Geometric Structures on Manifolds (12w5121)

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The geometry and topology of manifolds is a large research area, making connections with many flourishing specialties such as algebraic topology, symplectic geometry, gauge theory, knot and links, and differential geometry. The purpose of this meeting was to bring together a broad selection of researchers from many flourishing areas of current work in the more geometric aspects of topology, in order to promote awareness of new developments across the whole field.

This meeting was a sequel to our highly successful meetings ”Topology” 05w5067 and ”Topology” 07w5070 held at BIRS (August 27 - Sept. 1, 2005 and February 25 - March 2, 2007, respectively) both of which had similar objectives and scope. The strongly positive comments we received from the participants at that time encouraged us to think that a meeting with this broader scope was a valuable service to the mathematics research community. We limited the talks to 5 per day and 45 minutes each, allowing ample time for informal interactions. The speakers were asked to address a broad audience and most of them did this very successfully. A good number of the talks were given by younger mathematicians. In general the atmosphere was very creative, and also those who did not give a lecture had the chance to explain their ideas in numerous discussions in smaller groups. As organizers, we were very pleased with the high scientific level of the talks, and with the energy and enthusiasm of all the participants.

1 Overview of the Field

The overall problem is to classify manifolds of a fixed dimension in a variety of inter-related categories, e.g. topological, smooth, symplectic, complex, as well as those that support geometric structures reflected in its fundamental group. At our last meeting we focused on the recent work of Perleman on geometric structures on 3-manifolds. This year we focused on higher dimensions, with emphasis on the notoriously difficult classification of simply-connected smooth 4-manifolds, the topological classification of 4-manifolds with specific fundamental groups, geometric structures on 4-manifolds, and the study of mirror symmetry and the re-emerging study of G_2 and $Spin(7)$ manifolds.

2 Presentation Highlights

The format of the meeting was designed to promote interaction and discussion, as well as exposure of all the participants to the above-mentioned themes. We encouraged several of the speakers to provide a broad overview of their respective areas of geometry/topology. These included a broad overview by Robert Bryant of current research in the active area of G_2 and $Spin(7)$ manifolds. This area begins with the motivational observation (due to M. Wang and based on the famous classification of Riemannian holonomy due to Berger) that a connected, oriented Riemannian spin manifold (M^n, g) can have the dimension of its parallel spinor fields equal to 1 only when n is one of 1, 7, or 8.

The case $n = 1$ is, of course, trivial; the case $n = 7$ only occurs for Riemannian 7-manifolds whose holonomy is $G_2 \subset SO(7)$, the so-called 'proper' G_2-manifolds; and the case $n = 8$ only occurs for Riemannian manifolds whose holonomy is $Spin(7) \subset SO(8)$, the so-called 'proper' $Spin(7)$-manifolds. (If the holonomy of a Riemannian 7-manifold is a proper subgroup of G_2, then it is locally a product, and we say that it is an 'improper' G_2-manifold, with similar terminology for the 8-dimensional case.) Thus G_2-manifolds are characterized (and specified) by the choice of a 3-form $\sigma \subset \Omega^3_+(M^7)$ (where $\Omega^3_+(M^7) \subset \Omega^3(M^7)$ consists of the 'definite' 3-forms, which satisfy a point-wise non-degeneracy condition that is described in the talk). Such a form defines a canonical metric g_σ and orientation $\star_\sigma$ (i.e., Hodge star operation), and σ defines a (possibly improper) G_2-manifold structure if and only if σ and $\star_\sigma \sigma$ are closed forms on M. This system of differential equations is characterized in terms of its solvability and the known existence results are described, both in the cases of local solutions and closed 7-manifolds (particularly the spectacular results of Joyce and Kovalev proving existence in these cases). This leads to several obstructions.

There were several talks on the (possible and hoped-for) extensions of gauge theoretic techniques to the study of 7- and 8-manifolds. There is an enormous amount of work to be done there to make it into a useful theory in analogy with the application of geometric techniques that have been so successful in 3- and 4-dimensions. But it is clear that there are some talented young people working on this.

Mirror symmetry has taken a central role and there were two important contributions represented at this meeting. Tim Perutz provided insights into his work on the arithmetic aspects of homological mirror symmetry. Homological mirror symmetry (HMS) relates the Fukaya category of a degenerating Calabi-Yau manifold to the derived category of coherent sheaves on its mirror partner. Ordinarily, these categories are taken with coefficients in an algebraically closed field containing the complex numbers. It turns out that HMS for Calabi-Yau manifolds should have a refinement in which the mirror is a variety defined over a far smaller ring. In this way, the symplectic topology of one manifold is tied to the arithmetic geometry of its mirror. Joint work with Yankı Lekili proves a test case, arithmetic mirror symmetry for the 2-torus, which says in part that the Fukaya category of exact Lagrangian curves on the once-punctured 2-torus is equivalent, over the ring of integers, to the differential-graded category of perfect complexes on the nodal Weierstrass curve $y^2 + xy = x^3$.

Einstein 4-manifolds have taken a central role in 4-manifolds as a possible structure to place on many smooth 4-manifolds. This was highlighted by the beautiful talk of Claude LeBrun on the classification of Einstein 4-manifolds with complex structure. An Einstein metric is, by definition, a Riemannian metric of constant Ricci curvature. Not all smooth compact 4-manifolds admit such metrics, and the difference between existence and non-existence primarily depends on the smooth structure rather than the homeotype of the manifold in question. During this conference recent existence, uniqueness, and classification results concerning Einstein metrics smooth compact 4-manifolds were presented, contrasting these results with the wildly different results that have emerged regarding Einstein metrics in other dimensions.

To put the study of geometric structures on 4-manifolds in context: a *geometry* consists of a complete 1-connected Riemannian manifold X such that $G = Isom(X)$ acts transitively on X and has discrete subgroups Γ (lattices) which act freely on X, with finite covolume ($vol(X/\Gamma < \infty$).

Jonathan Hillman described some earlier work on the characterization of geometric 4-manifolds up to homotopy (or better). In dimension 4 there are 18 geometries with cocompact lattices. In 12 cases the model X is contractible, and a closed 4-manifold M is TOP s-cobordant to X/Γ if and only if $\pi_1(M) \cong \Gamma$ and $\chi(M) = \chi(\Gamma)$. (However the possible groups and values of χ are not known for the geometries H^4 and $H^2(C)$, and for the irreducible $H^2 \times H^2$-lattices.)

Three have compact models, and such manifolds are well understood. However complete algebraic invariants are not yet known when $X = S^2 \times S^2$ and $|\Gamma| = 4$ (i.e., $\chi = 1$). The key problem for the geometry $S^3 \times E^1$ is to show that if M is a closed 4-manifold with $\chi(M) = 0$ and $\pi_1(M) \cong Z/qZ \times_s Z$ then M is homotopy equivalent to a mapping torus. (This has been verified in the non-orientable cases, by Davis and Weinberger.)

Jonathan Hillman then went on to describe recent progress on the remaining two geometries $S^2 \times E^2$ and $S^2 \times H^2$. Every closed 4-manifold M with $\pi_2(M) \cong Z$ is either (i) homeomorphic to CP^2; (ii) TOP s-cobordant to the total space of an S^2-bundle; (iii) simple homotopy equivalent to the total space of an RP^2-bundle; or (iv) simple homotopy equivalent to the total space of an S^2-orbifold bundle over a flat or hyperbolic 2-orbifold with non-empty singular locus. (All the S^2- and RP^2-bundle spaces are geometric, as are most of the orbifold bundle spaces.) The recent work establishes case (iv). The key step involves a careful choice of model for the second Postnikov stage $f_M : M \to P_2(M)$, and comparison of the images of the fundamental classes of manifolds related by "Gluck reconstruction".

In a completely different direction, unusual and intriguing results were presented by Liviu Nicolaescu concerning the complexity of random smooth functions on compact manifolds. It turns out there is an interesting relationship between the distribution of critical values of a random smooth function on a compact m-dimensional and the distribution of eigenvalues of a random, real, symmetric $(m + 1) \times (m + 1)$-matrix.

3 Recent Developments and Open Problems

Given that this meeting covered a rather large portion of the overall research in the topology and geometry of manifolds, it is most concise to give the abstracts of the remaining talks and discussions presented at the meeting.

D. V. Alekseevsky: Compact cohomogeneity one Kähler and Kähler–Einstein manifolds

The talk is based on a joint work with A.Loi and F.Zuddas and also on a joint work with V.Cortes, K. Hasegawa and Y. Kamishima.

Let G be a compact semisimple Lie group. A cohomogeneity one compact G-manifold M with the orbit space $M/G = [0,1]$ is determined by a triple (H_0, L, H_1) of subgroups, where $H_i,\ i = 0,1$ are the stabilizers of the singular orbits , $L \subset H_0 \cap H_1$ is the stabilizer of a regular orbit and the coset spaces $S_i := H_i/L$ are spheres.

If $(g, J, \omega = g \circ J)$ is an invariant Kähler structure on M, then regular orbits $S = G/L$ carry an invarinat CR structure. We study Kähler cohomogeneity one G-manifolds under assumption that this CR structure is Levi non degenerate and unique, that it the distribution $(JT)^{\perp} \subset TS$ associated with the normal vector field T of S is the unique invariant contact structure on $S = G/L$. Such Kähler structures are called ordinary.

In this case, the image $F = \mu(S) = G/K$ of a regular orbit S under the moment map is a flag manifold with an invariant complex structure and $\pi : S = G/L \to F = G/K$ is the Sasaki principal S^1-bundle over F associated with the Reeb vector field Z of the contact form $\theta := g \circ JT$.

We describe cohomogeneity one manifolds $M = M(H_0, L, H_1)$ which admit an ordinary invariant Kähler structure in terms of painted Dynkin diagrams associated with the corresponding flag manifold $F = G/K$. Using results by F. Podesta and A. Spiro, we give an explicit description of invariant Kähler structures on M in terms of appropriate functions of one variable.

We classify cohomogeneity one G-manifolds which admit an invariant Kähler-Einstein metric under assumption that the associated flag manifold $F = G/K$ is defined by Dynkin diagram with ≤ 2 black nodes, or, equivalently, the center $Z(K)$ has dimension ≤ 2.

Scott Baldridge: Coisotropic Luttinger Surgery on Symplectic 6-Manifolds

We introduce a surgery operation on symplectic manifolds called coisotropic Luttinger surgery, which generalizes Luttinger surgery on Lagrangian tori in symplectic 4-manifolds. In this talk we will show how to use it to produce infinitely many distinct symplectic 6-manifolds X with $c_1(X) = 0$ which are not of the form $M^4 \times T^2$ and survey some of the recent results in this area. This work is joint with Paul Kirk.

Robert Bryant: G_2 and $Spin(7)$-manifolds

Interspersed among the talks, I had a number of interesting conversations with the other attendees. They inspired me to work out some interesting examples of special holonomy in the case of metric connections with skew-symmetric, closed torsion, a class of problems that has turned up in the physics literature, generalizing the usual background of a Riemannian metric to include a so-called 'H-field' as an extra feature of the theory. In particular, I was able to show, by constructing examples, that every connected subgroup of $SO(4)$ occurs as the holonomy of such a connection in dimension 4, (even the 1-dimensional 'skew lines' in a maximal torus, though it turns out that there is only a finite-parameter family of such examples). Also, I was able to classify the pairs (g, H), metric and 3-form, in dimension 3 for which the holonomy of both of the connections $\nabla_g + H^\sharp$ and $\nabla_g - H^\sharp$ reduces to $SO(2)$. This was asked of me by Gil Cavalcanti in connection with a problem he has been working on. It turns out that these exist and depend on one function of one variable, up to diffeomorphism. I am interested in pursuing this to see whether there are any useful or interesting restrictions on holonomy for such pairs (g, H) in higher dimension.

Olivier Biquard: Desingularization of Einstein orbifolds

Can one desingularize an Einstein orbifold ? the question is well studied in the Kähler case, but very little is known in general. For example, can one desingularize the quotient of the 4-sphere by an involution with 2 fixed points ? We will give a beginning of answer to this question.

Gil Cavalcanti: SKT geometry

An SKT structure on a manifold is a Hermitian structure for which the metric and complex structure are parallel for a connection whose torsion is skew and closed. If this torsion vanishes, this is simply a Kahler structure. At first, it appears that the introduction of torsion spoils several properties shared by Kahler manifolds. I will show how one can use the general framework from generalized complex geometry to develop the theory of SKT manifolds and show that these manifolds actually share several of the properties of (generalized) Kahler manifolds.

Weimin Chen: On Seifert fibered 4-manifolds

In the study of 4-dimensional S^1-manifolds, an important technique has been the so-called "Pao's replacement trick". Originally, Pao used this technique to construct nonlinear S^1-actions on the 4-sphere. Fintushel adapted Pao's trick to give a smooth classification of simply connected 4-dimensional S^1-manifolds. Later, Baldridge gave further applications of Pao's trick to the Seiberg–Witten invariants of S^1-manifolds.

Pao's trick requires that the circle group has a fixed point. In this talk, we will consider 4-dimensional S^1-manifolds without fixed points (called Seifert fibered 4-manifolds). Several results showing the rigidity of such smooth structures will be discussed. Main Theorems: (1) If the Seifert fibered 4-manifold has nonzero Seiberg–Witten invariant, then the fundamental group must have infinite center. (2) Given any finitely presented group with infinite center, there exist at most finitely many smoothly distinct orientable Seifert fibered 4-manifolds realizing the group.

Jim Davis: Topological Rigidity

A group G has a cocompact manifold model for $E_{fin}G$ if there is a G-manifold M with M/G compact, with M^F contractible for all finite subgroups F of G, and with M having the

G-homotopy type of a G-CW-complex. Examples of such arise from discrete subgroups of isometries of manifolds with non-negative curvature, or CAT(0) examples of Mike Davis.

Two such cocompact manifold models are G-homotopy equivalent. G satisfies equivariant rigidity if any such G-homotopy equivalence is G-homotopic to a G-homeomorphism. For G torsion-free, a group satisfies equivariant rigidity if and only if M/G satisfies the Borel Conjecture.

Frank Connolly, Qayum Khan, and I are systematically attacking the problem of equivariant rigidity, using, among other ingredients, the Farrell-Jones Conjectures in K- and L-theory. A sample result is:

Theorem: *All H_1-negative involutions on a torus T^n are conjugate to a smooth action. If $n \equiv 0, 1 \pmod 4$ or if $n = 2, 3$ then all H_1-negative involutions on T^n are topologically conjugate. Otherwise there are an infinite number of conjugacy classes of such actions.*

A involution on a space is H_1-negative if it induces multiplication by -1 on the first homology.

Matt Hedden: Recent progress on topologically slice knots

I'll discuss the world of topologically slice knots: those knots which bound topologically flat embedded disks in the 4-ball. These knots generate a particularly interesting subgroup of the smooth concordance group of knots which offers insight into the distinction between the smooth and topological categories in dimension 4. I'll discuss the first examples of (two-) torsion elements in this topologically slice subgroup. This is joint work with Se-Goo Kim and Charles Livingston.

Ludmil Katzarkov: From Higgs bundles to stability conditions

In this talk we will make an analogy between gauge theoretic and categorical invariants. We will outline possibility for constructing new algebro geometric and symplectic invariants.

Matthias Kreck: Codes from 3- and 4 - manifolds

This is a continuation of my previous work with Puppe where we studied the relation between self-dual codes and 3-manifolds with involution. Recently Manin showed interest in out work and asked for realization of non self-dual codes, the typical case, and also for an extension to codes over other fields. In both directions we proved several new results which might make up for a reasonable talk.

Yankı Lekili: Floer theoretically essential tori in rational blowdown

We compute the Floer cohomology of monotone tori in the Stein surfaces obtained by a linear plumbing of cotangent bundles of spheres, also known as the Milnor fibre associated with the complex surface singularity of type A_n. We next study some finite quotients of the A_n Milnor fibre which coincide with the Stein surfaces that appear in Fintushel and Stern's rational blowdown construction. We show that these Stein surfaces have no exact Lagrangian submanifolds by using the already available and deep understanding of the Fukaya category of the A_n Milnor fibre coming from homological mirror symmetry. On the contrary, we find Floer theoretically essential monotone Lagrangian tori, finitely covered by the monotone tori that we studied in the A_n Milnor fibre. We conclude that these Stein surfaces have non-vanishing symplectic cohomology. Joint work with Maksim Maydanskiy.

Tim Nguyen: Quantum Chern-Simons Theory and Perturbative Renormalization

Our goal is to give an informal discussion of the ideas involved in Kevin Costello's formalism for perturbatively quantizing gauge theories using BV geometry. For the sake of illustration, we will use Chern-Simons theory as a model example in this discussion. In this particular case, the path integral yields topological invariants generalizing those obtained by Axelrod-Singer and Kontsevich through Feynman diagramatic analysis.

Timothy Perutz: Arithmetic aspects of homological mirror symmetry

Homological mirror symmetry (HMS) relates the Fukaya category of a degenerating Calabi–Yau manifold to the derived category of coherent sheaves on its mirror partner. Ordinarily, these categories are taken with coefficients in an algebraically closed field containing the complex numbers. I'll explain why HMS for Calabi-Yau should have a refinement in which the mirror is a variety defined over a far smaller ring. In this way, the symplectic topology of one manifold is tied to the arithmetic geometry of its mirror. Joint work with Yankı Lekili proves a test case, arithmetic mirror symmetry for the 2-torus, which says in part that the Fukaya category of exact Lagrangian curves on the once-punctured 2-torus is equivalent, over the ring of integers, to the differential-graded category of perfect complexes on the nodal Weierstrass curve $y^2 + xy = x^3$.

Mihaela Pilca: Lowest eigenvalue of the Dirac operator on Kähler manifolds and special holonomy

On compact Kähler manifolds there exist lower bounds for the first eigenvalue of the Dirac operator (acting on the eigenbundles of the spinor bundle under the action of the Kähler form), which are given by the so-called refined Kirchberg inequalities. Our main result gives a complete description of the limiting manifolds, i.e. those on which this bound is attained. More precisely, we obtain Riemannian products of a Calabi–Yau manifold and a twistor space over a positive quaternion-Kähler manifold. One of the main tools used is the study of the corresponding eigenspinors, which turn out to be exactly the sections in the kernel of a natural first order operator adapted to the Kähler structure, namely the so-called Kählerian twistor (or Penrose) operator.

Nikolai Saveliev: Seiberg-Witten theory on a homology $S^1 \times S^3$

The usual count of solutions to the Seiberg-Witten equations on a 4-manifold X only gives a smooth invariant when $b^2_+(X) > 0$ (with wall crossing when $b^2_+(X) = 1$.) If X is a homology $S^1 \times S^3$, we provide an index-theoretic term which, when added to the count of Seiberg-Witten solutions, gives a smooth invariant of X. The definition of this term is based on our extension of the Atiyah–Patofi–Singer index theory for first order differential operators, from manifolds with cylindrical ends to manifolds with more general periodic ends. We will discuss some examples in which we have been able to make calculations of our invariant. This is joint work with Danny Ruberman and Tom Mrowka.

Andras Stipsicz: Knots in Lattice homology

We introduce a filtration on the lattice homology of a negative definite plumbing tree associated to a further vertex and show how to determine lattice homologies of surgeries on this last vertex. We discuss the relation with Heegaard Floer homology.

Nathan Sunukjian: The relationship between surgery and surface concordance in 4-manifolds.

Understanding embedded surfaces in 4-manifolds is a starting point to understanding smooth structures on 4-manifolds. One reason for this is because by performing some sort of surgery on a surface, it is sometimes possible to change the smooth structure of the 4-manifold without altering its homeomorphism class. A considerable hurdle to this approach is that it can be very difficult to find appropriate surfaces to perform the surgery on. Surface concordance is one way of trying to organize the surfaces in a 4-manifold. In this talk we will, (1) compute the concordance group for a class of surfaces, and (2) explain when surgery on concordant surfaces gives the same result.

Andrew Swann: Multi-moment maps and applications to special holonomy

Suppose a group acts on a manifold preserving a closed differential form. In work with Thomas Bruun Madsen we introduced a notion of multi-moment map that extends the established concept of moment map in symplectic geometry, and, in contrast to previous definitions, shares the basic existence properties. The concept applies particularly well to torus actions on manifolds of holonomy G_2 or $Spin(7)$. One application is an interesting correspondence between such structures and four-manifolds equipped with triples of symplectic forms.

Andrei Teleman: Gauge theoretical approach in the classification of class VII surfaces.

The classification of complex surfaces is not finished yet. The most important gap in the Kodaira–Enriques classification table concerns the Kodaira class VII, e.g. the class of surfaces X having $\mathrm{kod}(X) = -\infty$, $b_1(X) = 1$. These surfaces are interesting from a differential topological point of view, because they are non-simply connected 4-manifolds with definite intersection form. The main conjecture which (if true) would complete the classification of class VII surfaces, states that any minimal class VII surface with $b_2 > 0$ contains b_2 holomorphic curves. We explain a new approach, based on ideas from Donaldson theory, to prove existence of curves on class VII surfaces.

My method starts with the observation that the lack of curves (or more precisely the lack of a "cycle" of curves) implies the appearance of a smooth compact connected component in a certain moduli space of stable bundles ($\mathrm{PU}(2)$-instantons) over the given surface. For $b_2 \leq 2$ I showed that the presence of such a component leads to a contradiction, proving the existence of cycle for this class of surfaces. I have recentely showed that if a refined form of the Grothedieck–Riemman–Roch theorem for proper maps of (non-Kählerian) complex manifolds was true, the method would generalize to arbitrary b_2. Such refined forms of the Grothedieck–Riemman–Roch have been considered by Jean-Michel Bismut and Julien Grivaux. Extending my results to arbitrary b_2 (which is "work in progress") would completely solve the classification problem up to deformation equivalence.

References

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Thomas Walpuski: A conjectural G2 Casson invariant

Given a bundle over a G_2-manifold one can consider a special class of connections called G_2-instantons. Formally, G_2-instantons are rather similar to flat connection on 3-manifolds. In particular they can be understood as critical points of a G_2 Chern-Simons functional. It is conjectured that counting G_2-instantons will lead to a G_2 analogue of the Casson invariant. I will discuss some problems that arise in this project and how they could possibly be attacked. Finally, I will explain how one could hope to compute the conjectural Casson invariant in some rather special cases.

Katrin Wehrheim: How to construct 4-manifold invariants via the symplectic category

Combining Gay-Kirby's theory of Morse 2-functions and the theory of pseudoholomorphic quilts, we develop a procedure for constructing 4-manifold invariants depending on the choice of

- a homotopy class $X \to S^2$ (or in fact to a general Riemann surface);
- a "symplectization functor" from the 2+1 bordism category to the symplectic category (such as the ones used in constructions of 3-manifold invariants - arising from representation spaces or symmetric products).

The construction proceeds by

- interpreting the Cerf diagram of a Morse 2-function (with connected fibers of sufficiently high genus) as string diagram of X in the 2+1+1 bordism bicategory;
- applying the "symplectization functor" to the surface fibers and elementary 3-cobordisms, and associating canonical Floer classes to the 4-dimensional 2-morphisms labeling cusps and crossings of the Morse 2-function;
- evaluating the quilt invariant on the resulting string diagram in the symplectic 2-category.

Invariance of the resulting integer can be proven by translating the moves between Morse 2-functions into moves between quilt diagrams, where a fundamental strip shrinking isomorphism reduces them to verifying

- two axioms on the composition of Lagrangian correspondences;
- one nontrivial axiom on a local holomorphic curve count

In particular, this provides an approach for verifying the invariance of Perutz' Lagrangian matching invariants.

4 Outcome of the Meeting

Several new collaborations ensued during and after this meeting. In particular Tim Perutz talked through an idea with Tim Nguyen which might become the basis of a collaboration. Jim Davis made considerable progress with some of his collaborators present at the meeting. Andrew Swann met with Andrew Dancer and commented that the ability to meet away from their distractions of their departments allowed them to finish off a number of remaining points in a joint project they have and were pleasantly surprised by how much progress they made on a paper that is now very near completion. A collaboration between Matt Hedden and Nathan Sunujkian began as a direct result of the workshop, the outcome of which could be quite exciting. Later in the summer, Matthias Kreck and Ian Hambleton answered a question from the talk of Baldridge and Kirk, by showing how to "recognize" by explicit invariants which 6-manifolds are diffeomorphic to the products of surfaces and simply-connected 4-manifolds.

As pointed out earlier, another not well understood area in complex surface theory is the classification of Class VII surfaces. Andrei Teleman, using gauge theoretic input provided the best to date classification of these surfaces. Rather than using Seiberg-Witten theory the key new input is to use ideas from Gromov-Witten theory to count only rational curves in a given homology class. However, this talk engendered discussions with Nikolai Saveliev concerning his work with Ruberman concerning their use of Seiberg-Witten theory to study integral rational homology $S^1 \times S^3$ that may well finish off the classification.