RESEARCH IN TEAMS:

Stability and Computations for Stochastic Delay-Differential Equations

The goal of this research in teams was to bring together a group of researchers working on dynamics of stochastic delay differential equations (SDDE's) from different perspectives: theoretical aspects, nonlinear effects in stochastic dynamics, numerical methods, and mathematical modeling in applications. The goals of the group for the two weeks were to identify new directions in this research area and to begin preliminary work to determine productive directions to pursue. One motivating factor in this process is the increased use of models with memory in applications, resulting in a greater demand to have analysis and computations to understand the behavior of the solutions of these models.

The first direction was the weak convergence of numerical schemes for SDDE's. There have been a number of results for strong convergence, both for Euler-Maruyama methods and multi-step methods, but there are no proofs related to weak convergence. Weak convergence of the methods is important in applications since typically moments or the probability densities are of interest in understanding the behavior of the model. Also, the order of convergence should be better for weak schemes than for strong schemes. Difficulties identified for the proof include the fact that one can not rely on expressions for the generator, as one does for Markovian processes, so that a new approach is necessary. An approach was developed which borrowed from recent results for boundary value problems using Malliavin calculus, incorporated into a convenient formulation for the numerical error, thus making it possible to outline a method of proof which did not rely on the generator. Details in this outline are part of present research.

A second direction involved the development of stochastic models for metal cutting and the phenomenon of chatter. In these models, the position of the cutting tool is dependent on the previous part of the cut, so there is a delay in the forces affecting the tool dynamics. Models of these type are known to have oscillatory instabilities, which in this application is called chatter. The development of the model demonstrated that there is both additive and multiplicative noise, which results in resonance with the natural oscillations, thus amplifying the chatter. Here the goal is to determine how the operating parameter range is changed due to the noise sensitivity in the model.

A third direction identified was complementary empirical studies for the use of numerical methods for SDDE's. While convergence results for numerical methods are typically verified by solving test problems, they may not exhibit the same behavior as the problems observed in practice. For example, analysis for the metal cutting problem suggested several aspects that need to be considered: presence of both multiplication and additive noise, large delays, and oscillatory vs. exponential behavior.

In addition progress was made on some related projects: stochastic dynamics for problems with discontinuous coefficients and models for pricing options on assets with delayed memory. Some of the same ideas are necessary for these problems, since they are infinite dimensional systems with noise sensitivity.

Finally the group outlined possible ways to continue their collaboration. A Trans-Cooperation Program research grant proposal for collaboration between UBC and Humboldt University is being written presently. A number of possibilities were outlined for future meetings. Follow-up work in the directions mentioned above is continuing at individual institutions.

This Research in Teams wishes to thank their hosts at BIRS for a productive two weeks.