Foundation of Stochastic Analysis

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1 Scientific agenda of the conference

Over the years, the foundations of stochastic analysis included various specific topics, such as the general theory of Markov processes, the general theory of stochastic integration, the theory of martingales, Malliavin calculus, the martingale-problem approach to Markov processes, and the Dirichlet form approach to Markov processes. To create some focus for the very broad topic of the conference, we chose a few areas of concentration, including

- Dirichlet forms
- Analysis on fractals and percolation clusters
- Jump type processes
- Stochastic partial differential equations and measure-valued processes

Dirichlet form theory provides a powerful tool that connects the probabilistic potential theory and analytic potential theory. Recently Dirichlet forms found its use in effective study of fine properties of Markov processes on spaces with minimal smoothness, such as reflecting Brownian motion on non-smooth domains, Brownian motion and jump type processes on Euclidean spaces and fractals, and Markov processes on trees and graphs. It has been shown that Dirichlet form theory is an important tool in study of various invariance principles, such as the invariance principle for reflected Brownian motion on domains with non necessarily smooth boundaries and the invariance principle for Metropolis algorithm. Dirichlet form theory can also be used to study a certain type of SPDEs.

Fractals are used as an approximation of disordered media. The analysis on fractals is motivated by the desire to understand properties of natural phenomena such as polymers, and growth of molds and crystals. By definition, fractals are mathematical objects that are very rough and lack smoothness, so one can not use the standard analytic methods that were developed for Euclidean spaces and for manifolds. It turns out that Dirichlet form theory is well suited for studying fractals—significant progress has been made in this area in the last fifteen years. Detailed study of heat kernel estimates and parabolic Harnack principle on fractals require techniques both from probability and analysis. Stability of such estimates under perturbations of operators and spaces can be proved by translating the problem into some analytic and geometric conditions. Such equivalent conditions are often obtained in the framework of graphs and general metric measure spaces, and Dirichlet forms are one of the key tools for the analysis. As an example, we mention that this approach turned out to be very useful in the analysis of random walks on random media such as percolation clusters. We believe that Dirichlet forms can play a more important role in studying scaling limits of nearest neighbor

random walk and long range random walk on percolation clusters—this is an example of a concrete research project that was discussed at the conference.

In recent years there was an explosion of activity in the area of jump type processes. There are diverse reasons for this increased interest in the area. One is that many physical and economic systems are best modeled by discontinuous Markov processes. On the theoretical side, jump type processes provide a hard but elegant challenge for mathematical methods, because the infinitesimal generators of jump type Markov processes are non-local operators. Research on Markov processes generated many new results for non-local operators and for pseudo-differential operators as well, such as heat kernel estimates, parabolic Harnack principle and a priori Hölder estimates of parabolic functions. These new directions in the development of the de Giorgi-Moser-Nash theory for non-local operators made it possible to give general criteria for convergence of Markov chains with jumps, and they also provided a new approach to long-range random walk in random media.

A stochastic partial differential equation (SPDE) is a partial differential equation containing a random (noise) term. The study of SPDEs is an exciting topic which brings together techniques from probability theory in general and stochastic analysis in particular, functional analysis, and the theory of partial differential equations. SPDEs have many diverse applications: study of random evolution of systems with a spatial extension (random interface growth, random evolution of surfaces, fluids subject to random forcing), study of stochastic models where the state variable is infinite dimensional (for example, a curve or surface). The solution to a stochastic partial differential equation may be viewed in several manners. One can view the solution as a random field (set of random variables indexed by a multidimensional parameter). In the case when the SPDE is an evolution equation, the infinite dimensional point of view is to consider the solution at a given time as a random element in a function space and thus the SPDE is represented as a stochastic evolution equation in an infinite dimensional space. In the pathwise point of view, one can give a meaning to the solution for (almost) every realization of the noise and then view the solution as a random variable on the set of (infinite dimensional) paths thus defined. SPDEs are closely related to measure-valued processes, which are a class of strong Markov processes that model the spatial and temporal evolution of a population with reproductive mechanism.

The goal of the conference was to encourage the invited participants to bring their own perspectives on the foundations of stochastic analysis and help trigger activity in various exciting areas.

2 Presentation Highlights

The opening two lectures were related to the rough path theory. Y. Le Jan opened the conference and discussed his recent work showing that the signature of Brownian motion completely determines Brownian motion in the sense that the completions of the σ-fields of the two processes coincide. M. Hairer discussed SDEs driven by fractional Brownian motions. He discussed the existence of smooth densities of the solution of SDE under Hörmander's condition and uniqueness of the invariant measure.

The talks were diverse but included several well defined clusters centered on specific topics. The first cluster of talks was on jump-type processes, especially their potential theoretical aspects. M. Kassmann presented his recent works on a new formulation of the Harnack inequalities that implies regularity estimates for solutions of heat equations. P. Kim, R. Song and Z. Vondracek talked about subordinate Brownian motions; oscillation of harmonic functions and its application to Fatou's theorem, (boundary) Harnack inequalities and two-sided estimates of the Green functions for subordinate Brownian motions with and without Gaussian components. T. Kulczycki discussed the trace of reflected Brownian motion in D on a flat part of the boundary $F \subset \partial D$; the connection of the process with the Dirichlet to Neumann map and with the mixed Steklov problem, and some geometric conditions on D and F so that the hot spot conjecture is true. M. Kwasnicki discussed boundary Harnack inequalities for pure-jump processes under mild assumptions on metric measure spaces. R. Schilling discussed sample path properties of Feller processes such as estimates for maximal values of the process, transience/recurrence using the symbol of the semigroup. M. Takeda discussed Feynman-Kac penalisations for multi-dimensional symmetric α-stable processes. R. Banuelos summarized recent applications of the sharp martingale inequalities of Burkholder to various Fourier multipliers including some that arise from transformations of Lévy processes via the Lévy-Khintchine formula. A talk by S. Evans was also related to jump processes. He discussed Lipschitz minorants of Lévy processes and showed that if a

Lévy process indexed by the real line has a K-Lipschitz minorant, then the set of times where the process is in contact with its minorant is a stationary regenerative set and hence it is the closed range of a subordinator "made stationary". He also characterized this subordinator in many cases and determined properties such as whether or not its range is countable or has zero Lebesgue measure. T. Uemura presented some sufficient condition for a non-symmetric bilinear form to be a lower bounded semi-Dirichlet form so that there is a jump-diffusion associated with it.

Some of the talks on processes with jumps described above used tools from the Dirichlet form theory. Dirichlet forms played a significant role in a number of other presentations. P.J. Fitzsimmons discussed two problems related to quadratic variation and diffusions. One is a decomposition of CAF to a CAF determined by a quasi-continuous function and a local MAF. The other is that the infinite-dimensional Brownian motion constructed by Gross fails to satisfy the axiom of polarity, namely there exists a semipolar set that is not polar. M. Fukushima presented his recent results on how to apply Brownian motion with darning to conformal mappings of multiply connected planar domains and to the associated Komatu-Loewner differential equation. A. Winter constructed a continuum-tree-valued diffusion via a quasi-regular Dirichlet form. She showed that Aldous' Markov chain on N-cladogram (which is a semi-labeled, unrooted and binary tree with N leaves) suitably rescaled and started in the uniform cladogram converges in the weak Gromov topology to this diffusion. S. Feng discussed some progresses on the construction of a reversible diffusion process with the two-parameter Dirichlet process (which is a random measure generalizing Ferguson's Dirichlet process) as the reversible measure. In some cases, such a reversible diffusions can be constructed by using Dirichlet form method.

Another cluster of talks reported recent progress on SPDEs, measure-valued processes and SDEs. R. Bass considered the heat equation on an interval with random mechanism given by $A(u)(x)$ (with current temperature u) times space-time white noise, where A is Hölder continuous. He showed the uniqueness in law holds for the SPDE under some reasonable conditions on A. E. Perkins discussed non-uniqueness (in law and pathwise) of similar type SPDE where $A(u)(x)$ part is $|u(t,x)|^\gamma$. He showed that the SPDE with zero initial conditions has a non-zero solution for $0 < \gamma < 3/4$ (hence solutions are not unique in law or pathwise) whereas it is pathwise unique when $\gamma > 3/4$. D. Khoshnevisan described sensitiveness of randomly-forced heat equations on the choice of the initial function. A family of parabolic Anderson models is one such example, and he described some connection of the Anderson models to the so called KPZ relation. M. Röckner talked about recent extinction results for SPDE (with multiplicative noise) of porous media type and applications to self-organized criticality (SOC). He discussed recent results, where for the SOC-case asymptotic extinction was proved for all spatial dimensions and shown to be locally exponentially fast. K. Xian presented his recent results on a Schilder-type sample large deviation for super-Brownian motions with an explicit good rate function. J. Xiong discussed well-posedness of the martingale problem for superprocesses with interactions. Y. Ren discussed a supercritical Galton-Watson branching processes with immigration. She discussed the recent results on the small valued probabilities of the random variable arising as a non-degenerate limit for the normalized population size under some mild condition. X. Li discussed the construction of a stochastic process (called the derivative process) associated to the SDE whose coefficients are not differentiable. She presented a probabilistic representation for the derivative of the corresponding semi-group and the convergence of approximating SDEs.

The third cluster of talks was focused on stochastic processes on fractals and related topics. M. Barlow summarized the equivalent conditions for sub-Gaussian heat kernel estimates and discuss the role and application of the so-called cut-off Sobolev inequalities. D. Croydon introduced a notion of spectral Gromov-Hausdorff convergence and used it to prove the convergence of mixing times for a sequence of finite graphs. B. Hambly summarized recent results on quenched and annealed asymptotic order of eigenvalue counting functions for the generator on random fractals. J. Kigami discussed non-local Dirichlet forms on p-adic numbers and its heat kernels. Some heat kernel estimates were also derived. N. Kajino presented his recent results on Weyl's Laplacian eigenvalue asymptotics for the harmonic Sierpinski gasket.

Mass transport was also an important topic in the workshop. E.P. Hsu presented generalizations of Talagrand's transportation cost inequality to the heat kernel measure on a Riemannian manifold and the Wiener measure on the path space over a Riemannian manifold. S. Pal considered rank-based models which are certain interacting diffusion processes. Using transportation cost inequalities, he derived uniform Gaussian tail bounds for the empirical least-square estimate of the index over large intervals of time. K.-T. Sturm considered couplings of the Lebesgue measure and the Poisson point process. He showed that the minimal mean

L^p-transportation cost of the coupling is finite for all p provided $d \geq 3$, and when $d \leq 2$ it is finite if and only if $p < d/2$. When $p = 2$, the relation of the optimal coupling and a random tiling of $\mathbb{R}^d$ by convex polytopes of volume 1 was also discussed.

Some talks given during the week covered stochastic analysis themes harder to classify. Roughly speaking, they were related to heat kernel estimates and Gaussian processes. K. Bogdan presented recent results on Schrödinger perturbations of integral kernels. Under some integrability condition of the kernels, existence of the transition density for the perturbed operator and its upper bound were derived. J-D. Deuschel discussed Markov chain approximations to non-symmetric uniform elliptic divergence forms. He showed Gaussian heat kernel estimates for the centered random walks (a class of non-symmetric Markov chains), and as an application he proved the approximation by such random walks. M. Gordina talked about recent results on functional inequalities on infinite-dimensional curved spaces and applications. For instance, integrated Harnack inequalities for the heat kernel measures were discussed with an application to prove quasi-invariance of heat kernel measures on certain infinite-dimensional groups.

N. Eisenbaum discussed some properties of Gaussian processes. Given a Gaussian process $\{\eta_x\}_{x\in E}$ with continuous covariance, the equivalence of infinite divisibility of η^2 with other conditions such as η^2 satisfying FKG condition, the law of η^2 being Radon measure etc., were presented. J. Swanson discussed recent results on an explicit expression for the weak Stratonovich integral for a fractional Brownian motion with Hurst parameter $1/6$. Using this expression, an Itô-type formula for this integral was derived.

3 Participants

The participants spanned a wide spectrum by any measure. There were many participants from Canada and the US, but also a large number of probabilists from various European and Asian countries. There was a healthy mix of senior mathematicians and junior researchers. Although women were a minority at the conference, their number was significant and their contributions were among the most interesting.

4 Abstracts

Speaker: **Rodrigo Banuelos** (Purdue University)
Title: *Martingales and Fourier multipliers, sharp estimates*
Abstract: We shall discuss some recent applications of the sharp martingale inequalities of Burkholder to various Fourier multipliers including some that arise from transformations of Lévy processes via the Lévy–Khintchine formula. While these results are of interest on their own right, they are motivated by some well known open problems in analysis and PDE's.

Speaker: **Martin Barlow** (University of British Columbia)
Title: *Energy of cutoff functions and applications*
Abstract: A well known theorem of Grigoryan and Saloff Coste states that Gaussian heat kernels bounds are equivalent to volume doubling (D) plus Poincare inequalities(PI). In the sub-Gaussian case, which arises on various regular fractals, Rich Bass and I showed that these are equivalent to (D), (PI) and an additional condition, denoted CS(b). The condition CS(b) gives the existence of low energy cutoff functions. In this talk I will give some additional applications of CS(b), which hold without (PI). This is joint work with Sebastian Andres.

Speaker: **Richard F. Bass** (University of Connecticut)
Title: *Uniqueness in law for parabolic SPDEs and infinite dimensional SDEs*
Abstract: We consider the heat equation on an interval with heat being introduced according to a random mechanism. When the random mechanism is space-time white noise, this equation has been much studied. We look at the case where the white noise is modified by a function A(u)(x) of the current temperatures u and where A is Hölder continuous as a function of u. Unlike other work along these lines, A(u)(x) can depend on the temperatures throughout the interval. Our method involves looking at the Fourier coefficients,

which leads to an infinite dimensional system of stochastic differential equations. This is joint work with Ed Perkins.

Speaker: **Krzysztof Bogdan** (Wrocław University of Technology, Poland)
Title: *Schrödinger perturbations*
Abstract: I will report joint work in progress with Wolfhard Hansen, Tomasz Jakubowski and Sebastian Sydor on Schrödinger perturbations of integral kernels; ones which produce comparable integral kernels.

[1] W. Hansen. Global comparison of perturbed Green functions. Math. Ann. 334 (2006), no. 3, 643–678.

[2] K. Bogdan, W. Hansen, T. Jakubowski. Time-dependent Schrödinger perturbations of transition densities. Studia Math. 189 (2008), no. 3, 235–254.

[3] T. Jakubowski. On combinatorics of Schrödinger perturbations. Potential Anal. 31 (2009), no. 1, 45–55.

[4] T. Jakubowski, K. Szczypkowski. Time-dependent gradient perturbations of fractional Laplacian. J. Evol. Equ. 10 (2010), no. 2, 319–339.

Speaker: **David Croydon** (University of Warwick, UK)
Title: *Mixing time convergence for sequences of random walks on graphs*
Abstract: The main conclusion of this work, which is a joint project with Ben Hambly (University of Oxford) and Takashi Kumagai (Kyoto University), is that the mixing times of the simple random walks on a sequence of graphs converge to the mixing time of a limiting diffusion whenever the corresponding state spaces, invariant measures and heat kernels converge in a suitable Gromov-Hausdorff sense. In addition to presenting this result, I will describe how it can be applied to a number of interesting examples, including some simple lattice models, self-similar fractal graphs with random weights, critical Galton-Watson trees, the Erdos-Renyi random graph at criticality and the range of a random walk in high dimensions.

Speaker: **Jean-Dominique Deusche** (Berlin Technical University, Germany)
Title: *Markov chain approximations to non-symmetric diffusions with bounded coefficients*
Abstract: We consider a certain class of non-symmetric Markov chains and obtain heat kernel bounds and parabolic Harnack inequalities. Using the heat kernel estimates, we establish a sufficient condition for the family of Markov chains to converge to non-symmetric diffusions. As an application, we approximate non-symmetric divergence forms with bounded coefficients by non-symmetric Markov chains. This extends the results by Stroock-Zheng ([SZ]) to the non-symmetric divergence forms. Joint work with Takashi Kumagai.

[SZ] D.W. Stroock and W. Zheng. Markov chain approximations to symmetric diffusions. *Ann. Inst. Henri. Poincaré-Probab. Statist.* **33** (1997), 619-649.

Speaker: **Nathalie Eisenbaum** (Université Paris VI, France)
Title: *Some properties of Gaussian processes*
Abstract: We will describe and characterize some properties of Gaussian processes.

Speaker: **Steve Evans** (University of California at Berkeley)
Title: *Uplift under Lévystan: Lipschitz minorants of Lévy processes*
Abstract: Motivated by problems in statistical density estimation, Groeneboom 1983 established a number of remarkable properties of the greatest convex function dominated by the path of a one-dimensional Brownian motion. Pitman, Bass, Cinlar, Bertoin and others extended these results and showed how they could be derived more easily using techniques such as excursion theory and last exit decompositions. There has been a resurgence of interest in this area with a flurry of recent work on convex minorants of Lévy processes and random walks by various combinations of Abramson, Pitman, Ross and Uribe Bravo. Researchers in non-linear optimization, particularly optimal transport, have developed the notion of c-convexity that greatly generalizes the familiar definition of convex function while keeping many of its important features such as Legendre-Fenchel duality. Within this theory, the set of functions from the reals to the reals with a given upper bound K on their Lipschitz constant share many of the features of the convex functions. I will show that if a Lévy process indexed by the real line has a K-Lipschitz minorant, then the set of times where the

process is in contact with its minorant is a stationary regenerative set and hence it is the closed range of a subordinator "made stationary". It is possible to characterize this subordinator in many cases and determine properties such as whether or not its range is countable or has zero Lebesgue measure. If the Lévy process is a Brownian motion with drift, then there are explicit distributions for a number of random variables related to the minorant. This work is joint with Josh Abramson.

Speaker: **Shui Feng** (McMaster University)
Title: *An Open Problem Associated with the Two-parameter Dirichlet Process*
Abstract: The two-parameter Dirichlet process is a random measure generalizing Ferguson's Dirichlet process. This talk will discuss progresses and difficulties in constructing a reversible diffusion process with the two-parameter Dirichlet process as the reversible measure.

Speaker: **Patrick J. Fitzsimmons** (University of California at San Diego)
Title: *Zero-Energy Functions of a Symmetric Diffusion*
Abstract: Consider a continuous function f on the state space of a symmetric diffusion $X = (X_t : t \geq 0)$ such that $t \mapsto f(X_t)$ is locally of zero energy. Using time-reversal arguments, I will show that f must be constant along the paths of X. The space-time version of this result will also be discussed. Time permitting, I will examine a representation of general continuous additive functionals of X, extending a result of Tanaka for one-dimensional Brownian motion.

Speaker: **Masatoshi Fukushima** (Osaka University, Japan)
Title: *On Brownian motion with darning and Komatu-Loewner equation for multiply connected planar domains*
Abstract: Brownian motion with darning (BMD in abbreviation) is applied to the study of conformal mappings of multiply connected planar domains and the associated Komatu-Loewner differential equations. The notion of BMD and its basic properties are presented in Chapter 7 of a forthcoming book [CF] in a more general context. It is a diffusion process obtained from the absorbing Brownian motion on a multiply connected planar domain by rendering each hole of the domain into one point. Especially the zero period property of a general BMD-harmonic function and the conformal invariance of BMD will be used to derive the continuity of some fundamental quantities for the Komatu-Loewner equation. We are motivated by the works [BF1],[BF2],K and [L]. This talk is based on an ongoing joint work with Z.-Q. Chen and S. Rohde.

[BF1] R.O. Bauer and R.M. Friedrich, On radial stochastic Loewner evolution in multiply connected domains. *J. Funct. Anal.* **237** (2006), 565-588.

[BF2] R.O. Bauer and R.M. Friedrich, On chordal and bilateral SLE in multiply connected domains. *Math.* **Z. 258** (2008), 241-265.

[CF] Z.-Q. Chen and M. Fukushima, *Symmetric Markov Processes, Time Change, and Boundary Theory*. Princeton University Press, to appear.

[K] Y. Komatu, On conformal slit mapping of multiply-connected domains. *Proc.Japan Acad.* **26** (1950), 26-31.

[L] G. F. Lawler, The Laplacian-b random walk and the Schramm-Loewner evolution. *Illinois J. Math.* **50** (2006), 701-746.

Speaker: **Maria Gordina** (University of Connecticut)
Title: *Functional inequalities in infinite dimensions*
Abstract: we will talk about recent results on functional inequalities on infinite-dimensional curved spaces and applications of such inequalities. One type of such results concerns integrated Harnack inequalities for the heat kernel measures, and how these inequalities can be used to prove quasi-invariance of heat kernel measures on certain infinite-dimensional groups. These results use geometric data such as lower Ricci bounds. Therefore these techniques is not applicable to sub-elliptic Laplacians. Most of finite-dimensional methods for such heat kernels are dimension-dependent. Nevertheless, recently we were able to prove several types of functional inequalities for a sub-elliptic Laplacian on an infinite-dimensional Heisenberg group. The results of this talk are based on joint work with F. Baudoin, B. Driver, T. Melcher.

Speaker: **Martin Hairer** (University of Warwick, UK)
Title: *Hypoelliptic SDEs driven by rough paths*

Abstract: We present a number of recent results extending Hörmander's theorem to a larger class of SDEs driven by Gaussian stochastic processes. In particular, we focus on the case when the driving process is rougher than Brownian motion, using the theory of rough paths to give meaning to the equations. Our main ingredient in the implementation of Malliavin's programme is a deterministic version of Norris's lemma. This relies on a quantitative property of the driving process (a "modulus of Hölder roughness") ensuring that it is sufficiently non-degenerate. One application of the theory is to demonstrate ergodicity of such equations under conditions that are qualitatively similar to those in the classical ergodic theory for SDEs.

Speaker: **Ben Hambly** (University of Oxford)
Title: *Spectral asymptotics for random fractals*
Abstract: I will discuss some results concerning the high frequency asymptotics of the eigenvalue counting function for a variety of random fractals. The focus will mainly be on the continuum random tree where we can show the size of the second order term in the asymptotic expansion. We will also discuss other random recursive fractals where a variety of behaviors is possible.

Speaker: **Elton Hsu** (Northwestern University)
Title: *Brownian Motion and Transpotation Inequality on Path Spaces*
Abstract: We will show how to use synchronizing Riemannian Brownian motion to prove generalizations of Talagrand's transportation cost inequality to the heat kernel measure on a Riemannian manifold and the Wiener measure on the path space over a Riemannian manifold.

Speaker: **Naotaka Kajino** (University of Bielefeld)
Title: *Weyl's Laplacian eigenvalue asymptotics for the measurable Riemannian structure on the Sierpinski gasket*
Abstract: On the Sierpinski gasket K, Kigami [Math. Ann. 340 (2008), 781–804] has introduced the notion of the measurable Riemannian structure, with which the gradient vector fields of functions, the Riemannian volume measure μ and the geodesic metric ρ are naturally associated. Kigami has also proved in the same paper the two-sided Gaussian bound for the corresponding heat kernel, and I have further shown several detailed heat kernel asymptotics, such as Varadhan's asymptotic relation, in [Potential Anal., in press, doi: 10.1007/s11118-011-9221-5].

In the talk I will talk about the Weyl's Laplacian eigenvalue asymptotics for this case. The correct scaling order for the asymptotics of the eigenvalues is given by the Hausdorff dimension d of the metric space (K, ρ), and in the limit of the eigenvalue asymptotics we obtain a constant multiple of the d-dimensional Hausdorff measure H^d. Moreover, we will also see that this Hausdorff measure H^d is Ahlfors d-regular with respect to ρ but that it is singular to the Riemannian volume measure μ.

Speaker: **Moritz Kassmann** (University of Bielefeld, Germany)
Title: *Harnack's inequality: a new formulation and applications*
Abstract: We present a formulation of Harnack's inequality which is applicable to local and nonlocal operators at the same time. We show that this version of Harnack's inequality implies regularity estimates for solutions to several integrodifferential operators. We apply the method to some nonlocal symmetric Dirichlet forms and to generators of jump processes. We discuss how this approach extends known results. The talk is based on three works: [K. 2010], [K.-Mimica 2011] and [Dyda-K. 2011]

Speaker: **Davar Khoshnevisan** (University of Utah)
Title: *On the chaotic character of some parabolic SPDEs*
Abstract: I will describe how a family of randomly-forced heat equations depends sensitively on the choice of the initial function. Time permitting, we will see a surprising connection, and derivation, of the so called KPZ relation between the spatial and temporal fluctuation exponents of the solution to a family of parabolic Anderson models. This talk is based on joint works with D. Conus, M. Foondun, M. Joseph, and S. Shiu.

Speaker: **Jun Kigami** (Kyoto University, Japan)
Title: *Dirichlet forms on a noncompact Cantor set and random walks on its defining tree*
Abstract: First we will construct a class of Dirichlet forms on a noncompact Cantor set, which is a generalization of p-adic numbers, from prescribed sets of eigenvalues and measures. At the same time, we have

concrete expressions of the jump kernel and the transition density. Assuming the volume doubling condition, we construct an intrinsic metric under which estimates of transition density and jump kernel are obtained. Secondly transient random walks on the defining tree of the noncompact Cantor set are shown to induce a subclass of Dirichlet forms discussed in the first part on the noncompact Cantor set as traces.

Speaker: **Panki Kim** (Seoul National University)
Title: *Oscillation of unbounded harmonic functions for subordinate Brownian motion and its applications*
Abstract: In this talk we discuss the oscillation of unbounded harmonic functions for pure-jump subordinate Brownian motion. As an application, we give a probabilistic proof of relative Fatou's theorem for harmonic functions for such subordinate Brownian motion. This is a joint work with Yunju Lee.

Speaker: **Tadeusz Kulczycki** (Polish Academy of Sciences and Wrocław University of Technology, Poland)
Title: *On jump processes which are traces of reflected Brownian motion*
Abstract: Let B_t be a reflected Brownian motion in a bounded domain $D \subset R^d$. Assume that there exists a part F of a boundary of D which is flat (for example D can be a cylinder and F one of its bases or D can be a triangle and F one of its sides). Let us consider the process X_t which trajectories are traces of the process B_t on F. More formally $X_t = B_{\eta_t}$, where η_t is the inverse of a local time of B_t on F.

We will show the connection of the process X_t with the Dirichlet to Neumann map and with the mixed Steklov problem. Such Steklov problem appears very naturally in some problems of hydrodynamics.

It occurs that the jump process X_t on F shares many properties of a reflected Brownian motion on F. In particular we will show that under some geometric assumptions on D and F the hot spots property holds for the process X_t. That is, the first non-constant eigenfunction corresponding to the semigroup of the process X_t attains its maximum and minimum on the boundary of F. We will also study some properties of the process X_t under continuous deformation of the domain D. Some open questions will be also discussed.

Speaker: **Mateusz Kwasnicki** (Wrocław University of Technology, Poland)
Title: *Boundary Harnack inequality for jump-type processes*
Abstract: I will present the results of my recent work with Krzysztof Bogdan and Takashi Kumagai. We consider a Hunt process X_t in a metric measure space. Under relatively mild assumptions, we prove a boundary Harnack inequality for nonnegative functions harmonic with respect to X_t in an arbitrary open set. We require:

(1) existence of the dual process X_t^*,

(2) existence of bump functions in the domains of the Feller generators of X_t and X_t^*,

(3) boundedness (away from the diagonal) of the jumping kernel and the potential kernel.

Under these assumptions, we prove boundedness near the boundary point $z \in \partial D$ of the ratio $f(x)/g(x)$ of any two nonnegative harmonic functions f, g in an open set D, given that f and g vanish off D in some neighbourhood of z. More precisely, we prove that in this case, if $d(x,z) < r$ and $d(y,z) < r$, then

$$\frac{1}{C}\,\frac{f(x)}{g(x)} \le \frac{f(y)}{g(y)} \le C\,\frac{f(x)}{g(x)}\,,$$

where C does not depend on D, x, y, f and g, given that $f(x) = g(x) = 0$ whenever $x \notin D$ and $d(x,z) < 2r$.

When X_t is a symmetric (i.e. self-dual) process which admits local stable-like bounds for the transition density (i.e. heat kernel), then (1) and (3) are satisfied automatically. The only restrictive assumption is in this case (2). In Euclidean setting, (2) holds true if the jumping kernel is sufficiently regular. In general metric measure spaces, (2) can often be proved when there is a sub-Gaussian diffusion process. Under these assumptions, our boundary Harnack inequality is scale-invariant and, in some sense, stable under small perturbations.

The proof involves a new, probabilistic method for proving a priori supremum bounds for nonnegative harmonic functions, which may be of independent interest.

Speaker: **Yves Le Jan** (Université Paris-Sud, France)
Title: *The signature of Brownian paths*

Abstract: We show that in dimension larger than two, Brownian paths indexed by [0,T], are determined by their iterated integrals taken up to T.

Speaker: **Xue-Mei Li** (University of Warwick, UK)
Title: *The derivative process for SDEs with non-smooth coefficients*
Abstract: For an SDE with smooth coefficients, the solution depends continuously on the initial data. The latter is a basic assumption for numerical schemes. In this talk we discuss the construction of a stochastic process, which we call the derivative process, associated to the SDE in the case of the coefficients not differentiable. This is used to give a probabilistic representation for the derivative of the corresponding semi-group. For this construction we also studied the convergence of approximating SDEs and make sense of of an SDE with random coefficients. As a by product we have an existence theorem for the smooth solution flow.

Speaker: **Soumik Pal** (University of Washington)
Title: *Transportation Cost Inequalities for rank-based models*
Abstract: Transportation Cost inequalities are functional inequalities that compare the Wasserstein distance between two probability measures with their relative entropy. Via an argument by Marton these inequalities are powerful methods to show concentration of measure properties. We focus on applications to certain interacting diffusion processes called rank-based models. This is a multidimensional diffusion model where each particle (coordinate index) gets an instantaneous drift and diffusion coefficient depending on its rank among all the particles. It is known that if we exponentiate the coordinates and rescale to have a total sum of one, the ordered values exhibit power law decay in equilibrium. Using Transportation Cost Inequalities we derive uniform Gaussian tail bounds for the empirical least-square estimate of the index of this power law over large intervals of time. Part of this talk is based on joint work with Misha Shkolnikov.

Speaker: **Edwin Perkins** (University of British Columbia)
Title: *Nonuniqueness for a parabolic SPDE with $\frac{3}{4}-\epsilon$-Hölder diffusion coefficients*
Abstract: We prove an analogue of the Girsanov examples for SDE's for the parabolic stochastic partial differential equation (SPDE)

$$\frac{\partial u}{\partial t} = \frac{\Delta}{2}u(t,x) + |u(t,x)|^{\gamma}\dot{W}(t,x),$$

with zero initial conditions. Here $\dot{W}$ is a space-time white noise on $R_+ \times R$. More precisely, we show the above stochastic pde has a non-zero solution for $0 < \gamma < 3/4$, and hence solutions are not unique in law or pathwise unique. The case $\gamma = 1/2$ arises as a scaling limit point of a system of branching annihilating random walks. An analogue of Yamada-Watanabe's theorem for SDE's was recently shown by Mytnik and Perkins for SPDE's by establishing pathwise uniqueness of solutions to

$$\frac{\partial u}{\partial t} = \frac{\Delta}{2}u(t,x) + \sigma(u(t,x))\dot{W}(t,x),$$

where σ is Holder continuous of index $\gamma > 3/4$. The situation for the above class of parabolic SPDE's is therefore similar to their finite dimensional counterparts, but with the index $3/4$ in place of $1/2$. This is joint work with Carl Mueller and Leonid Mytnik.

Speaker: **Yanxia Ren** (Peking University)
Title: *Small value probabilities for supercritical branching processes with immigration*
Abstract: We consider a supercritical Galton-Watson branching process with immigration. It is well known that under suitable conditions on the offspring and immigration distributions, there is a finite, strictly positive and non-degenerate limit for the normalized population size, denoted as $\mathcal{W}$. The main purpose of this paper is to investigate the small value probabilities of $\mathcal{W}$, that is to estimate $P(\mathcal{W} \leq \varepsilon)$ for $\varepsilon > 0$ small. In comparison with the well-studied results for supercritical Galton-Watson branching process without immigration, precise effects of the balance between offspring and immigration distributions on small value probability of $\mathcal{W}$, are obtained. Several illustrative examples are analyzed carefully. They demonstrate the sharpness of our results and the significant effect of the immigration which can cause the near-constancy phenomena even when there is no oscillation in the setting without immigration.

Speaker: **Michael Röckner** (University of Bielefeld, Germany)
Title: *Recent extinction results for stochastic porous media equations and applications to self-organized criticality*
Abstract: The first part of the talk will recall extinction results for stochastic partial differential equations (with multiplicative noise) of porous media type. These include stochastic fast diffusion equations and more singular cases, where e.g. the nonlinearity is given by a Heaviside or sign function, so is multivalued. The latter describe certain continuum models for the phenomenon of self-organized criticality (SOC). These extinction results have been obtained in the past two years. In the SOC-case, however, extinction was only shown if the underlying spatial domain is one-dimensional and only with positive (though high) probability. The second part of the talk is devoted to very recent results, where for the SOC-case asymptotic extinction was proved for all spatial dimensions and shown to be locally exponentially fast. One main technique in the proofs is to transform the stochastic PDE into a deterministic PDE with a random parameter. The resulting deterministic PDE is of an entirely new type and new methods had to be invented for its analysis.

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[2] V. Barbu, G. Da Prato, M. Röckner *Finite time extinction for solutions to fast diffusion stochastic porous media equations,* C. R. Acad. Sci. Paris – Mathematics 347 (2009), no. 1–2, 81–84.

[3] V. Barbu, G. Da Prato, M. Röckner *Stochastic porous media equation and self-organized criticality,* Comm. Math. Phys. 285 (2009), no. 3, 901–923.

[4] V. Barbu, P. Blanchard, G. Da Prato, M. Röckner *Self-organized criticality via stochastic partial differential equations,* Theta Series in Advanced Mathematics, "Potential Theory and Stochastic Analysis" in Albac. Aurel Cornea Memorial Volume, 2009, pp. 11–19.

[5] V. Barbu, M. Röckner *On a random scaled porous media equation,* BiBoS–Preprint, publication in preparation, 22 pp., 2010.

[6] V. Barbu, M. Röckner *Stochastic porous media equations and self-organized criticality: convergence to the critical state in all dimensions,* BiBoS–Preprint, publication in preparation, 21 pp., 2011.

Speaker: **Rene Schilling** (Technical University at Dresden, Germany)
Title: *Sample Path Properties of Feller Processes*
Abstract: We present sufficient conditions for the transience and the existence of local times of a Feller process, and the ultracontractivity of the associated Feller semigroup. These conditions are sharp for Lévy processes, and they are based on the local symmetrization technique and a uniform upper bound for the characteristic function of a Feller process. As a byproduct, we obtain for stable-like processes (in the sense of R. Bass) on $\mathbb{R}^d$ with smooth variable index $\alpha(x) \in (0,2)$ a transience criterion in terms of the exponent $\alpha(x)$; if $d = 1$ and $\inf_{x\in\mathbb{R}} \alpha(x) \in (1,2)$, then the stable-like process admits local times.

This work is joint with Jian Wang (Fujian Normal University and TU Dresden)

Speaker: **Renming Song** (University of Illinois)
Title: *Potential theory of subordinate Brownian motions*
Abstract: A subordinate Brownian motion is a Levy process which can be by replacing the time parameter of Brownian motion by an independent subordinator. The subordinator can thought of as "intrinsic" time or "operational" time. Subordinate Brownian motions form a large subclass of Levy processes and they are widely used in applications.

Recently, a lot of progress has been made in the study of the potential theory of subordinate Brownian motions. In this talk, I will give a survey of some recent results on the potential theory of subordinate Brownian motions without Gaussian components. In particular, I will present a boundary Harnack principle and two-sided estimates on the Green functions of these processes in bounded smooth open sets.

This talk is based on joint works with Panki Kim and Zoran Vondracek.

Speaker: **Karl-Theodor Sturm** (University of Bonn, Germany)
Title: *Optimal Transport from Lebesgue to Poisson*

Abstract: We study couplings $q^\bullet$ of the Lebesgue measure and the Poisson point process $\mu^\bullet$, i.e. measure-valued random variables $\omega \mapsto q^\omega$ s.t. for a.e. ω the measure q^ω on $\mathbb{R}^d \times \mathbb{R}^d$ is a coupling of $\mathfrak{L}^d$ and μ^ω. For any given $p \in (0,\infty)$ we ask for a minimizer of the mean L^p-transportation cost

$$\mathfrak{C}(q^\bullet) \;=\; \sup_{B\subset\mathbb{R}^d} \frac{1}{\mathfrak{L}^d(B)} \mathbb{E}\left[\int_{\mathbb{R}^d\times B} |x-y|^p \, dq^\bullet(x,y)\right].$$

The minimal mean L^p-transportation cost turns out to be finite for all p provided $d \geq 3$. If $d \leq 2$ then it is finite if and only if $p < d/2$.
Moreover, in any of these cases we prove that there exist a unique translation invariant coupling which minimizes the mean L^p-transportation cost. In the case $p = 2$, this 'optimal coupling' induces a random tiling of $\mathbb{R}^d$ by convex polytopes of volume 1.

Speaker: **Jason Swanson** (Central Florida University)
Title: *The calculus of differentials for the weak Stratonovich integral*
Abstract: The weak Stratonovich integral is defined as the limit, in law, of Stratonovich-type symmetric Riemann sums. We derive an explicit expression for the weak Stratonovich integral of $f(B)$ with respect to $g(B)$, where B is a fractional Brownian motion with Hurst parameter 1/6, and f and g are smooth functions. We use this expression to derive an Itô-type formula for this integral. As in the case where g is the identity, the Itô-type formula has a correction term which is a classical Itô integral, and which is related to the so-called signed cubic variation of $g(B)$. Finally, we derive a surprising formula for calculating with differentials. We show that if $\mathbf{d}M = X\,\mathbf{d}N$, then $Z\,\mathbf{d}M$ can be written as $ZX\,\mathbf{d}N$ minus a stochastic correction term which is again related to the signed cubic variation.

Speaker: **Masayoshi Takeda** (Tohuku University)
Title: *Feynman-Kac Penalisations of Symmetric Stable Processes*
Abstract: B. Roynette, P. Vallois and M. Yor have studied limit theorems for Wiener processes normalized by some weight processes. K. Yano, Y. Yano and M. Yor studied the limit theorems for the one-dimensional symmetric stable process normalized by non-negative functions of the local times or by negative (killing) Feynman-Kac functionals. Our aim is to extend their results on Feynman-Kac penalisations to positive Feynman-Kac functionals of multi-dimensional symmetric α-stable processes.

Speaker: **Toshihiro Uemura** (Kansai University)
Title: *On multidimentional diffusion processes with jumps*
Abstract: Let a_{ij} and b_i be coefficients of a second order partial differenctial operator defined on an open set D of R^d and k a Levy-type kernel over D. We define a non-symmetric billinear form on $L^2(D;dx)$ having the dates above

Under some conditions on the datas, we will show the form becomes a lower bounded semi-Dirichlet form and there exists a diffusion process with jumps associated with the form.

Speaker: **Zoran Vondracek** (University of Zagreb)
Title: *Potential theory of subordinate Brownian motions with Gaussian components*
Abstract: In this talk I will look at a subordinate Brownian motion with a Gaussian component and a rather general discontinuous part. The assumption on the subordinator is that its Laplace exponent is a complete Bernstein function with a Lévy density satisfying a certain growth condition near zero. The main result that I will present is a boundary Harnack principle with explicit boundary decay rate for non-negative harmonic functions of the process in $C^{1,1}$ open sets. I will also discuss an example showing that the boundary Harnack principle fails for processes with finite range jumps. As a consequence of the boundary Harnack principle, one can establish sharp two-sided estimates on the Green function of the subordinate Brownian motion in any bounded $C^{1,1}$ open set D and identify the Martin boundary of D with respect to the subordinate Brownian motion with the Euclidean boundary.

Joint work with Panki Kim and Renming Song

Speaker: **Anita Winter** (Friedrich-Alexander-Universität Erlangen-Nürnberg)
Title: *Aldous move on cladograms in the diffusion limit*

Abstract: A N-cladogram is a semi-labeled, unrooted and binary tree with N leaves labeled $\{1, 2, ..., N\}$ and with unit edge lengths. Aldous constructed a Markov chain on cladograms and gave bounds on its mixing time.

In this talk we use Dirichlet form methods to construct a continuous tree-valued diffusion and show that Aldous move on cladograms suitably rescaled converges in the weak Gromov topology to this diffusion provided that started in the uniform cladogram.

Speaker: **Kai-Nan Xian** (Nankai University, China)
Title: *An Explicit Schilder-type Theorem for Super-Brownian Motions*
Abstract: Like ordinary Brownian motion, super-Brownian motion, a central object in the theory of superprocesses, is a universal object arising in a variety of settings. Schilder-type theorems and Cramér-type theorems are two of the major topics for large-deviation theory. A Schilder-type (which is also a Cramér-type) sample large deviation for super-Brownian motions with a good rate function represented by a variation formula was established in 1993 and 1994 (see [1]-[3]). There have been very valuable contributions for giving an affirmative answer to the question of whether this sample large deviation holds with an explicit good rate function since then. In [4], thanks to previous results on this issue and the Le Gall's Brownian snake, we established such a large deviation for nonzero finite initial measures. Then in [5], we proved the mentioned large deviation holds for infinite initial measures. Those concluded the attacking on the long-standing conjecture proposed in [1].

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[3] Schied, A. Sample large deviations for super-Brownian motion. *Prob. Th. Rel. Fields.* **104** (1996), no. 3, 319-347.

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Speaker: **Jie Xiong** (University of Tennessee)
Title: *Uniqueness problems for some measure-valued processes*
Abstract: A stochastic partial differential equation (SPDE) is derived for the super Brownian motion regarded as a distribution function valued process. The strong uniqueness for the solution to this SPDE is obtained by a connection between SPDEs and backward doubly stochastic differential equations. Similar results are also proved for the Fleming-Viot process. We also extend the uniqueness result to processes with interaction.