Recent Progress on the Moduli Space of Curves

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1 Overview of the Field

Mumford's 1983 paper [?] initiated the systematic study of the intersection theory of the moduli space of curves, in particular emphasizing the importance of tautological classes - Chow classes naturally arising from the intrinsic geometry of curves. Subsequently, Carel Faber explored the properties of the graded intersection ring generated by such classes - the so-called tautological ring of the moduli of curves [?]. About ten years later, based on a substantial amount of numerical data and the Witten conjectures (proved by Kontsevich), Faber conjectured that the tautological ring of curves in each $g \geq 2$ "*behaves like the algebraic cohomology ring of a nonsingular projective variety of dimension* $g-2$." Moreover, he conjectured surprising combinatorial formulas for the "intersection numbers" that have been shown ([?])to follow from a very deep result (the proof of the Virasoro conjecture) of Givental ([?]). The drawback of such a high-powered and indirect proof is that it doesn't shed much light on the fundamental geometric reasons for such formulas.

In the last few years, new ideas and techniques have been developed that offer some promising new lines of attack on Faber's conjectures. These include the following:

- *Relative stable maps and localization*. The Hurwitz spaces of branched covers of the Riemann sphere admit different compactifications by spaces of "relative stable maps", depending upon how one treats the collisions of branch points. The Hurwitz spaces have long been useful tools for studying the moduli spaces of curves, but somewhat surprisingly, recent applications of the Atiyah-Bott localization theorem to all compactifications at once have led to some important new insights. One of the most powerful of these is the Theorem "Star" of Graber-Vakil characterizing the support of tautological classes of codimension $> g$ [?].

- *Axiomatic Gromov-Witten theory.* Givental's approach to Gromov-Witten theory in higher genus, regarding the spaces of axiomatic Gromov-Witten theories as a sort of homogeneous space for a "quantized" loop group, led Y.P. Lee to make a series of intriguing conjectures on relations in the tautological rings. In principle, these conjectures ought to reduce the computation of all relations to linear algebra, but in practice the computations are prohibitively complicated. Nevertheless, this point of view has resulted in a systematic rediscovery of many subtle relations, as well as several new ones [?].

2 Recent Developments

In the last two years, the above circle of ideas have led to significant progress in the field. Some examples include:

- Lee's invariance conjectures constituted the backbone for the proof of *Witten's r-spin conjecture* by Faber, Shadrin and Zvonkine([?]).

- Work of Vakil with Goulden and Jackson has led to a new proof of the λ_g conjecture and of the intersection number part of Faber's conjecture for the moduli spaces of curves with rational tails, in the case of a top intersection number given by a small number of factors ([**?**], [**?**]).
- Only days before the workshop Liu and Xu posted yet another interesting proof for Faber's intersection number conjecture([**?**]).
- The analysis from the previous works led Goulden, Jackson and Vakil ([**?**]) to observe remarkable structure for double Hurwitz numbers: they are piecewise polynomial. Shadrin, Shapiro and Vainshtein ([**?**]) have described walls and wall crossing formulas in genus 0, and recent work of Cavalieri, Johnson and Markwig ([]) seems to have identified a very promising combinatorial framework to study the positive genus case.
- The study of Gromov-Witten theory for orbifolds has been extremely lively and exciting. We won't try to list specific works here for fear of being unfair and incomplete. On the one hand, orbifold Gromov-Witten theory can be shown to create yet another bridge between teh moduli space of curves and Hurwitz theory, and hence towards a combinatoruialization of questions about tautological classes. On the other hand, Givental machinery has been successfully adapted to the orbifold setting, providing an extremely powerful computational framework. Perhaps one of the most exciting open questions in the fiels is *Ruan's* (et al. by now) Crepant Resolution Conjecture, predicting a nontrivial equivalence of the orbifold Growov Witten theory of an orbifold with the ordinary Gromov-Witten of a crepant resolution.
- Recent work of Telemann([**?**]) is using a classification of families of semi-simple field theories to prove a conjecture of Givental, stating that higher genus Gromov-Witten theory can be recovered from the genus 0 case by a process of quantization.

3 Description of the Scientific Activity

The workshop "Recent progress in the moduli space of curves" counted forty three participants, with particular emphasis on encouraging interaction between estabilished researchers and young mathematicians: participants included fourteen graduate students and eight postdocs, from a variety of institutions all around the world.

The scheduled activities in this workshop consisted of:

- a three hour mini-course given by Y.P. Lee presenting recent work of Teleman([**?**]). This is a very technical work, and Lee provided an "as soft as possible" introduction to the circle of ideas used by Teleman.
- a three plus two hour mini-course coordinated between Ravi Vakil and Ian Goulden. Vakil gave an overview of the Faber conjectures and presented their history, particularly focusing on the geometric approaches using Gromov-Witten theory. Goulden presented a key combinatorial tool used in these arguments, explaining how and why a lot of the families of intersection numbers arising in this theory are organized by the KP integrable hierarchy.
- five half hour talks by advanced graduate students: Johnson, Pagani, Penev, Todorov and Wise.
- an open ended discussion session on open problems.
- nine one hour long research talks: the speakers were Bryan, Cadman, Edidin, Faber, Farkas, Kimura, Shapiro, Tseng, Yang.

4 Discussion and Questions Emerged

One afternoon of the conference was devoted to an open ended discussion session. The participants were encouraged to sumbit their questions and ideas. Aaron Bertram led the discussion and acted as a moderator.

The discussion was very lively, and several participants participated very actively. Several questions, ranging from basic to very advanced, emerged. This was indeed a crucial moment in the workshop. We wish to emphasize this by reporting here some of the questions and conclusions.

Sam Payne "spayne@stanford.edu" Here's a situation I find intriguing: the moduli space of stable n-pointed rational curves embeds naturally in a toric variety – the toric variety associated to the space of phylogenetic trees with n leaves. For small n, one can show that this toric variety has some exceptionally nice cohomological properties (e.g. the section ring of any ample line bundle is Koszul). Keel and Tevelev have shown similar properties for the kappa embedding of M_0, n. My question in two parts is then:

1. Does the toric variety associated to the space of phylogenetic trees with n leaves have Koszul homoegeneous coordinate rings for every n?

2. Can the Keel-Tevelev Theorem on the kappa embedding be deduced from the geometry of its embedding in the toric variety associated to the space of phylogenetic trees, and can such results be extended to other projective embeddings of M_0, n?

Angela Gibney "agibney@math.upenn.edu" There are some very basic questions about the birational geometry of $\overline{M}_{g,n}$ which by now are kind of old-ish, but that we shouldn't forget are still open.

One of the fundamental objects of study in Mori theory is the closed cone of curves. By understanding the cone of curves of a projective variety X, one could essentially describe what maps there are from X to any other projective variety. Carel Faber is responsible for identifying a natural collection of curves on $\overline{M}_{g,n}$ which he observed, for low g and $n = 0$, actually spanned the extremal rays of the Mori cone of curves. These curves are 1-dimensional boundary strata – they are numerically equivalent to the closures of the loci of points in $\overline{M}_{g,n}$ having $3g - 4 + n$ nodes. Faber's conjecture for the Mori cone is known in a number of intermediate cases between $n = 0$ and $g \leq 24$ to $g = 0$, and $n \leq 7$. Faber's conjecture would be true on $\overline{M}_{g,n}$ if it were true on $\overline{M}_{0,g+n}$, and so of course, knowing what happens in the genus zero case is very important.

In the genus zero case, Faber's conjecture on the cone of curves is equivalent to Fulton's conjecture for cycles. Fulton questioned whether the cycle structure on $\overline{M}_{0,n}$ is analogous to the cycle structure on a normal toric variety X_Δ, where Δ is the fan of cones. On X_Δ, every effective k-cycle would be equivalent to an effective combination of torus invariant cycles of dimension k. Fulton likened the boundary stratification of $\overline{M}_{0,n}$ to the fan of cones of a toric variety and wondered whether an effective cycle of dimension k on $\overline{M}_{0,n}$ might be numerically equivalent to an effective combination of k-dimensional boundary strata. This is actually true for 0-cycles since $\overline{M}_{0,n}$ is rational. Keel and others have shown it is false for cycles of dimension $k \geq 2$. The one remaining open case is for $k = 1$. Fulton's question is whether or not every effective curve is numerically equivalent to an effective combination of boundary curves. In other words, his question asks whether the extremal rays of the Mori cone are spanned by the curves that Faber predicts.

One can rephrase these questions to describe the cone of nef divisors on $\overline{M}_{g,n}$. As Farkas said in his talk, this is the cone spanned by divisors that nonnegatively intersect all curves. So Faber's conjecture can be rephrased as predicting that the cone of nef divisors $Nef(\overline{M}_{g,n})$ is equal to what people call the F-cone of divisors $F(\overline{M}_{g,n})$ which is spanned by those divisors that nonnegatively intersect the boundary curves. In other words, the F-cone is an upper bound for the Nef cone. Faber's conjecture is that the two are equal. Recently, Maclagan and Gibney have proved that the F-cone on $\overline{M}_{0,n}$ is actually the pull back of a cone of divisors on a toric variety X_Δ that contains $\overline{M}_{0,n}$. Gibney and Maclagan also define a lower bound $L(\overline{M}_{0,n})$ of the Nef-cone of $\overline{M}_{0,n}$ by pulling back another cone of divisors from the toric variety X_Δ. In other words, there is a chain of cones

$$L(\overline{M}_{0,n}) \subset Nef(\overline{M}_{0,n}) \subset F(\overline{M}_{0,n}),$$

and we know that for $n \leq 6$ all cones are the same. So a natural question is whether these cones can be distinguished from one another, and whether one can work on the ambient toric variety rather than on the moduli space itself. It also begs the question of whether there is an embedding of $\overline{M}_{g,n}$ in a toric variety in such a way as to explain the F-cone on those spaces for $g > 0$.

Andrew Morrison "andrewmo@math.ubc.ca" I was wondering if the bernoulli numbers that come up in the Hodge intergrals have any geometric meaning, do they count something good? My geuss is that they are just the residues of the Todd class used somewhere...

Someone also said at one point that Poincare' duality is not know for the case of $\overline{M}_{g,n}$ bar. I suppose I am a bit unfamilliar with the basics of its cohomology.

David Steinberg "dsteinbe@math.ubc.ca" Quotients in one form or another are ubiquitous in geometry, they form a basic tool for creating new spaces out of old ones. Of particular interest is a quotient of a space by a group action; here the quotient space is the often the set of orbits of the action. In algebraic geometry, however, one gets into trouble using this definition. The GIT quotient is NOT the collection of orbits, but it is what the quotient "should" be in algebraic geometry. Since GIT quotients are fundamental to the construction of the moduli space of curves, I would be interested to know more about them, in particular: why does the set of orbits fail to be a good quotient in the algebraic category, what is a GIT quotient, and why is the GIT quotient the right quotient.

In the simplest case, the MNOP conjecture states that the GW invariants of a Calabi-Yau 3-fold are related to its Donaldson-Thomas invariants; in particular, a change of variables of the DT (reduced) partition function yields the GW partition function. It has been stressed that this equality after change of variables does not hold for a fixed homology class, that one must work with all the invariants at once in order to obtain the above relation. An explanation of why this is the case would be very interesting.

Dave Anderson "dandersn@umich.edu" I've got a few questions, mainly related to symmetric functions. I'm not sure this is the desired format, but there's a *little* mathematical content at least...

Are the symmetric functions $P_{g,n}(alpha)$ occuring in the ELSV formulas Schur-positive? Or are they positive with respect to other bases (elementary, homogeneous, Q, etc)? If not, do the expansions have any predictable signs or combinatorial meaning? Symmetric functions that come up in geometry often have some such positivity, so it would be nice to know about this. (E.g., polynomials that integrate positively when evaluated at Chern classes of ample vector bundles are Schur-positive.)

Are the "y-augmented" Schur functions Goulden introduced related to factorial Schur functions (aka multi-Schur functions, shifted Schur functions, double Schur functions, ...)? (Okounkov and Olshanski studied them under the name "shifted Schur functions"; also, they represent equivariant Schubert classes in Grassmannians.) They can't be the same exactly, but possibly one gets the factorial Schurs after a substitution in the y's, or summing Goulden's functions appropriately.

What are the betti numbers of the tautological ring – do they have a combinatorial description? What is the dual Hopf algebra of the stable cohomology of M_g? Is there some basis nicer than monomials in psi- or kappa-classes with respect to the Hopf algebra structure? (Motivating these questions: Is there a meaningful rough analogy between the stable tautological ring and the ring of symmetric functions?)

Michael Shapiro "mchapiro@gmail.com" 1. Generalize the description of walls of polynomiality chambers and wall crossing formula for double Hurwitz numbers from genusC 0 to positive genus.

(During the meeting Paul Johnson and Renzo Cavalieri suggested a method to approach the problem using tropical geometry that sounds very promising).

2. Is there any " r-ELSV formula"C for the moduli space of r- spin structures (Zvonkine'sC program)?

Gavril Farkas "farkas@mathematik.hu-berlin.de" A couple of questions on the algebraic-geometry side of M_g:

1) What is really κ_1? We do not known a single explicit example of a very ample divisor class on the coarse moduli space of curves. Write down an explicit ample class on M_g, that is, describe its zero section as a geometric locus in M_g.

2) Find a lower bound on the slopes of effective divisors on M_g. Show that such a bound is independent of g.

3) What is the genus of the smallest curve passing through a general point in M_g. One should expect this genus to be at least $\log(g)$ (assymptotically).

Jonathan Wise "jonathan@math.brown.edu" What can you say about Hurwitz numbers allowing specified branching at an arbitrary collection of branch points? What do orbifold techniques tell you? Does it make life easier or harder to put a stack structure at infinity instead of using relative stable maps?

To what extent can you study FTFTs in purely algebraic terms (i.e., without parameterizing boundary circles)? What are the sources of semisimple FTFTs? Are there any that do not come from Gromov–Witten theory?

Yuan-Pin LEE "yplee@math.utah.edu" Are there other applications of the powerful facts in topology of moduli of curves, like Harer stability and Madsen–Wiess' theorem, to GW theory, besides Teleman's result?

Conversely, are there any implications of Teleman's result to topology on moduli of curves?

(As discussed during the lecture, Teleman's result implies the semisimple GW classes in $\bar{M}_{g,n}$ are tautological.)

Barbara Fantechi "fantechi@ias.edu" Can one give an algebraic definition of the morphism $M_{g,1} \to M_{g+1,1}$ (up to homotopy)?

More precisely, consider the DM stack Y parametrizing morphisms from $\overline{M}_{g,1}$ to $\overline{M}_{g+1,1}$. The gluing morphism $\overline{M}_{g,1} \times \overline{M}_{1,2} \to V_{g+1,1}$ defines a morphism (indeed a closed embedding) $\overline{M}_{1,2} \to Y$. Let X be the connected component of Y containing the image of $\bar{M}_{1,2}$, and let U be the intersection of X with the open substack parametrizing maps which map $M_{g,1}$ to $M_{g+1,1}$. The question above should be answered positively if U is nonempty and connected.

Gueorgui T Todorov "todorov@math.utah.edu" I would like to hear about people's opinion on the behavior of GW invariants under a general birational modification of the target space.

Arend Bayer "bayer@math.utah.edu" In a recent preprint, Constantin Teleman proved a theorem reconstructing higher-genus Gromov-Witten invariants from finitely many genus-zero invariants in the case of semisimple small quantum multiplication. The proof uses various topological results and methods. It would be interesting to try to understand to what extent these methods can be formulated in a completely algebraic geometric setting.

Hsian-Hua Tseng "tseng@math.wisc.edu" Connectedness of moduli spaces of twisted stable maps.

To the best of my knowledge (possibly due to my lack of knowledge), I don't seem to know answers to the following questions.

Part A: Let G be a finite group, and let $\overline{M}_{g,n}(BG)$ be the stack of n-pointed genus g twisted stable maps to BG. We know that it is not connected.

(1) I'd like to know whether the components parametrizing maps with fixed stack structures at marked points are connected. This may be easy or outright false.

(2) More generally, describe the connected components of $\overline{M}_{g,n}(BG)$.

Part B: We can ask the same questions for the stack of twisted stable maps to some nice stacks, say weighted projective stacks.

Melissa Liu "ccliu@math.northwestern.edu" Questions for Jim Bryan: You mentioned that you have been studying orbifold Donaldson-Thomas theory.

(1) Do you have a statement of the crepant resolution conjecture in the Donaldson-Thomas theory (relating the orbifold DT theory of the orbifold to the DT theory of its crepant resolution)?

(2) Do you have a statement of the orbifoldGW/orbifoldDT correspondence?

(3) If the answer to (2) is yes, can one proves it for toric orbifolds of dimension 3?

Questions for Yunfeng Jiang and Hsian-Hua Tseng:

You have stated the Virasoro constraints for orbifolds. Do you know that Virasoro conjecture hold for BG? (Paul Johnson seemed to say so in his talk today ...) If yes, can you modify Givental's proof of the Virasoro conjecture for toric Fano manifolds to obtain a proof of the Virasoro conjecture for toric Fano orbifolds?

Paul Johnosn "pdjohnso@umich.edu" Some questions that I've come up with/thought about while I was here: there are the ones mentioned in the talk. The moduli space $M_{g,\gamma_1,\ldots,\gamma_n}(BG)$ has an obvious forgetful map to $M_{g,n}$, forgetting the principal G bundle, and we have a pretty good understanding of how this interacts with the tautological class. But we also have various maps to $M_{g'}$ that remember just the total space of the principal G bundle, or do a change-fiber construction to replace the $G-$torsor fibers of the fiber with some other G space. The push-forward of tautological classes via these maps seems rather open. The simplest cases are just the images themselves: is the closure of the space of curves with isotropy group G tautological? Or how about the closure of the space of degree d ramified covers, where the monodromy does not generate all of S_d, but just some subgroup?

(Carl Faber comments): there is an example (by Gaber-Pandharipande arXiv:math/0104057) of an algebraic locus in $\mathcal{M}_{2,22}$ that is not tautological. The locus that is degree two covers of curves in $\mathcal{M}_{1,12}$ simply ramified over the last two points.

Constructions of non tautological classes on moduli spaces of curves Authors: T. Graber, R. Pandharipande

Stacky ELSV is known for $\mathbb{Z}_r$ and it can be deduced for an abelian group. 10 **Question:** What about for a general G?

Given a representation $\rho : G \to \mathbb{C}$ is there a formula of the following type.

$$\int_{\mathcal{M}_{g,n}} \frac{\lambda_g^\rho - t\lambda_{g-1}^\rho + \cdots}{(1-\mu_1\psi_1)\cdots(1-\mu_n\psi_n)} = \text{characteristic theoretic formula.}$$

Balazs Szendroi "szendroi@maths.ox.ac.uk" I have come here to find out more about orbifold GW and its possible relations to orbifold DT, algebraic structures on cohomology of moduli space of curves, cohomology theory interpretation of the virtual class, and have been intrigued by wall crossing behaviour of double Hurwitz numbers.

I haven't any explicit problem in mind that would interest the others; as you know I have been intrigued by $[C^3/Z_3]$ for a while.

Charles Cadman "cadman@math.ubc.ca" I think it would be interesting to extend the ELSV formula in some way to higher genus targets. This could provide more interesting relations between Hodge integrals. The original ELSV formula, as well as the extensions of it, use ramified covers of P^1 to compute the relations. Since the target is P^1, it is possible to use localization to quickly obtain the formula. If the target is a higher genus curve, then localization would not work (at least not in any obvious way) and so one would need a different approach. It might be a matter of finding the right compactification of the space of smooth ramified covers, and using some kind of virtual excess intersection method to transform the degree of the branch map into an integral over something which lies in the boundary. I should note that the original ELSV approach used a non-standard compactification of the space of ramified covers of P^1, together with some delicate analysis of the boundary.

Greg Smith "ggsmith@mast.queensu.ca" Can one prove Faber's conjectural presentation of the tautological ring of $\mathcal{M}_g$? Is there a conjectural presentation of the tautological rings if $\mathcal{M}_{g,n}^{\mathrm{rt}}$, $\mathcal{M}_{g,n}^{\mathrm{ct}}$, or $\overline{\mathcal{M}}_{g,n}$?

For $\mathcal{M}_{g,n}(BG)$ or $\overline{\mathcal{M}}_{g,n}(BG)$ describe the connected components. For a fixed conjugacy class, how is this ramified. (Look at Hurwitz papers by Mike Freed et. al)

Is there an analog of Faber's conjecture for $\mathcal{M}_{g,n}(BG)$? How would one even define tautological classes in this case?

Note: Mumford's conjecture for $\mathcal{M}_{g,n}(BG)$ has recently been proved by Ralph Cohen and Soren Galatius.

Renzo Cavalieri "crenzo@umich.edu" Note that λ_g is the evaluation class for curves of compact type and $\lambda_g\lambda_{g-1}$ is the evaluation class for curves with rational tails. A similar statement seems possible for λ_i or $\lambda_g\lambda_i$ for varying i. Note that λ_i "kills" all curves whose dual graph has more than $g-i$ loops; similarly $\lambda_g\lambda_i$ "kills" all curves whose dual graphs have more than $g-i$ vertices of positive genus.

One would hope that a Faber-type statement be made for these loci, but perfect pairing fails already in genus 4 with the class $\lambda_4\lambda_2$, as well as with the class λ_3. One may try to recover Renzo's dream with classes other than λ_i or $\lambda_g\lambda_i$: for example, look at the homogeneous pieces of the polynomial $c(\mathbb{E})c(\mathbb{E})$.

Carel's suggestion for a little sisters: The class $\lambda_g\lambda_{g-1}\lambda_{g-2}$ is the class of a fixed curve C in $\mathcal{M}_g$, and it yields a Fulton-MacPherson space $C[n]$ in $\mathcal{M}_{g,n}$. What is the tautological ring of this space? One possible definition is to look at the subalgebra of $A^*(C[n])$ generated by diagonals (no κ classes!).

Another little sister: Look at the n-fold fiber product of C_g over M_g, denoted $(C_g^n)_{\mathcal{M}_g}$.

5 Conclusions

As organizers, we feel very satisfied with the outcome of the workshop. Our goal of "massaging" the usual structure of a research conference in order to increase the level of participation among participants seems to have been achieved. The mini-courses provided some solid reference points for all participants to focus on. The graduate student talks provided an excellent opportunity of interaction. In particular, during one if the graduate students talks, Carel Faber and Gabi Farkas identified a flaw in the student's thesis problem planned strategy. After the talk, Faber and Farkas discussed the issue at length with the student, and were able to suggest other more promising strategies to go about the same problem. It's needless to point out how valuable such an experience has been for the student. Having a good number of standard research talks helped avoiding "over-focusing", and gave the workshop a significant breadth. Several speakers, such as Cadman and Tseng, chose to give a survey talk on their research rather than focusing on a specific result. This turned out to be extremely pleasant and useful for many of the participants.

Several new collaborations were activated thanks to this workshop. Charles Cadman activated collaborations with Greg Smith and YP Lee, Tyler Jarvis with Dan Edidin; Paul Johnson and Renzo Cavalieri have much to discuss with Misha Shapiro.

We are pretty confident in saying that all participants were very satisfied with this workshop. Of course we organizers like to claim part of the credit for this, but it's no doubt that the amazing work enviroment provided by the Banff Center, and the efficient organization of BIRS were instrumental to such a success of this activity. We would also like to acknowledge the Clay mathematical institute, which provided us with a $10000 grant to subsidize travel expenses for graduate students.

6 Testimonials

Paul Johnson This conference has been by far the favorite I've attended. I've found that I've been doing a lot more math and, for lack of a better work, schmoozing, between talks. Maybe I'm just further along and starting to understand things and know people and have results, but I also think being here at Banff as a part of it: I'm used to everyone coming to a conference knowing who they wanted to talk to, and rushing off immediately when they have spare time to talk. Having all the meals together at the same place, and all staying at the same place and sharing the common space have made interacting a lot easier. A part of this could just be that I'm one of the people rushing off to talk to people now, but if that's the case it's wonderful that they're all in the same place to talk to.

Misha Shapiro I want to join Paul's email that it is really great conference!

Joro Todorov Thank you so much for a very enjoyable conference and also for the opportunity to give a talk.

Yunfeng Jiang It is my second time to attend conferences in Banff. The working and studying environment here is amazing and I really made some progress on my research.

The workshop is on the hot subject "Moduli of curves" in algebraic geometry in modern mathematics. It not only covers the Gromov-Witten side of the moduli of curves on the talks and discussions, on which I am working, but also contains the birational geometry of the moduli of curves, which is another important subject in algebraic geometry. I have learnt a lot from the talks.

At last, I think that it would be more fascinating if there were some talks on Donaldson-Thomas theory and Pandharipande-Thomas theory, which these theories also encode the enumerative geometry of curves in Calabi-Yau 3-folds.

Tyler Jarvis I just want to thank you all for an excellent conference. This was certainly one of the best mathematics conferences I have attended.

I learned a lot and was also able to meet some people that are likely to prove very helpful to my research. Specifically, I have begun a new collaboration with Dan Edidin that seems likely to produce some interesting results in the near future.