Competing Species, Predator-Prey Models and Measured-valued Diffusions

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In [5] Evans and Perkins introduced a class of measure-valued branching diffusions which modelled two populations undergoing migration and near critical reproduction which compete for resources. The mathematically challenging part of the model was that competition only occurs when members of the two populations are zero distance apart. This produces a singular interaction involving the collision local time of the two processes. The processes were constructed as solutions of a singular martingale problem in dimensions 3 or less and it was shown that solutions do not exist in higher dimensions. Uniqueness of solutions was established in one dimension and in some special cases–e.g. when only one population feels the competitive effect. In 1998 Durrett conjectured that in these special cases these processes should arise as the weak limit of a space-time rescaling of a model studied by Durrett and Levin [3] in which two contact processes interact through one having a linear competitive effect on the other. At about the same time Mytnik [7] proved uniqueness of solutions to the original model in the more interesting case of symmetric competition. Earlier Evans and Perkins [6] had shown uniquenss of solutions to an associated hitorical martingale problem. This is an enriched setting in which one keeps track of the genealogical histories of the interacting populations.

In 1999, Durrett, Mytnik and Perkins began to work on a general project to show a more general class of competing and cooperating contact processes under rescaling will converge to a more general class of singularly interacting measure-valued diffusions in dimensions 3 or less. The interactions included both competing species and predator-prey type models. We worked on this pairwise for many years and a lengthy manuscript began circulating between the three of us in the spring of 2003. There were some technical issues to work out still just to get tightness of the approximating systems and to identify the limit points as solutions of the natural martingale problems. A full uniqueness theorem was also missing. A two week meeting at BIRS was proposed as a venu where this project could be completed. It allowed all three of us to work on the problem simultaneously for the first time.

The second week of the Research in Teams period coincided with the related 5-day workshop on "Stochastic processes in evolutionary and disease genetics" which was of great interest to all of us and to which Rick Durrett was already committed. We met daily during the first week. During the second week Rick Durrett attended the workshop lectures while Leonid Mytnik and myself attended about a third of the lectures. The three of us continued to meet daily. Early in the first week a serious error was found in our earlier work and correcting it took several days. It was very fortunate that we were all together for this period. We extended the known uniqueness results to some additional cases in the predator-prey setting including the general one-dimensional setting and special cases in the higher dimensional setting. The full uniqueness theorem still remains an open problem although we know how to obtain a general convergence result in the competing species setting by using [6]. A fair amount of work was done on the write-up as well and a completed (as of two days ago) 74-page manuscript is now ready for submission [4].

Several other problems were also discussed including an ongoing project between Mytnik and myself on weak uniqueness for signed solutions of the one-dimensional stochastic pde for super-Brownian motion.

Having the 5-day workshop overlap with our meeting worked out very well. First it was a superb meeting and a pleasant diversion from our technical problems. Secondly, Ted Cox was attending the meeting and Cox, Durrett and myself had some very fruitful discussions on another rescaling limit theorem for high density Lotka-Volterra models. The issue here was to obtain a limit theorem in the regime where both populations are of the same order of magnitude. In earlier work Cox and myself ([1], [2]) had establised a limit theorem in the case where one population is rare and used it to get information on coexistence and survival in the regime where the populations prefer each other to their own type. We were interested in proving an analogous result for the other parameter regime and in particular in showing that the critical survival curve has a discontinuous derivative at the phase transition point where you go from preferring the other type to preferring your own type. Durrett's expertise on related rapidly stirring limits proved to be instrumental in laying out a game plan for getting information on the survival and co-existence questions in this other parameter regime. It involves first showing that in the regime when both populations are large, the appropriate limit is the solution of a particular nonlinear pde. We believe this leads to a valid approach for establishing the above derivative discontinuity. We feel we only need a bit of time to flesh out the proofs. Perhaps we would be fortunate enough to secure another period at BIRS next year? It was an exciting development in a spectacular setting.

Ed Perkins, UBC.

References

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