Geometric Inequalities, Convexity and Probability
Activity Report

BIRS-IMAG Workshop, Granada,

Organizers:
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Dario Cordero-Erausquin (Sorbonne Université)
Max Fathi (Universite Paris-Cité)
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1 Synopsis

The workshop brought together researchers and students from several areas covering classical convex geometry, high dimensional phenomena in convexity, complex analysis, differential equations, optimal transport and theoretical computer science. This resulted in a vibrant exchange of ideas and a successfully interactive workshop. Participants were excited about the new location and as many traveled mostly from Europe (some of the North American participants, almost exclusively Canadian-based, were also in the region for other events) the travel contributed less to the carbon footprint.

More than a quarter of the participants were women, and more than a quarter were early career researchers including students, making one of the youngest and most diversely represented audience among recent workshop in convexity.

We were also happy and honored that many members of the Spanish, informally called, convexity community attended the workshop.

2 Presentation of the field, with a tale

The subject of Asymptotic Geometric Analysis originated in Functional Analysis, mainly infinite dimensional. After a few transformations it became mostly a finite dimensional theory, but with the dimension typically very high. It is an asymptotic theory, asymptotic by the increasing to infinity of the dimensions of the objects of our study, say normed spaces, convex bodies or convex functions. The asymptotic approach reveals many very novel phenomena which also influence other fields in mathematics, especially where a large data set is of main concern, or a number of parameters which becomes uncontrollably large. One of the important features of this new field of mathematics is in developing tools which allow to study high parametric families. The tools then become immediately also central for a number of adjacent fields, such as complexity theory in computer science, "high dimensional" combinatorics, probability theory, analysis of large biological or medical data, and so on.

In a nutshell, one notes that on the one hand, high dimension means many variables and many configurations, so one may expect an increase in the diversity and complexity as dimension increases. However, the concentration of measure phenomenon and similar effects caused by the convexity assumption imply in fact a reduction of the diversity with increasing dimension, and the collapse of many different possibilities into one, or, in some cases, a few possibilities only (a simple example of this is the Central Limit Theorem from probability, which roughly states that when there are many independent random variables, the average behaviour is very predictable).

Once upon a time, in a faraway land,
There existed a workshop, where knowledge did expand.
It dwelled at the crossroads of Geometry, Analysis, and Probability,
In the realms of mathematics and physics, it did thrive,
And theoretical computer science kept it alive.
A field of growth, with results both exciting and new,
Interacting with adjacent domains, its influence grew.
The scientific community, with profound interest, did await,
Eager to partake in the workshop's enlightening state.
Now let me regale thee with the tale of this premise grand,
When faced with high-dimensional geometry, one might understand,
The abundance of configurations, diverse and vast,
Would deem it impossible to formulate theorems that last.
But lo! Contrary to expectations, order and simplicity prevailed,
High dimensionality, when rightly perceived, unveiled.
From this unexpected order, wisdom was derived,
Low dimensional projections, knowledge did thrive.
Data compression and simplified algorithms, a wondrous sight,
And structural theorems, connecting dimensions with might.
Amidst this enchanting realm, a jewel of old,
Dvoretzky's theorem, its brilliance manifold.
It showed that high-dimensional convex shapes,
Contained nearly-Euclidean sections, like gleaming capes.
Many theorems, born since the seventies' light,
Influenced the realms of math, shining ever bright.
The quotient of a subspace, limits central and true,
Isomorphic slicing and thin-shell miracles grew.
But a recent breakthrough, by Chen under Eldan's sway,
Solved the isoperimetric problem in a high-dimensional way.
Partitioning convex realms with equal volume, a feat so grand,
A hyperplane bisection, the solution did command.
In statistics, its applications did bloom,
Data science and sampling, where its power consumed.
Quantitative estimates for the central limit's might,
And Bourgain's slicing problem, a triumph in sight.
In the realm of high-dimensional geometry, motifs held sway,
Compensating for possibilities, vast in array.
The most famous motif, concentration of measure divine,
A scalar Lipschitz function behaved, as if by magic, a constant sign.
Imagine seventeen points, from a unit sphere plucked,
In a Lipschitz function, their values tightly tucked.
Those seventeen numbers, harmonious and near,
Like the high-dimensional Euclidean sphere, their grace clear.
For most mass gathers near the equator's line,
A geometric truth, wondrous and fine.
Spherical isoperimetric might, strengthening this tale,
In three dimensions, unimaginable, but not to fail.
Isoperimetric inequality in Gaussian realm did shine,

Concentration of measure, in many forms it did define.
Spectral gap and functional inequalities, a feast for the mind,
In discrete spaces, their analogues, enchanting and kind.
Convexity's assumption, a companion so dear,
Unleashed the power of motifs, far and near.
With its help, non-trivial theorems took flight,
Isoperimetric inequalities and concentration estimates, shining bright.
No longer bound by strong regularity's decree,
Independence and spherical symmetry set them free.
Elegant geometric conditions, their reach did extend,
Log-concavity and positive Ricci curvature, a transcendent blend.
And so, the fairy tale of this workshop concludes,
Where Geometry, Analysis, and Probability abounds.
A world where order triumphs over complexity's wiles,
And high-dimensional realms reveal their secret styles.
May this tale inspire and captivate your mind,
To explore the realms where mathematical wonders unwind.

3 Recent Developments and Open Problems

It has become apparent in recent years that the effects of convexity and curvature, for example the local mixing that they imply, are a great source of regularity, especially in high dimensions. Isoperimetric inequalities and concentration estimates that were previously known to hold only under strong regularity assumptions such as independence or spherical symmetry, are now available under elegant geometric conditions such as log-concavity. For instance, the quantitative estimates in the central limit theorem for convex sets now nearly match those of the classical central limit theorem for independent random variables. The geometric inequalities that power this engine are developed in the Brunn-Minkowski framework, and constitute a body of knowledge whose influence resonates in remote disciplines such as Statistics, Machine Learning and Theoretical Computer Science and Statistical Physics, as well as Classical and Functional Analysis, Differential Geometry and Combinatorics. A multitude of high impact results have been obtained in recent years, such as the near resolution of the Kannan, Lovász and Simonovits (KLS) conjecture and the Bourgain slicing problem, the proof of the Gaussian Correlation Conjecture, advances in concentration of measure (e.g., the Bernoulli conjecture was solved), just to name a few. The field is developing quickly, and some think that it is entering a transformative period. Ideas that were born or developed in the cradle of convexity and geometric inequalities permeate throughout Analysis and beyond, and vice versa: mathematicians and scientists from other areas have contributed directly to this field in recent years.

In the workshop, many results in this vein have been discussed and disseminated. The many open problems in the field, ranging from the Mahler conjecture (a.k.a. exact constants in the Bourgain Milman theorem), general B-conjecture and log Brunn Minkwoski inequality, to name a few, remain a source of exciting new advances.

During the workshop, participants engaged in discussions on the relevance of the complex Brunn-Minkowski theory to the classical aims and goals of Convexity theory, both in fixed dimensions and in high dimensions. The complex viewpoint was presented by Berndtsson, a distinguished contributor in this field.

Among the recent achievements of the complex approach is the simplification of Kuperberg's bound for the Bourgain-Milman inequality. Additionally, it provides a natural framework (circled convex bodies in even dimensions) in which the log Brunn-Minkowski theory is validated. Some attendees believe that the complex viewpoint sheds new light on the subject by offering a fresh perspective on the previously intricate proofs of the Alexandrov-Fenchel inequalities.

The deep and precise connections between mixed volumes and convexity inequalities in Algebraic Geometry can be traced back to Bernstein and Kusnirenko, and possibly even Minding. Berndtsson's approach, linking Brunn-Minkowski theory to the curvature of holomorphic vector bundles, represents a significant advancement in complex geometry and algebraic geometry. It remains to be seen how influential these developments will be for the still open classical problems of convex geometry. Only time will reveal their true impact.

Details of the achievements reported on in the workshop are given in the following sections.

4 Highlights

The workshop brought together researchers and topics at the confluence of geometry, analysis and probability focusing on recent developments of geometric flavor appearing in several branches of mathematics and theoretical computer science.

The talks orbited around three series of lectures intended to be mini-courses for early career participants while meant to connect the advanced participants from different areas by identifying a set of ideas that arise repeatedly in the investigation of geometric, or certain random, structures across different fields. A first such course was delivered by Berndtsson who viewed many standard results in convex geometry in a complex setting that generalize the real theory. As such, he presented a general inequality for Bergman kernels of Bergman spaces defined by convex weights in $\mathbb{C}^n$ and discussed how this can be used in Nazarov?s proof of the Bourgain-Milman theorem and Mahler's conjectured inequality. Though different in nature, a functional approach to geometric inequalities resonated in other talks (Luwdig, Haddad).

A second series of lectures was centred on optimal transport which encompassed several points such as techniques for the numerical resolution of optimal transport problems and mathematical analysis of the algorithms to solve such discretized problems (Mérigot), but also optimal transport as tool for obtaining new proofs of Poincaré type inequalities and Santaló inequality (Fradelizi, Kolesnikov), including such inequalities in the discrete setting (Eskenazis). The third theme, addressed jointly by Dadush and Regev, aimed at applications to integer programming, as well as applications of the reverse Minkowski theorem to integer lattices, the latter related to important questions in the geometry of numbers.

Phenomena related to the behavior of singular values of random matrices as well as estimates of the behavior of marginals and other connections between random strcutures and convexity were adressed in talks by Livshyts, Paouris (who presented work still in progress), Valettas, Litvak.

Finally, without being exhaustive, we want to emphasize the impact of the isoperimetric results presented here starting with updates on results and remaining open problems on the famous multi-bubble conjecture by E. Milman. Other geometric extremal characterizations followed in talks by Ryabogin, Tatarko, Yaskin.

The references below illustrate an accelerated period of growth of the field whose state-of-the-art recent results were presented at the workshop while the abstracts of the talks listed in the next section speak for the diversity and the high quality of the talks.

References:

[1] Bo Berndtsson, *Complex integrals and Kuperberg's proof of the Bourgain-Milman theorem*, to appear Adv. in Math.

[2] Daniel Dadush, Friedrich Eisenbrand, Thomas Rothvoss, *rom Approximate to Exact Integer Programming*, International Conference on Integer Programming and Combinatorial Optimization IPCO 2023: Integer Programming and Combinatorial Optimization, 100?114, 2023.

[3] Alex Delalande, Quentin Mérigot, *Quantitative Stability of Optimal Transport Maps under Variations of the Target Measure*, to appear in Duke Math. Journal.

[4] Emanuel Milman, Joe Neeman, *The Gaussian Double-Bubble and Multi-Bubble Conjectures*, Annals of Math. 195, 89-206, 2022.

[5] Oded Regev, Noah Stephens-Davidowitz, *A reverse Minkowski theorem*, to appear in Annals of Math.

5 Summary of talks

Discrete Rogers-Shephard and Zhang-type inequalities

David Alonso Gutiérrez

Universidad de Zaragoza

On the one hand, the classical Rogers-Shephard inequality states that given a convex body $K \subseteq \mathbb{R}^n$ the volume of the difference body $K - K$ verifies that

$$|K - K| \leq \binom{2n}{n} |K|.$$

On the other hand, Zhang's inequality states that given a convex body $K \subseteq \mathbb{R}^n$, its polar projection body $\Pi^* K$ verifies that $|K|^{n-1} |\Pi^* K| \geq \frac{\binom{2n}{n}}{n^n}$, with equality if and only if K is a simplex. This inequality can be obtained from the inclusion relation

$$\binom{2n}{n}^{\frac{1}{n}} K_n(g_K) \subseteq n|K|\Pi^*(K),$$

where $K_n(g_K)$ is the n-the ball body of the covariogram function g_K, defined by its radial function

$$\rho^n_{K_n(g_K)}(\theta) = \frac{n}{|K|} \int_0^\infty r^{n-1} |K \cap (r\theta + K)| dr, \quad \forall \theta \in S^{n-1}.$$

In this talk we will provide discrete analogues of the inequality and inclusion above, where we will consider the lattice point enumerator measure instead of the Lebesgue measure. Such discrete analogues still imply the continuous classical Rogers-Shephard and Zhang inequalities.

This is a joint work with Eduardo Lucas, Javier Martín Goñi, and Jesús Yepes Nicolás.

Brunn Minkowski inequalities for path spaces on Riemannian surfaces

Rotem Assouline

Weizmann Institute of Science, Israel

The notion of a Minkowski average of two sets extends to Riemannian manifolds by replacing straight lines with geodesics. The Brunn Minkowski inequality is then equivalent to nonnegative Ricci curvature. We propose a generalization of this operation in which geodesics are replaced by an arbitrary family of curves. We show that horocycles in the hyperbolic plane satisfy the Brunn Minkowski inequality, in stark contrast to geodesics. Our main tool is needle decomposition. Joint work with Bo'az Klartag.

A quick estimate for the volume of a polyhedron

Alexander Barvinok

University of Michigan (Ann Arbor)

Let P be a bounded polyhedron defined as the intersection of the non-negative orthant in $\mathbb{R}^n$ and an affine subspace of codimension m. In a joint work with Mark Rudelson, we prove a simple and computationally efficient formula that approximates the volume of P within a multiplicative factor of c^m, where $c > 0$ is an absolute constant. The formula allows us to obtain asymptotic formulas for volumes of some combinatorially interesting families of polytopes. Let P be a bounded polyhedron defined as the intersection of the non-negative orthant in $\mathbb{R}^n$ and an affine subspace of codimension m. In a joint work with Mark Rudelson, we prove a simple and computationally efficient formula that approximates the volume of P within a multiplicative factor of c^m, where $c > 0$ is an absolute constant. The formula allows us to obtain asymptotic formulas for volumes of some combinatorially interesting families of polytopes.

Bo Berndtsson
Chalmers University of Technology, Gothenborg

Lecture 1: Superforms and the volume of convex bodies.

Abstract: The calculus of superforms is a real variable analog of the ddbar-calculus in complex analysis. Although basically very simple, it is a practical tool in the study of Monge-Ampere measures and variants thereof. I will sketch the basic theory and illustrate its use in connection with the Alexandrov-Fenchel theorem.

Lecture 2: Complex Brunn-Minkowski theory.

Abstract: The (non-standard !) term 'Complex Brunn-Minkowski theory' refers to a collection of inequalities in the complex analytic setting. The analogy with the real variable theory comes both from the method of proofs and the fact that the classical B-M inequalities can be viewed as a special case of the complex ones. I will explain the basic ideas and some applications.

Lecture 3: Bergman kernels and Paley-Wiener spaces.

Abstract: The background of this lecture is Nazarov's proof of the Bourgain-Milman theorem, where the basic ingredient is an estimate from below of the Bergman kernel of certain spaces of entire functions. I will give a proof of this estimate that uses the methods from the second lecture and explain the relation to the Bourgain-Milman theorem.

An Introduction to the Subspace Flatness Theorem and its Application to Integer Programming

Daniel Dadush
Centrum Wiskunde & Informatica

In a recent breakthrough, Reis & Rothvoss '23 showed that the problem of deciding whether an n-dimensional convex body contains an integer point can be solved in $2^{O(n)}(\log n)^{4n}$ time, improving on the prior best $2^{O(n)}n^n$ complexity bound. At the heart of this result is the resolution of the subspace flatness conjecture of Kannan & Lovasz (Ann. of Math '88), which posits that the covering radius of a lattice with respect to any convex body can be controlled up to a polylogarithmic factor in dimension by volumetric lower bounds.

Given a convex body K in $\mathbb{R}^n$, the covering radius $\mu(K, Z^n)$, with respect to the integer lattice $\mathbb{Z}^n$, is the minimum scaling $s > 0$ such that $sK + \mathbb{Z}^n = \mathbb{R}^n$, i.e. the minimum scaling such that all integer translates cover space. Precisely, Reis & Rothvoss showed that for some $d \in \{1, ..., n\}$, there exists a rank d integer projection matrix $P \in \mathbb{Z}^{d\times n}$, such that $\mu(K, \mathbb{Z}^n) \leq O(\log n)^3 vol(PK)^{-1/d}$. This complements the lower bound, due to Kannan & Lovasz, that $\mu(K, \mathbb{Z}^n) \geq vol(PK)^{-1/d}$ for any integer projection P.

In this talk, I will explain the application of this result to integer programming, as well as its relation to the reverse Minkowski theorem of Regev & Stephens-Davidowitz (STOC '17). Time permitting, I will sketch a proof of the l_2 version of this result, where K is restricted to be an ellipsoid.

Discrete logarithmic Sobolev inequalities in Banach spaces

Alexandros Eskenazis
CNRS, Sorbonne Université

We shall discuss certain aspects of vector-valued harmonic analysis on the discrete hypercube. After presenting the geometric motivation behind such investigations, we will survey known results on the Poincaré inequality and Talagrand's influence inequality. Then we will proceed to present a new optimal vector-valued logarithmic Sobolev inequality in this context. The talk is based on joint work with D. Cordero-Erausquin (Sorbonne).

Functional forms of Blaschke-Santaló's inequality and Mahler's conjecture for s-concave functions and link with transport-entropy inequality

Matthieu Fradelizi

Université Gustave Eiffel, Paris

Starting from the functional form of Blaschke-Santaló's inequality for log-concave functions, Max Fathi established a symmetrized form of Talagrand transport-entropy inequality. In a similar way, Nathaél Gozlan showed that the functional form of Mahler's conjecture has a fascinating transport-entropy reformulation. In this talk, I will present extensions of these results, in particular to s-concave functions. In the way, I will show new forms of the Blaschke-Santaló's inequality for s-concave functions when $s > 0$, a new study of duality for s-concave functions when $s < 0$ and an application to establish Mahler's conjecture for unconditional s-concave functions, when $s < 0$. The linearisation of our transport-entropy inequality gives a new proof of a Poincaré inequality of Brascamp-Lieb type recently proved by Cordero-Erausquin and Rotem. We also obtain a new symmetrized forms of log-Sobolev inequality for even probability measures on the sphere, in connection the log-Minkowski problem. Based on a joint work with Nathaél Gozlan, Shay Sadovsky and Simon Zugmeyer.

Affine Hardy-Littlewood-Sobolev inequalities

Julián Haddad

Universidad de Sevilla

The HLS inequality

$$\iint \frac{f(x)f(y)}{|x-y|^{n-\alpha}} dxdy \le \|f\|^2_{\frac{2n}{n+\alpha}}$$

is a powerful tool in analysis and geometry, with important applications, specially in the study of fractional PDEs and singular integrals. We define a convex body $S_\alpha f$, associated to the Riesz potential of a function f, and prove an isoperimetric-type inequality for $S_\alpha f$. The result is a sharp, affine-invariant integral inequality that is stronger, and directly implies the sharp HLS inequality.

Dual volumes and dual polynomials

Maria A. Hernandez Cifre

Universidad de Murcia

Dual (mixed) volumes and dual Steiner polynomials are central notions in the well-known dual Brunn-Minkowski theory. Many interesting questions and generalizations arise around these two concepts, and our aim in this talk is to present some of them, mainly focusing in a general family of n-th degree polynomials closely related to that of dual Steiner polynomials of star bodies. The contrast of these results with the corresponding ones in the classical Brunn-Minkowski setting will be also shown.

Transportational proof of the functional Blaschke-Santaló inequality and related questions

Alexander Kolesnikov

Faculty of Mathematics, Higher School of Economics, Moscow

We discuss how to prove the functional Blaschke-Santaló inequality using mass transportation arguments and related questions: stability of functional inequalities, conjectured Blaschke-Santaló inequality for many bodies, Pogorelov-type theorems.

Volume ratio between projections of convex bodies

Alexander Litvak

University of Alberta

We discuss volume ratios between convex bodies and their projections. We recall known results and then show that for every n-dimensional convex body K there exists a centrally-symmetric convex body L such that for any two projections P, Q of rank $k \leq n$ the volume ratio between PK and QL is large. Our result is sharp (up to logarithmic factors) when $k \geq n^{2/3}$. This is a joint work with D. Galicer, M.Merzbacher, and D. Pinasco.

On the smallest singular value of inhomogeneous random matrices

Galyna Livshyts

Georgia Institute of Technology

We discuss small-ball probability estimates of the smallest singular value of a rather general ensemble of random matrices which we call "inhomogeneous". One of the novel ingredients of our family of universality results is an efficient discretization procedure, applicable under unusually mild assumptions, while another new ingredient is the notion of the so-called randomized Least Common Denominator of a vector and of a matrix, and a double-counting method. Most of the talk will focus on explaining the ideas behind the proof of the first ingredient. Partially based on the joint work with Tikhomirov and Vershynin, and an ongoing joint work with Fernandez and Tatarko.

Monge–Ampère Measures and Valuations

Monika Ludwig

Technische Universität Wien

A functional Z defined on a space of real-valued functions $\mathcal{F}$ is called a *valuation* if

$$\mathrm{Z}(f \vee g) + \mathrm{Z}(f \wedge g) = \mathrm{Z}(f) + \mathrm{Z}(g)$$

for all $f, g \in \mathcal{F}$ such that the pointwise maximum $f \vee g$ and the pointwise minimum $f \wedge g$ are in $\mathcal{F}$. The important classical notion of valuations on convex bodies in $\mathbb{R}^n$ is a special case of the rather recent notion of valuations on function spaces.

We present new results on valuations on the space of convex functions on $\mathbb{R}^n$. In particular, a classification of valuations on convex functions on $\mathbb{R}^n$ with values in the space of signed Radon measures on $\mathbb{R}^n$ is established. As a consequence, a characterization of the Monge–Ampère operator is obtained.

(Based on joint work with Jin Li.)

Computational aspects of (quadratic) optimal transport

Quentin Mérigot

Université Paris-Saclay

Optimal transport has emerged as a suitable tool in various numerical applications, particularly in statistics, inverse problems, and PDE discretization. In the first part of this mini-course, I will present numerical methods to solve approximately optimal transport problems, focusing on methods base on Kantorovich duality. I will concentrate on entropy-regularized optimal transport and on semi-discrete optimal transport. The presentation will emphasize algorithmic considerations such as convergence speed.

The second part of the presentation will delve into recent advancements regarding the stability of optimal transport solutions. The question of (quantitative) stability has been largely overlooked, despite its necessity to justify most of the numerical and statistical methods relying on optimal transport. I will show how classical functional inequalities (e.g. Prékopa-Leindler and Brascamp-Lieb) can shed light on this question.

Multi-Bubble Isoperimetric Problems - Old and New

Emanuel Milman

Technion – Israel Institute of Technology

The classical isoperimetric inequality in Euclidean space $\mathbb{R}^n$ states that among all sets of prescribed volume, the Euclidean ball minimizes surface area. One may similarly consider isoperimetric problems for more general metric-measure spaces, such as on the n-sphere $\mathbb{S}^n$ and on n-dimensional Gaussian space $\mathbb{G}^n$ (i.e. $\mathbb{R}^n$ endowed with the standard Gaussian measure). Furthermore, one may consider the "multi-bubble" isoperimetric problem, in which one prescribes the volume of $p \geq 2$ bubbles (possibly disconnected) and minimizes their total surface area – as any mutual interface will only be counted once, the bubbles are now incentivized to clump together. The classical case, referred to as the single-bubble isoperimetric problem, corresponds to $p = 1$; the case $p = 2$ is called the double-bubble problem, and so on.

In 2000, Hutchings, Morgan, Ritoré and Ros resolved the double-bubble conjecture in Euclidean space $\mathbb{R}^3$ (and this was subsequently resolved in $\mathbb{R}^n$ as well) – the boundary of a minimizing double-bubble is given by three spherical caps meeting at $120°$-degree angles. A more general conjecture of J. Sullivan from the 1990's asserts that when $p \leq n+1$, the optimal multi-bubble in $\mathbb{R}^n$ (as well as in $\mathbb{S}^n$) is obtained by taking the Voronoi cells of $p+1$ equidistant points in $\mathbb{S}^n$ and applying appropriate stereographic projections to $\mathbb{R}^n$ (and backwards).

In 2018, together with Joe Neeman, we resolved the analogous multi-bubble conjecture for $p \leq n$ bubbles in Gaussian space $\mathbb{G}^n$ – the unique partition which minimizes the total Gaussian surface area is given by the Voronoi cells of (appropriately translated) $p+1$ equidistant points. In the present talk, we describe our recent progress with Neeman on the multi-bubble problem on $\mathbb{R}^n$ and $\mathbb{S}^n$. In particular, we show that minimizing bubbles in $\mathbb{R}^n$ and $\mathbb{S}^n$ are always spherical when $p \leq n$, and we resolve the latter conjectures when in addition $p \leq 5$ (e.g. the triple-bubble conjectures when $n \geq 3$ and the quadruple-bubble conjectures when $n \geq 4$).

On marginals of uniform measure on p-Schatten balls

Grigoris Paouris

Texas A & M University

We provide sharp estimates for the tail behaviour of marginals of the uniform measures on the unit balls defined by the p-Schatten norms on the set of (real or complex) matrices. Based on a joint work with Kavita Ramanan.

On bodies with symmetric sections, floating bodies and related problems

Dmitry Ryabogin
Kent State University

Christos Saroglou and Sergii Myroshnychenko proved that a convex origin-symmetric body in $\mathbb{R}^n, n \geq 3$, with central sections having symmetries of a cube, must be a Euclidean ball. We will discuss several results on floating bodies related to this problem.

The Reverse Minkowski Theorem

Oded Regev
Courant Institute of Mathematical Sciences, New York University

We will describe recent progress on "reverse Minkowski" results on the geometry of lattices. Such results provide upper bounds on the number of short vectors a lattice can have, assuming that it does not have any sublattice of low determinant. We also briefly describe the proof ideas, and mention some open questions.

Based on joint work with Daniel Dadush and Noah Stephens-Davidowitz.

On refinements of classical inequalities under projection assumptions

Eugenia Saorin Gomez
University of Bremen

Based on projection assumptions, we will discuss some classical and new "linear" refinements of inequalities within the Brunn-Minkowski and elliptic Brunn-Minkowski theory, with the aim of exploring parallelisms and differences of both theories.

Jointly with Nico Lombardi and Christian Richter.

Isobarycentric inequalities

Boaz Slomka
The Open university of Israel

Consider the following problem: Given a finite Borel measure on $\mathbb{R}^n$, which sets have maximal measure among all subsets with prescribed barycenter? We shall describe the solution to this problem under mild assumptions on the measure. As an application, we partially answer a question of Henk and Pollehn, which is equivalent to a special case of the Log-Minkowski inequality.

Joint work with Shoni Gilboa and Pazit Haim-Kislev.

Reverse isoperimetric problem under curvature constraints

Kateryna Tatarko
University of Waterloo

The well-known classical isoperimetric problem states that among all convex bodies of fixed surface area in $\mathbb{R}^n$, the Euclidean ball has the largest volume. In this talk, we will discuss the question of reversing the classical isoperimetric inequality in the class of λ-convex bodies, i.e., convex bodies with curvature at each point of their boundary bounded below by some positive parameter λ. This is a joint work with Kostiantyn Drach.

Conditional concentration for functions of high-dimensional random arrays

Petros Valettas
University of Missouri

The use of martingale methods for establishing concentration properties occupy central role in algorithmic discrete mathematics, in particular in situations that the functionals of study lack smoothness assumptions (e.g., Lipschitz conditions, bounds for the L_2 norm of the gradient, etc.) A prototypical example which highlights this utility is the so-called conditional concentration. In this talk we will discuss how we can extend this more combinatorial (rather than standard) form of concentration to functions of high-dimensional random arrays, which do not have necessarily independent entries but enjoy some symmetries. Based on a joint work with P. Dodos and K. Tyros (University of Athens).

On the Illumination Conjecture for convex bodies with many symmetries

Beatrice-Helen Vritsiou
University of Alberta

We will show how to verify the Hadwiger-Boltyanski Illumination Conjecture (along with its equality cases) for 1-symmetric convex bodies of all dimensions and some cases of 1-unconditional convex bodies as well. This is joint work with Wen Rui Sun.

An analogue of polynomially integrable bodies in even-dimensional spaces

Vladyslav Yaskin
University of Alberta

For a convex body K in $\mathbb{R}^n$ its parallel section function is given by $A_{K,\xi}(t) = |K \cap (\xi^\perp + t\xi)|$, where $\xi \in S^{n-1}$ and $t \in \mathbb{R}$. We say that K is polynomially integrable if $A_{K,\xi}(t)$ is a polynomial of t on its support. It was shown by Koldobsky, Merkurjev, and Yaskin that the only polynomially integrable bodies are ellipsoids in odd dimensions. In even dimensions such bodies do not exist. In this talk we will discuss an analogue of polynomially integrable bodies in even dimensions: these are the bodies for which the Hilbert transform of $A_{K,\xi}(t)$ is a polynomial of t (on an appropriate interval). We will see that ellipsoids in even dimensions are the only convex bodies satisfying this property. Joint work with M. Agranovsky, A. Koldobsky, and D. Ryabogin.

Weighted Brunn-Minkowski Theory

Artem Zvavitch
Kent State Univeristy

The Brunn-Minkowski Theory concerns the behaviour of convex bodies in $\mathbb{R}^n$ (compact, convex sets with non-empty interior), by studying their properties e.g. volume, surface area, projections, and Minkowski sum. We shall discuss a generalization of this theory to the measure theoretic setting (replacing volume with some Borel measure with density). In particular, we defined the mixed measures of three convex bodies, and present an integral formula for such mixed measures. We will show inequalities for this quantity, such as Minkowski's First and Second inequality and as well as Fenchel's inequality. As applications, we study log-submodularity and supermodularity of the measure of Minkowski sums of symmetric convex bodies, inspired by recent investigations of these properties for the volume. This is a part of a joint project with Matthieu Fradelizi, Dylan Langharst and Mokshay Madiman.
