Research in photonics: Modeling, analysis, and optimization

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1 Overview of the Field

The focus of this research team is to study mathematical problems arising in photonics. Photonics is the science of manipulating light, including its generation, transmission, and processing. The phenomena in photonics are modeled by Maxwell's equations. Specifically, we are interested in the behavior of the solutions of Maxwell's equations in microstructured materials, where the microstructure is modeled by changes in material property.

One problem we considered has to do with efficient localization of light. The fundamental issue in this problem is a mathematical topic of resonances of wave equations. We are particularly interested in developing novel computational methods that directly approximate the resonances associated with the optical device.

Another problem of interest is the propagation of light in a micro-structured plasmonic material. These are made by creating periodic cells consisting of metallic and dielectric components. The mathematical issue that arises involves characterizing the dispersion relation associated with waves propagating in the structure.

2 Recent Developments and Open Problems

In the case of the resonance problem, the team has recently discovered numerical evidence for the dependence of the resonances in a 1-D Schroedinger equation on the width of the potential walls. One goal of the team is to develop a full mathematical understanding of the resonances and to devise an efficient general numerical method for solving for resonances that are based on theory.

Raman and Fan [1] performed calculations of dispersion curves of a 2-D plasmonic band-gap structure using a direct finite difference time-domain method. Their calculation revealed so-called plasmonic modes which concentrate energy in the neighborhood of the metal-dielectric interfaces. We set out to develop a mathematical understanding of this phenomenon as well as a theory for wave propagation in periodic structures with metallic components.

3 Scientific Progress Made

The team made some progress on a number of problems during the visit. The wonderful care-free atmosphere of BIRS contributed to the success of our visit.

1. **Behavior of resonances in a 1-D Schroedinger equation** We were able to make progress on understanding why when a true potential well is truncated at some distance L, the resonances are close to the bound states of the Schroedinger equation. Moreover, we began to investigate numerically the properties of the resonances for frequencies with large real parts. We were able to obtain asymptotics of these resonances.
2. **Dispersion curves of plasmonic band-gap structure** We spent some time discussing plasmonic phenomena in wave propagation through media with metals. We formulated a simple model problem from which we hope to gain fuller understanding. We plan to continue working on this problem.

4 Outcome of the Research in Team

Two projects resulted from the visit to BIRS. The first project has already yielded a manuscript about the behavior of resonances in a 1-D Schroedinger equation [2]. Several other questions are being investigated including the extension of the said result to the case of acoustic wave and Maxwell's equations and problems in multiple dimension.

The second project is an investigation of the dispersion curves of a layered plasmonic band-gap structure, and this is on going. Numerical results obtained during the visit are providing clues to the unusual behavior of waves in such structures.

We are also contemplating writing a joint proposal in this topic.

References

[1] A. Raman and S. Fan, Photonic band structure of dispersive metamaterials formulated as a Hermitian eigenvalue problem, *Phys. Rev. Letters* **104** (2010), 087401-1–4.

[2] D. Dobson, F. Santosa, S. Shipman, M. Weinstein, Resonances of a one-dimensional Schroedinger equation with a potential well of finite extent, *Preprint* (2011).