Sampling and Reconstruction: Applications and Advances 10w5122

Alireza Entezari (University of Florida),
Dimitri Van De Ville (University of Geneva & EPFL)

Nov 28 – Dec 3, 2010

The workshop gathered researchers with diverse backgrounds with an interest in sampling theory. In particular, we had electrical engineers, computer scientists and mathematicians who presented their work and discussed their research challenges throughout the event. The workshop also hosted a number of PhD students, post-doctoral researchers and junior faculty as well as established researchers in their respective disciplines.

1 Overview of the Field and Recent Developments

Sampling and reconstruction are pervasive in virtually all engineering applications and widely employed in the digital world. There has been tremendous interest and research in this area due to the instrumental role it plays in a wide range applications such as communications, signal and image processing, scientific computing, medical imaging, scientific visualization and computer graphics.

The problem of sampling and reconstruction has its roots in communication theory. The cornerstone is Shannon's celebrated result in sampling theory that showed the sufficient conditions for accurately reconstructing functions based on its samples using the *cardinal series*:

$$f(x) = \sum_{k=-\infty}^{\infty} f(k)\mathrm{sinc}(2\pi(x-k)) \tag{1}$$

where $\mathrm{sinc}(t) := \sin(t/2)/(t/2)$ and f is a *bandlimited* function. While Shannon is credited with this reconstruction and applying it in communication theory, this relationship has been the focus of interest to mathematicians like Nyquist, Whittaker, Borel and Kotelnikov.

Active research is ongoing to overcome the limitations of the signal representation of Eq. (1):

- While the traditional sampling theorem provides a guarantee for signals with frequencies upto the Nyquist rate, modern developments in sampling theory exploits sparsity to enable sampling at sub-Nyquist rates and efficient reconstruction. The Compressive Sensing (CS) and Finite Rate Of Innovation (FRI) paradigms precise exploit the sparsity (in Fourier or another transform domain) to enable such a sub-Nyquist rate sampling. Sampling on a uniform grid with (at least) the Nyquist sampling is still considered as the golden standard as it does not require sparsity assumptions. The research efforts in CS and FRI domains try to enable the transformation of these ideas into practical applications.

- The bandlimited assumption is not realistic for real-world signals; i.e., the typical measurement process integrates (e.g., over space or time) some physical property and thus corresponds to a non-ideal low-pass filter. Furthermore, bandlimitedness brings along badly localized basis function in the original domain. Alternative representations using spline-based shift-invariant spaces have received a tremendous amount of attention and are the key to high-quality representations.

- Sinc basis functions are badly localized, and therefore they do not render a efficient representation for common transient features in signals such as discontinuities (1-D) and edges (2-D, 3-D). Multiscale representations and wavelet transforms in particular can provide well-localized basis functions that represent well transients in signals. Some notable successes have been in coding (FBI finger prints, JPEG2000), denoising, and regularization.

- Multi-dimensional extensions have often been pursued using the separable approach, which applies the 1-D theory dimension-wise. Such an approach goes naturally with classical (multidimensional) Cartesian sampling lattices. Both theoretical and practical arguments indicate than non-Cartesian sampling schemes can lead to important advantages to represent the data.

2 Presentation Highlights

The talks in the workshop were organized in four tracks each being organized by a track-leader. The topics in the tracks were:

- Compressive Sensing and Finite Rate of Innovation Techniques which was lead by Yonina Eldar and Dimitri Van De Ville.
- Multiscale methods and wavelets which was lead by Dimitri Van De Ville and Ivan Selesnick .
- Biomedical and scientific image processing and visualization lead by Ross Whitaker and Alireza Entezari.
- Splines and subdivision modeling which was lead by Alireza Entezari and Jorg Peters.

Our speakers covered an impressive array of topics concerning recent developments in sampling theory, signal processing and various applications. In the following we report on the highlights of each presentation.

Speaker: **Akram Aldroubi** (Vanderbilt University)
Title: *Non-linear signal representations, subspace clustering and some applications*
Summary: There are new paradigms for signal representations that considers signals as elements of a union of subspaces in an ambient Hilbert space. This type of models is inherent is the theory of compressed sensing and signals with finite rates of innovation. Prototypical examples in which signals can be well modeled by a union of subspaces are the signals that are acquired from moving objects in video sequences and those acquired from facial views of a set human subjects in various positions and under various illuminations. Learning the models from the data allows us to track objects in video sequences and recognize faces from images. In this talk, we will explain how to learn the model from the data, and give some mathematical results showing the existence of optimal signal models. We then show how the models can be used to track moving objects in video sequences.

Speaker: **Usman Alim** (Simon Fraser University)
Title: *On the Solution of Poisson's Equation on a Rectangular Domain in a Shift-Invariant Space*
Summary: We propose a method for solving Poisson's equation with Dirichlet boundary conditions on a unit hypercube. Our solution lies in an approximation space generated by shifting a periodic reconstruction function to the nodes of an integration lattice that are within the unit hypercube. We obtain the relevant coefficients by orthogonally projecting the sampled Laplacian to the space generated by the shifts of the Laplacian of the periodic reconstruction function. This yields a solution methodology that can be efficiently implemented via the multi-dimensional discrete Sine Transform. Additionally, we analyze the L2 approximation error of our proposed scheme in the Fourier domain using a reformulation of the error kernel of Blu et al. Preliminary results on the Cartesian Cubic and the Body-centered Cubic lattices are also presented.

Speaker: **Chandra Bajaj** (University of Texas at Austin)
Title: *A Variational Approach for 3D Reconstruction in Electron Microscopy*

Summary: I shall present a variational approach for reconstructing a three dimensional density function from a set of two dimensional electron microscopy (EM) images. By minimizing an energy functional consisting of a data fidelity term and a choice of regularization terms, an L2-gradient flow is derived and efficiently solved. An iterative solution for the flow is constructed using tensor product B-spline finite elements in the spatial direction, and an explicit Euler scheme in the temporal direction. Experimental tests show that this variational method is efficient and effective and provides higher resolution EM reconstructions, compared to existing methods.
This is joint work with G. Xu, and M. Li of Academia Sinica, China.

Speaker: **Nat Beagley** (Pacific North West National Lab)
Title: *Some Challenges We Face: Sensor Detection at PNNL*
Summary: Pacific Northwest National Laboratory (PNNL) in Washington State is a multi-science national laboratory with a broad portfolio of sensor technologies aimed at detecting threats to national and personal security. From bio-forensics to explosives detection to radar meteorology for atmospheric science, I will give a description of some of our laboratories sensor capabilities and a few challenges we face that we hope to solve using sampling and reconstruction approaches. I will also describe how we plan to leverage recent research successes in data intensive computing and a new thrust aimed at developing methods to discover new signature of threat scenarios.

Speaker: **Rui M. Castro** (Eindhoven University of Technology)
Title: *Distilled Sensing: Adaptive Sequential Experimental Designs for Sparse Signal Recovery*
Summary: In the present work I will describe a novel sensing procedure for the estimation and detection of sparse signals. This procedure, called Distilled Sensing (DS), is a sequential and adaptive method for recovering sparse signals in noise. Non-adaptive (passive) sensing approaches, currently the most widespread data collection methods, involve non-adaptive data collection procedures that are completely specified before any data is observed. In contrast, DS collects data in a sequential and adaptive manner, using information gleaned from previous observations to guide the collection of future data. Often such procedures are known as sequential experimental designs or active sensing. The added flexibility of active sensing, together with a sparsity assumption, has the potential to enable extremely efficient and accurate inference. Specifically we show that certain weak, sparse patterns are imperceptible from passive measurements, but can be recovered perfectly using the proposed DS selective sensing procedure. This type of procedure can also be viewed as an adaptive experimental design approach to large-scale multiple hypothesis testing problems. Finally these same ideas can also be casted in the context of compressive sensing, where one can adaptively construct projective measurements in order to greatly enhance reconstruction performance and robustness to observation noise. This is joint work with Jarvis Haupt and Robert Nowak.

Speaker: **Hyun Sung Chang** (MIT)
Title: *Informative sensing*
Summary: Compressed sensing is a recent set of mathematical results showing that sparse signals can be reconstructed from a small number of linear measurements. Interestingly, for ideal sparse signals with no measurement noise, random measurements allow perfect reconstruction while measurements based on principal component analysis (PCA) or independent component analysis (ICA) do not. At the same time, for other signal and noise distributions, PCA and ICA can significantly outperform random projections in terms of enabling reconstruction from a small number of measurements. In this paper we ask: given the distribution of signals we wish to measure, what are the optimal set of linear projections for compressed sensing? We consider the problem of finding a small number of linear projections that are maximally informative about the signal. Formally, we use the InfoMax criterion and seek to maximize the mutual information between the signal x and the projection y=Wx. We show that in general the optimal projections are not the principal components of the data nor random projections, but rather a seemingly novel set of projections that capture what is still uncertain about the signal, given the knowledge of distribution. We present analytic solutions for certain special cases. In particular, for natural images, the near-optimal projections are bandwise random, i.e., incoherent to the sparse bases at a particular frequency band but with more weights on the low-frequencies, which has a physical relation to the multi-resolution representation of images.

Speaker: **Peter Chin** (Johns Hopkins University APL)
Title: *Application of Compressive Sensing to Cognitive Radio and Digital Holography*
Summary: One of the key aspects of cognitive radio is the concept of dynamic spectrum access, where a radio searches for a (temporarily) unused white space in order to transmit and receive its data. To enable such dynamic spectrum utilization, it is critical to detect the existence/absence of primary users, and furthermore understand the spectrum usage pattern of primary users. Currently, this is done by periodically switching the radio from the operation mode (transmit/receive) to sensing mode, during which the radio tries to sense for the existence of other (incumbent) signals. It is thus of utmost importance to make the sensing period as small as possible, which can not only increase the operational utilization, but also enable detection of more white spaces (including finer ones that can't be otherwise detected). We show that the nascent theory of compressive sensing, a revolutionary sampling paradigm which enables sampling at a much lower rate than the Nyquist rate by exploiting the sparsity of a given signal, can offer new opportunities for DSA. In particular, we show that compressive sensing theory is able to reduce significantly the amount of time and size of data that a cognitive radio needs in order to sense the existence of incumbent signals. Furthermore, We show that the recent progress on compressive sensing is also applicable and useful in a digital hololographic system, especially in that it greatly reduces the number of pixels (up to 80%) in hologram that is necessary to reconstruct the image without losing essential features of the image. This offers a path to sparsely sample the hologram and produce images with resolution comparable to the fully populated array, which in turn effects an holographic system to capture images at longer ranges using an array of sparsely populated smaller CCD arrays.

Speaker: **Laurent Condat** (ENSICAEN)
Title: *Evaluation and design of linear reconstruction methods with the frequency error kernel*
Summary: Linear reconstruction in a shift-invariant space of a function or its derivatives from uniform linear measurements is a classical operation in signal and image processing. Applications include all resampling tasks and gradient evaluation for volume rendering. In this general framework, we show that the frequency error kernel is a particularly useful tool to evaluate and design reconstruction schemes.

Speaker: **Mike Davies** (University of Edinburgh)
Title: *Compressive Power Spectral Density Estimation*
Summary: We consider the power spectral density estimation of bandlimited, wide-sense stationary signals from sub-Nyquist sampled data. This problem has recently received attention within the emerging field of cognitive radio for example, and solutions have been proposed that use ideas from compressed sensing as well as the theory of digital alias-free signal processing. Here we develop a compressed sensing based technique that employs multi-coset sampling and produces power spectral estimates at arbitrarily low average sampling rates. The technique applies to spectrally sparse and non-sparse signals alike, but we show that when the wide sense stationary signal is spectrally sparse, compressed sensing is able to enhance the estimator. The estimator does not require signal reconstruction and can be directly obtained from a straightforward application of nonnegative least squares.
This is joint work with Michael Lexa and John Thompson

Speaker: **Laurent Demanet** (MIT)
Title: *Fitting matrices from applications to random vectors, with some help from multiscale analysis*
Summary: What can be determined about the pseudoinverse A^{-1} of a matrix A from one application of A to a vector of random entries? If the n-by-n pseudoinverse A^{-1} belongs to a specified linear subspace of dimension p, then come to the talk to hear which assumptions on this subspace, its dimension p, and the size n guarantee an accurate recovery of A^{-1} with high probability. Multiscale directional techniques – curvelets in particular – enter the picture to give an a priori description of the null-space of the matrix A. This randomized fitting method provides a compelling preconditioner for the wave-equation Hessian (normal operator) in seismic imaging. Joint work with Pierre-David Letourneau (Stanford) and Jiawei Chiu (MIT).

Speaker: **Edward Di Bella** (University of Utah)
Title: *Sampling and Reconstruction for Cardiac MRI*
Summary: MRI is a very interesting application in part due to the flexibility of sampling Fourier space.

Recent advances in terms of different sampling strategies in Fourier space, along with multi-coil acquisitions and reconstruction methods using constrained reconstruction and compressed sensing are having an impact on what can be done with MRI. I will introduce and show examples of ways that MRI is benefitting from these advances. Cardiac MRI in particular will be emphasized as it is widely used for a number of very important clinical assessments, as well as to track response to therapies. I will also discuss how we have adapted single-input multiple output blind estimation methods for the quantification of cardiac perfusion.

Speaker: **Yonina Eldar** (Technion, Israel Institute of Technology)
Title: *Structured Compressed Sensing: From Theory to Hardware*
Summary: Compressed sensing (CS) is an exciting and emerging field that has attracted considerable research interest over the past few years. Much of the original work in this area was limited to standard discrete-to-discrete measurement architectures using matrices of randomized nature and discrete signal models based on standard sparsity. In recent years, CS has worked its way into several exciting new application areas. This, in turn, necessitates a fresh look on many of the basics of CS. The random matrix measurement operator must be replaced by more structured measurement systems that can be implemented efficiently in hardware. The standard sparsity prior has to be extended to include a much richer class of signals including continuous-time signals to allows sub-Nyquist sampling. In this talk, we discuss some of these extensions as well as recent applications including ultrasound imaging, sub-Nyquist analog-to-digital converters, subwavelength imaging, time delay estimation, source separation, and superresolution radar.

Speaker: **Alireza Entezari** (University of Florida)
Title: *Voronoi Splines and Shift-Invariant Spaces For Multivariate Approximation*
Summary: Non-Cartesian lattices have been studied in the context of sampling theory for efficient spectral packing. In particular, sphere packing and sphere covering lattices have been shown to outperform the sampling efficiency of the Cartesian lattice in multi-dimensions. However, the reconstruction problem on non-separable lattices has not been well-studied. We present a multivariate (non-separable) spline construction that generate a lattice-shift-invariant spline space for approximation. Experiments are presented based on common 3-D lattices for sampling and reconstruction of volumetric data. We finally present some computational challenges for dealing with this class of splines that can be considered as a generalization of uniform B-splines to the multivariate setting.

Speaker: **Jalal Fadili** (CNRS-ENSICAEN)
Title: *Block Stein sparse restoration*
Summary: In this work, we propose fast image denoising and deconvolution algorithms that use adaptive block thresholding in sparse representation domain. Our main theoretical result investigates the minimax rates of Stein block thresholding in any dimension d over the so-called decomposition spaces. These smoothness spaces cover the classical case of Besov spaces for which wavelets are known to provide a sparse representation, as well as smoothness spaces corresponding to the second generation curvelet-type construction. We shows that block estimator can achieve the optimal minimax rate, or is at least nearly minimax (up to a log factor) in the least favorable situation. The choice of the threshold parameter is theoretically discussed and its optimal value is stated for the white Gaussian noise. We provide simple, fast and easy to implement algorithms. We also report a comprehensive simulation study to support our theoretical findings. The practical performance of our block Stein denoising and deconvolution algorithms compares very favorably to state-of-the art algorithms on a large set of test images.

Speaker: **Bart Goossens** (Ghent University)
Title: *Solving Realistic Image Restoration and Reconstruction Problems Through Shearlet-based L1-Regularization*
Summary: Image restoration techniques recover original images from observed degraded images. Very often, only a subset of the data is observed and then a reconstruction technique is needed to obtain the original image. In general, the recovery is a challenging task due to the ill-posedness of the inversion problem. Many techniques therefore rely on regularization to obtain an acceptable solution. The regularization is usually applied in a transform domain, in which the images have a sparse representation. In this work, we investigate a few realistic restoration and reconstruction problems, such as the removal of signal-dependent noise from images, image demosaicing and parallel MRI reconstruction. We use the Shearlet transform, which provides

a multiresolution analysis (such as the wavelet transform) and is at the same time an optimally sparse image-independent representation for images that are smooth away from discontinuities along curves. We consider L1-regularization in the discrete Shearlet domain and we adopt split-Bregman algorithms (which is in case of linear transforms also known as split-augmented Lagrangian shrinkage algorithms, SALSA) to solve the inversion problem. We will show that this approach allows for a lot of flexibility in realistic applications, as the framework inherently decouples the image model from the degradation model. Results show that our specific design of the discrete shearlet transform thereby offers a vast performance improvement compared to related transforms, both in visual quality and in PSNR.

Speaker: **Bin Han** (University of Alberta)
Title: *Refinable Function Vectors with Interpolation Property*
Summary: Shift invariant spaces generated by refinable function vectors are of interest in sampling theorems, numerical algorithms, and construction of balanced multiwavelets. In this talk, we shall discuss a particular family of refinable function vectors with interpolation property. Such interpolating refinable function vectors include many known families of refinable function vectors in the literature as special cases and generalize several results of I. W. Selesnick and D.-X. Zhou in the setting of sampling theorems. We present a complete characterization of such interpolating refinable function vectors as well as a general procedure for constructing such interpolating refinable functions. Some univariate and bivariate examples are presented to illustrate the results and the construction. This is joint work with Xiaosheng Zhuang.

Speaker: **Ali Hormati** (EPFL)
Title: *Multi-channel finite rate of innovation sampling: algorithms and performance bounds*
Summary: We consider the finite rate of innovation sampling scheme in a sparse common-support scenario (SCS-FRI), in which input signals contain Diracs with common locations but arbitrary weights. We first derive the best theoretical performance for the SCS-FRI setup using the Cramér-Rao lower bound. Our results show that for a set of well-separated Diracs, the theoretical lower bound for each common position is a function of the total power of the Diracs at that particular location and therefore, the bound is not perturbed by the choice of power allocation scheme among the channels. We then propose a multi-channel reconstruction algorithm and compare its performance with the Cramér-Rao lower bound. Our numerical results clearly demonstrate the effectiveness of the proposed multi-channel reconstruction scheme in low SNR regimes.

Speaker: **Tao Hu** (Howard Hughes Medical Institute)
Title: *Super-resolution Reconstruction of Brain Structure Using Sparse Representation over Learned Dictionary*
Summary: A central problem in neuroscience is reconstructing neuronal circuits on the synapse level. Large range of scales in brain architecture requires imaging that is both high-resolution and high-throughput. Existing electron microscopy (EM) techniques possess required resolution in the lateral plane and either high-throughput or high depth resolution but not both. Here, we exploit recent advances in unsupervised learning and signal processing to obtain high depth resolution EM images computationally without sacrificing throughput. First, we show that the brain tissue can be represented as sparse linear combination of localized basis functions that are learned using high-resolution datasets. We then develop reconstruction techniques inspired by compressive sensing that can reconstruct the brain tissue from very few (typically 5) tomographic views of each section. This enables tracing of neuronal processes and, hence, high throughput reconstruction of neural circuits to the level of individual synapses.

Speaker: **Aleksandar Ignjatovic** (UNSW)
Title: *Some signal processing applications of numerically robust differential operators (chromatic derivatives)*
Summary: Chromatic derivatives are linear differential operators with constant coefficients which correspond to certain families of orthogonal polynomials. Their action on a band limited signal can be approximated in a very accurate and noise robust way using samples of the signal taken at a rate somewhat higher than the usual Nyquist rate. We show how one can use such operators for signal interpolation and for frequency estimation of several sinusoids in the presence of noise which is not necessarily white. The methodology is intended to enhance the standard, global signal processing methodology based on Nyquist rate sampling, by combining it with local signal processing methods based on chromatic derivatives.

Speaker: **Anna Jezierska** (Université Paris-Est)
Title: *Multichannel image quantization under spatial smoothness constraints*
Summary: Quantization, defined as the act of attributing a finite number of levels to an image, is an essential task in image acquisition and coding. It is also intricately linked to essential image analysis tasks, such as denoising and segmentation. We investigate vector quantization combined with regularity constraints, a little-studied area that is of interest, in particular, when quantizing in the presence of noise or other acquisition artifacts. We present an optimization approach to the problem involving a novel two-step, iterative, flexible, joint quantizing-regularization method featuring both convex and combinatorial optimization techniques. We show that for both color and grey-scale images, our approach may yield better quality images in terms of SNR, with lower entropy, than conventional optimal quantization methods.

Speaker: **Qingtang Jiang** (University of Missouri - St. Louis)
Title: *Highly Symmetric Affine Frames for Surface Multiresolution Processing*
Summary: In this talk we will discuss the construction of highly symmetric biorthogonal affine frames for surface multiresolution processing. A (high resolution) triangle or quad surface to be processed has in general an arbitrary topology, namely, it consists of not only regular vertices but also extraordinary vertices. To process such a surface, the multiresolution algorithms for the regular vertices must have high symmetry so that one can design the corresponding algorithms for the extraordinary vertices. We consider the construction of bi-frames with 6-fold and 4-fold axial symmetry. These frames result in multiresolution algorithms with desired symmetry for triangle and quad surface processing. The multiresolution algorithms are given by templates (stencils) so that it is easy to implement them for surface processing applications such as surface progressive transmission, surface sparse representation and surface shrinkage. In this talk we will also discuss the correspondence between frame shrinkage and high order nonlinear diffusion.

Speaker: **Minho Kim** (University of Seoul)
Title: *Symmetric Box-Splines on Root Lattices*
Summary: Root lattices are efficient sampling lattices for reconstructing isotropic signals in arbitrary dimensions, due to their highly symmetric structure. One root lattice, the Cartesian grid, is almost exclusively used since it matches the coordinate grid; but it is less efficient than other root lattices. Box-splines, on the other hand, generalize tensor-product B-splines by allowing non-Cartesian directions. They provide, in any number of dimensions, higher-order reconstructions of fields, often of higher efficiency than tensored B-splines. But on non-Cartesian lattices, such as the BCC (Body-Centered Cubic) or the FCC (Face-Centered Cubic) lattice, only some box-splines and then only up to dimension three have been investigated. In this talk, I present efficient symmetric box-spline reconstruction filters on all irreducible root lattices that exist in any number of dimensions $n >= 2$ ($n >= 3$ for D_n and D_n^* lattices). In all cases, box-splines are constructed by convolution using the lattice directions, generalizing the known constructions in two and three variables. For each box-spline, we document the basic properties for computational use: the polynomial degree, the continuity, the linear independence of shifts on the lattice and optimal quasi-interpolants for fast approximation of fields.

Speaker: **Demetrio Labate** (University of Houston)
Title: *Analysis and detection of multidimensional singularities using the continuous shearlet transform*
Summary: It is well known that the continuous wavelet transform is able to identify the singular support of functions and distributions through its decay at fine scales. However, it lack the ability to provide additional information about the geometry of the singularity set. In this talk, we show that the shearlet transform, a recently introduced directional multiscale transform, is able to overcome the limitations of the traditional wavelet approach in dealing with geometric features. In particular, we show that the shearlet approach provides a precise characterization of edge discontinuities for planar regions, including corner points, and that it can be generalized to deal with general singularities in higher dimensions. This investigation is motivated by applications in edge detection and feature extraction for 2-D and 3-D data.

Speaker: **Kieran G. Larkin** (Canon Information Systems Research Australia (CiSRA))
Title: *Multiplicative Decomposition in 2-D as a Prototype for Irregular Sampling: Application to the Extreme Compression of Fingerprints*
Summary: Much has been written over the last decade on the sparse sampling theory of additive decompositions of signals and images. We instead consider the less well-known multiplicative decompositions

and concentrate on two dimensions. It has become apparent that certain classes of images have extremely sparse representations in the modulation (or multiplicative) domain. Although these images are not directly amenable to sparse sampling strategies such as random projection, they do have explicit sparse sample structures. The structure is most apparent as a classic Hadamard-Weierstrass-Osgood product in the setting of a complex function of several complex variables.

Images of human fingerprints exhibit a fringe structure which can be closely modelled by 2-D phase and amplitude modulation. By defining an isotropic quadrature image it is possible to create a complex modulation image which naturally decomposes as the product of complex basis functions. Conveniently the basis functions coincide with the unique Helmholtz-Hodge decomposition of the phase into two parts: the smooth and the spiral phases. The decomposition permits extreme parsimony in the coding of the image. For example, a fingerprint image can be encoded by the bytes equivalent to a few lines of pixels, so the compression reduces the image complexity to just one dimension! Although the decomposition is quite tractable, it involves a sequence of unfamiliar processing steps. In contrast, the reconstruction from irregularly spaced bases is simple and direct.

Speaker: **Peter Lindstrom** (Lawrence Livermore National Laboratory)
Title: *Topological Analysis of High-Dimensional Functions*
Peter Lindstrom talked about current challenges in topological analysis of high dimensional functions. In particular the topology extraction methods using k-nearest neighbor (KNN) graphs were presented and their shortcomings were discussed. Moreover the advantages of empty region graphs were discussed for extraction of topological information for sparsely sampled high dimensional functions.

Speaker: **Olgica Milenkovic** (University of Illinois, Urbana-Champaign)
Title: *Single-Sample Sequential Compressive Sensing*
Summary: We consider the problem of non-linear filtering of sparse signals with sparse innovations which falls into the general framework of sequential compressive sensing. We show that the signal can be tracked and locked on using only one observation sample per step via a simple modification of the subspace pursuit algorithm. This mitigates the problem of known filtering approaches, which require that the set of sampling matrices obey the restrictive isometry property. We also compare our approach to unscented Kalman filtering and sequential Monte-Carlo simulation methods and show that the subspace pursuit-based method outperforms the aforementioned standard techniques both in terms of complexity and tracking accuracy.
This is a joint work with Wei Dai, University of Illinois, Urbana-Champaign, and Dino Sejdinovic, University of Bristol.

Speaker: **Karl Ni** (MIT)
Title: *Multi-resolution Representation of Fused 3-Dimensional Dataspaces*
Summary: The increasing collection capability of imaging sensors from 3D modalities exposes a need to fuse various and diverse data sets in a scalable, efficient, and compressible manner. Exploitation of such a multi-modal fusion effort calls for a compact representation. While algorithms to store and transport 2D (image) and 2D + time (video) media have been standardized, extensions to 3D data spaces scale exponentially and are nontrivial. Spanning sparse LADAR collections to dense volumes in tomographic data sets, issues that arise in multi-resolution 3D include complexity, parallelization, ease in access, multi-layer storage sizes, quick and incremental rendering, and compression ratio performance. We observe the issues and ideas that arise in hierarchically incorporating multi-modal 3D image data into a single voxel space. We discuss the hurdles that the community faces in the overall representation itself. Finally, we explore the exploitation potential in a proposed testbed that investigates desired attributes of a multi-resolution 3D data space.

Speaker: **Jorg Peters** (University of Florida)
Title: *Splines on manifolds?*
Summary: This talk discusses the challenge and a promising approach to defining smooth tensor-product B-spline functions on manifolds of genus other than 1. Such splines should be useful both for sampling and reconstructing data on manifolds and also for representing the manifolds.

Speaker: **Javier Portilla** (CSIC)
Title: *A unified Bayesian approach for L0-based estimation using Parseval frames*

Summary: Sparsity (in the strict sense of the term)-based techniques have emerged during the last years as an efficient alternative to more classical approaches to signal restoration. Whereas the typical involved cost function optimizations can be naturally interpreted as maximum a posteriori (MAP) Bayesian estimation, the direct optimization in a general case is not easy, especially in the L0 pseudonorm case. In this talk we present a convenient (non-MAP) Bayesian formulation which, through using a hidden sparse vector and a hidden Gaussian correction term, translates into an efficient estimation, especially for the Analysis-based Sparsity (AbS) case. In addition, by using a dynamic weight for the contribution of data and prior to the cost function we can drastically reduce computation. We will show some examples of very fast (0.1 s. for 256x256) high quality image deblurring.

Speaker: **Holger Rauhut** (University of Bonn)
Title: *Compressive sensing with partial random circulant matrices*
Summary: While Gaussian random matrices are known to provide optimal sampling matrices for compressive sensing, they are of limited use for practical problems due to lack of structure. Subsampled convolutions appear in a number of applications, and they are described by partial circulant matrices. We study such matrices generated by random vectors in connection with compressive sensing. We present a near-optimal non-uniform recovery result for l1-minimization, and an estimate of the restricted isometry constant of such partial random circulant matrices.

Speaker: **Naoki Saito** (UC Davis)
Title: *Harmonic/Wavelet Analysis on Graphs and Networks with Applications*
Summary: More and more data are collected in a distributed and irregular manner. They are not organized such as familiar digital signals and images sampled on regular lattices. Examples include from sensor networks, social networks, web-pages, biological networks, and so on. Moreover, constructing a graph from a usual signal or image and analyzing it can also be very useful (e.g., the nonlocal mean denoising algorithm of Buades, Coll, and Morel). Hence, it is very important to transfer harmonic and wavelet analysis techniques originally developed on the usual Euclidean spaces to graphs and networks. In this talk, I will mainly discuss my own work with Ernest Woei on analysis of the eigenvalue distribution of the Laplacian of graphs representing neuronal dendritic trees and the surprising behavior of the eigenfunctions: some of them are global oscillations like Fourier modes and the others are localized wiggles like wavelets. If time permits, I will also review some of the very recent works by: 1) Hammond, Vandergheynst, and Gribonval on constructing wavelets from graph Laplacians; and 2) Coifman, Gavish, and Nadler on tensor-product Haar-like analysis of digital databases.

Speaker: **Ivan Selesnick** (Polytechnic University)
Title: *Signal restoration using non-dyadic wavelet transforms with tunable Q-factors*
Summary: In signal restoration using sparse wavelet representations, the Q-factor of the wavelet transform should, in principle, be chosen so as to match the resonance properties of the signal to be restored. A 'high-resonance' signal is one in which oscillations are sustained, while a 'low-resonance' signal is one comprised mostly of non-oscillatory transients. This talk describes a new family of non-dyadic wavelet transform where the Q-factor is continuously tunable and its application to signal restoration. The transform is fully discrete, self inverting (with the exact perfect reconstruction property), modestly overcomplete, and can be implemented with radix-2 FFTs.

Speaker: **Jorge Silva** (Duke University)
Title: *Dictionary Learning for One–Block–Sparse Signals with Compressive Measurements*
Summary: The problem of simultaneous signal recovery and dictionary learning based on compressive measurements with multiple sensing matrices is addressed, under the assumption of structured sparsity. In particular, it is assumed that the unknown signals of interest obey one-block sparsity. This problem is important, for instance, in image inpainting applications. This work constitutes an extension of Dictionary Learning (DL) and Block-Sparse Dictionary Optimization by considering compressive measurements; previous work on Blind Compressed Sensing (Blind CS) is also extended by considering multiple sensing matrices and relaxing some of the restrictions on the learned dictionary. It is proven that, under one-block sparsity and other conditions, and with high probability, the dictionary and signals can be uniquely recovered up to block permutations and certain linear transformations of the dictionary atoms, albeit with high computational effort.

The recovery is contingent on the number of measurements per signal and the number of signals being sufficiently large; bounds are derived for these quantities. In addition, a computationally practical algorithm that performs dictionary learning and signal recovery is presented, and conditions for its convergence to a local optimum are found. Experimental results with image inpainting demonstrate the capabilities of the algorithm.

Speaker: **Antonio Tristan Vega** (Harvard Medical School)
Title: *Diffusion MRI: State of the art in sampling and reconstruction of the diffusion propagators*
Summary: Diffusion MRI aims to infer the macroscopic properties of nerve fibers through the characterization of the diffusion process of the water molecules restricted by neural axons. More concretely, the challenge is to describe the 3-D Probability Density Function (PDF) of the displacements of water molecules driven by restricted Brownian motion, the so-called Diffusion Propagator (DP). It is known that the signal acquired by the MRI scanner can be modeled as the 3-D Fourier transform of such DP, so the most direct approach consists of sampling the whole 3-D Fourier domain and then numerically compute the Fourier Transform. Unfortunately, this requires a dense sampling which is not feasible in clinical times. Diffusion Tensor (DT) MRI works around this pitfall by assuming a zero-mean Gaussian PDF, so that only the six degrees of freedom of its covariance matrix (the DT) have to be estimated from six or more samples in the transformed domain. The Gaussian model turns out to be quite limited to describe certain neural anatomies, which has driven the appearance of new general models for diffusion. Such models were originally designed to be determined by a regular sampling in the transformed domain on the surface of a sphere of given radius q. With the advent of new machinery and acquisition protocols, new schemes based on arbitrary sparse samplings of the 3-D transformed space are now possible, so the problem statement reads: given the hard restriction on the number of available samples, imposed by scanning times, what is the best way to sample and reconstruct the DP, and how good are the results we can expect?

Speaker: **Michael Unser** (EPFL)
Title: *FUN-SP: A functional framework for sparse, non-Gaussian signal processing*
Summary: We introduce an extended family of continuous-domain stochastic models for sparse, piecewise-smooth signals. These are specified as solutions of stochastic differential equations, or, equivalently, in terms of a suitable innovation model; this is analogous conceptually to the classical interpretation of a Gaussian stationary process as filtered white noise. The non-standard aspect is that the models are driven by non-Gaussian noise (impulsive Poisson or alpha-stable) and that the class of admissible whitening operators is considerably larger than what is allowed in the conventional theory of stationary processes. We provide a complete distributional characterization of these processes. We also introduce signals that are the non-Gaussian (sparse) counterpart of fractional Brownian motion; they are non-stationary and have the same 1/f-type spectral signature. We prove that our generalized processes have a sparse representation in a wavelet-like basis subject to some mild matching condition. Finally, we discuss implications for sampling and sparse signal recovery.

Speaker: **Dimitri Van De Ville** (EPFL - HUG)
Title: *Analytic Sensing: Localization of (Sparse) Sources using Finite Rate of Innovation*
Summary: I will present analytic sensing as a new EEG source imaging technique for multiple dipole localization. This method is based on two main concepts. First, the sensing principle relates the boundary measurements to the sources within the volume. Second, the annihilation principle deploys recent advances in signal processing known as “finite rate of innovation” to find the dipoles’ positions using a non-iterative procedure. I will show multiple extensions to this framework for denoising, dealing with multi-layer spherical head models, and local application of analytic sensing. (This is joint work with Djano Kandaswamy and Prof. Thierry Blu)

Speaker: **Joe Warren** (Rice University)
Title: *Harmonic B-splines and beyond*
Summary: Univariate B-splines are the workhorse of approximation theory in one variable. Many attempts have been made to generalize B-splines to two dimensions with varying levels of success. Interpolation via radial basis functions (especially using biharmonics) is one particularly fruitful area. However, previous efforts here while yielding useful interpolants have not replicated the elegant theory that underlies univariate B-splines.

In this talk, I will describe an approach to approximation in the plane with harmonic functions that replicates much of the theory of univariate B-splines including partition of unity, localized support and knot insertion. I will conclude with some remarks on the challenges of extending this approach to the more practical case of approximation using biharmonic functions and demonstrate some preliminary results.

Speaker: **Ross Whitaker** (University of Utah)
Title: *On the Manifold Structure of the Space of Brain Images*
Summary: This talk describes a set of technologies for manifold learning and several new developments that make manifold learning more useful and practical for analysis of large datasets. We present several examples and applications of manifold learning in data analysis. Finally, we present a more comprehensive study of using manifolds to model the variability of shapes in a relatively large set of MRI brain images. These results include comparisons against linear models and evaluations in the prediction of clinical parameters (in collaboration with S. Gerber).

Speaker: **Rebecca Willett** (Duke University)
Title: *Compressed Sensing with Poisson Noise*
Summary: Compressed sensing has profound implications for the design of new imaging and network systems, particularly when physical and economic limitations require that these systems be as small and inexpensive as possible. However, several aspects of compressed sensing theory are inapplicable to real-world systems in which noise is signal-dependent and unbounded. In this work we discuss some of the key theoretical challenges associated with the application of compressed sensing to practical hardware systems and develop performance bounds for compressed sensing in the presence of Poisson noise. We develop two novel sensing paradigms, based on either pseudo-random dense sensing matrices or expander graphs, which satisfy physical feasibility constraints. In these settings, as the overall intensity of the underlying signal increases, an upper bound on the reconstruction error decays at an appropriate rate (depending on the compressibility of the signal), but for a fixed signal intensity, the error bound actually grows with the number of measurements or sensors. This surprising fact is both proved theoretically and justified based on physical intuition.

Speaker: **Ahmed I. Zayed** (DePaul University)
Title: *Sampling Spaces in the Fractional Fourier Transform Domain*
Summary: Shift-invariant spaces play an important role in sampling theory, multiresolution analysis, and many other areas of signal and image processing. A special class of the shift-invariant spaces is the class of sampling spaces in which signals are determined by their values on a discrete set of points. In this talk we extend some of these notions to the fractional Fourier transform domain. First, we introduce two definitions of the discrete fractional Fourier transform and two semi-discrete fractional convolutions associated with them. We employ these definitions to derive necessary and sufficient conditions pertaining to the fractional Fourier transform domain, under which integer shifts of a function form an orthogonal basis or a Riesz basis for a shift-invariant space.

We also introduce the fractional Zak transform, which is an extension of the Zak transform, and derive two different versions of the Poisson summation formula for the fractional Fourier transform. These extensions are used to obtain new results concerning sampling spaces.

3 Outcome of the Meeting

The meeting was considered by many (if not all) of our participants a success. The most frequent comment that was echoed by a large number of participants was that the mix of folks from CS, EE and Math brought a multi-disciplinary flavor to the meeting that was appreciated by many. The special events on the state of compressive sensing, finite rate of innovation, sparse signal processing, multi-scale methods and harmonic analysis on graphs brought about discussions on the challenges that need to be tackled before these emerging theories can actually impact applications. As a result of these discussions, some folks managed to start new collaborations. Some identified research interest in hosting institutions for their sabbatical. A number of folks who were introduced to the BIRS for the first time decided to write proposals to BIRS for other workshops.