Final Report on First Canadian Summit on Applications of Partial Differential Equations in the Sciences

Thomas Hillen (University of Alberta),
Jun-Cheng Wei (University of British Columbia)
August 28, 2015–August 30, 2015

1 PARTICIPANTS

Cheviakov, Alexei; University of Saskatchewan
de Vries, Gerda; University of Alberta
Fetecau, Razvan; Simon Fraser University
Hillen, Thomas; University of Alberta
Iron, David; Dalhousie University
Kolokolnikov, Theodore; Dalhousie University
Kuske, Rachel; University of British Columbia
Kutz, J. Nathan; University of Washington
Muir, Paul; Saint Mary’s University
Ou, Chunhua; Memorial University of Newfoundland
Spiteri, Ray; University of Saskatchewan
Szmigielski, Jacek; University of Saskatchewan
Ward, Michael; University of British Columbia
Wei, Jun Cheng; University of British Columbia
Williams, JF; Simon Fraser University
Wittenberg, Ralf; Simon Fraser University
Yi, Yingfei; University of Alberta

2 Schedule of Summit

Friday, August 28, 2015: Arrival of participants. 19:00 onward: Informal meeting in the BIRS lounge (2nd Floor, Corbett Hall)
Saturday, August 29, 2015: 9:00-12:00 and 14:00-17:00 Short presentations of delegates; 15 talks arranged (see below)
Sunday, August 30, 2015: 9:00-12:00 Discussion of CRG activities.
3 First Day of Talks

On August 29th, 15 talks were arranged.

As part of the PIMS Collaborative Research Group on Applied PDEs, the purpose of this summit meeting is to get to know each other and their current research activities in order to identify common interests and to discuss upcoming events and activities.

Each participant was invited to present a 20 minute talk. The focus of these talks was to provide an introduction of their interests, a summary of typical methods used or developed and a discussion of interesting open questions.

There are seven talks in the morning, starting from 9am and ending at 12:30pm. This was chaired by Thomas Hillen. There are eight additional talks in the afternoon, starting from 2pm and ending at 5:20pm. This was chaired by Jun-Cheng Wei.

Nathan Kutz (University of Washington): Data-driven dimensionality-reduction in PDEs

Nathan introduced several basic data-driven methods in PDEs and discussed applications in nonlinear Schrodinger equations, compressive sensing for PDEs in fluids, etc. This is a completely new inter-disciplinary field and creates enormous opportunity for understanding data.

Razvan Fetecau (Simon Fraser University): Nonlocal aggregation models: dynamics and equilibria

Razvan presented aggregation model of self-collective behavior with potentials having short attractive range and long replusive range. He gave a complete characterzation of steady states and their stability in some parameter regions. He mentioned several open problems including anisotropic model as well as nonlocal models with nonlinear diffusion.

Rachel Kuske (University of British Columbia): Stochastic facilitation through interactions of delays and noise

Pattern formations with white noise have gain widespread interests in recent years. Rachel described noise-driven patterns and oscillations and sustained transients "stabilized" by noise. She gave an example of Swift-Hohenberg model with Paragas controls and noise. Then she mentioned several challenging open problems including understanding the effect of the noise on bifurcation structure of SPDES, sustained transients, stochastic bifurcations, etc.

David Iron (Dalhousie University): Models of processes in cell physiology

David proposed reaction-diffusion systems to model polarity formation in nematode development and aggregation of cell surface receptors. He showed several numerical and partial results on possible patterns. Some open conjectures are raised.

Paul Muir (St. Mary’s University): B-spline adaptive Gaussian collocation error control software for the numerical solutions of PDEs

Paul discussed several old and new numerical PDE-solvers: BACOL, HPNEW, MOVCOL, EPDCOL, BACOLR, BACOLI, etc. He mentioned future work on spatial adaptivity/error control based on transformations involving Moving Mesh PDEs.

Ray Spiteri (University of Saskatchewan): Efficient time-stepping

Ray’s topics is an interface talk between computer science, math and numerical analysis. He introduced time-stepping method in differential equations and applications in optimization and optimal control. Then he mentioned current projects on heart simulation, optimal design and operation of PEM fuel cells, etc.
Jacek Szmigielski (University of Saskatchewan): Wave breaking and shocks: an integrable systems perspective

Jacek started introduced a shallow water wave equation known as the Camassa-Holm PDE, and related equations, and and discussed the role of peakons in existence and blowing ups. He then discussed the Hamiltonian structure of peakons and perspectives from integrable systems. He mentioned generalized $b$--systems and several open problems associated with it.

Theodore Kolokolnikov (Dalhousie University): Vortex nucleation in a dissipative GPE with rotation

Theodore analyzed the stability and instability of boundary layer solutions of GPE with large rotations. A weakly nonlinear analysis is performed for the ground states and the instability mode is analyzed. He mentioned several open problems on the analysis of nonlinear vortex patterns.

Yingfei Yi (University of Alberta): PDE method in stochastic dynamics

Yingfeng introduced an ODE system with Ito white noise perturbation. Using the distribution-based approach, the transition probability density is shown to satisfy Fokker-Planck equation which can be solved using geometric and PDE methods. General open questions on dynamical systems with noise perturbations are raised.

Jeff Williams (Simon Fraser University): Proof by construction: an overview of rigorous numerics

Jeff gave an overview of rigorous numerics computing proofs. He started with the general scheme of such methods and then gave several examples of this scheme. Applications of this new method include the existence and nondegeneracy of nonlinear Schrodinger equations, and the energy cascade of diblock copolymer models.

Chun-Hua Ou (Memorial University of New Foundlan): Research on two fields: applied dynamical systems and PDE

Chun-Hua investigated the traveling wave problems for nonlocal population models. The new characteristics of the model is the combination of delays and nonlocal interactions. He introduced on-going work on traveling wave solutions for SI model. Another topic of his interest is the asymptotics of PDEs in higher dimensions.

Ralf Wittenberg (Simon Fraser University): PDEs with spatiotemporal complex behavior

Ralf discussed patterns and spatiotemporal chaos on stabilized Kuramoto-Sivashinsky equation. He gave several interesting behavior of the so-called Matthew-cox equation. A challenging question is to rigorous analysis.

Alexei Cheviakov (University of Saskatchewan): Systematic construction of conservation laws, symmetries for PDEs and ODEs

Alex started with general definitions of local conservation laws and the associated Euler-Lagrange equation to find multipliers. He discussed several applications in mathematical modelling. He expressed willingness to co-supervise graduate students.

Michael Ward (University of British Columbia): Pattern formations: new directions and challenges

Michael presented two new directions in pattern formations for reaction-diffusion systems. The first is the dynamics of localized spots on the sphere. A new DAE system is derived and its equilibria, dynamics and Fekette points are discussed. The second direction is diffusion induced synchronous oscillations. This is an ODE model coupled with PDE bulk diffusion. Many new features of this system including Hopf bifurcations are discovered.
Thomas Hillen (University of Alberta): Mathematical modelling with fulling anisotropic diffusion


4 Second Day of Discussions

On Sunday morning (Aug. 30), Michael Ward, representing the organizers of the CRG, presented an overview of the CRG proposal and proposed activities in the next three years. The participants had extensive discussions on possible collaborations and suggested several new initiatives including possible graduate visiting and co-supervision.

5 Scientific Progress Made and Outcome of the Meeting

All of the work that was presented was very recent. After each talk there was always long discussions and interactions. Many participants saw new ideas arising from interaction with others who brought their own perspectives. A number of new collaborations appear to have arisen from the meeting, and the work of other collaborations was advanced. For example Michael mentioned possible link of Thomas’s sea turtle model with the calculation of the first mean pass time. Theodore took deep interest in Razavan’s nonlocal models. Ray was interested in numerical computations of Michael’s DAE systems with large number of particles. Jeff’s rigorous computing may solve several open problems of Wei. Nathan’s data-driven methods bring new ideas to pattern formations. Alexei’s conservation laws may be applied to Toda system studied by Wei. Yingfeng’s dynamical system with white noise resonates with Rachel’s noise-induced pattern formation. Jasek’s peakon analysis resonates with spiky patterns and singularity formation studied by Michael and Wei.