Algebraic and Statistical ways into Quantum Resource Theories

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1 Overview of the Field

Quantum information science is one of the most active, intellectually stimulating, and technologically promising areas in science. It offers a unique opportunity to engage in a wide variety of topics, such as the mathematical and logical foundations of quantum theory, the theory of quantum computation and quantum Shannon theory, and practical applications like quantum cryptography and quantum sensing. Within quantum information science, an increasingly important role is played by *quantum resource theories* (QRTs), a collective name accounting for the fact that some distinctive features of quantum mechanics, like entanglement and coherence, are not just qualitative traits of quantum systems, but are tangible resources that can be extracted, transformed, traded for one another, and transferred from one system to another [1]. It is quite natural to apply a resource-theoretic outlook to the study of quantum systems since processes like decoherence rapidly eliminate most quantum behavior of a system. Like an oil digger, one must exert considerable experimental effort to witness and control the subtle effects of quantum mechanics.

The basic idea of a quantum resource theory is to study quantum information processing under a restricted set of physical operations. The permissible operations are called "free," and because they do not encompass all physical processes that quantum mechanics allows, only certain physically realizable states of a quantum

Figure 1: In a quantum resource theory, the precious commodity is some physical property or phenomenon that emerges according to the principles of quantum mechanics. The paradigmatic example is quantum entanglement.

system can be prepared. These accessible states are likewise called "free," and any state that is not free is called a resource state. Thus a quantum resource theory identifies every physical process as being either free or prohibited, and similarly it classifies every quantum state as being either free or a resource.

The most celebrated example of a quantum resource theory is the theory of entanglement. For two or more quantum systems, entanglement can be characterized as a resource when the allowed dynamics are local quantum operations and classical communication (LOCC). For example, as depicted in Fig. 2, Alice and Bob may be working in their own quantum laboratory while being separated from each other by some large distance. Due to current technological limitations, the only communication channel connecting their laboratories is classical, such as a telephone. Hence Alice cannot directly send quantum states to Bob and vice versa, and the free operations in this resource theory consists of LOCC. While the classical communication channel allows for the preparation of classically correlated states between the two laboratories, not every type of joint quantum state can be realized for Alice and Bob's systems using LOCC. A state is said to be entangled, and therefore a resource, precisely when it *cannot* be generated using the free operations of LOCC. For instance, if Alice and Bob each control a single spin-1/2 quantum system, the singlet state $\sqrt{1/2}(|01\rangle - |10\rangle)$ cannot be created by LOCC and it is therefore called an entangled state.

Inspired by the success of entanglement theory, researchers have adopted the resource theory framework within many other areas of quantum information and physics. For example, asymmetry and quantum reference frames, quantum thermodynamics, quantum coherence and superposition, secret correlations in quantum and classical systems, non-Gaussianity in bosonic systems, "magic states" in stabilizer quantum computation, non-Markovianity in multi-part quantum processes, nonlocality, and quantum correlations have all been studied as resource theories. Even more foundational objects such as contextuality and Bell non-locality have been envisioned as resources within quantum information theory.

At the same time, operator theory and mathematical statistics represent two very deep, extremely active, and intimately interconnected areas of mathematics that have provided the formal basis for the development of statistical mechanics, quantum theory, and quantum field theory. Notions like algebra of observables, complete positivity, or quantum hypothesis testing have appeared very soon after the inception of quantum theory and have been used ubiquitously ever since. Developments in quantum physics have often served as inspiration for new results in operator theory and statistics. These fields have largely benefited from mutual influences, cross-breeding, and feedbacks.

It has recently been discovered how generalized resource theories carry many similarities with the theory of statistical comparisons in mathematical statistics. In the latter, the statistician is interested in answering questions like, Is one statistical test more informative than another one in deciding between alternative hypotheses? or Which statistical test, chosen among a set of alternatives, is the most informative one? The theory of statistical comparison was established in the 1950s by work of Blackwell, Sherman, and Stein (BSS), as a generalization of the theory of majorization.

The theory of quantum statistical comparison has advanced as an emerging area in quantum statistics, with important contributions that extends initial quantum generalizations of the classical BSS theory [6] to the approximate case [29] and the case of infinite dimensional quantum systems [7]. Extremely important connections with the theory of operator Schur-convexity have also been explored [42]. The program of quantum statistical comparison has become intertwined with the program of characterizing generalized entropy for quantum systems via the notion of reverse tests [9].

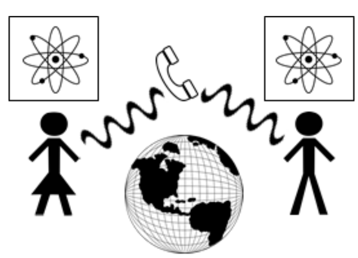

Figure 2: Quantum entanglement is a quantum resource in the "distant-lab" scenario where the free operations are LOCC.

Recent work has shown how the theory of statistical comparison can provide a new insight into quantum resource theories, in particular, quantum nonlocality [10], and quantum thermodynamics and the resource theories of asymmetry and coherence [2, 3]. Indeed, the only (to date) known complete set of necessary and sufficient conditions for arbitrary quantum state transformation under thermodynamic processes [4] has been obtained using the framework of quantum relative majorization [5] and quantum statistical comparison [6]. This workshop aimed to create new links between operator theory, mathematical statistics, and the burgeoning field of quantum information science in particular, quantum thermodynamics and generalized resource theories.

2 Presentation Highlights and Scientific Progress Made

This workshop united over forty international researchers to present their work on QRTs, discuss recent results, and stimulate new research directions. The results and scientific work covered in the workshop are summarized in the following.

- **General Structures of QRTs**

  One of the advantages to adopting a resource-theoretic approach to studying some quantum phenomenon is that it allows one to leverage techniques and analytic tools that apply to general QRTs. Recent work has focused on identifying key structural properties shared by all QRTs that satisfy certain mild conditions. In this workshop, some general features that emerge when casting QRTs in terms of von Neumann subalgebras were covered [11]. Complementary findings for the task of one-shot resource cost and distillation in general QRTs were also discussed [12]. It was further shown how the resource objects in any QRT with convex structure have an operational interpretation of being advantageous in some channel discrimination task [13].

- **Quantum coherence and thermodynamics**. The QRT of quantum coherence analyzes the operational utility of superposition and off-diagonal elements in the density matrix. In this workshop, techniques for computing the robustness measure of coherence were demonstrated [14]. Recent work on extending coherence theory to the level of quantum operations and superchannels was also presented [15]. Applications to clock synchronization via coherence distillation were discussed [16]. In the QRT of thermodynamics, an application to molecular transitions and their thermodynamics costs was described [17].

- **Entanglement and nonlocality** Quantum entanglement and nonlocality are two quintessential quantum resources that emerge in multipartite systems. In the workshop, recent efforts to understand the structure of multipartite entanglement from a QRT perspective were described [18]. Quantum entanglement is the key resource in performing quantum teleportation, and new bounds in the asymptotic cost of port-based teleportation were presented [19]. Entanglement is also used for "embezzling" state transformations, and a rigorous analysis of quantum embezzlement was conducted within the context of a QRT [20]. A resource related to entanglement is quantum nonlocality, and its presence is detected through the violation of a Bell Inequality. Recently, such violations have been shown to certify the type of quantum state shared between the different parties, and some results in this direction were presented [21].

- **Quantum computation**. QRTs for quantum computation attempt to identify the properties of quantum systems that enable them to perform certain computations seemingly faster than the best classical algorithms. New measures of quantum computational "magic" were discussed as well as new bounds on magic state convertibility [22, 23]. In addition, an updated analysis of the matchgate quantum computational model and its classical simulation were presented [24].

- **Pairwise conversion of resources**. A particular problem arising in all QRTs is to determine when one pair of resource states (ρ_1, σ_1) can be transformed into another (ρ_2, σ_2) using a free operation in the QRT. An analysis of this problem in general majorization-based QRTs was presented [25, 26]. These results included the case of converting pairs of states from one to the other, and variations to this central problem were analyzed both in the one-shot and asymptotic settings [27, 28]. The problem was also extended to the pairwise convertibility of quantum channels [29].

- **Quantum Shannon Theory** Quantum Shannon Theory exemplifies a QRT in that it studies the convertibility between different information-theoretic resources in the presence of restrictions. New tools in the study of quantum information systems were presented including entropic continuity bounds [30], improved decoupling techniques [31], and De Finetti Theorems for Quantum Channels [41].

- **Quantum channels and superchannels**. A dynamical QRT studies properties of quantum channels and quantum measurements. Work was presented showing how the structure of devices can be inferred from the study of classical output data [33, 34, 35]. The general theory of quantum superchannels and quantum combs was surveyed [36]. In addition, a framework for quantifying the resourcefulness of quantum channels was described in considerable detail [37, 38]. Specific applications discussed include the Quantum Zeno Effect [39].

3 Open Problems

A highlight of the workshop was the two open problem sessions. All participants were encouraged to pose an interesting problem to the community related to quantum resource theories. Here we record the open problems that were presented.

- **Parallel versus sequential strategies for quantum channel discrimination** - Presenter: Mark Wilde. **Comment: A solution to this problem has recently been given in arXiv:quant-ph/1909.05826 with an acknowledgement of the workshop.**. An experimenter has access to one of two unknown quantum channels, $\mathcal{N}_1$ or $\mathcal{N}_2$, that both act on system S. The goal is to identify which of the two channels is given in the many-copy setting. A *parallel discrimination strategy* involves applying n copies of the channel to an entangled state ρ^{RS^n} and them performing a joint measurement on the outcome state $\mathrm{id}^{\otimes n} \otimes \mathcal{N}_i^{\otimes n}(\rho^{RS^n})$. Based on the measurement outcome, a guess is made to the channel's identity i. In the language of hypothesis testing, it is known that the optimal rate for the type-two error exponent is given by a regularized channel relative entropy [40]: $D^\infty(\mathcal{N}_1\|\mathcal{N}_2) = \lim_{n\to\infty}\frac{1}{n}D(\mathcal{N}_1^{\otimes n}\|\mathcal{N}_2^{\otimes n})$, where
$$D(\mathcal{N}_1\|\mathcal{N}_2) = \sup_{\psi^{RS}} D(\mathrm{id}^R\otimes\mathcal{N}_1(\psi^{RS})\|\mathrm{id}^R\otimes\mathcal{N}_2(\psi^{RS})).$$
In contrast, a *sequential discrimination strategy* does not involve using all n channels at once; rather, the output of the j^{th} channel can be used in the $j^{th}+1$ input. In the sequential setting, the optimal discrimination rate is given by the amortized relative entropy [41]:
$$D^A(\mathcal{N}_1\|\mathcal{N}_2) = \sup_{\rho^{RS},\sigma^{RS}} D(\mathrm{id}^R\otimes\mathcal{N}_1(\rho^{RS})\|\mathrm{id}^R\otimes\mathcal{N}_2(\sigma^{RS})) - D(\rho^{RS}\|\sigma^{RS}).$$
In general the adaptive strategy is no worse that the parallel strategy, in the sense that
$$D^\infty(\mathcal{N}_1\|\mathcal{N}_2) \le D^A(\mathcal{N}_2\|\mathcal{N}_2). \tag{1}$$
For classical channels and quantum-classical channels it is known that this inequality is tight. The open problem is to determine whether there exists quantum channels in which this is a strict inequality, that is, whether adaptive discrimination can be strictly more powerful than parallel discrimination.

- **Second-order asymptotics in pairwise state convertibility** - Presenter: Marco Tomamichel. Given two pairs of quantum states (ρ_1,σ_1) and (ρ_2,σ_2), a general problem is to decide whether there exists a quantum channel $\mathcal{E}$ such that $\mathcal{E}(\sigma_1)=\sigma_2$ and $\mathcal{E}(\rho_1)\approx_\epsilon \rho_2$. Such a transformation can be denoted as
$$(\rho_1,\sigma_1)\to_\epsilon(\rho_2,\sigma_2),$$
this question arises in QRTs defined by some fixed point constraint on the allowed maps $\mathcal{E}(\sigma_1)=\sigma_1$, such as Gibbs-preserving maps in thermodynamics. In the special case of $\epsilon=0$, the solution is known [42, 5]. The asymptotic version of this problem considers the largest rate R such that for all $\epsilon>0$ there exists a sufficiently large n such that
$$(\rho_1^{\otimes n},\sigma_1^{\otimes n})\to_\epsilon(\rho_2^{\otimes Rn},\sigma_2^{\otimes Rn}).$$

It is known that the optimal R_∞ is given by [28, 43]

$$R_\infty = \frac{D(\rho_1\|\sigma_1)}{D(\rho_2\|\sigma_2)}. \tag{2}$$

However, the second-order terms in the rate are not known, and the open problem is to characterize, for a given n and ϵ, the exact achievable values $R_{n,\epsilon}$ such that $(\rho_1^{\otimes n}, \sigma_1^{\otimes n}) \to_\epsilon (\rho_2^{\otimes R_{n,\epsilon} n}, \sigma_2^{\otimes R_{n,\epsilon} n})$.

- **Sequential channel simulation** - Presenter: Andreas Winter. The Reverse Shannon Theorem addresses the problem of simulating a given quantum channel $\mathcal{N}$ using one-way classical communication channels plus local operations with unlimited shared entanglement (LOSE) [44, 45]. In terms of resource transformation, this can be expressed as

$$lR \cdot [c \to c] + (LOSE) \to \mathcal{N}^{\otimes l},$$

which says that lR bits of classical communication with LOSE can simulate l copies of $\mathcal{N}$. In this setting, LOSE represents the free operations in the QRT, and the goal is to find the minimal rate R for which this transformation is possible. Notice that this describes a *parallel* simulation of $\mathcal{N}$ in the sense that $\mathcal{N}^{\otimes l}$ is an object that acts on l input spaces all at once. A more general simulation involves reproducing l uses of $\mathcal{N}$ that may be applied in a sequential manner. Figure 3 provides an example of three sequential uses of $\mathcal{N}$. The goal is to simulate such a dynamical resource using

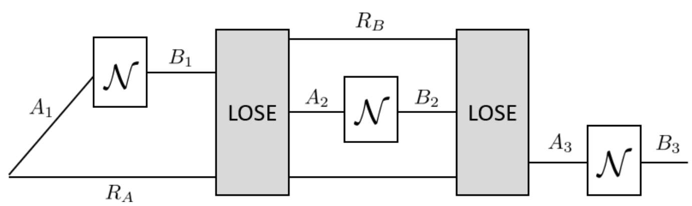

Figure 3: Three sequential uses of $\mathcal{N}$.

classical communication and LOSE, and the sequential simulation cost is the smallest rate of classical communication needed to faithfully simulate n sequential uses of $\mathcal{N}$, as $n \to \infty$. The open question is whether there exists channels in which the sequential simulation cost is strictly larger than the parallel simulation cost.

- **Catalytic entropy conjecture** - Presenter: Paul Boes. The catalytic entropy conjecture proposes necessary and sufficient conditions for the catalytic convertibility of one state ρ_1^S into another ρ_2^S by unitary evolution. Namely, it says that when the spectrum of ρ_1 and ρ_2 are inequivalent, there exists a catalytic state σ^C and joint unitary U^{SC} such that

$$\mathrm{Tr}_C\left[U(\rho_1^S \otimes \sigma^C)U^\dagger\right] = \rho_2 \quad \text{and} \quad \mathrm{Tr}_S\left[U(\rho_1^S \otimes \sigma^C)U^\dagger\right] = \sigma \tag{3}$$

if and only if $S(\rho_1) > S(\rho_2)$ and $rk(\rho_1) \geq rk(\rho_2)$. Eq. (3) characterizes the catalytic convertibility of ρ into ρ', and such a problem appears naturally in the QRTs of thermodynamics and entanglement [46]. The conjecture here is that the von Neumann entropy is unique measure for deciding catalytic convertibility. If true, it would provide an operational interpretation of the von Neumann entropy in the single-shot setting, in contrast to standard i.i.d. interpretations of the von Neumann entropy. The open problem is to prove or disprove the catalytic convertibility conjecture, and recent work in this direction can be found in Refs. [46, 47], as well as a formal statement of the problem at [48].

- **Realization of Completely PPT-Preserving Superchannels** - Presenter: Gilad Gour. A bipartite quantum channel $\mathcal{N}^{A_0B_0\to A_1B_1}$ is called PPT if it remains completely positive when composed with partial transpose maps. That is, $\mathcal{N}^{A_0B_0\to A_1B_1}$ is PPT if $T^{A_1} \circ \mathcal{N}^{A_0B_0\to A_1B_1} \circ T^{A_0} \geq 0$, where T is the transpose operation on the given system. Such maps play an important role in the study of entanglement since they provide a mathematically-friendly relaxation on the class of LOCC [49]. A superchannel $\Theta^{AB\to A'B'}$ is called completely PPT-preserving if $1^{\overline{AB}} \otimes \Theta[\mathcal{N}^{A\overline{A}B\overline{B}}]$ is PPT for any PPT channel

$\mathcal{N}^{A\overline{A}B\overline{B}}$, where $\overline{A}$ and $\overline{B}$ are arbitrary auxiliary systems and $1^{\overline{AB}}$ is the identity supermap [50]. A general superchannel can be realized by pre-processing quantum channel that is connected to some post-processing channel via an auxiliary memory system. If these pre- and post-processing channels are themselves PPT, then the resulting superchannel is completely-PPT preserving (see Fig. 4). The open problem is whether *every* completely PPT-preserving superchannel can be realized in this way.

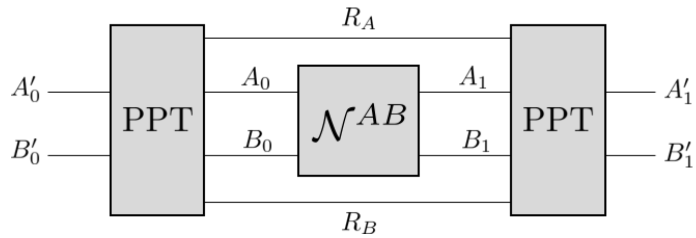

Figure 4: One type of completely PPT-preserving superchannels is composed of pre- and post-processing PPT channels. Can all completely PPT-preserving superchannels be built in this way?

- **Entanglement Distillation using Local Incoherent Operations** - Presenter: Eric Chitambar. In the quantum resource theory of coherence, one is restricted to performing some family of quantum operations that cannot generate coherence. The most commonly studied are the so-called incoherent operations [51]. These operations can be extended to bipartite systems, and when additional locality constraints are placed on the operations, one arrives at a resource theory in which only local incoherent operations and classical communication (LIOCC) are free. The canonical resource states are local maximally coherent bits (cobits), $|\phi_+\rangle^A$ and $|\phi_+\rangle^B$ where $|\phi_+\rangle = \sqrt{1/2}(|0\rangle + |1\rangle)$, as well as a maximally entangled coherent bit (ecobit) $|\Phi^+\rangle = \sqrt{1/2}(|00\rangle + |11\rangle)$. A general distillation protocol then involves converting a given bipartite state ρ^{AB} into a triple of cobits and ecobits (see Fig. 5). The problem of asymptotic distillation for a pure state $|\Psi\rangle^{AB}$ has been studied in Ref. [52], and an optimal point in the rate region has been identified as

$$(R^A_{co}, R^B_{co}, R^{AB}_{eco}) = (0, S(Y|X)_{\Delta(\Psi)}, I(X:Y)_{\Delta(\Psi)}),$$

  where $\Delta(\Psi)^{XY}$ is the fully classical state obtained from locally dephasing $|\Psi\rangle^{AB}$. However, it is known that higher entanglement rates R^{AB}_{eco} are achievable at the cost of reducing the coherence rate R^B_{co}, but optimal rate has not been solved. The open problem is to determine the largest achievable rate of ecobit distillation from an arbitrary pure state $|\Psi\rangle^{AB}$ using LIOCC.

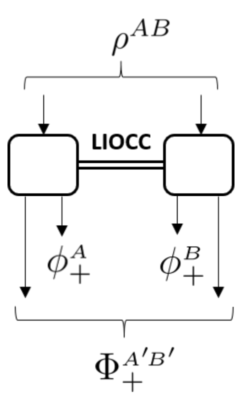

Figure 5: In the resource theory of distributed coherence, an operational task is to distill local coherent bits $\phi+^A$ and ϕ_+^B as well as entangled coherent bits Φ_+^{AB}. In general there will be a trade-off in distillation rates.

- **Tightening of the Alicki-Fannes Inequality** - Presenter: Mark Wilde. The Alicki-Fannes Inequality puts a bound on the difference of conditional von Neumann entropies [53], and it reads

$$|S(A|B)_\rho - S(A|B)_\sigma| \leq 4\epsilon \log_2 |A| + 2h(\epsilon), \tag{4}$$

where $\epsilon \geq \|\rho - \sigma\|_1$ and $h(\epsilon) = -\epsilon \log_2 \epsilon - (1-\epsilon)\log_2(1-\epsilon)$. This is a useful inequality in quantum information theory as it establishes a uniform continuity bound on the conditional von Neumann entropy. Recently, the RHS has been tightened to $2\epsilon \log_2 |A| + (1+\epsilon)h(\frac{\epsilon}{1-\epsilon})$ [54], however it is not known whether this is optimal in the sense that it can be satisfied by certain pairs of states. For classical-quantum (CQ) states, an improvement can be made by replacing the RHS with $\epsilon \log_2(|A|-1) + h_2(\epsilon)$, and this is known to be optimal [55]. The open problem is whether the Alicki-Fannes Inequality can be improved in the fully quantum case to the following form, which would be tight using the pair of states used in Remark 3 of [54]:

$$|S(A|B)_\rho - S(A|B)_\sigma| \leq \epsilon \log_2(|A|^2 - 1) + h(\epsilon). \tag{5}$$

4 Outlook

A common theme in physics is the unification of theories and models that at first glance may seem completely unrelated. Most notable in this regard is the successful unification of the three non-gravitational forces in nature. Such an amalgamation not only leads to new discoveries, but it also has the potential to profoundly change the way we perceive the world around us. With the advent of quantum information science, many seemingly unrelated properties of physical systems, such as entanglement, asymmetry, and athermality, have now become recognized as resources. This recognition is profound as it allows them to be unified under the same roof of quantum resource theories. Entanglement, athermality, and asymmetry, are no longer regarded as just interesting physical properties of a quantum system, but they now emerge as resources that can be utilized and manipulated to execute a variety of remarkable tasks, such as quantum teleportation.

This BIRS workshop has focused on the interface between quantum resource theories, operator theory, and (quantum) mathematical statistics. We believe the results presented at the workshop and the discussions shared by its participants will have a lasting impact on all the fields involved. It is an exciting time for quantum resource theories, and we thank BIRS for providing the opportunity to further advance this important subject.

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