Geometrical Degrees of Freedom in Topological Phases

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I. OVERVIEW OF THE FIELD

Geometry has long enjoyed deep connections with physics. In the context of quantum many-particle systems, one is typically interested in how the geometry of space affects the behavior of the interacting electron fluid, either via the underlying crystalline lattice or through the application of external spatial perturbations. In the absence of a lattice or of spatially varying perturbations, the electron fluid enjoys the continuous translational/rotational invariance of free space. In this context geometry is synonymous with spatial symmetry, which can be broken explicitly by the lattice (down to a residual discrete space group symmetry) or by external fields, or spontaneously, under the influence of sufficiently strong electron-electron interactions. The effect of space group symmetry on the quantum many-electron problem is at the core of conventional solid state physics, while spontaneously broken spatial symmetries in the interacting electron fluid have been the object of intense study for the past few decades, largely inspired by their classical analogs in liquid crystal systems.

Likewise, topology has had profound applications in several areas of physics. In condensed matter physics, and as illustrated by this year's Nobel Prize in Physics, topology has illuminated our understanding of and ability to classify defects ("topological defects") in phases with spontaneously broken symmetries, such as kinks, vortices, and hedgehogs. In particular, phases with spontaneously broken spatial symmetries such as smectics and nematics support interesting topological defects, dislocations and disclinations. Here topology refers to the global shape of a defect in real space, and connections between geometry and topology abound in these types of systems (e.g., the Gauss-Bonnet theorem). Since the 1980s, it has however become clear that topology plays another role besides the classification of defects: it can also classify quantum (zero-temperature) gapped phases of matter, where distinct phases of matter can be loosely understood as distinct topological equivalence classes of many-body ground-state wavefunctions. Phases in distinct equivalence classes must be separated by quantum phase transitions. Nontrivial equivalence classes are known as "topologically ordered phases" and exhibit exotic properties such as multiple degenerate ground states on noncontractible closed spatial manifolds (with the precise degeneracy depending on the homotopy type of the manifold and the particular topological phase), excitations with fractional statistics, and (sometimes) protected gapless boundary modes. The relevant topology here applies to the space of quantum ground states of gapped local Hamiltonians, and can be thought of as "quantum topology" (by contrast with the "classical topology" of topological defects in broken symmetry phases). In its most basic incarnation, topological order is insensitive to spatial geometry, and it would seem that there is no interesting interplay between geometry and quantum topology.

II. RECENT DEVELOPMENTS AND OPEN PROBLEMS

A number of recent developments, however, have challenged this basic premise and revealed a number of unexpected connections between geometry and topological order. The first is the discovery of topological insulators – gapped phases of matter that, despite not being topologically ordered in the sense described above (e.g., their ground state on a closed manifold is unique regardless of its topology), can be grouped into distinct topological equivalence classes and exhibit protected gapless boundary modes. The relevant notion of topological order here is weaker than its predecessor, in that it exists only in the presence of a symmetry, but (in the case of fermionic systems) can be realized in noninteracting systems and thus is amenable to simple theoretical treatments. This is by contrast with "intrinsic" topological order whose study typically requires the knowledge of nonperturbative solutions to a complicated many-body problem. The second development is the growing realization that the integer and fractional quantum Hall liquids, an important class of topological phases, seem particularly susceptible to the formation of phases of matter with spontaneously broken spatial symmetries. There is thus a need to understand the reason (if any) why there exists a special interrelation between topological and spatial order in these systems. Finally, various developments on the theoretical front have addressed the interplay between geometrical degrees of freedom (background/classical or dynamical in nature) and topological order, especially in the context of the fractional quantum Hall effect.

Below we summarize these developments, which served as the main inspiration for this workshop.

A. Topological phases protected by spatial symmetries

The concept of topological phase of matter was first introduced in the context of fractional quantum Hall states and spin liquids, where topological order requires strong correlations but exists in the absence of any symmetry. The discovery of topological insulators demonstrated the existence of a new class of topological phases symmetry-protected topological phases, which do not necessarily require strong correlations but whose existence is tied to a particular symmetry of the system (time-reversal symmetry in the case of topological insulators). There currently exists a fairly exhaustive classification of symmetry-protected topological phases of fermions and bosons in the case that the protecting symmetry is an on-site symmetry, i.e., a symmetry that acts locally in space. Much less is known if the symmetry is a spatial symmetry such as a mirror reflection or a spatial rotation that relates spatially separated points. Important progress has been made in this area for free-fermion systems, leading to the notion of topological crystalline insulator, a topological phase protected by crystallographic point group symmetries. Remarkably, this exotic topological phase was recently found experimentally in the semiconductor material SnTe. The problem becomes more difficult for symmetry-protected phases of bosons, which are intrinsically interacting. While the classification of bosonic topological phases protected by on-site symmetries is believed to be essentially complete, the problem of spatial symmetries remains largely unsolved. Another outstanding challenge is the classification of symmetry-enriched topological phases, where the intrinsic topological order characteristic of strongly correlated systems coexists with certain symmetries, which leads to exotic phenomena such as the fractionalization of quantum numbers. Some work has been done in this area for spatial symmetries, especially in the context of spin liquids, but a general classification is still lacking. Specific goals of the workshop for this theme of research are to extend the classification of (i) free-fermion topological phases protected by arbitrary spatial symmetries, (ii) topological phases of bosons (and interacting fermions) protected by arbitrary spatial symmetries, and (iii) topologically ordered phases of bosons or fermions enriched by arbitrary spatial symmetries.

B. Interplay of spatial and topological order

The prototypical topological phases that exhibit the integer and fractional quantum Hall effects are found in two-dimensional electron gases (2DEGs) confined in semiconductor heterostructures under high magnetic fields. The phase diagram of these 2DEGs is quite intricate. Owing to the delicate balance of competing interactions and electron degeneracies such as those due to intrinsic spin or discrete spatial symmetries these systems host a multitude of phases where topological order coexists or competes with spatial ordering. Besides the featureless states of matter that exhibit the quantum Hall effects in their basic form, 2DEGs at low magnetic fields hold bubble and stripe phases. These are novel types of electronic crystalline phases where crystallization is driven by the combination of field-induced electron motion and strong electron-electron interactions. Another line of study has focused on internal symmetries due to the crystalline lattice, that lead to electronic states with different spatial structure; when forming a quantum Hall state, the 2DEG must choose among different possibilities that break the orientational symmetry of space. The resulting interplay of spatial ordering, the edge states that are an inevitable consequence of quantum Hall order, and the local spatial anisotropies generated by frozen-in impurities leads to a plethora of new phases that combine many of the most interesting aspects of disordered systems with those of topological phases. In addition, recent experiments in GaAs quantum wells have observed anisotropic longitudinal transport coexisting with a fractionally quantized Hall conductivity. This suggests that the topological order characteristic of fractional quantum Hall states may coexist with broken spatial (translational and/or orientational) symmetries. These experimental observations have prompted theoretical studies of the quantum Hall fluid based on analogies to classical liquid crystals, and the corresponding interpretation of experimental data in terms of quantum Hall smectic and nematic phases. The possibility of phase transitions between spatially disordered, isotropic quantum Hall states and spatially ordered, anisotropic versions thereof requires a theoretical description that goes beyond the pure topological limit. Of particular interest are topological defects in the spatially ordered phases, such as disclinations and dislocations, which may acquire fractional charge and/or statistics as a result of the coupling between spatial and purely topological degrees of freedom. Related developments include the study of extrinsic topological defects in more general topological phases, which, surprisingly, can acquire non-Abelian statistics even though the topological order of the host topological phase is purely Abelian. Topics along those lines to be addressed in the workshop include (i) finding a general strategy to determine the coupling between spatial and topological degrees of freedom in a given topological phase, (ii) constructing specific theoretical models that exhibit quantum and/or thermal phase transitions between spatially disordered and spatially ordered topological phases, and (iii) suggesting experimental protocols to realize or engineer spatial topological defects in topological phases.

C. Gravitational response of topological phases

While much of the early work on the response of topological phases was motivated by the fractional quantum Hall effect and focused on their electromagnetic response as a result, much recent theoretical work has been devoted to the elastic and thermal response properties of topological phases. Unlike electromagnetic response properties, these response properties can be defined even in topological phases that do not have a conserved electric current, such as topological superconductors. As shown by Luttinger in the mid 1960s, thermal transport coefficients can be computed as the response of a system to an applied gravitational potential. More recent work has shown that other response properties such as the Hall viscosity are related to the response of a system to an applied geometry with torsion. In the context of topological phases, one is thus led to consider the coupling of topological degrees of freedom to fairly complex background geometries. Theoretical frameworks such as nonrelativistic diffeomorphism invariance and Newton-Cartan geometry with torsion have recently been applied to the study of the response properties of fractional quantum Hall systems, and may also apply to other types of topological phases. Finally, the gravitational response of topological phases is often related to the topic of gravitational anomalies in high-energy physics. Such anomalies are in turn are connected to the existence of exotic gapless boundary degrees of freedom in the topological phase. The workshop will seek to bring together these concepts in a unified description, and will address questions such as: Which gravitational response properties of a topological phase are universal, and which are not? How can one measure such properties? Can one develop a hydrodynamic description of topological phases that goes beyond the strict topological limit? Can one develop a description of the boundary degrees of freedom of topological phases that goes beyond the pure topological limit and takes geometry into account?

D. Emergent dynamical geometry in topological phases

Recent theoretical work in the context of the fractional quantum Hall effect suggests that topological phases may not only couple to a background geometry, but may also support emergent geometric degrees of freedom that are truly dynamical. Specifically, it has been proposed that fractional quantum Hall liquids admit a dynamical spatial metric tensor, unrelated to the (static) metric tensor of physical space. This intrinsic metric tensor is subject to quantum-mechanical fluctuations that are identified with the well-known Girvin-MacDonald-Platzman neutral collective mode in the fractional quantum Hall effect. Reversing the analogy, the neutral collective mode can then be given an interpretation in terms of a fluctuating metric. These ideas provoke many pressing questions. What is the correct quantum gravity theory that describes this fluctuating geometry? Can one devise experiments that confirm this interpretation of the neutral collective mode in the fractional quantum Hall effect? Is the emergence of a dynamical geometry specific to the fractional quantum Hall states, or can it apply to other topological phases? How does this dynamical geometry affect boundary degrees of freedom?

III. PRESENTATION HIGHLIGHTS

Our workshop was structured around eleven sessions, each with three presentations of fifteen minutes each, plus five minutes for questions. Here we briefly summarize the scientific content of these presentations.

A. Emergent geometry in topological phases (chaired by E. Fradkin)

- F. Duncan M. Haldane (Princeton University): "Geometry of flux attachment to composite bosons and fermions in a partially-filled Landau level". This talk discussed the emergence of a new geometrical degree of freedom, the guiding-center metric, that appears in fractional quantum Hall systems when projected to a single Landau level.

- Matthew Roberts (University of Chicago): "Neutral excitations of fractional quantum Hall states and the composite fermion liquid". Fractional quantum Hall liquids exhibit a rich set of excitations, the lowest-energy of which are the magneto-rotons with dispersion minima at finite momentum. We propose a theory of the magneto-rotons on the quantum Hall plateaux near half filling, namely, at filling fractions $\nu = N/(2N+1)$ at large N. The theory involves an infinite number of bosonic fields arising from bosonizing the fluctuations of the shape of the composite Fermi surface. At zero momentum there are $O(N)$ neutral excitations, each carrying a well-defined spin that runs integer values 2,3,... The mixing of modes at nonzero momentum q leads to the characteristic bending down of the lowest excitation and the appearance of the magneto-roton minima. A purely algebraic argument show that the magneto-roton minima are located at $q\ell_B = z_i/(2N+1)$, where ℓ_B is the magnetic length and z_i are the zeros of the Bessel function J_1, independent of the microscopic details. We argue that these minima are universal features of any two-dimensional Fermi surface coupled to a gauge field in a small background magnetic field.

- Zhengcheng Gu (Perimeter Institute): "The emergence of geometry on the interface of topological phases". This talk discussed the emergence of gapless degrees of freedom on domain walls between nonchiral topological phases, and how exactly souble lattice models can be constructed for them.

B. Band topology and geometry (chaired by S. Parameswaran)

- Marcel Franz (University of British Columbia): "Quantum oscillations without a magnetic field". When magnetic field B is applied to a metal, nearly all observable quantities exhibit oscillations periodic in $1/B$. Such quantum oscillations reflect the fundamental reorganization of electron states into Landau levels as a canonical response of the metal to the applied magnetic field. We predict here that, remarkably, in the recently discovered Dirac and Weyl semimetals quantum oscillations can occur in the complete absence of magnetic field. These zero-field quantum oscillations are driven by elastic strain which, in the space of the low-energy Dirac fermions, acts as a chiral gauge potential. We propose an experimental setup in which the strain in a thin film (or nanowire) can generate pseudomagnetic field b as large as 15 T and demonstrate the resulting de Haas-van Alphen and Shubnikov-de Haas oscillations periodic in $1/b$.
- Titus Neupert (University of Zürich): "Exotic fermions in topological metals". This talk discussed new classes of topological gapless Fermi systems: topological metals protected by 2D nonsymmorphic space group symmetries, 3D nodal line semimetals, and 3D nexus fermions.
- Rahul Roy (University of California, Los Angeles): "Role of quantum band geometry in the fractional quantum Hall effect in periodic systems". This talk discussed the role of band geometry in the fractional quantum Hall effect in systems without Landau levels (fractional Chern insulators).

C. Entanglement and transport (chaired by J. Maciejko)

- Tami Pereg Barnea (McGill University): "Topological transport out of equilibrium". This talk discussed the transport properties of topological edge states in the presence of time-periodic potentials, both for naturally occurring edge states and driven ones.
- William Witczak-Krempa (Harvard University): "Universal shape dependence of entanglement in gapless systems". This talk discussed the geometry of entanglement in correlated 2D gapless quantum systems, in particular for tori and cylinders.
- Jay Sau (University of Maryland): "Transport signatures of topological superconducting junctions: subtleties of the fractional Josephson effect". This talk discussed experimental signatures of the fractional Josephson effect (dc and ac) in topological superconductors.

D. Symmetries and anyon models (chaired by X. Chen)

- Bela Bauer (Microsoft Station Q): "Ising anyons in frustration-free Majorana-dimer models". Dimer models have long been a fruitful playground for understanding topological physics. Here we introduce a new class – termed Majorana-dimer models – wherein bosonic dimers are decorated with pairs of Majorana modes. We find that the simplest examples of such systems realize an intriguing, intrinsically fermionic phase of matter that can be viewed as the product of a chiral Ising theory, which hosts deconfined non-Abelian quasiparticles, and a topological $p_x - ip_y$ superconductor. While the bulk anyons are described by a single copy of the Ising theory, the edge remains fully gapped. Consequently, this phase can arise in exactly solvable, frustration-free models.
- Fiona Burnell (University of Minnesota): "Topological phases enhanced by global anyon-permuting symmetries: constructing solvable lattice models". Some anyon models admit anyon permuting symmetries meaning that species of anyons can be interchanged or permuted without changing the mutual statistics and other fundamental topological data. For example, exchanging the two bosons (usually called e and m) in the Toric code results in a theory that is topologically identical. However, this symmetry is not manifest in Kitaevs Toric code Hamiltonian. I will describe a procedure that yields manifestly symmetric solvable lattice Hamiltonians given a model with an underlying anyon-permuting symmetry (together with certain additional topological data).
- Sagar Vijay (Massachusetts Institute of Technology): "Fracton topological orders from a generalized lattice gauge theory". We introduce a generalization of conventional lattice gauge theory to describe fracton topological phases, which are characterized by immobile, point-like topological excitations, and sub-extensive topological degeneracy. We demonstrate a duality between fracton topological order and interacting spin systems with symmetries along extensive, lower-dimensional subsystems, which may be used to systematically search for and characterize fracton topological phases. Commutative algebra and elementary algebraic geometry provide

an effective mathematical toolset for our results. Our work paves the way for identifying possible material realizations of fracton topological phases.

E. Symmetries, topological phases and transitions (chaired by K. Shtengel)

- Yizhi You (University of Illinois at Urbana-Champaign): "Stripe melting, a transition between weak and strong symmetry protected topological phases". For a gapped disordered many-body system with both internal and translation symmetry, one can define the corresponding weak and strong Symmetry Protected Topological (SPT) phases. A strong SPT phase is protected by the internal symmetry G only while a weak SPT phase, fabricated by alignment of strong SPT state in a lower dimension, requires additional discrete translation symmetry protection. In this paper, we construct a phase transition between weak and strong SPT phase in strongly interacting boson system. The starting point of our construction is the superconducting Dirac fermions with pair density wave(PDW) order in 2d. We first demonstrate that the nodal line of the PDW contains a 1d boson SPT phase. We further show that melting the PDW stripe and condensing the nodal line provoke the transition from weak to strong SPT phase in 2d. The phase transition theory contains an $O(4)$ non-linear-σ-model with topological Θ-term emerging from the proliferation of domain walls bound to an SPT chain. Similar scheme also applies to weak-strong SPT transition in other dimensions and predicts possible phase transition from 2d to 3d topological order.

- Michael Hermele (University of Colorado Boulder): "Topological phases protected by point group symmetry". We consider symmetry protected topological (SPT) phases with crystalline point group symmetry, dubbed point group SPT (pgSPT) phases. We show that such phases can be understood in terms of lower-dimensional topological phases with on-site symmetry, and can be constructed as stacks and arrays of these lower-dimensional states. This provides the basis for a general framework to classify and characterize bosonic and fermionic pgSPT phases, that can be applied for arbitrary crystalline point group symmetry and in arbitrary spatial dimension. We develop and illustrate this framework by means of a few examples, focusing on three-dimensional states. We classify bosonic pgSPT phases and fermionic topological crystalline superconductors with $\mathbb{Z}_2^P$ (reflection) symmetry, electronic topological crystalline insulators (TCIs) with $U(1) \times \mathbb{Z}_2^P$ symmetry, and bosonic pgSPT phases with C_{2v} symmetry, which is generated by two perpendicular mirror reflections. We also study surface properties, with a focus on gapped, topologically ordered surface states. For electronic TCIs we find a $\mathbb{Z}_8 \times \mathbb{Z}_2$ classification, where the $\mathbb{Z}_8$ corresponds to known states obtained from non-interacting electrons, and the $\mathbb{Z}_2$ corresponds to a "strongly correlated" TCI that requires strong interactions in the bulk. Our approach may also point the way toward a general theory of symmetry enriched topological (SET) phases with crystalline point group symmetry.

- Ying Ran (Boston College): "Anyon condensation and generic tensor-network constructions of (either on-site or spatial) SPTs". This talk discussed an approach to construct generic wavefunctions for symmetry-protected topological (SPT) phases using tensor-network wavefunctions, in particular for spatial symmetries, and discussed a connection between SPT phases and symmetry-enriched topological phases (SET) via anyon condensation.

F. Adiabaticity and topological response (chaired by J. Sau)

- Kai Sun (University of Michigan Ann Arbor): "Adiabatic continuity, wavefunction overlap and topological phase transitions". In this talk, we discuss the relation between wavefunction overlap and adiabatic continuity in gapped quantum systems. We show that for two band insulators, a scalar function can be defined in the momentum space, which characterizes the wavefunction overlap between Bloch states in the two insulators. If this overlap is nonzero for all momentum points in the Brillouin zone, these two insulators are adiabatically connected, i.e. we can deform one insulator into the other smoothly without closing the band gap. In addition, we further prove that this adiabatic path preserves all the symmetries of the insulators. The existence of such an adiabatic path implies that two insulators with nonzero wavefunction overlap belong to the same topological phase. This relation, between adiabatic continuity and wavefunction overlap, can be further generalized to correlated systems. The generalized relation cannot be applied to study generic many-body systems in the thermodynamic limit, because of the orthogonality catastrophe. However, for certain interacting systems (e.g. quantum Hall systems), the quantum wavefunction overlap can be utilized to distinguish different quantum states. Experimental implications are also discussed.

- Sergej Moroz (TU München): "Topological order, symmetry, and Hall response of two-dimensional spin-singlet superconductors". Fully gapped two-dimensional superconductors coupled to dynamical electromagnetism are known to exhibit topological order. In this work, we develop a unified low-energy description for spin-singlet paired states by deriving topological Chern-Simons field theories for s-wave, $d+id$, and chiral higher even-wave superconductors. These theories capture the quantum statistics and fusion rules of low-energy excitations and incorporate global continuous symmetries – specifically, spin rotation and conservation of magnetic flux – present in all singlet superconductors. We compute the Hall response for these symmetries and investigate the physics at the edge. In particular, the weakly-coupled phase of a chiral state paired in the even kth partial wave has a spin Hall coefficient $\nu_s = k$ and a vanishing Hall response for the magnetic flux symmetry. We argue that the latter is a generic result for two-dimensional superconductors with gapped photons, thereby demonstrating the absence of a spontaneous magnetic field in the ground state of chiral superconductors.

- Masaki Oshikawa (University of Tokyo): "Polarization and gauge invariance". Quantum systems on a non-simply connected space possess a "large" gauge invariance. Laughlin utilized this to explain quantum Hall effect. Later, it was applied to elucidate a universal relation between filling factor and energy spectrum in quantum many-body systems on periodic lattices (Lieb-Schultz-Mattis-M.O.-Hastings). Somewhat surprisingly, the large gauge invariance is also deeply related to modern theory of electric polarization developed by Resta et al. I will give an overview of applications of the large gauge invariance to condensed matter physics, and also discuss most recent results obtained by combining it with the theory of polarization.

G. Geometry and FQHE I (chaired by M. Zaletel)

- Andrey Gromov (University of Chicago): "Geometric defects in quantum Hall states". We describe a geometric (or gravitational) analogue of the Laughlin quasiholes in the fractional quantum Hall states. Analogously to the quasiholes these defects can be constructed by an insertion of an appropriate vertex operator into the conformal block representation of a trial wavefunction, however, unlike the quasiholes these defects are extrinsic and do not correspond to true excitations of the quantum fluid. We construct a wavefunction in the presence of such defects and explain how to assign an electric charge and a spin to each defect, and calculate the adiabatic, non-abelian statistics of the defects. The defects turn out to be equivalent to the genons in that their adiabatic exchange statistics can be described in terms of representations of the mapping class group of an appropriate higher genus Riemann surface. We present a general construction that, in principle, allows to calculate the statistics of $\mathbb{Z}_n$ genons for any "parent" topological phase. We illustrate the construction on the example of the Laughlin state and perform an explicit calculation of the braiding matrices. In addition to non-abelian statistics geometric defects possess a universal abelian overall phase, determined by the gravitational anomaly.

- Barry Bradlyn (Princeton University): "Geometric response of anisotropic quantum Hall states". This work discussed geometric response functions (e.g., Hall viscosity) in integer/fractional quantum Hall systems with broken rotational invariance.

- Gil Young Cho (Korea Advanced Institute of Science and Technology): "Geometric responses of fractional quantum Hall effects". This talk discussed the geometry of fractional quantum Hall fluids from the perspective of composite boson/fermion effective field theories.

H. Geometry and FQHE II (chaired by I. Sodemann)

- Tankut Can (Simons Center for Geometry and Physics): "Probing quantum Hall states with flux tubes, cones, and cusps". My talk will concern the Laughlin wave function in the presence of magnetic field and spatial curvature singularities. I will discuss both local observables and adiabatic transport of singularities. The latter leads to an interpretation of singularities as localized coherent states whose charge, spin, and statistics turn out to be controlled by the Hall viscosity (charge) and the central charge (spin and statistics).

- Zlatko Papić (University of Leeds): "Geometry and anisotropy in the fractional quantum Hall effect". Haldane pseudopotentials have played a key role in the study of the fractional quantum Hall (FQH) effect as they allow an arbitrary rotationally-invariant interaction to be expanded over projectors onto the two-particle eigenstates of relative angular momentum. Here we introduce a more general class of pseudopotentials that form a complete basis in the cases where rotational symmetry is explicitly broken, e.g., due to tilted magnetic field or anisotropic band structure. Similar to the standard isotropic pseudopotentials, the generalized pseudopotentials are also parametrized by a unimodular metric, which groups the effective interactions into equivalence classes, and is particularly useful in determining optimal model Hamiltonians of the anisotropic FQH fluids. We show that purely anisotropic pseudopotentials lead to new types of bound "molecular" states for a few particles in an infinite plane. We discuss realizations of the generalized pseudopotentials in systems with tilted magnetic field, as well as fractional Chern insulators harboring intrinsic anisotropy due to the underlying lattice structure.

- Nicolas Regnault (École Normale Supérieure): "Evidence of a fractional quantum Hall nematic phase in a microscopic model". At small momenta, the Girvin-MacDonald-Platzman (GMP) mode in the fractional quantum Hall (FQH) effect can be identified with gapped nematic fluctuations in the isotropic FQH liquid. This correspondence would be exact as the GMP mode softens upon approach to the putative point of a quantum phase transition to a FQH nematic. Motivated by these considerations as well as by suggestive evidence of an FQH nematic in tilted field experiments, we have sought evidence of such a nematic FQHE in a microscopic model of interacting electrons in the lowest Landau level at filling factor 1/3. Using a family of anisotropic Laughlin states as trial wave functions, we find a continuous quantum phase transition between the isotropic Laughlin liquid and the FQH nematic. Results of numerical exact diagonalization also suggest that rotational symmetry is spontaneously broken, and that the phase diagram of the model contains both a nematic and a stripe phase.

I. Dualities (chaired by S. Parameswaran)

- Max Metlitski (Perimeter Institute): "Particle-vortex duality of Dirac fermion in 2+1D from electric-magnetic duality of topological insulators in 3+1D". Particle-vortex duality is a powerful theoretical tool that has been used to study bosonic systems. Here we propose an analogous duality for Dirac fermions in 2+1 dimensions. The physics of a single Dirac cone is proposed to be described by a dual theory, QED3 with a dual Dirac fermion coupled to a gauge field. This duality is established by considering two alternate descriptions of the 3d topological insulator (TI) surface. The first description is the usual Dirac cone surface state. The second description is accessed via an electric-magnetic duality of the bulk TI coupled to a gauge field, which maps it to a gauged topological superconductor. This alternate description ultimately leads to a new surface theory - dual QED3. The dual theory provides an explicit derivation of the T-Pfaffian state, a proposed surface topological order of the TI, which is simply the paired superfluid state of the dual fermions. The roles of time reversal and particle-hole symmetry are exchanged by the duality, which connects some of our results to a recent conjecture by Son on particle-hole symmetric quantum Hall states.

- Chong Wang (Harvard University): "A duality web in 2+1 dimensions". Building on earlier work in the high energy and condensed matter communities, we present a web of dualities in 2+1 dimensions that generalize the known particle/vortex duality. Some of the dualities relate theories of fermions to theories of bosons. Others relate different theories of fermions. For example, the long distance behavior of the 2+1-dimensional analog of QED with a single Dirac fermion (a theory known as $U(1)_{1/2}$) is identified with the O(2) Wilson-Fisher fixed point. The gauged version of that fixed point with a Chern-Simons coupling at level one is identified as a free Dirac fermion. The latter theory also has a dual version as a fermion interacting with some gauge fields. Assuming some of these dualities, other dualities can be derived. Our analysis resolves a number of confusing issues in the literature including how time reversal is realized in these theories. It also has many applications in condensed matter physics like the theory of topological insulators (and their gapped boundary states) and the problem of electrons in the lowest Landau level at half filling. (Our techniques also clarify some points in the fractional Hall effect and its description using flux attachment.) In addition to presenting several consistency checks, we also present plausible (but not rigorous) derivations of the dualities and relate them to 3+1-dimensional S-duality.

- Itamar Kimchi (Massachusetts Institute of Technology): "Fermion duality and CT-symmetry in a quantum Hall bilayer". We study the interplay of particle-hole symmetry and fermion-vortex duality in multicomponent half-filled Landau levels, such as quantum Hall gallium arsenide bilayers and graphene. For the $\nu = 1/2 + 1/2$ bilayer, we show that particle-hole-symmetric interlayer Cooper pairing of composite fermions leads to precisely the same phase as the electron exciton condensate realized in experiments. This equivalence is easily understood by applying the recent Dirac fermion formulation of $\nu = 1/2$ to two components. It can also be described by Halperin-Lee-Read composite fermions undergoing interlayer $p_x + ip_y$ pairing. An RG analysis showing strong instability to interlayer pairing at large separation $d \to \infty$ demonstrates that two initially-decoupled composite Fermi liquids can be smoothly tuned into the conventional bilayer exciton condensate without encountering a phase transition. We also discuss multicomponent systems relevant to graphene, derive related phases including a $\mathbb{Z}_2$ gauge theory with spin-half visons, and argue for symmetry-enforced gaplessness under full $SU(N_f)$ flavor symmetry when the number of components N_f is even.

J. Twists in topological phases (chaired by F. Burnell)

- Maissam Barkeshli (Microsoft Station Q): "Realizing modular transformations in physical systems". The ground state subspace of a topological phase of matter forms a representation of the mapping class group of the space on which the state is defined. We show that elements of the mapping class group of a surface of genus g can be obtained through a sequence of topological charge projections along at least three mutually intersecting non-contractible cycles. We demonstrate this both through the algebraic theory of anyons and also through an analysis of the topology of the space-time manifold. We combine this result with two observations: (i) that surfaces of genus g can be effectively simulated in planar geometries by using bilayer, or doubled, versions of the topological phase of interest, and inducing the appropriate types of gapped boundaries; and (ii) that the required topological charge projections can be implemented as adiabatic unitary transformations by locally tuning microscopic parameters of the system, such as the energy gap. These observations suggest a possible path towards effectively implementing modular transformations in physical systems. In particular, they also show how the Ising $\otimes$ $\overline{\text{Ising}}$ state, in the presence of disconnected gapped boundaries, can support universal topological quantum computation.

- Abolhassan Vaezi (Stanford University): "A DMRG study of topological domain walls in fractional quantum Hall states". This talk discuss the numerical simulation of parafermion zero modes and genons using the density-matrix renormalization group.

- Peng Ye (University of Illinois at Urbana-Champaign): "Charles symmetry and extrinsic twist defect in three spatial dimensions". While two-dimensional symmetry-enriched topological phases (SETs) have been studied intensively and systematically, three-dimensional ones are still open issues. We propose an algorithmic approach of imposing global symmetry G_s on gauge theories (denoted by GT) with gauge group G_g. The resulting symmetric gauge theories are dubbed "symmetry-enriched gauge theories" (SEG), which may be served as low-energy effective theories of three-dimensional symmetric topological quantum spin liquids. We focus on SEGs with gauge group $G_g = \mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \times \cdots$ and on-site unitary symmetry group $G_s = \mathbb{Z}_{K_1} \times \mathbb{Z}_{K_2} \times \cdots$ or $G_s = U(1) \times \mathbb{Z}_{K_1} \times \cdots$. Each SEG$(G_g, G_s)$ is described in the path integral formalism associated with certain symmetry assignment. From the path-integral expression, we propose how to physically diagnose the ground state properties (i.e., SET orders) of SEGs in experiments of charge-loop braidings (patterns of symmetry fractionalization) and the *mixed* multi-loop braidings among deconfined loop excitations and confined symmetry fluxes. From these symmetry-enriched properties, one can obtain the map from SEGs to SETs. By giving full dynamics to background gauge fields, SEGs may be eventually promoted to a set of new gauge theories (denoted by GT*). Based on their gauge groups, GT*s may be further regrouped into different classes each of which is labeled by a gauge group G_g^*. Finally, a web of gauge theories involving GT, SEG, SET and GT* is achieved. We demonstrate the above symmetry-enrichment physics and the web of gauge theories through many concrete examples.

K. Novel correlated phases (chaired by V. Gurarie)

- Tim Hsieh (Kavli Institute for Theoretical Physics): "All Majorana models with translation symmetry are supersymmetric". We establish results similar to Kramers and Lieb-Schultz-Mattis theorems but involving only translation symmetry and for Majorana modes. In particular, we show that all states are at least doubly degenerate in any one and two dimensional array of Majorana modes with translation symmetry, periodic boundary conditions, and an odd number of modes per unit cell. Moreover, we show that all such systems have an underlying $\mathcal{N} = 2$ supersymmetry and explicitly construct the generator of the supersymmetry. Furthermore, we establish that there cannot be a unique gapped ground state in such one dimensional systems with anti-periodic boundary conditions. These general results are fundamentally a consequence of the fact that translations for Majorana modes are represented projectively, which in turn stems from the anomalous nature of a single Majorana mode. An experimental signature of the degeneracy arising from supersymmetry is a zero-bias peak in tunneling conductance.

- Kirill Shtengel (University of California, Riverside): "Fractional Weyl semimetals". Formulating consistent theories describing strongly correlated metallic topological phases is an outstanding problem in condensed matter physics. In this work we derive a theory defining a fractionalized analogue of the Weyl semimetal state: the fractional chiral metal. Our approach is to construct a 4+1D quantum Hall insulator by stacking 3+1D Weyl semimetals in a magnetic field. In a strong enough field the low-energy physics is determined by the lowest Landau level of each Weyl semimetal, which is highly degenerate and chiral, motivating us to use a coupled-wire approach. The one-dimensional dispersion of the lowest Landau level allows us to model the system as a set of degenerate 1+1D quantum wires that can be bosonized in the presence of electron-electron interactions and coupled such that a gapped phase is obtained, whose response to an electromagnetic field is given in terms of a Chern-Simons field theory. At the boundary of this phase we obtain the field theory of a 3+1D gapless fractional chiral state, which we show is consistent with a previous theory for the surface of a 4+1D Chern-Simons theory. The boundary's response to an external electromagnetic field is determined by a chiral anomaly with a fractionalized coefficient. We suggest that such anomalous response can be taken as a working definition of a fractionalized strongly correlated analogue of the Weyl semimetal state.

- Liang Fu (Massachusetts Institute of Technology): "Electron teleportation in Majorana islands". We study a topological superconductor island with spatially separated Majorana modes coupled to multiple normal metal leads by single electron tunneling in the Coulomb blockade regime. We show that low-temperature transport in such Majorana island is carried by an emergent charge-e boson composed of a Majorana mode and an electron from the leads. This transmutation from Fermi to Bose statistics has remarkable consequences. For noninteracting leads, the system flows to a non-Fermi liquid fixed point, which is stable against tunnel couplings anisotropy or detuning away from the charge-degeneracy point. As a result, the system exhibits a universal conductance at zero temperature, which is a fraction of the conductance quantum, and low-temperature corrections with a universal power-law exponent. In addition, we consider Majorana islands connected to interacting one-dimensional leads, and find different stable fixed points near and far from the charge-degeneracy point.

IV. OUTCOME OF THE MEETING

One of our chief objectives was to bring together researchers with a broad range of expertise representing both the mathematical and more applied ends of the theoretical physics spectrum, but with a common interest in geometrical aspects of topological phases of matter. As can be seen from the range of topics covered in the presentations summarized in the previous, we believe this goal has been achieved. Various approaches to geometrical aspects of topological aspects were discussed, ranging from exactly soluble lattice models to effective field theories and numerical simulations.

Allowing ample time for discussions also made it possible to reach our goal of hopefully fostering future collaborations among the participants. While there were no presentations by experimentalists, a number of recent experimental advances (e.g., those relating to the discovery of materials exhibiting topological crystalline order, novel topological metallic/semimetallic phases, nematic quantum Hall/composite Fermi liquid phases in gallium arsenide, multicomponent quantum Hall systems such as gallium arsenide bilayers and graphene) were the object of several discussions among the participants.

Collectively, we look forward to witnessing the ramifications of this workshop in this community's future work.