Probabilistic and Statistical Properties of Stochastic Volatility Models

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1 Overview of the Field

One of the standardized features of financial data is that log-returns of stock prices are uncorrelated and possibly heavy tailed, but their squares, or absolute values, are (highly) correlated. Furthermore, data exhibit heteroskedasticity and leverage. In the financial time series context, leverage is understood to mean negative dependence between previous returns and future volatility (i.e. a large negative return will be followed by a high volatility).

To model such phenomena, Robert Engle introduced the ARCH (AutoRegressive Conditionally Heteroscedastic) family, further extended to G(eneralized)ARCH. His work was rewarded with the Nobel Prize in Economics (2003). This model is now well understood but fails to capture the long memory property of squares. However, recent applications are focused on models with long memory in squared log-returns. Therefore we deal in our project with models of the form

$$Y_i = \sigma(X_i)Z_i \ , \qquad i \geq 1 \ ,$$

where Z_i are independent, identically distributed random variables, and X_i is a long memory process with a memory parameter (Hurst parameter) H. If X_i, $i \geq 1$, and Z_i, $i \geq 1$, are independent, then the model is called the Long Memory in Stochastic Volatility (LMSV) process. In general, there is no independence assumption and the models are referred to as the Stochastic Volatility with Leverage process.

In order to model the high variability of financial data, it is assumed that Z_i are in the domain of attraction of the Fréchet law, i.e. as $x \to \infty$,

$$P(|Z| > x) = x^{-\alpha}L(x),$$

where $L(\cdot)$ is a function that is slowly varying at infinity.

Although models such as the LMSV process introduced above, and also EGARCH, LARCH, FIGARCH processes, are widely used in practice, there is little understanding of their probabilistic structure, especially if heavy tails are involved, whereas, in contrast, there is a wealth of results on partial sums, sample covariances or parameter estimation for linear processes with finite or infinite moments, with short or long memory. There is also a number of publications on ARCH-type models, which exclude long memory.

2 Recent Developments and Open Problems

Tools used in this area involve the deep theory of dependent sequences, point process techniques and time series analysis. Davis and Mikosch (2001) established point process convergence for the stochastic volatility process without long memory. This convergence yields the convergence of partial sums and sample covariances. Surgailis and Viano (2002), using specific techniques of strongly dependent time series, dealt with partial sums in a special case of stochastic volatility model with leverage and finite variance. Surgailis (2008) studied partial sums of LARCH(∞) processes with heavy tails.

As for statistical inference, Hurvich et al. (2005) studied estimation of the memory parameter, whereas Kulik and Soulier (2011) established limit theorems for the tail empirical process based on heavy tailed LMSV models. As a consequence, they obtained the asymptotic normality of the Hill estimator of the tail index of the marginal distribution.

However, to the best of our knowledge, there are no results for sample covariances, in the case of stochastic volatility models with both long memory and heavy tails, and possible leverage. The only available result on partial sums is the one established in Surgailis (2008).

3 Scientific Progress Made

During the meeting we focused on two topics, finishing the paper entitled *Limit theorems for long memory stochastic volatility models with infinite variance: Partial Sums and Sample Covariances*.

We established limit theorems for partial sums and sample covariances for stochastic volatility models with long memory, heavy tails and possible leverage. The basic idea is to decompose these statistics into a martingale part and a long memory part. Convergence of the first part is treated using the point process methodology. As for the long memory part, we apply existing tools available for subordinated long memory processes. In due course, however, one has to establish many new technical results, which are of independent interest.

Interestingly, although the leverage does not have any effect on the asymptotic behaviour of partial sums, it may have a significant effect on the sample covariances. The rates of convergence and limiting distributions may differ in a non trivial way between the models with or without leverage.

4 Outcome of the Meeting

Based on the scientific progress described in the previous section, we were able to finish our paper on sample covariances for stochastic volatility models, see Kulik and Soulier (2011b). The paper has been submitted to *Advances in Applied Probability* and posted on arxiv, `arXiv:1109.5298`.

As mentioned above, the presence of leverage is a significant property of the data. From a theoretical point of view, it has an effect on the asymptotic behaviour of certain statistics. Hence, during the meeting, we started to investigate tests for leverage effects based on sample covariances-type statistics.

5 Note

The participants would like to thank BIRS for hospitality. It was for both of us a great opportunity to focus on research for the entire week.

References

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