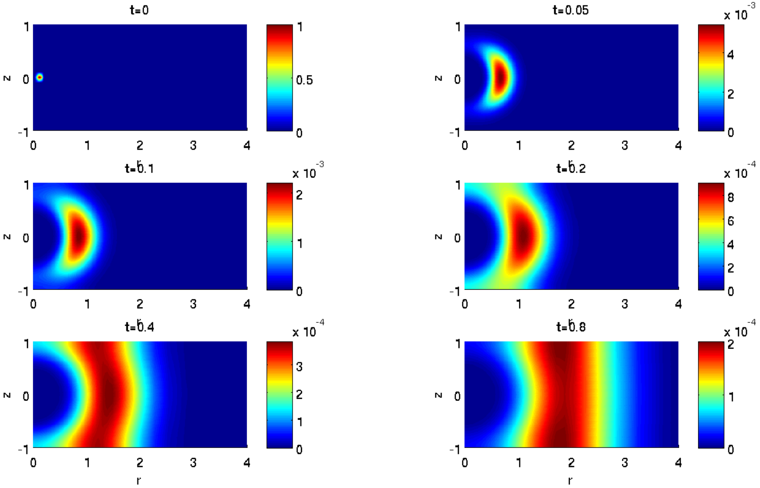
Tracer mass inside the system



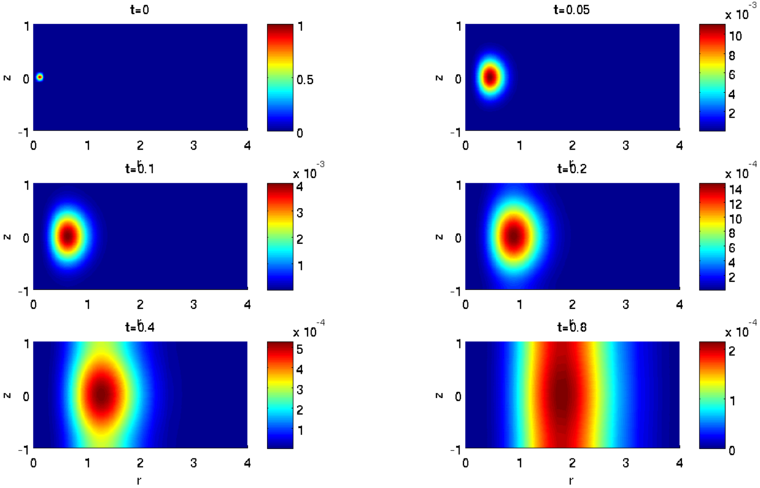
D	Final Mass	Error
0.05	0.9930	0.7045 %
0.075	0.9659	3.41 %
0.1	0.9219	7.88 %

The pulse dynamics

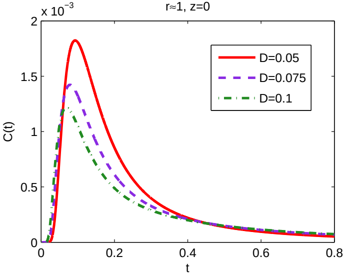
Tracer pulse dynamics in the dimensionless r-z plane for $D = 0.05$ and $h = 2.5$



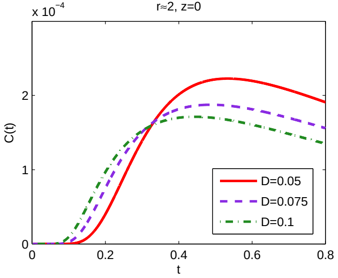
Pulse dynamics for D = 0.05 and purely radial velocity (fully penetrating well)



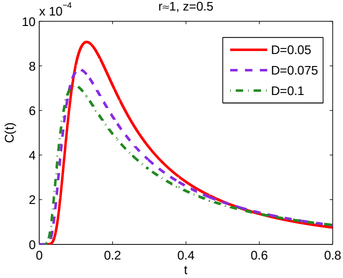
The tracer breakthrough curves



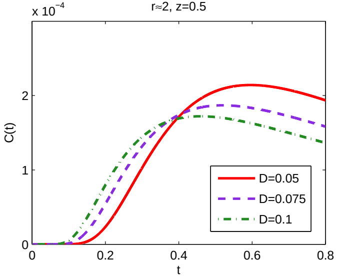
(g) $r = 1$



(h) $r = 2$



(i) $r = 1$



(j) $r = 2$

Conclusions

- The effect of the vertical component of the flow appearing in the partial penetrating well case, and not present the fully penetrating case, has been examined.
- We have found that the tracer concentration at the screening section central plane moves faster along the radius and attains smaller values when the vertical divergent velocity is taken into account.
- In the partially penetrating case the tracer accumulates at the top and the bottom of the porous layer, and reduces in the central zone.
- it was found that at large observation radii the breakthrough time and the breakthrough curve shape become similar in both penetration cases. However the differences can impact the interpretation of tracer tests.
- Crank-Nicolson method and Least Squares formulation have stabilized successfully the convective term.

Tracer tests in vertical bipolar flow

Oil fields in horizontal layers

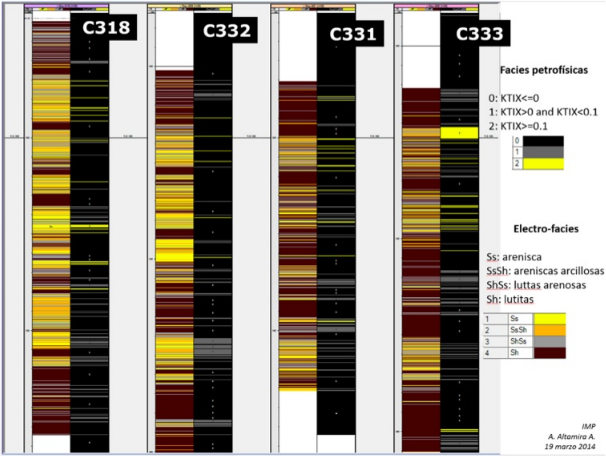
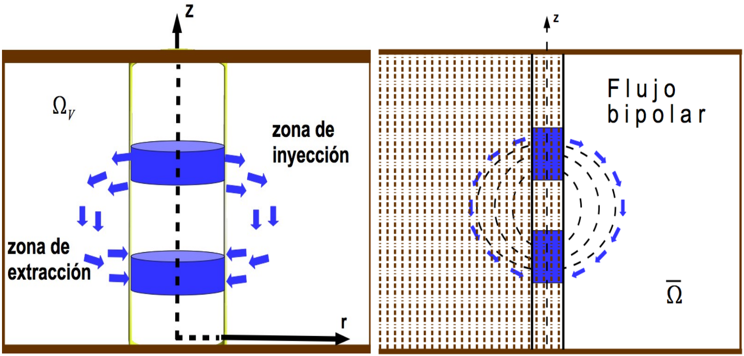


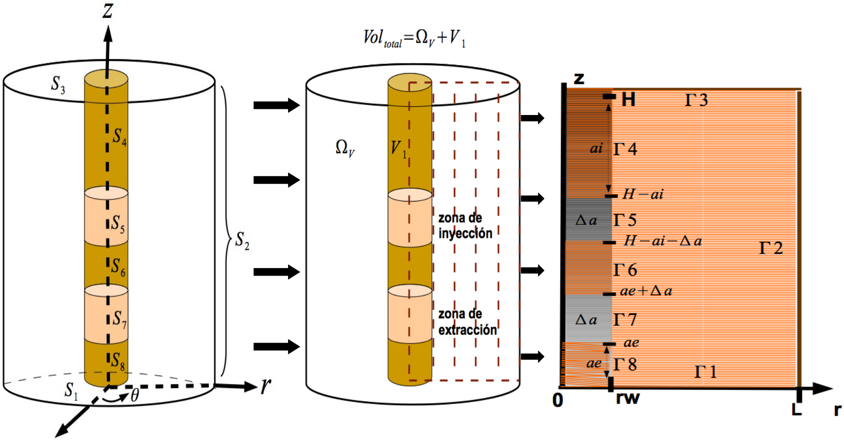
Figura: Records of rock material and permeability ranges.

Tracer tests in vertical bipolar flow



Two screening sections are drilled by a well seal. A tracer pulse is introduced into the fluid by the injection screening section, and is extracted through the other zone.

The system domain.



V_1 is the well volume with radius r_w,
Ω_V is the volume between the cylinders of radius L and r_w

Mathematical modeling

Darcy equation and continuity equation.

1. Fluid pressure.
2. Velocity field.

Advection - dispersion equation.

- Tracer concentration.

The purpose is to research if the breakthrough curve in a stratified reservoirs is sensitive to the thickness of the layers and their permeability, porosity or dispersion.

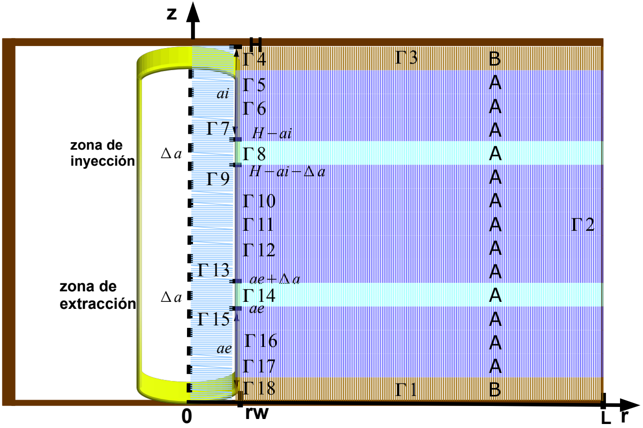
Numerical results

- Case 1-layer: A single layer between the injection zone and the extraction zone.
- Case 3-layers: Tree Three layers between the injection zone and the extraction zone.
- Case 5-layers: Five layers between the injection zone and the extraction zone.

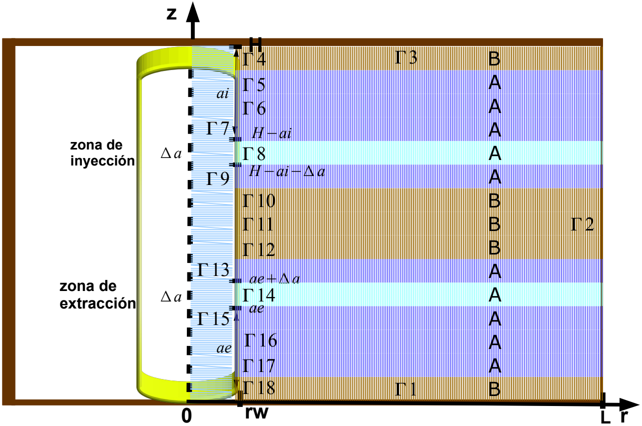
High permeability zones A and low permeability zones B are considered:

medium properties	zone A (high)	zone B (low)
permeability (mD)	100	10
porosity (%)	30	10
dispersion	1	0.1

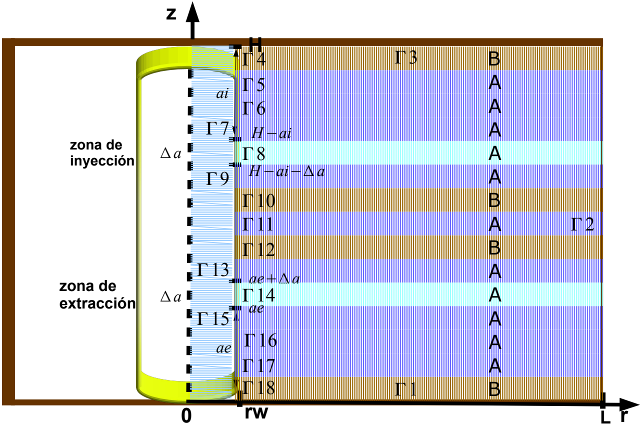
- Case 1-layer. The tracer transport phenomenon takes place in a homogeneous formation of high permeability A, in the region between the screening sections.



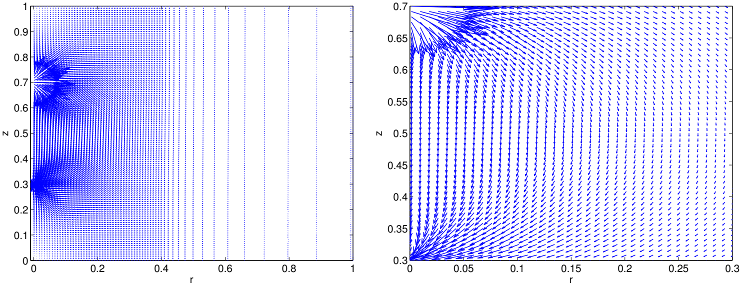
- Case 3-layers. In the zone of interest regions of high-low-high permeability are defined respectively.



- Case 5-layers. In the zone of interest regions of high-low-high-low-high permeability are defined respectively. This case is similar to the 3-layer case, adding a layer of high permeability

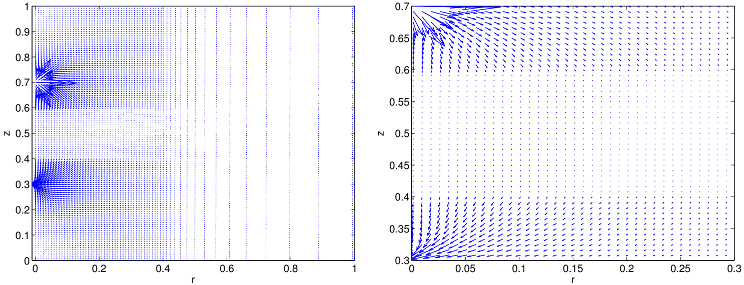


Velocity Field (Case 1-layer)



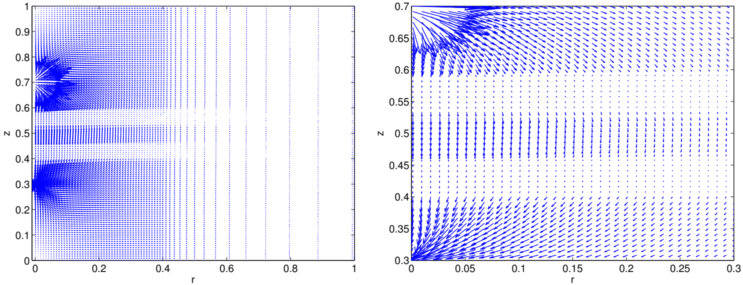
Case 1-layer. The figures on the left consider $r \in [0, 1]$ and the figures on the right $r \in [0, 0.3]$.

Velocity field (Case 3-layers)



Caso 3-capas. The figures on the left consider $r \in [0, 1]$ and the figures on the right $r \in [0, 0.3]$.

Velocity field (Case 5-layers)



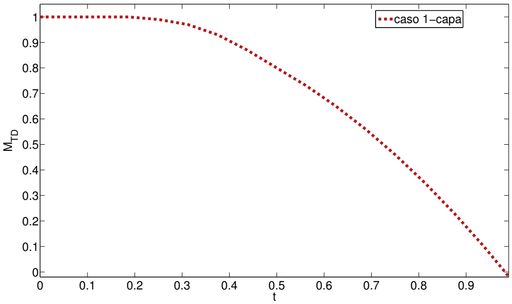
Case 5-layers. The figures on the left consider $r \in [0, 1]$ and the figures on the right $r \in [0, 0\ mbox.3]$

Tracer concentration

The total tracer mass inside the domain is defined as a function of time

$$M_{TD} = 2\pi r_w \int\int C\,(r,z,t)\ r\ dr\ dz, \qquad (1)$$

for any fixed $t \in [0, T]$.

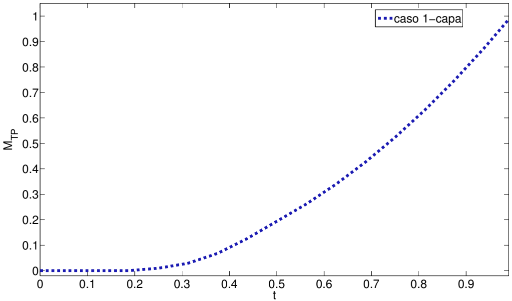


Total tracer mass for $t \in [0, T]$.

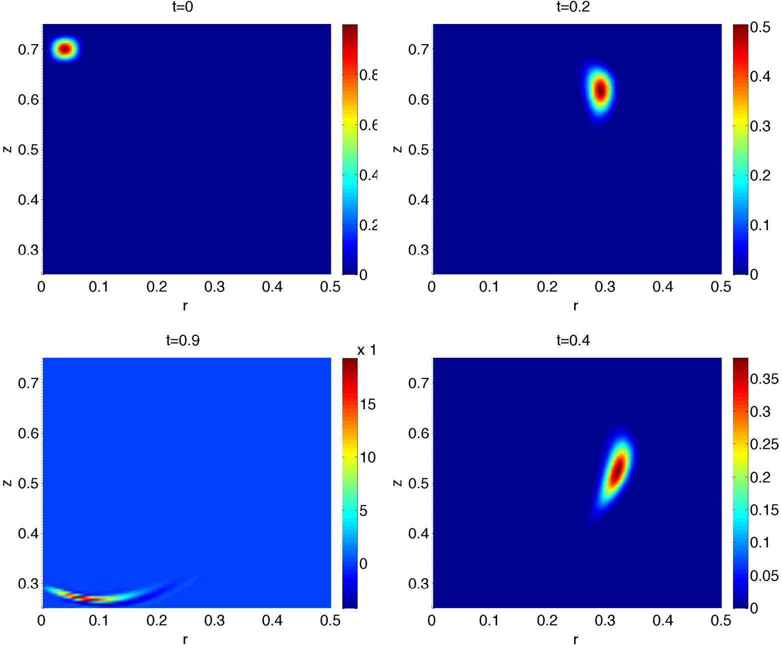
The amount of accumulated tracer mass that has left the well at time t is calculated as follow:

$$M_{TP} = 2\pi r_w \int \int U(r_w, z, t)\, C(r_w, z, t)\; dz\, dt, \qquad (2)$$

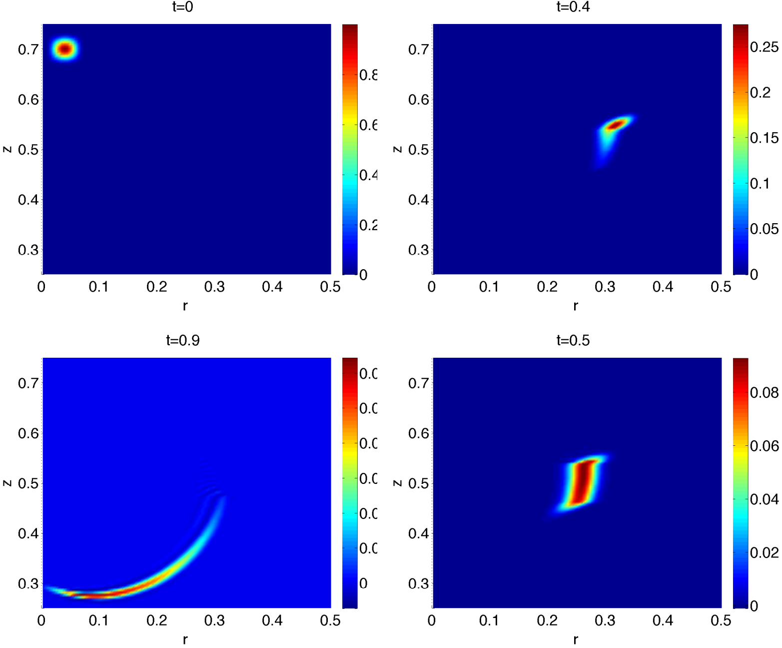
for any fixed $t \in [0, T]$.



Accumulated tracer mass that has left the well, $t \in [0, T]$.

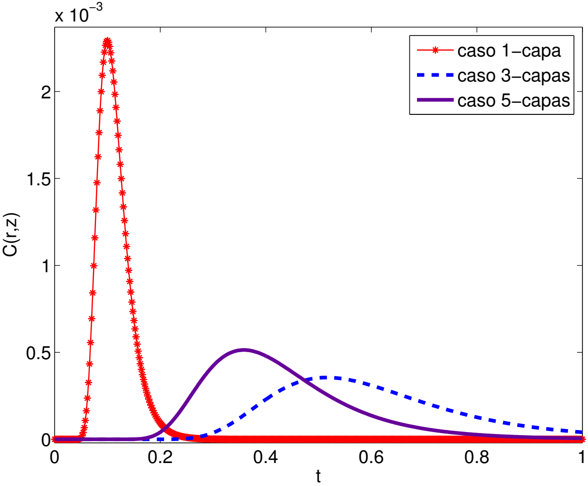


Behavior of the 2D tracer for the case 1-layer in a time interval $[0, T]$.



Behavior of the 2D tracer for the case 5-layer in a time interval $[0, T]$.

The tracer breakthrough curves



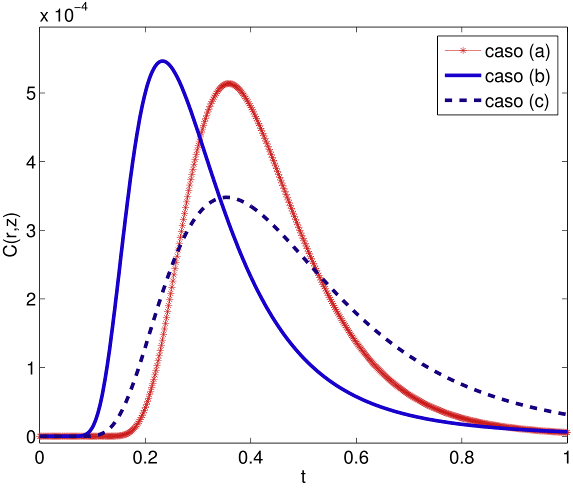
Concentration profiles at the midpoint of the extraction zone.

Sensitivity to permeability of the layers

Values for the permeability parameter k.

case (a)	case (b)	case (c)
100 mD	100 mD	100 mD
10 mD	30 mD	3 mD
100 mD	100 mD	100 mD
10 mD	30 mD	3 mD
100 mD	100 mD	100 mD

Sensitivity to permeability



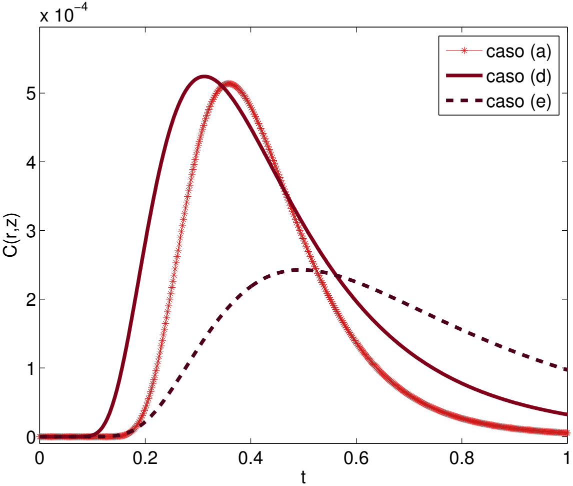
Concentration profiles at the midpoint of the extracted zone that show the sensitivity to increase or decrease in permeability.

Sensitivity to thickness of the layers

Values for the permeability parameter k.

case (a)	case (d)	case (e)
100 mD	10 mD	10 mD
10 mD	100 mD	10 mD
100 mD	100 mD	100 mD
10 mD	100 mD	10 mD
100 mD	10 mD	10 mD

Sensitivity to thickness of the layers



Concentration profiles at the midpoint of the extracted zone that show the sensitivity to thickness of the layers.

- The parameters involved in the model are worked by finite element and are remarkable in the tracer pulse dynamics.
- The breakthrough curves reflect the heterogeneity of the medium, however the specific cause-effect can not be determined. Therefore, the application of the model developed here to the estimation of parameters will require additional information from other sources that reduces the number of adjustment parameters.
- As future work, modeling fines detachment and migration induced by low salinity water injection in oil reservoirs

Thanks for your attention...

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