PERMUTATION GROUPS (16w5087)

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November 13 - November 18, 2016

1 Overview of the Field

Permutation groups are a mathematical approach to analysing structures by studying the rearrangements of the elements of the structure that preserve it. Finite permutation groups are primarily understood through combinatorial methods, while concepts from logic and topology come to the fore when studying infinite permutation groups. These two branches of permutation group theory are not completely independent however because techniques from algebra and geometry may be applied to both, and ideas transfer from one branch to the other. Our workshop brought together researchers on both finite and infinite permutation groups; the participants shared techniques and expertise and explored new avenues of study.

The permutation groups act on discrete structures such as graphs or incidence geometries comprising points and lines, and many of the same intuitions apply whether the structure is finite or infinite. Matrix groups are also an important source of both finite and infinite permutation groups, and the constructions used to produce the geometries on which they act in each case are similar.

Counting arguments are important when studying finite permutation groups but cannot be used to the same effect with infinite permutation groups. Progress comes instead by using techniques from model theory and descriptive set theory. Another approach to infinite permutation groups is through topology and approximation. In this approach, the infinite permutation group is locally the limit of finite permutation groups and results from the finite theory may be brought to bear.

The progress in "modern" times on what one would traditionally call "permutation groups" is characterised by the interplay of these rich and varied areas and approaches. The workshop brought together people working in (1) finite permutation groups, acting on graphs, geometric objects, *i.e.* more classical areas of the theory, (2) a model theoretic approach to infinite permutation groups and related questions, (3) topological groups, in particular, totally disconnected locally compact (t.d.l.c.) groups and, (4) matrix groups, particularly simple groups of Lie type and their primitive permutation actions.

There is no single perspective from which an overview of the workshop can be taken because it brought together researchers from four fields, each with its own methods and goals. The next section discusses the current state of research on these topics at a general and fairly high level. The remainder of the report discusses recent progress on the topics which were emphasized during the workshop and indicates connections made between the different fields.

2 Recent Developments and Open Problems

The four fields of research on permutation groups have reached different levels of maturity, as indicated by the fact that while those studying finite permutation groups and algebraic groups speak of 'classical' results and even of 'the classical groups', those studying infinite permutation groups and model theory or totally disconnected locally compact groups usually do not. A significant part of the benefit of bringing researchers from these fields together derives from this disparity. Those investigating totally disconnected local compact groups and infinite permutation groups seek to emulate the approaches successful for finite permutation and algebraic groups. Moreover, classical results about finite and linear groups underpin further work not only in those fields but also form the basis of current work in all four fields and, in the other direction, questions about totally disconnected locally compact or infinite permutation groups may lead back to unsolved problems in the more established areas. Researchers in these fields therefore can learn much from each other.

Much of the ongoing success of the study of finite permutation groups and matrix groups is based on decompositions of these groups into simple factors and on classifications of the simple groups in these classes. A decomposition theory for t.d.l.c. groups is only now emerging, following the appearance of [6], and techniques that would aid a classification of simple groups, including [7, 21], are still being developed. It is known [22] that compact generation is a necessary additional hypothesis if there is to be hope of a classification. Several classes of compactly generated simple groups are known, among them Lie groups over local fields and groups of automorphisms of trees [20], an up-to-date list of the known sources of simple groups is given in [7]. It is possible that other sources of large numbers of simple groups remain to be found. Gaps remain and it is not known, for example, whether simple groups exist with certain of the local structures identified in [7] or whether there can be compactly generated simple groups having flat-rank 0 (where the flat-rank is an invariant for t.d.l.c. groups associated with the scale [21] and equal to the familiar rank in the case of simple Lie groups). To resolve these difficulties either new examples of simple t.d.l.c. groups or new theorems are needed.

Developing a decomposition theory and classifying simple groups is not the end of the story of permutation groups however. Recent results (many still unpublished) suggest that the finite simple groups still hold secrets about simply stated properties of finite permutation groups and the structures on which they act. It has been shown, for example, that in most primitive permutation groups there is an unexpectedly close relationship between the orders of elements in the group and the size of the set on which it acts (at least four times the order) and the size of its orbits (there is an orbit of size equal to the order). These relationships are all the more remarkable for the fact that there are a few exceptions, which can all be described.

Primitive permutation actions of a group can be studied by classifying maximal subgroups of the group. Although much progress has been made with the maximal subgroups of almost simple finite groups, this program is still ongoing. Work on maximal subgroups of infinite groups must of necessity be confined to particular groups or classes of groups because no decomposition theory or classification of simple groups is possible for general infinite groups. One such natural class under consideration are the closed subgroups of $\mathrm{Sym}(\mathbb{N})$. These are exactly the automorphism groups of countable homogeneous structures and the maximal closed subgroups correspond to structures having no proper reducts in the model theoretic sense. There has been a lot of interest in this question even from the point of view of theoretical computer science.

Transitive permutation actions of groups may be studied by analysing the subgroup structure of the group in general. One approach to doing that is to seek information about various generating sets for the groups or for its subgroups. This approach can be applied to both finite and infinite groups. For this question too, the investigation is more advanced in the case of finite groups. There has been considerable progress in recent years, particularly for simple groups, where knowledge of good generating sets is useful for representing groups by permutations or matrices in computer algebra packages. In the case of infinite groups it is often useful to consider topological groups and topological generating sets.

3 Presentation Highlights

Many participants reported their immense appreciation at being able to interact with colleagues, to explore new questions to make progress on existing major research projects. In particular, they mentioned seeing unexpected connections between the various research directions that were represented, and finding discussions with the speakers after their lectures very inspiring and thought-provoking. Here are a few highlights and examples. More details may be found in the section on scientific progress.

Csaba Schneider's exposition of his theory of Cartesian decompositions left invariant by finite (and some families of infinite) permutation groups led to vigorous discussions, initiated by Alejandro Garrido, on whether there was a useful inverse limit concept for Cartesian decompositions and the wreath products which act on them.

Simon Smith's structure theory for subdegree-finite permutation groups, and his new fundamental box product construction/decomposition related to trees is a significant addition to the theory of t.d.l.c. groups. In his talk he showed how, as one outcome, his construction produces the first known uncountable family of compactly generated, non-discrete simple t.d.l.c. groups.

Pierre-Emmanuel Caprace gave a clear survey of recent work on automorphism groups of trees from the point of view of 2-transitive group actions on compact spaces. These groups are one of the key topics in the theory of t.d.l.c. groups and he described an emerging systematic analysis of them.

Zoë Chatzidakis presented a remarkable link between the theory of difference fields and t.d.l.c. groups in which the scale function on these groups helped to motivate the formulation of a new invariant for the fields. Her talk sparked a lot of interest and discussion among participants.

Colin Reid introduced the theory of decompositions of t.d.l.c. groups which he and Phillip Wesolek are developing. This theory can be expected to become a standard part of the analysis of t.d.l.c. groups and his talk helped to promote awareness of important new technique.

Alejandra Garrido's talk described the maximal subgroups in a class of branch groups and showed the their number is countable. This description for groups of intermediate growth breaks new ground because groups for which it had been done previously were either solvable or contained a non-abelian free subgroup.

André Nies spoke about a new direction of research inspired by the work of Dugald Macpherson and others on bi-interpretability of structures and their reducts. He addressed the question of the logical difficulty of determining conjugacy or topological isomorphism of two groups (represented as permutation groups) within a particular class. This is the same question as addressed by Caprace for automorphism groups of trees.

Dugald MacPherson described work with Cheryl Praeger and Simon Smith on subgroups of the full symmetric group on a countable set equipped with the permutation topology, which is totally disconnected. These included locally compact groups and groups which are maximal with respect to various properties and as a result his talk provoked discussions with a number of the participants.

Tim Burness gave an account of his current joint work with Martin Liebeck and Aner Shalev on generating sets for maximal and second maximal subgroups of finite almost simple groups which is yet to appear in print. His talk thus provided participants with tools and up-to-date knowledge for their own research. The talk by Inna Capdeboscq addressed similar questions for (infinite) Kac-Moody groups.

David Craven's survey of the current state of knowledge of the classification of subgroups of exceptional algebraic groups also brought participants up-to-date with that endeavour.

Gabriel Verret spoke his work on finite vertex-primitive graphs of valency 5 and 6 and there is a productive interaction between that work and David's resulting from the difficult problem of determining the conjugacy classes of extremely small maximal subgroups of the finite almost simple exceptional groups of Lie type. This problem about the nonabelian simple groups is forming the back-bone of David's recent research program (involving simple groups theory and representation theory), and some of David's results have already fed into the classification which Gabriel spoke about. A bonus from having such a diverse audience was the questions which led to research bridging the areas. In this case questions from Simon Smith about infinite vertex-primitive graphs having vertices with similar neighbourhoods prompted Gabriel to take a new look at the problem. He discovered that methods which had seemed only to work in the finite case could be modified to apply to infinite groups.

4 Scientific Progress and Interactions

The advances discussed and interactions which occurred at the workshop will now be reported in more detail from the points of view of the four subfields represented.

Finite permutation groups, graphs, geometries, and simple groups

Questions concerning groups, or the structures on which they act, often reduce to fundamental questions about primitive permutation groups. A pertinent example is the theme of the talk given by Michael Aschbacher inspired by the Palfy-Pudlak question: is each finite lattice an 'intermediate subgroups lattice', that is, is it isomorphic to the lattice of subgroups of a some group G which contain some subgroup H? At the conference Aschbacher considered the question for the lattice of overgroups of a given subgroup in an exceptional Lie type simple group.

In previous work of Aschbacher [1, 2] on this problem, some in collaboration with Shareshian, the context considered was sub-lattices of the overgroup lattice of a finite primitive permutation group. A major question was to decide when a given finite primitive group G is 'product indecomposable', in other words, to determine whether or not G preserves a Cartesian decomposition of the underlying point set. Knowledge of the lattice of overgroups of a given permutation group can also be important in combinatorics or geometry, for example, if the given group is a group of automorphisms of some structure and we wish to determine the full automorphism group. Csaba Schneider's talk addressed the fundamental question of finding all Cartesian decompositions preserved by a given permutation group G, in the case where G has a transitive minimal normal subgroup [18]. This theory is most powerful in the case where G is finite, as it exploits the finite simple group classification. However it was shown also to apply to a large family of infinite permutation groups; this extension was inspired by recent work of Smith [19] and by Neumann, Praeger and Smith [15].

Recent results about finite permutation groups (many still unpublished) suggest that the finite simple groups still hold secrets about simply stated properties of finite permutation groups and the structures on which they act: for example, in most primitive permutation groups each element has a regular cycle (of maximum possible length, equal to the element order), and in most primitive groups each element has order less than a quarter of the number of points; and in both cases we can describe the exceptions.

New studies of symmetric structures demand strong new theoretical tools from permutation groups. This was illustrated in Michael Giudici's talk which reported on application of the structure theory of quasiprimitive permutation groups to study s-arc transitive graphs, and showed how the subgroup structure of certain finite simple groups yielded new constructions of s-arc transitive digraphs with almost simple automorphism groups. This answered a 30 year old question of Ito and Praeger. In a similar vein, Gabriel Verret's talk explained how detailed knowledge of small maximal subgroups of simple groups of Lie type was required to complete a classification of primitive valency 5 graphs - the classification applied recent work of David Mason on maximal A_5 subgroups of $E_8(q)$. A different example was provided by Joanna Fawcett's talk on partial linear spaces. These are point–line incidence structures in which each point-pair is incident with at most one line. The most symmetrical partial linear spaces admit a group transitive on both collinear point-pairs and non-collinear point-pairs. Thus, provided the space is non-degenerate, in the sense that non-collinear point-pairs exist, such symmetrical spaces admit an automorphism group acting as a rank 3 permutation group on points. The work of Alice Devillers (2005, 2008) reduced classification of primitive rank 3 linear spaces to the case where the group is of affine type. Joanna Fawcett reported on an almost completed classification of the remaining case, the affine primitive rank 3 partial linear spaces.

From the opposite point of view, breakthroughs in combinatorics and geometric group theory lend new methods for studying group actions, and throw up new group theoretic challenges. For instance, Cai Heng Li's studies of edge-transitive embeddings of vertex-primitive graphs in compact Riemann surfaces pose new factorisation problems for finite almost simple groups. Also the exciting results of Helfgott on expansion in groups, extended by Pyber and Szabó, and independently by Breuillard, Green and Tao, have shown not only that Cayley graphs of bounded rank finite simple Lie type groups are expanders, but have also yielded proof of the Weiss Conjecture of 1978 for locally primitive graphs involving only bounded rank composition factors, [17]. This highlights the classical Lie type groups of unbounded rank as those for which greater understanding is needed.

The Weiss and Praeger conjectures for locally primitive and locally quasiprimitive graphs predict bounds, in terms of the valency, on the number of graph automorphisms fixing a vertex. More recent studies showed that the only local actions which could possibly result in such a bound are semiprimitive actions: a permutation group is semiprimitive if each normal subgroup is either transitive or semiregular. Potočnik, Spiga and Verret [16] conjecture that, for a locally semiprimitive graph of valency d, the vertex stabiliser order should be bounded in terms of d. Studies by Bereczky and Maróti [3] on semiprimitive groups give some information about them, but more information is needed about the general structure of semiprimitive groups: this is on-going work of Giudici and Morgan [9], and Luke Morgan's lecture gave a helpful report on recent progress.

Thus the talks, even in this central area of permutation groups, addressed both progress with the fundamental theory of permutation groups, and many of their applications. There were obvious bridges spanning the different areas with new questions posed, and we saw collaborations newly formed or strengthened.

Algebraic groups, subgroup structure and group actions

An effective approach to solving problems in finite group theory is to reduce to the case of quasisimple groups. The large majority of these are finite groups of Lie type, which in turn are obtained as fixed point subgroups of certain endomorphisms of reductive algebraic groups. Hence one is led to include this family of infinite groups in the study of finite groups.

One of the themes of current research on finite and infinite reductive groups aims at understanding the subgroup lattice of these groups, knowledge of which is required for the study of permutation actions of the groups. The by-now classical reduction theorems of Aschbacher and Liebeck-Seitz have motivated the study of so-called *irreducible triples* (X, Y, V), where X is a subgroup of Y and V is an absolutely irreducible module for Y on which X acts irreducibly. Recent progress on the classification of such triples has been achieved in the work of Husen, Hiss and Magaard (an AMS Memoir on imprimitive actions for finite groups), Burness, Ghandour, Marion and Testerman, (two AMS Memoirs on irreducible triples for classical algebraic groups), and further work of Tiep, Kleshchev, Magaard, Röhrle, Testerman, and others for certain configurations of finite irreducible triples. Magaard and Testerman have an ongoing project to extend this work and the workshop provided an occasion for discussing this work. In a related project, Liebeck, Seitz and testerman are studying multiplicity free actions of simple algebraic groups and the meeting gave Seitz and Testerman the opportinuty to make progress and to discuss their work with other participants.

In a different direction, there has been significant progress in recent work of Stewart and Thomas, Thomas and Litterick, and Craven on understanding the subgroup lattice of the exceptional type algebraic groups. Stewart and Thomas consider *irreducible subgroups*, that is, those not lying in any proper parabolic subgroup, and *non completely reducible* subgroups, that is, those which lie in a proper parabolic subgroup but do not lie in any Levi factor of the parabolic. The techniques here involve some delicate calculations in the first and second cohomology groups of the embedded subgroup, techniques which have been developed and exploited in Stewart's work on the subgroups of F_4 and the non completely reducible subgroups of the exceptional groups. Thomas and Litterick have a joint project whose aim is to classify the non-completely reducible subgroups of the exceptional groups. Litterick reported on their progress in the case of good characteristic.

During the workshop Aschbacher reported on work on the Palfy-Pudlak question for exceptional groups, concerning the lattice of overgroups of a given subgroup in an exceptional type Lie group. Craven reported on work on Lie primitive subgroups of exceptional type groups. Further work on the subgroup structure of the exceptional type algebraic groups is being carried out by Craven and independently Burness and Testerman, in particular, the existence and uniqueness of certain A_1 type subgroups. The workshop provided an opportunity for several of these people to discuss this work.

Further work on the subgroup lattice of classical type groups is related to questions about the modular representation theory of these groups. Malle reported on recent joint work with Robinson giving a conjectural upper bound for the number of irreducible characters in a p-block of a group is evidence for some of the progress in this area. This very active area of research has seen considerable progress in the past 5-10 years, and has been the topic of varous conferences, workshops and research semesters. While there is a deep connection to the subgroup structure of Lie type groups, this was not the central theme of the meeting.

There is continuing work being done on the monodromy groups of compact, connected Riemann surfaces of genus at most 2, beyond the Frohardt-Magaard proof of the Guralnick-Thompson conjecture. In particular, Frohardt, Guralnick, and Magaard have recently shown that any such configuration arising from the action of a classical group on Ω, the set of 1-spaces of its natural modules, must satisfy $|\Omega| \leq 10^4$. In two recent papers Magaard and Waldecker analyze the structure of transitive permutation groups that have trivial four point stabilizers, but some nontrivial three point stabilizer and consider permutation groups that act nonregularly, such that every nontrivial element has at most two fixed points. The applications they have in mind are to the monodromy groups problem.

A further geometric construction which provides a rich family of interesting examples is that of letting a finite group act on a product of Riemann surfaces and quotienting out by this action to obtain a so-called *Beauville surface*. This then translates to studying goups with a *Beauville structure*, which is also related to the study of triangle groups. The recent work of Larsen, Marion and Lubotzky using deformation theory essentially proves Marion's conjecture concerning triangle generation of low-rank groups of Lie type. Their work led to the notion of "saturation"; roughly, a given hyperbolic triangle group is "saturated" with finite quotients of type Φ (here Φ is an irreducible root system) if it has finite quotients isomorphic to $\Phi(p^\ell)$ for infinitely many p and ℓ. While they have established a very strong result, there remain some interesting open questions. This is related to the question of generation in finite groups; here Burness, Liebeck and Shalev have recently completed some interesting work on the number of generators of maximal, second maximal, and third maximal subgroups in finite simple or quasisimple groups. This also has some interesting connections to number theoretic questions and may open new research directions. This also highlights one of the "cross-overs" between the different topics of the conference: Capdeboscq's talk on generation questions in Kac-Moody type groups took as a starting point the classical results on generation in finite quasisimple groups.

Infinite permutation groups and model theory

What might have been somewhat surprising, but certainly hoped for, was the discovery that even though the participants belonging to different groups in many cases did not previously know each other at all, it turned out during the talks that there were many interconnections and inspiring recurrences of themes in different settings.

Over the last years there have been major developments in infinite permutation group theory. One aspect here is the interaction between permutation group theory, combinatorics, model theory, and descriptive set theory, typically in the investigation of first order relational structures with rich automorphism groups. The connections between these fields are seen most clearly for permutation groups on countably infinite sets which are closed (in the topology of pointwise convergence) and oligomorphic (that is, have finitely many orbits on k-tuples for all k); these are exactly the automorphism groups of ω-categorical structures, that is, first order structures determined up to isomorphism (among countable structures) by their first order theory.

The use of group theoretic means (O'Nan-Scott, Aschbacher's description of maximal subgroups of classical groups, representation theory) to obtain structural results for model-theoretically important classes (totally categorical structures, or much more generally, approximable structures, finite covers of well-understood structures). Recent progress by Macpherson, Kaplan and Simon towards showing that natural classes of automorphism groups are maximal closed subgroups of the symmetric group give rise to hope for a complete classification in certain settings. These questions were addressed in both the talks by Macpherson and Simon. It was very inspiring to see the progress combining aspects from very different fields. Simon presented his joint work with Itay Kaplan, showing that $AGL_n(Q), n > 1$ and $PGL_n(Q), n > 2$ are maximal amongst closed proper subgroups of the infinite symmetric group.The proof relies on Adeleke and Macpherson's classification of infinite Jordan groups.

Properties which the full symmetric group S on a countable set shares with various other closed oligomorphic groups. We have in mind such properties as: complete description of the normal subgroup structure; uncountable cofinality (that is, the group is not the union of a countable chain of proper subgroups); existence of a conjugacy class which is dense in the automorphism group, or, better, comeagre (or better still, the condition of 'ample homogeneous generic automorphisms'); the Bergman property for a group (a recently investigated property of certain groups G, which states that if G is generated by a subset S, then there is a natural number n such that any element of G is expressible as a word of length at most n in $S \cup S^{-1}$); the small index property. These questions were also addressed in Macpherson's talk.

Reconstruction of a first order structure (up to isomorphism, up to having the same orbits on finite sequences, up to 'bi-interpretability') from its automorphism group, typically, presented as an abstract group.

Partially successful techniques here include the description of subgroups of the automorphism group of countable index (the 'small index property'), and first order interpretation of the structure in its automorphism group. This leads to a new, yet very natural direction of research which was presented in Nies' talk. He addressed the question of how complicated it is for a class $\mathcal{C}$ of closed subgroups of S to determine whether $G, H \in \mathcal{C}$ are conjugate or topologically isomorphic. He explained how to consider such classes of groups as Borel sets and how to study Borel reducibility in this context. Using a result of Lubotzky's he showed that the isomorphism relation between finitely generated profinite groups is Borel-equivalent to id_R and that for a prime $p \geq 3$, graph isomorphism can be Borel reduced to isomorphism between profinite nilpotent class 2 groups of exponent p.

In a more unexpected vein, Chatzidakis' talk presented yet a different connection between model theory and totally disconnected locally compact groups: if (K, f) is a difference field with a distinguished automorphism f, and a is a finite tuple in some difference field extending K, and such that $f(a)$ is algebraic over $K(a)$, then we define $dd(a/K) = \lim[K(fk(a), a) : K(a)]^{1/k}$, the distant degree of a over K. This is an invariant of the difference field extension of the algebraic closure $K(a)^{alg}$ of $K(a)$ over K. They show that there is some b in the difference field generated by a over K which is equi-algebraic with a over K, and such that $dd(a/K) = [K(f(b), b) : K(b)]$. This means that for every $k > 0$, we have $f(b)$ in $K(b, f^k(b))$. Viewing $Aut(K(a)^{alg}/K)$ as a locally compact group, this result is connected to results of Willis on scales of automorphisms of locally compact totally disconnected groups.

Totally disconnected locally compact groups

The inspiration for a number of talks at the workshop was a link between totally disconnected locally compact groups and permutation groups first seen in the paper [4] by M. Burger and S. Mozes, in which they related the structure of the group of tree automorphisms to its "local action" on the neighbours of vertices. Properties of the finite permutation group formed by this local action were thus seen to control properties of the infinite topological group of tree automorphisms.

Automorphism groups of trees that are 2-transitive on the boundary of the tree were the subject of Caprace's talk. Infinitely many topologically simple t.d.l.c. groups with flat-rank 1 arise as closed subgroups of the automorphism groups of semiregular trees and he surveyed recent classification and characterization results for these groups obtained by himself and his students Radu and Stulemeijer. The valency of the tree and properties of the local actions such as primitivity are important features in these results and establish a link with finite permutation groups. He also presented his approach to proofs of simplicity of groups through the existence of what he has called *micro-supported actions* of a group. This approach unifies and provides a clear insight into numerous simplicity proofs obtained in previous decades and will be an important method for future proofs.

Smith showed how to construct an uncountably infinite number of topologically simple compactly generated t.d.l.c. groups, all having flat-rank 1, using a generalization of the ideas of Burger-Mozes to invent what he calls a "box product" of permutation groups. He also described his classification of infinite primitive permutation groups having finite sub-orbits. As a variation of Cayley's Theorem shows, each compactly generated t.d.l.c. group may be represented as a permutation group with finite sub-orbits and since each such permutation group embeds densely in a t.d.l.c. group, and the topologically simple groups are an important part of his classification.

In his talk, Reid outlined a theory of decompositions of t.d.l.c. groups developed in collaboration with Wesolek. An important conclusion of the theory is that each compactly generated t.d.l.c. group may be broken down into a *finite number* of chief factors. The key idea, which again is based on the paper of Burger and Mozes [4] and its extension to the general situation by Caprace and Monod [6], is to represent the group as acting on a graph and to consider its local action. The point is then that, when the group is decomposed, the valency of this local action strictly decreases and the number of pieces in the decomposition must therefore be finite. Reid and Wesolek also show how to compare the factors occurring in any two decompositions of the same group.

Ramagge's talk concerned a direct link between t.d.l.c. groups and permutation groups established by Möller when he gave a new proof of basic results about the scale function and established the *spectral radius formula* for the scale which was referred to in the talk by Chatzidakis. She explained this link and then went on to describe current joint work with Praeger and Willis which is extending an equivalence found by Möller for a single group element to flat subsemigroups of elements. This extension involves yet another cross-over, this time to the field of operator algebras, because the relevant structure needed to establish the equivalence turns out to be that of a P-graph, a generalization of the notion of graph originally made in that field. The collaboration continued at the workshop and this work is now near completion.

Other talks on t.d.l.c. groups described recent advances on several fronts. Castellano spoke about a cohomology theory [8]. Wesolek and Garrido both gave talks about branch groups: Garrido characterizing primitive permutation actions of these groups and Wesolek showing the rigidity theorem that just-infinite groups in this class have no non-trivial commensurated subgroups. Capdeboscq talked about the size of generating sets for Kac-Moody groups over finite fields. Kac-Moody groups are closely related to algebraic groups and this work, which is related to corresponding investigations for finite simple groups and groups of Lie type, is another aspect of the connections between the different types of permutation groups.

Links between finite permutations, algebraic groups and t.d.l.c. groups were anticipated. Less anticipated were the links with model theory and infinite permutation groups which emerged. Chatzidakis described Galois groups which are t.d.l.c., thus providing a new class of these groups which remains to be explored, and how the scale function inspired results about field extensions. MacPherson also talked about how potentially new examples of t.d.l.c. groups that are infinite permutation groups might be constructed using model theoretic methods. Nies' talk on the complexity of classes of groups is relevant to attempts to classify simple t.d.l.c. groups up to isomorphism and other discussions regarding the complexity of computation in profinite groups should be relevant to attempts to evaluate the effectiveness of certain algorithms and arguments in t.d.l.c. groups with a view towards automating them.

5 Outcome of the Meeting

The workshop brought together both senior and mid- to early-career scientists, from North and South America, Europe, the Middle East and Oceania. Notably, of the 24 talks, 10 were given by post-doctoral fellows or early-career academics. Participants were drawn from four distinct, albeit closely related, fields and there was a chance that there would be little interaction between them. The risk that the organisers took in bringing together researchers from these distinct fields paid off in terms of transfer of knowledge between the fields and the potential for the formation of new collaborations.

Because of the nature of the conference, the majority of the audience for each talk had only a general interest in the field and only a broad knowledge of it. The talks presented significant recent advances in their respective fields, informing the experts who were present but also giving the non-experts an insight into the field. Verret, Fawcett, Schneider, Morgan and Giudici spoke about primitivity and classifications of finite permutation groups and geometries; Aschbacher, Craven, Litterick, Burness and Malle about classifications and structure of finite and, in particular, matrix groups; MacPherson, Smith, Garrido, Simon and Nies about infinite permutation groups and model theory; and Caprace, Wesolek, Castellano, Ramagge and Reid about totally disconnected, locally compact groups. There was often lively discussion following a talk, with participants from different specialties asking questions from their own perspectives. For example, questions asked by Smith about infinite graphs prompted Verret to apply methods used for finite graphs with some success, and MacPherson's talk on closed subgroups of the topological group $\mathrm{Sym}(\mathbb{N})$ provoked a lot of questions from the t.d.l.c. group community. In addition to these talks in particular fields, talks by Chatzidakis, Capdeboscq, Tiep and Segev bridged the interests of many participants, in some cases transferring ideas from one field to another and in other cases simply delivering an excellent talk for the general audience. The interest aroused by these talks is ongoing and will likely lead to future work. Chatzidakis' talk has prompted a series of working seminars on Galois groups to be held in Newcastle, Australia, for example.

Ample time was provided in the schedule of the workshop for participants to collaborate. This opportunity was taken up to established collaborations to continue existing projects, as with Testerman and Magaard for example, or to plan future work, as with Caprace, Reid and Willis for example. There were many discussion following talks, as mentioned in the previous paragraph, and new collaborations were initiated or are envisaged. Examples here are a planned collaboration between Ramagge and Garrido, and proposed exchange visits between Nies and Willis.

It is probably true to say that every participant, including the organisers, met several mathematicians, and their work, for the first time and learned many new ideas which will influence their future research.

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