05w5078 Workshop in Homotopical Localization and the Calculus of Functors

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Overview and Introduction to the Subject

This workshop focused on two relatively recent developments in homotopy theory: homotopical localization, and the calculus of homotopy functors. An effort was made to promote the, as of yet, sparsely explored interrelationship between these two subjects. To develop a sense of purpose and perspective, let us mention a few evolutionary highlights of algebraic topology/homotopy theory, and observe how its concerns and viewpoints progress over time (we use present day terminology throughout):

1. Early activity in the subject centered around combinatorial invariants of polyhydra, such as the Euler characteristic, Betti numbers, etc. These were adequate to classify the members of certain families of spaces, such as connected surfaces which are compact and without boundary. More generally, they provided a tool for distinguishing spaces.

2. Next followed a functorial approach to invariants for the disconnectivities in general topological spaces: homotopy groups, various species of (co-)homology theories, etc. As a 'biproduct' the homotopy invariance of the earlier invariants was obtained.

3. The next evolutionary layer came with the notion of a homotopy functor (one which preserves homotopy equivalences). This provided a unifying platform for all of the specific and geometrically motivated constructs which characterized the previous stage. In addition, it set the stage for a systematic comparison of such functors; e.g. which functors detect a homotopy theoretical property in a given space? which homotopy functor factors through another? etc.

4. With homotopy functors in the center of view, the need for tools to study such resulted in the study of functors on the category of homotopy functors.

Each step further in this development was motivated by the prospect of gaining insight in earlier steps. As history testifies, each step has been successful in this regard.

How do homotopical localization and the calculus of homotopy functors fit in? Homotopy localization of spaces or spectra generates homotopy functors with certain predictable properties. Such functors fit naturally into framework of 3 above. Building on ideas and the groundwork provided by the works of Adams [1], Bousfield [5, 6], Bousfield-Kan [9], Sullivan [23], and others a flurry of activity over the 1990's culminated in a fully developed theory which permits implementations in suitable model categories; see the works of Farjoun [13] and Hirschhorn [17].

The calculus of homotopy functors belongs to level 4. above. It aims to study a homotopy functor F by a tower of homotopy functors

$$\cdots \to T_nF \longrightarrow T_{n-1}F \to \cdots \to T_1F \to T_0F.$$

This tower is strikingly analogous with Taylor polynomial approximations of a smooth function as we'll describe below.

At this point we'd like to describe homotopical localization and the Goodwillie Calculus in more detail.

1 Mathematical Background

We will be working in categories where it is possible to do homotopy theory or something related to homotopy theory. The most basic example of such a category is the category $\mathcal{T}$ of topological spaces.

There are many variations on this category, some of which are considered in Goodwillie's work, and some of which have been considered in the work of other authors. One can do homotopy theory in the category of topological spaces with distinguished basepoints, $\mathcal{T}_*$ (where all functions must preserve the basepoint), topological spaces *over* some fixed base space Y, and the category of spectra, $\mathcal{S}$. We will use $\mathcal{T}_*$ in the succeeding and take this opportunity to describe three basic constructions. Let X be a space with a distinguished basepoint x_0, and I be the unit interval. The *suspension* of X is

$$\Sigma X = (I \times X)/(\{0,1\} \times X \cup I \times \{x_0\}).$$

The *based loop space* on X is

$$\Omega X = \text{Maps}((I, \{0,1\}), (X, x_0))$$

in other words, all continuous maps from the interval to X which take the endpoints of the interval to the basepoint. The *smash product* of X with Y is

$$X \wedge Y = (X \times Y)/(\{x_0 \times Y \cup X \times \{y_0\}\}).$$

Since $\mathcal{S}$ features prominently, and may not be familiar, we also describe it briefly, taking liberties with the definition for the sake of conciseness. A **spectrum** may be thought of as being a sequence of topological spaces with basepoints

$$\{X_0, X_1, \dots\}$$

together with continuous functions (preserving basepoints)

$$s_i \Sigma X_i \to X_{i+1}$$

which are nice inclusions.

It is not important to elaborate the details of the morphisms (the functions) in $\mathcal{S}$, these being somewhat technical but to note the most germane properties of this category. There is a functor

$$S^\infty : \mathcal{T}_* \to \mathcal{S}$$

which takes a space X with distinguished basepoint to the spectrum

$$\{X, \Sigma X, \Sigma^2 X \dots\}.$$

In $\mathcal{S}$, the function $\Sigma(\cdot)$ is invertible, and fibers of maps (equivalent to) desuspensions of cofibers.

There is also a smash product in $\mathcal{S}$ which is also denoted $\wedge$ and which is determined by wanting

$$S^\infty(X \wedge Y) = S^\infty(X) \wedge S^\infty(Y).$$

$\wedge$ in $\mathcal{S}$ plays a role similar to $\otimes$ in a category of modules.

Our interest in $\mathcal{S}$ arises because any functor

$$X \mapsto h_*(X)$$

from $\mathcal{T}_*$ to graded groups which satisfies the axioms of a homology theory is actually given by

$$X \mapsto \pi_*(S^\infty(X) \wedge H)$$

for an appropriately chosen spectrum H. So spectra represent homology theories.

2 Goodwillie's Calculus

We begin by considering a functor

$$F : \mathcal{T} \to \mathcal{T}$$

such that F preserves weak homotopy equivalences. For purposes of simplicity, we also assume that $F(*)$ is contractible (F is *reduced*). There is a special class of such functors which are referred to as *excisive*. An excisive functor is a functor which takes homotopy pushout squares to homotopy pullback squares. Loosely this condition can be though of as taking cofiber sequences of spaces to fiber sequences of spaces. In other words, if F is excisive, then the functor

$$X \mapsto \pi_*(F(X))$$

satisfies the axioms of a *homology theory*. (It is a consequence as discussed above that linear functors are represented by spectra; in fact an excisive reduced functor from based spaces is represented by the spectrum $F(S^0)$.)

One reason excisive functors have a special role is that in many cases homology theories are computable, so that even if we can't always identify $F(X)$ precisely, we can at least compute its homotopy groups. Goodwillie considers excisive functors to be analogous to linear functions in single variable calculus.

One way to think about the beginning of the functor calculus is to imagine searching for an algorithm which allows one to approximate an arbitrary (reduced homotopy) functor by an excisive one. In ordinary calculus, the analogy is to finding a linear approximation to an arbitrary function.

Goodwillie solves this problem in [14]. Given an arbitrary reduced homotopy functor F, Goodwillie gives an algorithm for computing a linear (excisive) functor

$$P_1F : \mathcal{T} \to \mathcal{T}$$

which comes with a natural transformation $\eta : F \to P_1F$ which is *initial* among natural transformations from F to linear functors. That is, given any natural transformation $\nu : F \to G$ where G is linear, ν factors through η. With the restrictions we've given, it is easy to describe the algorithm for making P_1F. With the restrictions we've given, there is a natural map

$$F(X) \to \Omega F(\Sigma X).$$

The target functor (as a functor of X) is also a reduced homotopy functor, so the construction can be iterated. Then P_1F is (loosely) the limit of

$$F(X) \to \Omega F(\Sigma X) \to \Omega^2 F(\Sigma^2 X) \to \dots$$

The notion of a linear approximation to a functor turns out to be just the beginning of an analogy between Taylor polynomials and Taylor series. Goodwillie calls an excisive functor is "1-excisive." Goodwillie gives a definition of *n-excisive*: roughly speaking, a functor is n-excisive if it takes any $n+1$-cubes of spaces in which every square is a pushout to some $n+1$ cubes of spaces in which the initial corner is the pullback of the rest of the cube. From this definition it is obvious that it is easier to be $n+1$ excisive than n excisive (that is, n-excisive functors are automatically $n+1$-excisive).

For each (reduced, homotopy) functor F, there is an n-excisive approximation P_nF and a natural transformation $\eta_n : F \to P_nF$ which is initial among natural transformations from F to n-excisive functors. Just as 1-excisive functors are to be thought of as analogous to linear functions, n-excisive functors should be thought of as analogous to polynomial functions of degree n. So P_nF can be though of as the degree n polynomial approximation to F. Because $n-1$-excisive implies n-excisive, the universal property of the natural transformation

$$\eta_n : F \to P_nF$$

implies there is a functor $\pi_n : P_nF \to P_{n-1}F$ so that

$$\pi_n \circ \eta_n = \eta_{n-1}.$$

There are two important structural observations to make here. First the natural transformations π_n give us a tower of functors $\{P_nF\}$ and the natural transformation η_n give compatible maps from F into this tower. One can ask what the relationship is between F and the homotopy inverse limit of this tower. In particular, one hopes that for any particular space X, F is *analytic* at X (that is, $F(X) = \lim(P_nF)(X)$).

Second, recall that linear functors are described by spectra. Polynomial functors of degree greater than 1 don't have such a simple description, but for each n, the fiber of the natural transformation $\pi_n : P_nF \to P_{n-1}F$ is completely describe by a spectrum with the nth symmetric group, Σ_n, acting on it, and techniques for determining what this spectrum actually is are described in [16]. This functor should be though of as a homogenous functor of degree n. So while excisive functors of degree n may be somewhat complicated, they are described by a finite number of extensions of functors which are themselves determined by equivariant spectra. In principle, this leads to descriptions of (analytic) functors from spaces to spaces in terms of equivariant stable data together with extension information.

This is already interesting in the case where F is the identity functor. In this case the functor is, of course, understood, but because homotopy groups are extremely difficult to compute for most topological spaces, the homotopy groups of the functor evaluated at most interesting spaces are not understood. The homogenous layers are discussed in [16] and [19], and the entire tower is discussed in [2]. This work is further developed for particular values of the space X in[3] where the homotopy groups of the spaces in the Goodwillie tower shed light on the homotopy groups of X.

3 Homotopical localization

Homotopical localization has its roots in algebraic localization. Serre introduced C-theory as a tool that allowed him to prove local versions of classical theorems like the Hurewicz theorem. Some years later the implicit ideas are developed in different directions by Quillen and Sullivan.

Quillen, in [22], gives a development of localization in "model categories". At its most fundamental, this gives conditions where a new category can be constructed from an old category by "inverting" some collection of morphisms which are to be thought of as equivalences (in the new category). A specific and commonly used example is to take the old category to be the category of topological spaces and the equivalences to be maps which induce isomorphisms on $H_*(-;\mathbf{Q})$. (More examples can be easily produced by substituting other coefficients for **Q**.)

Sullivan, in [23] takes a different approach. He describes for a set of primes S and sufficiently nice CW complexes X a construction X_S which "inverts" primes in S. That is, if $X \to Y$ is a map which induces an isomorphism in $H_*(-;\mathbf{Z}[S^{-1}])$, then the induced map $X_S \to Y_S$ will be an equivalence.

Bousfield in [5] generalized these ideas considerably. A *homology theory* $E_*(-)$ is a homotopy invariant functor from spaces to graded abelian groups which satisfies the usual properties of singular homology except that if $*$ represents the one point space, the graded group $E_*(*)$ is not required to be concentrated in dimension 0. Given such a homology theory, Bousfield constructs a functor L_E from the category of spaces to itself which he calls E-localization, and a natural transformation, η from the identity functor to E. E-localization is determined up to homotopy by the following two properties:

1. L_EX is E-local.

2. The natural transformation evaluated at X gives a map $X \to L_EX$ which is initial (up to homotopy) among maps from X to E-local spaces.

Here by Y is *E-local*, we mean that if $E_*(A) = E_*(*)$, then $[A, Y] = *$, the one point set. So all maps from A to Y are homotopic to the constant map.

Fundamental to the construction of Bousfield's localization functors are the class of maps which are to localize to homotopy equivalences. Bousfield ([8]), Dror ([12]) and other authors study more general localizations based on collections of maps which are to become equivalences.

There is a sequence of homology theories related to cobordism known as Johnson-Wilson theories

$$E(0)_*(-) = H_*(-;\mathbf{Q}), E(1)_*(-), E(2)_*(-), \ldots$$

(here $E(1)_*(-)$ is closely related to complex K-theory). Since work of Morava as expanded by Miller, Ravenel and Wilson [20] and the celebrated Nilpotence Theorem [11, 18] localization with respect to these theories has become one of the central organizing principles of stable homotopy, and to a lesser extent, unstable homotopy. Localization with respect to the homology theory $E(n)$ is generally denoted $L_n(-)$, and this family of localizations are referred to as the chromatic localizations.

4 Scope of workshop

The workshop was intended to center on areas where the calculus of functors meets homotopical localizations.

Let L be a homotopical localization functor on some category. L is guaranteed to come with an important structure; a natural transformation from the identity functor to L:

$$\eta_X : X \to L(X)$$

such that

$$\eta_{LX} : LX \to L(LX)$$

is a homotopy equivalence (L is homotopy idempotent).

This is also a property satisfied by the functors in Goodwillie's Taylor Tower when interpreted suitably. Consider the category whose objects are homotopy functors from (for example) $\mathcal{T}$ to $\mathcal{T}$. Then P_n applied to this category of functors is idempotent and comes with a natural transformation from the identity functor. In Dwyer's presentation at the workshop, he described how to produce P_n as a homotopical localization.

One of the more fascinating results in these area is that of Arone and Mahowald in [3]. This paper analyzes the Goodwillie tower of the identity functor from spaces to spaces. One of the main results is that for certain spaces (at least for spheres) the layers in the Goodwillie tower for the identity functor are essentially the chromatic localizations, L_n. While the implications of this fact are far from completely understood, Michael Ching's work presented at this workshop displays these same objects (the derivatives of the identity functor) arising as the spaces in an operad.

A second place where an interaction between chromatic localizations and Goodwillie's techniques was demonstrated at this meeting was in Kuhn's report on his work. If X is a spectrum, it determines a certain infinite loop space (written $\Omega^\infty X$). Kuhn is able to use a number of techniques including Goodwillie calculus to compute $E_*(\Omega^\infty X)$ in terms of $E_*(X)$ for homology theories $E_*(-)$ related to chromatic localizations.

While initially the calculus of homotopy functors was designed for functors on spaces or spectra, the theory has in the mean time found parallel instances in a number of other categories, such as chain complexes, vector spaces and the category of open subsets of a manifold. This begs for an eventual full bodied framework for the calculus of homotopy functors on suitable model categories.

There were two main goals to this conference. First, we sought to introduce researchers in the calculus of functors or homotopical localization to each other's subject. Second, we sought to develop an overlap of these two research areas by exploring current research in both areas. Towards the first objective, Tom Goodwillie provided a series of expository lectures which laid out the foundations of the calculus of functors. A complementary series of lectures were provided by Bill Dwyer, who gave an excellent introduction to localizations and explained how to construct Goodwillie's Taylor stages as homotopy localizations within a suitable category of diagrams of spaces as mentioned above. These lectures laid the groundwork for what followed.

Outcome of the meeting

While it is unfair to categorize the contributions of the participants of this conference into such a short list of topics, it is beneficial to enumerate those topics which form current trends in the calculus of functors and homotopical localizations. What follows is a short compilation of those topics which pertain most to the intended goals of this meeting.

- Manifold Calculus: As mentioned in the introduction, the calculus of functors has applications to areas reaching beyond homotopy theory. In particular, Goodwillie's machinery can be applied to functors from the category of open sets of a manifold to the category of topological spaces. In tandem lectures, Ismar Volic and Brian Munson gave a gentle introduction to manifold calculus. The talks pertained to research in both the machinery of calculus (Munson's results address the lifting problem from the second stage of the tower to the third stage of the tower), and applications of this machinery to the study of embeddings (Volic described joint work with Pascal Lambrechts and Greg Arone related to finite type invariants of knots).

- Calculus and Operads: Recently, there has been a flurry of activity trying to understand an apparent operad structure on the layers of the Goodwillie tower of the identity functor from spaces to spaces. One of the great testimonies to the beauty of the calculus of functors is complexity of the Goodwillie tower of the identity functor, which is seemingly innocuous. In particular, this complexity is the main obstacle to obtaining a chain rule. Motivated by our instinct from the calculus of real variables, we would expect that the layers of the tower for $F \circ G$, where F and G are homotopy functors of spaces, would be the composition of the layers of F with the layers of G. However, the expected formulation fails. Rather, the identity functor plays a critical role. In his talk, Michael Ching showed that the layers of the identity functor form an operad, and conjectured a solution to the chain rule problem, relying on the left and right module structures of the layers of F and the layers of G over this operad. An alternative approach to understanding the operad structure of the layers of any homotopy functor of spaces equipped with a natural transformation $F \circ F \to F$ was suggested in the talk of Andrew Mauer. Mauer's approach relies on the formulation of the layers of such a functor in terms of the cross effects of this functor. This is also related to Dev Sinha's talk, in which he presented another formulation of the operad structure on cross effects of the identity of functors, at least for spheres. The relationship between Sinha's work and Ching's work can be seen by relating both of their operads to the Lie operad.

- Tensor calculus of homotopy functors: ad hoc special session by Tom Goodwillie with an outline of an obstacle toward a 'theory of differential forms' of homotopy functors (spaces) to (spectra).

- Relationships between calculus of functors and localizations: Nick Kuhn's work with $K(n)$ localizations and calculus, Taylor stages in the calculus of homotopy functors are homotopy localizing functors in a suitable category of diagrams of spaces: Bill Dwyer

Abstracts of Talks

M. Ching *Operads and calculus of functors*
I'll talk about some aspects of the relationship between the calculus of homotopy functors and the theory of operads. In particular, I'll describe the operad structure on the derivatives of the identity functor and try to explain how the derivatives of other functors might fit into this framework.

C. Casacuberta *Continuity of homotopy idempotent functors*
A functor L in a simplicial model category is called simplicial or continuous if it defines a map from $\mathrm{map}(X, Y) \to \mathrm{map}(LX, LY)$ for all X, Y, which is natural and preserves composition and identity. As shown by Farjoun and Hirschhorn, f-localizations can be constructed as continuous functors. Thus, a necessary condition for a homotopy idempotent functor to be equivalent to some f-localization is that it be equivalent to a continuous functor.

In joint work with different coauthors, we discuss continuity of homotopy functors in several model categories, with emphasis on simplicial sets, spectra, and groupoids. In the latter, remarkably, continuity is automatic.

W.G. Dwyer *Localization and Calculus I and II*
A general discussion of the idea of localization in homotopy theory. Followed in part II by specialization to the localization of diagram categories, and further specialization to the case of a particular diagram category associated to the Goodwillie tower.

E. Farjoun *Open problems and some recent progress in localization and cellularization theory*
The talk will revisit some of the progress made recently in understanding localization and co-localization functors. We shall list some interesting problems and describe related partial progress. The talks will concentrate mostly on general properties of localization with respect to a map in both algebraic homological algebra and topological categories.

T. Goodwillie *Introduction to the Calculus of Homotopy Functors, I,II, and III*
Overview of basic definitions and results (excisive and n-excisive approximations of functors, classification of homogeneous functors, chain rule); key examples; matrix notation. Followed in part II by: more about homogeneous functors, with an emphasis on results which require no information about connectivity.

A geometric view of the functor/function analogy. In this view, Top is a variety and functors Top $\rightarrow$ Spectra are global functions. I will say which categories are the tangent spaces of Top. I will discuss tangent vector fields and more generally tensor fields, in both a coordinate-free way and a coordinate-dependent way. I will show that there are two tangent connections, both of which are flat, and that their difference is the tensor field known as smash product of spectra. I will say something about higher-order jets and about differential operators. I cannot make much sense of differential forms (except 0-forms and 1-forms), but I may talk about them anyway. Applications are work in progress, but I will make sure to at least say something trivial about some nontrivial examples, and maybe something nontrivial about some trivial examples.

M. Hovey *$E(n)_* - E(n)$-comodules*
I will recap my results with Neil Strickland about the structure of the category of $E(n)_*E(n)$-comodules (e.g. the Landweber filtration theorem works there as well). I will describe why we need to know more about comodules (derived functors of product in the category of comodules form the E_2-term of a spectral sequence converging to the $E(n)$-homology of a product of spectra; this is relevant for the chromatic splitting conjecture). Then I will describe some new results I have about the honest injective $E(n)_*E(n)$-comodules. There are only $n+1$ isomorphism classes of indecomposable injectives, and most interestingly, the endomorphism ring of the k-th one is $(E(k)^\wedge)^*(E(k)^\wedge)$, where $E(k)^\wedge$ is the completion of $E(k)$ at I_k.

So in the category of $E(n)_*E(n)$-comodules, you are seeing all the $E(k)^\wedge$ operations for $0 \leq k \leq n$, and therefore seeing all the different stabilizer groups S_k for $0 \leq k \leq n$. This is a good thing, since the relation between the different stabilizer groups is basically what the chromatic splitting conjecture is about.

N. Kuhn *Periodic homology of infinite loop spaces*
If E_* is a homology theory, one can ask to what extent the E_*-homology of an infinite loop space is determined by the E_*-homology of the associated spectrum. Using a combination of the Hopkins-Smith Periodicy Theorem, as packaged in the telescopic functors of Bousfield and me, and Goodwillie calculus, I can give a quite definitive answer to this question when the homology theory is Morava K-theory. There are calculations still to be done that may inform on the Telescope conjecture.

A. Mauer-Oats *An operad from the derivatives of a monad*
McClure and Smith have a simple idea that explains how to produce an operad from a functor operad by evaluating on the unit of the smash product. The cross effects of a (reasonably good) monad F are a functor-operad of spaces. We explain the proper way to prolong a multivariate functor to spectra, and use this to produce an operad of symmetric spectra. If a certain problem of cofibrancy can be overcome, the spectra in the operad will be the derivative spectra of F.

B. Munson *The layers of the embedding tower*
I will discuss the layers of the embedding tower and their relationship to the obstructions to finding embeddings.

D. Sinha *A pairing between graphs and trees*
We give an elementary pairing between graphs and trees, which facilitates the study of the Lie operad and free Lie algebras. It arises in topology through both homology of configuration spaces and (conjecturally) in studying Hopf invariants and Whitehead products. We sketch its possible application in using the embedding calculus to define knot invariants, and hope that it might be of interest in the homotopy calculus as well.

D. Stanley *Complete invariants of t-structures*
Let R be a Noetherian ring. We give a classification of Bousfield classes on the bounded derived category of R. This also gives complete invariants of t-structures on the same category. We also show that the t-structures on the unbounded derived category of Z-modules do not form a set.

I. Volic *Embedding calculus and formality of the little cubes operad*
I will first give a brief introduction to embedding calculus and say how a certain Taylor tower can be assigned to an isotopy functor. Then I will describe joint work with Greg Arone and Pascal Lambrechts in which the central observation is that the stages of the Taylor tower in the case of Emb(M, V), the space of embeddings of a manifold in a vector space (up to immersions), have the structure of maps of certain modules over the little cubes operad. Using Kontsevich's formality of this operad, one then concludes that the cohomology spectral sequence for Emb(M, V) arising from the Taylor tower collapses rationally. In the special case of spaces of knots, this was conjectured by Vassiliev. Additionally, using the interplay between embedding and orthogonal calculus, one also deduces that the rational cohomology of Emb(M, V) only depends on the rational homotopy type of M when 2dim$(M) + 1 <$ dim(V).

M. Weiss *Stratifications and homotopy colimit decompositions*
This talk will discuss the art of converting stratifications into homotopy colimit decompositions, perhaps with applications to the theory of surface bundles. Every well behaved stratified space has a homotopy colimit decomposition indexed by a certain topological category in which all endomorphisms are invertible up to homotopy. In many cases one can do better and match the stratification with a homotopy colimit decomposition indexed by a discrete category in which all endomorphisms are invertible. The matching property means roughly that the strata correspond to the isomorphism classes of the indexing category.

List of participants

A determined effort was made to ease the entry into these subjects by young researchers. Specifically, out of 34 participants, 3 were graduate students and a number of 5 were within the first 3 years of their postdoctoral career. We had talks from one of the graduate students and from three of the postdocs.

Arlettaz, Dominique (Universite de Lausanne)
Bauer, Kristine (University of Calgary)
Casacuberta, Carles (University of Barcelona)
Chebolu, Sunil (University of Washington)*
Ching, Michael (Massachusetts Institute of Technology)*
Chorny, Boris (University of Western Ontario)**
Dover, Lynn (University of Alberta)*
Dror-Farjoun, Emmanuel (Hebrew University of Jerusalem)
Dwyer, William (Notre Dame University)
Goodwillie, Tom (Brown University)
Gutierrez, Javier (University of Barcelona)
Hovey, Mark (Wesleyan University)
Krause, Eva (University of Alberta)
Kudryavtseva, Elena (University of Calgary/Moscow State University)
Kuhn, Nick (University of Virginia)
Lambrechts, Pascal (Louvain-la-Neuve)
Mauer-Oats, Andrew (Northwestern University)**
McCarthy, Randy (University of Illinois at Urbana-Champaign)
Munson, Brian (Stanford University)**

Nicas, Andrew (McMaster University)
Nikolaev, Igor (University of Calgary)
Palmieri, John (University of Washington)
Peschke, George (University of Alberta)
Prince, Tom (University of Alberta)
Ravenel, Douglas (University of Rochester)
Sadofsky, Hal (University of Oregon)
Scull, Laura (University of British Columbia)
Sinha, Dev (University of Oregon)
Stanley, Don (University of Regina)
Varadarajan, Kalathoor (University of Calgary)
Volic, Ismar (University of Virginia)**
von Bergmann, Jens (University of Calgary)**
Weiss, Michael (University of Aberdeen)
Zvengrowski, Peter (University of Calgary)

References

[1] J.F. Adams. *Localisation and Completion*. Univ. of Chicago Press 1975.

[2] G. Arone, A generalization of Snaith-type filtration, *Trans. Amer. Math. Soc.* **351** (1999), 1123-1150.

[3] G. Arone, M. Mahowald. The Goodwillie tower of the identity functor and the unstable periodic homotopy of spheres. *Invent. Math.* **135** no. 3 (1999) 743–788.

[4] M. Bökstedt, G. Carlsson, R. Cohen, T. Goodwillie, W.C. Hsiang, I. Madsen. On the algebraic K-theory of simply connected spaces. *Duke Math J.* **84** no. 3 (1996) 541–563.

[5] A.K. Bousfield. The localization of spaces with respect to homology. *Topology* **14** (1975) 133–150.

[6] A.K. Bousfield. Construction of factorization systems in categories. *J. Pure Appl. Algebra* **9** (1977) 207–220.

[7] A.K. Bousfield. Localization and periodicity in unstable homotopy theory. *J. AMS* **7** (1994) 831–873.

[8] A.K. Bousfield. Homotopical localization of spaces. *American J. Math.* **119** no. 6 (1997) 1321–1354.

[9] A.K. Bousfield, D.M. Kan. *Homotopy limits, Completions, and Localizations*. Springer-Verlag LNM 304, Berlin New York 1972.

[10] G. Carlsson, R. Cohen, T. Goodwillie, W.C. Hsiang. The free loop space and the algebraic K-theory of spaces. *K-theory* **1** no. 1 (1987) 53–82.

[11] E.S. Devinatz, M.J. Hopkins, J.H. Smith. Nilpotence and stable homotopy theory, I, *Ann. of Math.* **128** (1988), no.2, 207–241.

[12] E. Dror Farjoun. Homotopy localization and v_1-periodic spaces. Algebraic topology (San Feliu de Guíxols, 1990) *Lecture Notes in Math.* **1509** (1992) 104–113.

[13] E. Dror Farjoun. *Cellular Spaces, Null Spaces, and Homotopy Localization*. Springer-Verlag Lect. Notes in Math. 1622, Berlin New York 1995.

[14] T.G. Goodwillie. Calculus I. The first derivative of pseudoisotopy theory. K-theory **4** no. 1 (1990) 1–27.

[15] T.G. Goodwillie. Calculus II. *K-theory* **5** no. 4 (1991) 295–332.

[16] T.G. Goodwillie. Calculus III, Taylor series. *Geometry and Topology* **7** (2003) 645 – 711.

[17] P.S. Hirschhorn. *Model Categories and Their Localizations*. AMS Math. Surveys **99** 2003.

[18] M.J. Hopkins, J.H. Smith. Nilpotence and stable homotopy theory, II, *Ann. of Math.* **148** (1998), no.1, 1–49.

[19] B. Johnson, The derivatives of homotopy theory, *Trans. Amer. Math. Soc.* **347** (1995), no. 4, 1295-1321.

[20] H. R. Miller, D. C. Ravenel, and W. S. Wilson. Periodic phenomena in the Adams–Novikov spectral sequence. *Annals of Mathematics* **106** (1977), 469–516.

[21] N. Kuhn. Tate Cohomology and periodic localization of polynomial functors.. *Preprint* (2005) .

[22] D.G. Quillen. *Homotopical Algebra*. Springer-Verlag Lect. N Math. 20, Berlin New York 1963.

[23] D.P. Sullivan. Genetics of homotopy theory and the Adams conjecture. *Annals Math.* **100** no. 2 (1974) 1–79.