Homology stability of moduli of vector bundles over a curve
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1 Overview

Arapura and Dhillon spent an intensive and productive week at BIRS, under the auspices of the *Research in Teams* programme. During this period, they were able to produce an outline of a research project, described below, on the moduli of bundles over a curve. Although some more work will be needed to flesh out the details, a finished paper is expected to result from this research in the near future.

2 Mathematical Details

Let C be a smooth projective curve of genus $g \geq 2$ over field of complex numbers, and let $G = G_n$ be a classical group (i.e. one of GL_n, SL_n, SO_n, Sp_n). The research project involves the study of the moduli stack $Bun_G(C)$ (respectively moduli space $M_G(C)$) of (stable) principal G-bundles over C. When $G = GL_n$, these objects can be identified with the moduli stack or space of vector bundles. The basic goal is understand the Hodge structure, and the underlying motive, on the cohomology of $Bun_G(C)$ and $M_G(C)$ as C varies. This can be reduced to a series of subproblems:

1. Construct a relative theory of motives in the spirit of André [1].

2. Use an Atiyah-Bott type isomorphism [3, 6] to "compute" the motive of $Bun_G(C)$ in terms of the motive of C. Apply this to the universal curve.

3. Find good estimates to relate the cohomology and motive of $Bun_G(C)$ to that of $M_G(C)$. For vector bundles, suitable estimates have been found in [2, 5]. In general, some of the basic tools are contained in [4].

Since the estimates in 3 should grow with n, this can be used to show that $H^*(M_{G_n}(C))$ stabilizes, as expected.

References

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