Third Northwest Functional Analysis Symposium

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Functional Analysis plays an important role in mathematics. It is a major strength of Canada's, particularly in western Canada, where, whithin the PIMS network, there are several strong groups with internationally recognized standing.

In our workshop such groups were well-presented. We had also participants who work in related areas where methods of Functional Analysis are used. Altogether 6 western Universities were represented, and many postdoctoral fellows and advanced graduate students participated. There were 11 talks on various aspects of Functional Analysis and related topics. There was also time for informal communication between participants.

Geometric Functional Analysis

Geometric Functionals Analysis deals with geometry of Banach spaces in both finite-dimensional and infinite dimensional cases. In particular, it studies asymptotic properties of high dimensional convex bodies and related topics in convex and combinatorial geometry.

More than 60 years ago Paul Erdos raised two questions which became the best known problems in combinatorial geometry. (i) How often can the same distance occur among N points in the plane? (ii) At least how many distinct distances must occur among N points in the plane? He conjectured that for (i) the upper bound is $N^{1+\epsilon}$ and in (ii) the lower bound is $N^{1-\epsilon}$. The first conjecture would imply the second one, however both conjectures are widely open. Even less is known for point sets of general normed spaces. **József Solymosi** described some related results and listed several problems and conjectures.

Márton Naszódi presented his joint work with Károly Bezdek and Balázs Visy on Petty numbers of normed spaces. In 1983, P. Erdős raised the following question: What is the largest number $g(n)$ with the property that every system of n non-overlapping unit disks in the Euclidean plane has an independent subsystem with at least $g(n)$ members; i.e., there are $g(n)$ members no two of which are tangent. In 1971, C.M. Petty proved that the maximum number of pairwise touching non-overlapping translates of a convex body in d-space is 2^d. Motivated by Erdős' problem and Petty's Theorem, Márton with his coauthors defined two quantities in a real normed space M^d of finite dimension d. For $m \geq 2$, the mth *Petty number* $P_{\text{pack}}(m, M^d)$ of M^d is the largest cardinality of a set A in M^d with the property that among any m points of A there are two at distance one. The *packing number* $P_{\text{pack}}(m, M^d)$ of M^d is the largest integer k such that there are k non-overlapping unit balls in M^d with the property that among any m of them, there are two touching ones. The main challenge is to find upper bounds for the Petty- and the packing numbers. Márton showed general upper bounds in terms of m and the dimension of M^d, and bounds specifically for ℓ_p spaces, and for the space $l^d_\triangle$ with unit ball $T - T$, where T is the regular simplex. These problems, apart from being intriguing questions in the theory of finite dimensional normed spaces, also have applications in coding theory.

It was shown by Milman in 1970 that the product of any two strictly singular operators on $L_p[0,1]$ is a compact operator. In his talk **Vladimir Troitsky** presented his joint work with G.Androulakis, P.Dodos, A.Popov, and G.Sirotkin. It was shown by Milman in 1970 that the product of any two strictly singular operators on $L_p[0,1]$ is a compact operator. Vladimir with coauthors show extension of this approach to other Banach spaces. It was proved that if X is a Banach space such that the number n of pair-wise non-equivalent seminormalized weakly null Schreier spreading basic sequences is finite, then the product of any $n+1$ strictly singular operators on X is compact. Further, for every countable ordinal ξ the Schreier family S_ξ (introduced by D. Alspach and S.A. Argyros) was used to define the class of S_ξ-singular operators. It was proved that the classes of S_ξ-singular operators are nested increasing in ξ and exhaust the class of all strictly singular operators. These classes are norm-closed, and stable under left and right multiplications by bounded operators. It was also shown that if the number n_ξ of pair-wise non-ξ-equivalent seminormalized weakly null Schreier spreading basic sequences is finite, then the product of any $n+1$ S_ξ-singular operators on X is compact. However, in general, S_ξ-singular need not even be polynomially compact. Finally, analogously to HI-spaces, HI$_\xi$-spaces we introduced. It was shown that the classes of HI$_\xi$-spaces are nested increasing in ξ and exhaust the class of all HI-spaces, and that every operator on an HI$_\xi$-space is of the form "scalar plus S_ξ-singular". Here are some related open questions: (i) Whether there exists an HI$_\xi$-space X such that $n_\xi(X)$ is finite? A positive answer to this question would imply a positive answer to the following long-standing question: does there exist a Banach space on which every operator has an invariant subspace? (ii) Whether the classes of S_ξ-singular operators are operator ideals?

Abstract Harmonic Analysis

Abstract Harmonic Analysis concerns investigations of Banach algebras and spaces of measures or functions associated with a locally compact group. Two talks were given on this subject.

Hung Le Pham delivered a talk on the structure of (discontinuous) homomorphisms from $C_0(X)$ into Banach algebras. In particular, it was shown that for most locally compact metrizable space X the continuity ideal of a homomorphism from $C_0(X)$ (the largest ideal on which the homomorphism is continuous) is not always a finite intersection of prime ideals. The examples of such spaces include all uncountable σ-compact locally compact metrizable spaces. The method of construction is algebraic, involving results from algebraic geometry. Another consequence of the construction is a general result on norming commutative semiprime algebras; extending the class of algebras known to be normable. The problem of describing the detailed structure of (the continuity ideals of) homomorphisms from $C_0(X)$ into Banach algebras is still open.

In his talk **Faruk Uygul** proved that every completely contractive dual Banach algebra is completely isometric to a w^*-closed subalgebra the operator space of completely bounded linear operators on some reflexive operator space.

C^*-algebras and non-commutative geometry

Three talks on current research projects within several areas of non-commutative geometry were delivered.

Heath Emerson reported on aspects of joint work with R. Meyer concerning a noncommutative extension to an operator algebra context of the classical Lefschetz invariant associated with a group acting on manifold or simplicial complex. This is accomplished by connecting equivariant KK theory to Lefschetz fixed point theory.

The irrational rotation algebras were among the first significant examples of noncommutative operator spaces associated with topologically intractable dynamical systems. **Zhuang Niu** reported on progress of recent joint work with Elliott concerning C*-algebras generated by the irrational rotation algebra and certain spectral projections, so called extended rotation algebras. This is the first step in a programme to find a canonical C*-algebraic way to embed the rotation algebra in an AF algebra. In certain cases these are shown to be simple and nuclear, so of interest to the Elliott classification programme. The Elliott classification programme is one of the major threads of research in C*-algebras over the past 15 years, with a goal of finding a classification functor on nuclear, separable, simple C*-algebras with range involving K-groups with order structure, and topological convex cones with natural pairings between them.

E_0 semigroups of endomorphisms and its attendant formal classifying structure of product systems is a significant area of current research interest. **Remus Floricel** gave a talk on classes of essential representations of product systems: Essential representations of a product system E give rise to E_0-semigroups whose associated concrete product systems are isomorphic to E. He first constructed essential representations of a product system $E = \{E(t)|t > 0\}$ with respect to a given unital vector $u_t \in E(t)$, $t > 0$. His constructions employes techniques developed by M. Skeide, W. Arveson and V. Liebscher. He then showed that the E_0-semigroups associated with the essential representations of E constructed out of two unital vectors $u_t \in E(t)$, respectively $u_s \in E(s)$, are conjugate if and only if $s = t$, producing in this way uncountably many non-conjugate, cocycle conjugate E_0-semigroups. Secondly, he constructed essential representations of E with respect to measurable sections of E, and shows that the associated E_0-semigroups are conjugate if and only if the group of automorphisms of E acts transitively on the set of measurable sections. He also described the structure of the tail algebras of these E_0-semigroups.

Ergodic Theory

Two talks we delivered on Ergodic theory and relations to Analysis and Functional Analysis.

Ergodic theory provides a number of powerful tools for analysis of the asymptotic behavior of measure preserving dynamical systems. Some systems come with a natural invariant measure; many more do not. In his talk **Chris Bose** discussed both classical and new results about how to find invariant measures in some of these cases, and how to compute explicit approximations where there may be no tractable formula for the measure.

Many authors have considered “subsequence ergodic theorems”, where measurements (i.e. the value of an L^p function) of a measure-preserving dynamical system are taken at a fixed sequence of times. A sequence of times is called good if the averages converge. In his talk **Anthony Quas** considered bad sequences of times and asked for the maximal rate of badness.

Applications of Operator Theory

Pedro Massey gave a talk on The Schur-Horn theorem for operators and frames with prescribed norms and frame operator (joint work with J. Antezana, M. Ruiz and D. Stojanoff). In recent years, the concept of frame in s Hilbert space has gained attention due to its important applications to data transmission in channels with noise. Roughly speaking, an ordered set of vectors $\mathcal{F} = \{f_k\}_{k\in\mathbf{N}}$ is a frame for a separable (real or complex) Hilbert space $\mathcal{H}$ if linear combinations with coefficients in $\ell^2(\mathbf{N})$ generate $\mathcal{H}$ in a stable way. It turns out that the canonical way in which the vectors in $\mathcal{H}$ are generated by a frame $\mathcal{F}$ depends on inverting the so called frame operator of $\mathcal{F}$, which is typically a hard problem. A way out to this problem is to consider frames with simple frame operators, but then the existence of such frames is no longer evident. Given a bounded positive definite operator $S \in B(\mathcal{H})$, and a bounded sequence $c = \{c_k\}_{k\in\mathbf{N}}$ of non negative numbers, the pair (S, c) is called *frame admissible* if there exists a frame $\{f_k\}_{k\in\mathbf{N}}$ for $\mathcal{H}$ with frame operator S and such that $\|f_k\|^2 = c_k$, $k \in \mathbf{N}$. We relate the existence of such frames with the problem of determining the principal diagonals of an operator in $B(\mathcal{H})$. This is a key tool, since this last problems has been recently considered by several researchers. Massey gave a reformulation of the extended version of Schur-Horn theorem due to A. Neumann that characterizes the (closure of the set of) possible diagonals of selfadjoint operators in $B(\mathcal{H})$ and used it to get necessary conditions, and to generalize known sufficient conditions, for a pair (S, c) to be frame admissible. We also describe some results for diagonals of finite rank operators determining completely when a pair (S, c) is frame admissible when dim $\mathcal{H}$ is finite.