Theoretical and Applied Stochastic Analysis

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Sep 9 –14, 2018

1 Overview of the Field

Stochastic analysis is a term which usually encompasses both stochastic integration of Itô type and Malliavin calculus techniques. It is certainly one of the most active areas of probability theory, with ramifications in stochastic PDEs, limit theorems in probability, rough paths analysis, Dirichlet forms, stochastic differential geometry, Lévy processes on the one hand, and applications to theoretical physics, finance modeling, biophysics and engineering on the other hand. Within this framework, our proposal focuses on both theoretical and applied aspects of stochastic analysis. Indeed, Malliavin's vision of analysis on Wiener space has evolved (in its almost 40 years of existence) into a huge and solid theory, which has developed fertile connections with numerous fields within and outside the probability realm. With this conference, we tried to give a broad overview and gather many active experts in the area, favoring interdisciplinary exchanges and possibly yielding new developments in the field. The following lines detail some of the aspects we have focused on during the workshop.

(1) Stochastic PDEs. A growing attention has been devoted recently to track physically relevant phenomenon displayed by certain classes of stochastic PDEs. Among the most exciting and challenging situations is the one involving the so-called parabolic Anderson model. This model can be described by a simple linear PDE which can be written formally as follows:

$$\frac{\partial u}{\partial t} = \Delta u + u\,\dot{W}\,, \quad t \geq 0, \quad x \in \mathbb{R}^d\,, \tag{1}$$

where $\dot{W}$ is a noisy term, usually described as a Gaussian field. In spite of its simple form, equation (1) exhibits all sorts of non classical behaviors, in terms of moment estimates, growth rate in time and space, or energy landscape.

(2) Rough paths techniques. The theory of rough paths has been originally developed in the mid-nineties by T. Lyons (see, e.g., [4] and [3]). It is based on the profound insight that stochastic differential equations can be solved pathwise and that the solution map is continuous in suitable rough path metrics. The topic has now grown into a mature and widely applicable mathematical theory. Its scope of applications is twofold: (i) By establishing a continuous relation between noise and solutions to differential equations, it sheds a new light on diffusion processes and other differential systems driven by a Brownian motion. (ii) It extends widely the class of processes which can be considered as driving noises of differential equations. One particularly popular example of application is fractional Brownian motion, and the rough paths method is the only one allowing to properly define and solve general equations driven by this noise. A lot of the current effort of the community is devoted to an in-depth study of equations driven by fractional Brownian motions $B^1, \dots, B^d$, of the form

$$Y_t = a + \int_0^t V_0(Y_s)\, ds + \sum_{j=1}^{d} \int_0^t V_j(Y_s)\, dB_s^j,$$

for a given initial condition $a \in \mathbb{R}^n$ and some smooth vector fields $V_1, \dots, V_d$ on $\mathbb{R}^n$. This is precisely one of the instances where stochastic analysis enters into the play, and has to be mixed with rough paths in order to produce fruitful results.

(3) Dirichlet forms. Dirichlet form theory is the most convenient and useful tool to construct diffusion processes in non smooth settings, like graphs, fractals, or Gromov-Hausdorff limits of Riemannian manifolds. In the last few years, there have been tremendous efforts to use Dirichlet forms theory to prove geometric or analytic theorems on singular spaces, that before only were available in smooth spaces. A crowning achievement in non-smooth geometry is the Lott-Villani-Sturm theory of synthetic Ricci lower bounds on metric measure spaces and the most recent Riemannian curvature dimension theory by Ambrosio-Gigli-Savaré. Both of those theories are nowadays the object of intensive studies by geometers, analysts and probabilists, showing the ubiquity of Dirichlet forms techniques.

On a more analytic side, Dirichlet forms have also been a powerful tool in the analysis of partial differential equations, where intrinsic properties of the Dirichlet form (like the volume doubling property or the scale invariant Poincaré inequality) have been proven equivalent to Harnack type inequalities for solutions of relevant parabolic partial differential equations. Recently, the range of those techniques has been extended to cover more and more singular situations like graphs or fractals.

(4) Stochastic and rough differential geometry. Despite the tremendous interest in applying techniques from stochastic analysis to non smooth spaces, stochastic analysis on smooth manifolds is still a rich area in which many open and challenging questions are being addressed. A recent trend has been the stochastic analysis of sub-Riemannian manifolds. Despite the smoothness of the underlying manifold, in sub-Riemannian geometry, the space of geodesics is extremely singular and poorly understood. Classical geometric methods generally fail and stochastic analysis provide a set of robust tools that have successfully applied in the last few years. We mention for instance the generalized curvature dimension inequalities by Baudoin-Garofalo. Another topic covered by the conference is the study of rough paths on manifolds. As mentioned before, rough paths theory techniques are the only tools available to make sense to solutions of differential equations driven by very irregular paths. It turns out that those techniques extend to the framework of differential manifolds. This opens the door to a theory of rough signals on manifolds. The ongoing efforts in this direction will be presented by B. Driver and T. Cass.

(5) Limit theorems. The recent discovery by Nourdin and Peccati of the connection between Malliavin calculus and Stein's method, has led to a burst of new research in stochastic analysis. Subjects like dimension free estimates for the rate of convergence in limit theorems, Sudakov-Fernique theorem or Slepian inequality can now be generalized to Wiener chaos by methods of Malliavin calculus. They represent important tools when investigating a wide number of demanding contexts, such as central and non-central limit theorems, stochastic PDEs, spin glasses or polymer measures. In another direction, estimates of Malliavin-Stein type can be successfully applied to geometric problems, involving in particular random graphs defined on the points of a homogeneous Poisson process. A number of new applications include Poisson-Voronoi approximation and Boolean models. We should also mention a deep and abstract work by Ledoux, generalizing certain bounds of Nourdin and Peccati to random variables living in the chaos of a general Markov operator.

(6) Applications. In spite of being inspired by theoretical considerations, stochastic analysis tools have been applied in a wide range of situations, and have also generated a large array of interdisciplinary research topics. We have already mentioned some links with challenging models of theoretical physics, such as KPZ or Φ^4 equations, spin glasses or polymer measures. Finance is also an effervescent area where rough paths techniques are fruitfully applied, either for stochastic volatility models or for numerical methods such as cubature on Wiener's space. We should also mention that signatures (that is iterated integrals of paths or processes) are central objects in the rough paths theory. These objects have been recently exploited to characterize written (and in particular Chinese) characters, which paves the way for new real world applications.

2 Recent Developments and Open Problems

Stochastic PDEs. D. Khoshnevisan talked about the asymptotic behavior of the heat contents in a stochastic context. Y. Hu gave an account on recent developments concerning pathwise definitions of the Brownian motion in a random environment. This is closely related to stochastic partial differential equations with singular coefficients. Other exciting aspects of the theory, such as the geometry of the fields for solutions to stochastic PDEs or multiple points problems, have been addressed by R. Dalang and C. Mueller. As far as open problems are concerned, S. Tindel gave a conjecture about the localization of eigenfunctions for the parabolic Anderson operator. X. Chen explained how a basic continuity result could help to solve a challenging open problem related to the asymptotic behavior of the multiplicative stochastic heat equation.

Rough paths techniques. Bruce Driver gave a presentation on the global existence of rough differential equations on manifolds, and gave a necessary condition for the global existence [2]. The aim is to deal with unbounded vector fields. Thomas Cass gave a presentation on the Ito-Stratonovich conversion formula using rough paths [1]. Applications to the computation of expected signature have been pointed out and discussed among several participants during the workshop. Xi Geng and his collaborators made some progress on the length conjecture and reported their progress in his talk. But the conjecture in its full generality still remains open.

Other open problems in this field include, for example, a sharp upper bound for the density of hypo-elliptic SDEs driven by fractional Brownian motions (in the rough case) and a matching lower bound.

Dirichlet forms. K.T. Sturm, one of the worlwide leaders in the theory of Dirichlet forms and metric spaces, presented his recent works on those topics. He presented several characterizations of Ricci curvature lower bounds on domains of metric spaces which may not be convex. Those characterizations can be expressed in terms of gradient bounds for the heat semigroup. Several open problems and recent develoments have been discussed in his talk.

Stochastic and rough differential geometry. J. Wang presented some of the most recent developments in the theory of heat contents for domains in the Heisenberg group, this was part of a joint work with Jeremy Tyson. Research directions toward more general sub-Riemannian spaces have been pointed out. Yuzuru Inahama gave a presentation on heat trace asymptotics for equiregular sub-Riemannian manifolds. His approach involved ideas from Malliavin calculus (Watanabe distribution theory) and sub-Riemannian geometry. Elton Hsu presented some ideas of stochastic differential geometry in order to recover some well-known Hamilton's type gradient estimates for non negative solutions of the heat equation on a Riemannian manifold. He pointed several possible further applications of the methods he was using and this initiated a discussion among some of the participants. Tai Melcher discussed some recent works in collaboration with E. Meckes about convergence to equilibrium for Brownian motion in U(n). The method she presented is mostly based on functional inequalities like log-Sobolev and concentration inequalities. Research developments to other of the classical compact Lie groups have been pointed out.

Limit theorems. I. Nourdin give us a survey on several aspects of the fourth moment theorem and of its application to quantitative versions of the central limit theorem in the context of random matrices. He pointed several research directions and actually started a research collaboration with D. Nualart during this workshop. D. Nualart also presented several recent developments around the fourth moment theorem and explained how those ideas could be applied to obtain functional versions (similar to Donsker-Varadhan) of the central limit theorems.

3 Presentation Highlights

- Elton Hsu – Brownian Motion and Hamilton's Gradient Estimate

  In this talk, Elton Hsu used Hamiton's gradient estimate for the solution of the heat equation on a compact Riemannian manifold and its generalizations as an example to show how Brownian motion and stochastic analysis on manifolds can be used as a powerful tool to study such estimates. This approach can often leads to improvements and new results which may not be reached by the more well known semigroup method.

- Tai Melcher – Convergence Rates for Paths of the Empirical Spectral Distribution of Unitary Brownian Motion

  In this talk, Tai Melcher talked about convergence rates for the empirical spectral measure of a unitary Brownian motion. She gave explicit bounds on the 1-Wasserstein distance of this measure to both the ensemble-averaged spectral measure and to the large-N limiting measure identified by Biane. She were then able to use these bounds to control the rate of convergence of paths of the measures on compact time intervals. The proofs use tools developed by the first author to study convergence rates of the classical random matrix ensembles, as well as recent estimates for the convergence of the moments of the ensemble-average spectral distribution.

- Jing Wang – Heat content on the Heisenberg group

  In this talk Jing Wang presented a joint work (with J. Tyson) in studying small time asymptotic of the heat content for a smoothly bounded domain with non-characteristic boundary in the Heisenberg group, which captures geometric information of the of the boundary including perimeter and the total horizontal mean curvature of the boundary of the domain. She used probabilistic method by studying the escaping probability of the horizon- tal Brownian motion process that is canonically associated to the sub-Riemannian structure of the Heisenberg group.

- Bruce Driver – Global Existence of RDEs on Manifolds

  In this talk Bruce Driver discussd a theorem guaranteeing the existence of global (in time) solutions to rough path differential equations on a smooth manifold.

- Qi Feng – Geometric and Stochastic Analysis on Totally Geodesic Foliations under Ricci Flow

  In this talk, Qi Feng briefly introduced the connections between geometric and stochastic analysis on Riemannian (and sub-Riemannian) manifolds. In particular, on totally geodesic foliations under transverse Ricci flow, he proved differential Harnack inequalities and gradient estimates for the positive solutions of heat equation, he proved time dependent generalized curvature dimension inequality and also proved monotonicity property for basic functions. The long term goal is to solve interesting sub-Riemannian geometry problems by using new techniques from aforementioned areas.

- Mariana Prez Rojas – Excursions of the Brox diffusion

  The Brox diffusion is a stochastic process in random environment often considered the scale and time continuous analogue of Sinai's walk. In this talk, Mariana Pérez Rojas applied Excursion theory to the Brox diffusion in order to obtain the distribution of certain important random variables.

- Daniel Kelleher – Differential one-forms on Dirichlet spaces and Bakry-Emery estimates on metric graphs

  A general framework on Dirichlet spaces is developed to prove a weak form of the Bakry-Emery estimate and study its consequences. This estimate may be satisfied in situations, like metric graphs, where generalized notions of Ricci curvature lower bounds are not available. Daniel Kelleher also dicussed about current research directions by taking limits.

- Li Chen – Gundy-Varopoulos martingale transforms and their projection operators

  Based on a joint work with R. Bañuelos and F. Baudoin, Li Chen discussed about the dimension-free L^p boundedness of operators on manifolds obtained as conditional expectations of martingale transforms la Gundy-Varopoulos. Applications on Lie groups of compact type and the Heisenberg group was also introduced.

- Patricia Alonso-Ruiz – Heat kernels and functional inequalities on generalized diamond fractals

  Generalized diamond fractals constitute a parametric family of spaces that arise as scaling limits of so-called diamond hierarchical lattices. The latter appear in the physics literature in the study of random polymers, Ising and Potts models among others. In the case of constant parameters, diamond fractals are self-similar sets. This property was exploited in earlier investigations by Hambly and Kumagai to study the corresponding diffusion process and its heat kernel. These questions are of interest in this setting in particular because the usual assumption of volume doubling is not satisfied. For general parameters, also the self-similarity is lost. Still, a diamond fractal can be regarded as an inverse limit of metric measure graphs and a canonical diffusion process obtained through a general procedure proposed by Barlow and Evans. This approach will allow us to provide a rather explicit expression of the associated heat kernel and deduce several of its properties. As an application, Patricia Alonso-Ruiz discussed some functional inequalities of interest.

- Karl-Theodor Sturm – Optimal transport and heat flow on metric measure spaces with lower bounded Ricci curvature – and beyond

  The fundamental concept of synthetic lower Ricci bounds for metric measure spaces was introduced, illustrated by striking consequences for optimal transports and for heat flows. New approaches were presented for the heat flow with Neumann boundary conditions on non-convex domains as well as for the heat flow with Dirichlet boundary conditions. Moreover, Theodor Sturm studied the heat flow on time-dependent metric measure spaces and its dual as gradient flows for the energy and for the Boltzmann entropy, resp. Monotonicity estimates for transportation distances and for squared gradients were shown to be equivalent to the so-called dynamical convexity of the Boltzmann entropy on the Wasserstein space which is the defining property of super-Ricci flows. Moreover, the speaker showed the equivalence with the monotone coupling property for pairs of backward Brownian motions as well as with log Sobolev, local Poincaré and dimension free Harnack inequalities.

- Victor Manuel Rivero Mercado – Deep factorisation of the stable process: Radial excursion theory and the point of closest reach.

  In this talk, Victor Rivero Mercado provided some explicit results for stable processes obtained from the perspective of the theory of self-similar Markov processes. In particular, he turned their attention to the case of d -dimensional isotropic stable process, for $d \geq 2$. Using a completely new approach he considered the distribution of the point of closest reach. This leads to a number of other substantial new results for this class of stable processes. He engaged with a new radial excursion theory, never before used, from which he develop the classical Blumenthal–Getoor–Ray identities for first entry/exit into a ball, to the setting of n-tuple laws. He identified explicitly the stationary distribution of the stable process when reflected in its running radial supremum. Moreover, he provided a representation of the Wiener–Hopf factorisation of the MAP that underlies the stable process through the Lamperti–Kiu transform.

- Yuzuru Inahama – Heat trace asymptotics for equiregular sub-Riemannian manifolds

  Yuzuru Inahama discussed a "div-grad type" sub-Laplacian with respect to a smooth measure and its associated heat semigroup on a compact equiregular sub- Riemannian manifold. He showed a short time asymptotic expansion of the heat trace up to any order. His main result holds true for any smooth measure on the manifold, but it has a spectral geometric meaning when Popp's measure is considered. His proof is probabilistic. In particular, he used S. Watanabe's distributional Malliavin calculus.

- Thomas Cass – Generalisations of the Ito-Stratonovich conversion formula using rough paths

  Lyons' theory of rough paths allows one to solve stochastic differential equations driven by a Gaussian processes X under certain conditions on the covariance function. The rough integral of these solutions against X again exist, and a natural question is to find a closed-form conversion formula between this rough integral and the Skorohod integral of the solution which generalises the classical Stratonovich-Ito conversion formula. Previous works in the literature assumes the integrand to be the gradient of a smooth function of X; Based on a joint with with Nengli Lim, Thomas Cass presented a formula that recovers these results as special cases.

- Xi Geng – Path Development and the Length Conjecture

  Given a (rough) path $x : [0, 1] \to V$ in some Banach space, one can associate it with a sequence of iterated integrals known as the *signature* of x. One can think of the signature as a deterministic version of moments of a random variable. It is a fundamental result in rough path theory that the signature uniquely determines the path up to tree-like pieces. It is then a natural question to ask whether we could explicitly recover some quantitative information of the path from its signature. A very simple and elegant conjecture asserts that, when x has bounded variation, its length can be recovered from the tail asymptotics of the signature. The length conjecture is established for C^1 or piecewise linear paths, and remains open for general BV paths.

While the general BV case is currently out of scope, the speaker and his collaborators investigated the rough path analogue for the length conjecture but in the simplest rough path case–a rough line segment. More precisely, they considered rough paths of the form $\mathbf{X}_t = \exp(tL)$ where L is a Lie polynomial of degree m. The speaker proposed a general method of algebraic development to attack this problem. More precisely, an *algebraic development* consists of an embedding F of V into some complex Lie algebra $\mathfrak{g}$ and a representation ρ of $\mathfrak{g}$ over some complex Banach space W:

$$V \xrightarrow{F} \mathfrak{g} \xrightarrow{\rho} \mathrm{End}(W).$$

It turns out that, in the finite dimensional setting, by choosing $\mathfrak{g}$ to be a complex semisimple Lie algebra and mapping the space of m-th degree homogeneous Lie polynomials into a Cartan subalgebra, the tail asymptotics of the signature is closely related to spectral properties of L when viewed as a linear transformation over W under the given development. Explicit calculations can be done by designing F and ρ properly. In particular, in the case when $m = 2, 3$ and $V = \mathbb{R}^2$, the conjecture (**??**) can be proved by choosing $\mathfrak{g} = \mathfrak{sl}(m;\mathbb{C})$. A general non-sharp estimate for arbitrary roughness m can also be proved within this scheme, and the speaker was optimistic that this method would lead us further beyond that.

The talk is based on a joint working project with Horatio Boedihardjo and Nikolaos-Panagiotis Souris.

- David Nualart – Central limit theorems for functionals of Gaussian Processes

  In this talk David Nualart presented some new results for the convergence rate of the total variation distance in the framework of the Breuer-Major theorem, assuming some smoothness properties of the underlying function. The results were proved by applying new bounds for the total variation distance between a random variable expressed as a divergence and a standard Gaussian random variable, which were derived by a combination of techniques of Malliavin calculus and Stein's method. Some applications to the asymptotic behavior of power variations of the fractional Brownian motion was discussed.

- Frederi Viens – Wiener chaos and Berry-Esseen consistency for variations estimators of general Gaussian processes

  This talks is based on a joint work with Luis Barboza (U. Costa Rica), Khalifa es-Sebaiy (U. Kuwait), and Soukaina Douissi (U. Cadi Ayyad, Morocco), and focuses on the application of stochastic analysis to statistics. In this talk, the class of all Gaussian processes observed at regularly spaced discrete times is considered. For stationary processes, when the spectral density is parametrically explicit, a Generalized Method of Moments estimator that satisfies consistency and asymptotic normality was defined, using the Breuer-Major theorem which applies to long-memory processes. This result is applied to the joint estimation of the three parameters of a stationary fractional Ornstein-Uhlenbeck (fOU) process driven for all Hurst parameters. For general non-stationary processes, no matter what the memory length, the state-of-the-art Malliavin calculus tools were used to prove Berry-Esseen-type and other speeds of convergence in total variation, for estimators based on power variations.

- Ivan Nourdin – Asymptotic Behavior of Large Gaussian Correlated Wishart Matrices

  In this talk, Ivan Nourdin considered high-dimensional Wishart matrices associated with a rectangular random matrix $X_{n,d}$ whose entries are jointly Gaussian and correlated. The main focus was on the case where the rows of $X_{n,d}$ are independent copies of a n-dimensional stationary centered Gaussian vector of correlation function s. When s is 4/3-integrable, he showed that a proper normalization of the corresponding Wishart matrix is close in Wasserstein distance to the corresponding Gaussian ensemble as long as d is much larger than n^3, thus recovering the main finding of Bubeck et al. and extending it to a larger class of matrices.

- Jorge A. Leon – Semilinear fractional differential equations driven by a fractional Brownian motion with $H > 2/3$.

  In this talk, Jorge Leon used the techniques of fractional calculus and the fix-point theorem to show that a semilinear fractional differential equation driven by a gamma-Holder continuous noise, $\gamma > 2/3$, has a unique solution. The initial condition could be not defined at zero and the involve integral is in the Young sense.

- Carl Mueller – Hitting questions and multiple points for stochastic PDE in the critical case

  Hitting questions play a central role in the theory of Markov processes. For example, it is well known that Brownian motion hits points in one dimension, but not in higher dimensions. For a general Markov process, we can determine whether the process hits a given set in terms of potential theory. There has also been a huge amount of work on the related question of when a process has multiple points. For stochastic partial differential equations (SPDE), much less is known, but there has been a growing number of papers on the topic in recent years. Potential theory provides an answer in principle. But unfortunately, solutions to SPDE are infinite dimensional processes, and the potential theory is intractible. As usual, the critical case is the most difficult. In this talk, Carl Mueller gave a brief survey of known results, followed by a discussion of an ongoing project with R. Dalang, Y. Xiao, and S. Tindel which promises to answer questions about hitting points and the existence of multiple points in the critical case.

- Raluca Balan – Second order Lyapunov exponent for hyperbolic Anderson model

  In this talk, Raluca Balan examined the connection between the hyperbolic and parabolic Anderson models in arbitrary space dimension d, with constant initial condition, driven by a Gaussian noise which is white in time. She considered two spatial covariance structures: (i) the Fourier transform of the spectral measure of the noise is a non-negative locally-integrable function; (ii) d = 1 and the noise is a fractional Brownian motion in space with index $1/4 < H < 1/2$. In both cases, she showed that there was striking similarity between the Laplace transforms of the second moment of the solutions to these two models. Building on this connection and the recent powerful results of Huang, Le and Nualart (2017) for the parabolic model, she computed the second order (upper) Lyapunov exponent for the hyperbolic model. In case (i), when the spatial covariance of the noise is given by the Riesz kernel, she presented a unified method for calculating the second order Lyapunov exponents for the two models.

- Yaozhong Hu – Brownian motion in noisy environment

  In this talk, Yaozhong Hu discussed the weak and strong solutions to the stochastic differential equation $dX(t) = -1/2W'(X(t))dt + dB(t)$, where $(B(t), t \geq 0)$ is a standard Brownian motion and $W(x)$ is a two sided Brownian motion, independent of B. It is shown that the Ito?McKean representation associated with any Brownian motion (independent of W) is a weak solution to the above equation. It is also shown that there exists a unique strong solution to the equation. Ito calculus for the solution is developed. For dealing with the singularity of drift term $W'(X(t))$, the main idea is to use the concept of local time together with the polygonal approximation of the Brownian motion. The talk is based on a joint work with Khoa Le and Leonid Mytnik.

- Hakima Bessaih – Stochastic lattice models driven by fractional Brownian motion

  The focus of this talk was on some stochastic lattice dynamical systems driven by a fractional Brownian motion with Hurst parameter $1/2 < H < 1$. First of all, the speaker investigated the existence and uniqueness of pathwise mild solutions to such systems by the Young integration setting and proved that the solution generates a random dynamical system. Furthermore, she analyzed the exponential stability of the trivial solution.

- Davar Khoshnevisan – Dissipation and Parabolic Stochastic PDE

  Davar Khoshnevisan presented in his talk that, for a large class of semi-linear parabolic PDEs, driven by space-time white noise, the solution converges exponentially rapidly to zero either as time tends to infinity or as the noise input is increased. All of this is based on a moment decay inequality that is a counterpart to standard intermittency-type moment estimates for the solution. The said inequality is based on an L^1/L^∞ interpolation method which is of independent interest. This talk is based on joint work with Carl Mueller (U.Rochester-USA), Kunwoo Kim (POSTECH-Korea), and Shang-Yuan Shiu (NCU-Taiwan).

- Yimin Xiao – Regularity Properties of Gaussian Random Fields and Stochastic Heat Equation on the Sphere

  This talk is concerned with sample path regularities of isotropic Gaussian fields and the solution of the stochastic heat equation on the unit sphere $\mathbb{S}$. In the first part, Yimin Xiao established the property of strong local nondeterminism of an isotropic spherical Gaussian field based on the high-frequency behavior of its angular power spectrum; he then applied this result to establish an exact uniform modulus of continuity for its sample paths. Yimin Xiao also discussed the range of values of the spectral index for which the sample functions exhibit fractal or smooth behavior. In the second part, he considered the stochastic heat equation driven by an additive infinite dimensional fractional Brownian noise on $\mathbb{S}^2$ and established the exact uniform moduli of continuity of the solution in the time and spatial variable, respectively. This talk is based on joint works with Xiaohong Lan and Domenico Marinucci.

- Jingyu Huang – Dense blowup for parabolic SPDEs.

  The aim of this talk is to give an example of stochastic PDE such that for each (t, x), the paths of the solution $u(t, x)$ are discontinuous with probability one.

- Le Chen – Density properties of the stochastic heat equations with degenerate conditions. In this talk, Le Chen studied the stochastic heat equation on $\mathbb{R}^d$ driven by a multiplicative Gaussian noise which is white in time and colored in space. The diffusion coefficient ρ can be degenerate, which includes the parabolic Anderson model $\rho(u) = u$ as a special case. The initial data is rough in the sense that it can be any measure, including the Dirac delta measure, that satisfies some mild integrability conditions. Under these degenerate conditions, for any given $t > 0$ and distinct m points $x_1, ...x_m$ in $\mathbb{R}^d$, Le Chen established the existence, regularity, and strict positivity of the joint density of the random vector $(u(t, x_1), ...u(t, x_m))$. The talk is based on a recent jointwork with Yaozhong Hu and David Nualart for the spatial dimension case, and an ongoing research project with Jingyu Huang for the higher spatial dimension case.

4 Scientific Progress Made & Outcome of the Meeting

The workshop brought together world leading experts in various fields of stochastic analysis. In addition to a large number of talks, we had Informal Discussion sessions on Thursday afternoon and Friday, which gave sufficient time for scientific interaction among the participants. Many discussions and projects sprung from the exchanges we had in Oaxaca. Among the fruitful discussions we had, let us mention the following ongoing scientific projects on which significant progress were made:

- D. Nualart and I. Nourdin on limit theorems thanks to Malliavin calculus methods.
- J. León, C. Ouyang and S. Tindel on their project concerning the law of solutions to rough differential equations.
- H. Bessaih and F. Delgado on stochastic shell models.

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