Scattering and Inverse Scattering

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March 23–March 27

In the fields of scattering and inverse scattering theory techniques of microlocal analysis, including the use of eikonal equations and of complex geometrical optics solutions to Schrödinger and other equations, has led to substantial progress in recent years. The purpose of the workshop was be to bring together people working on different aspects of these fields, to appraise the current status of development and to encourage interaction between mathematicians and scientists and engineers working directly with the applications of scattering and inverse scattering.

Despite close mathematical connections between the fields of scattering and inverse scattering, there has not always been a strong interaction between these fields. Part of the rationale of this workshop was to bring together workers who might not ordinarily interact, but could benefit from sharing ideas.

1 Basic ideas in direct and inverse scattering

Scattering theory, in the time dependent formulation, is the study of the long time behaviour of solutions of an evolution equation that move out to infinity. The evolution equation might be the time dependent Schrödinger equation, (quantum scattering), the scalar wave equation (acoustical scattering), Maxwell's equations (electromagnetic scattering) or even a non-linear evolution equation. The underlying space might be Euclidean space or a Riemannian manifold. In each problem there is a localized scattering target. Moving in space away from the target to infinity, the equations, or the geometry, become simpler. The idea is that in the distant past and in the far future, the scattered wave will be located in the region where the equation or geometry is simple. It then becomes possible to compare distant past input to the far future output.

Inverse scattering is how we obtain a large part of our information about the world. An everyday example is human vision: from the measurements of scattered light that reaches our retinas, our brains construct a detailed three-dimensional map of the world around us. Dolphins and bats are able to do the same thing from listening to scattered sound waves.

We know about the interior structure of the Earth by solving the inverse problem of determining the sound speed by measuring travel times of seismic waves, the structure of DNA by solving inverse X-ray diffraction problems and the structure of the atom and its constituents from studying the scattering when materials are bombarded with particles.

Medical imaging uses scattering of X-rays, ultrasound waves and electromagnetic waves to make images of the human body which is of invaluable help with medical diagnosis. The oil exploration industry uses the reflection of seismic waves in oil prospecting. Inverse scattering is used in non destructive evaluation of materials to find cracks and corrosions.

In summary the task of direct scattering theory is to determine the relation between the input and output waves, given the details about the scattering target. The task of inverse scattering theory is to determine properties of the target, given sufficiently many input output pairs.

In fact, most often scattering problems are stated in a time independent formulation that results after taking a Fourier transform in the time variable. Although the immediate connection to the original scattering experiments is obscured, it can be easier to state and study scattering problems in the this formulation. Here is the time independent formulation of the obstacle scattering problem, which is relevant to applications in radar and sonar.

The underlying space is $\mathbb{R}^n$ and the obstacle $\Omega \subset \mathbb{R}^n$ is a closed bounded set. We consider solutions to the Helmholz equation

$$\Delta u(x) + k^2 u(x) = 0 \quad \text{for } x \in \mathbb{R}^n \backslash \Omega$$

with either Dirichlet (for a soft obstacle) or Neumann (for a hard obstacle) boundary conditions on $\partial\Omega$. We look for a family of solutions, indexed by ω in the sphere S^{n-1} with the following asymptotic behaviour:

$$u(x;\omega,k) = u(r\xi;\omega,k) \sim e^{ik\langle\omega,x\rangle} + r^{-(n-1)/2} e^{ikr} A(\xi,\omega,k)$$

as $r \to \infty$. Here $r = |x|$ and $\xi = x/|x|$, and $\sim$ denotes equality up to terms of order $r^{-(n+1)/2}$.

For a given obstacle, these equations uniquely determine the function $A(\xi,\omega,k)$, called the *scattering amplitude* or the *far field pattern.*.

The *scattering operator* can be defined in the following way. If we consider any solution $u(x) = u(r\xi)$ of the Helmholz equation with asymptotic expansion for large r of the form

$$u(r\xi) \sim r^{(n-1)/2} e^{ikr} a(\xi) + r^{(n-1)/2} e^{-ikr} b(\xi)$$

it turns out that the coefficients $a(\xi)$ and $b(\xi)$ are not independent. Specifying either a or b determines the solution u uniquely and thus the other coefficient. The scattering operator $S(k)$, acting on functions (or distributions) on the sphere describes the relationship between a and b via

$$a = S(k)b$$

To find the relationship between the scattering amplitude and the scattering operator, think of $e^{ik\langle\omega,x\rangle} = e^{ikr\langle\omega,\xi\rangle}$, for fixed ω and k as a family of distributions on the the sphere. The asymptotics of these distributions as $r \to \infty$ are obtained by a stationary phase calculation. As a function of $\xi \in S^{n-1}$ the phase $k\langle\omega,\xi\rangle$ has two critical points, one where $\xi = \omega$ and the other when $\xi = -\omega$. This results in

$$\int_{S^{n-1}} e^{ikr\langle\omega,\xi\rangle} \varphi(\xi) ds(\xi) \sim \left(\frac{2\pi}{r}\right)^{(n-1)/2} \left\{ e^{-i\pi(n-1)/4} e^{ikr} \varphi(\omega) + e^{i\pi(n-1)/4} e^{-ikr} \varphi(-\omega) \right\}$$

or

$$e^{ikr\langle\omega,\xi\rangle} \sim r^{-(n-1)/2} e^{ikr} (2\pi)^{(n-1)/2} e^{-i\pi(n-1)/4} \delta_\omega(\xi) + r^{-(n-1)/2} e^{-ikr} (2\pi)^{(n-1)/2} e^{i\pi(n-1)/4} \delta_{-\omega}(\xi)$$

Comparing asymptotic expansions, we conclude that

$$S(k) = i^{-(n-1)} R + (2\pi)^{-(n-1)/2} e^{-i\pi(n-1)/4} A(k) R$$

where R is the reflection operator sending $\varphi(\xi)$ to $\varphi(-\xi)$ and A is the operator with integral kernel $A(\omega,\xi,k)$.

In this formulation, the direct problem for obstacle scattering is to determine properties of the scattering operator $S(k)$ (or equivalently the scattering amplitude $A(k)$) given the shape of the obstacle Ω and the type of boundary condition imposed. An example of a property of $S(k)$ that has been studied extensively is its meromorphic continuation in k. Poles in the this continuation are called scattering poles or resonances. The goal is to estimate the location of these resonances, depending on the geometry of Ω.

The inverse problem for obstacle scattering is to determine the shape of the obstacle from knowledge of $S(k)$ (or $A(k)$). It turns out that that only partial information about $S(k)$ is required. However, when inverse scattering is used in practical applications, even this partial information is never all available, since only finitely many measurements can be made. Therefore there are many difficult computational issues that arise.

Similar problem direct and inverse scattering problems can be considered for the Helmholtz equation with a variable coefficient index of refraction $n(x)$ which is a positive function equal to one outside a ball. The equation in this case is

$$(\Delta + n(x)k^2)u = 0$$

in Euclidean space.

2 Recent developments and open problems

2.1 Geometric Scattering Theory

In the early 1990's Melrose proposed a far-reaching and novel program to study "geometric scattering theory". The investigations in this area and the main lines of study are influenced both by the classical formulation of mathematical scattering theory and also by many results and methods in the study of the spectral theory of noncompact complete Riemannian manifolds. The point is to develop 'nonrelative' techniques to study questions of a scattering-theoretic nature; even when applied back to classical Euclidean scattering, this perspective has led to some dramatic advances and insights for the direct and inverse problem , e.g. for quantum N-body scattering. and hyperbolic manifolds. The exploration of the various ramifications of this theory, in particular its connection to Hodge theory and index theory, has occupied much of the efforts of Melrose and his coworkers and students over the past decade. The lectures by Mazzeo and Sa Barreto were concerned with this topic.

2.2 Inverse scattering in anisotropic media

The medium parameters in anisotropic media depend on direction and position. The inverse scattering problem of determining such media from the scattering amplitude is difficult and closely related to rigidity questions in differential geometry. The index of refraction of the Earth for instance depends on direction; with the fastest the direction of the axis of ration of the Earth. The problem of recovering this anisotropic index of refraction from the singularities of the scattering operator leads to the problem of determining a Riemannian metric on a bounded domain from the lengths of geodesics joining points in the boundary. This corresponds physically to the first arrival times of sound waves. In Riemannian geometry this is known as the boundary rigidity problem. The two dimensional case has recently been solved by Pestov and Uhlmann but the higher dimensional case remains largely open. Stefanov reported progress on this question in his lecture. Hansen considered in his talk an inverse anisotropic scattering problem arising in the mechanics of materials in particular isotropic elastic materials with residual stress. Residual stresses (or initial stresses) in solids and structures generally result from the manufacturing process.

2.3 Scattering poles

The scattering operator $S(z)$ is initially defined for real z, but often has a meromorphic continuation to the complex plane. Poles in this continuation are called scattering poles, or resonances. The term resonance comes from physics where they are interpreted as long-lived or resonant states. The lifetime of the resonant state is inversely proportional to its imaginary part. Thus, one is mostly interested in resonances that lie close to the real axis. Often one looks to the corresponding classical mechanical system as a guide to the location of resonances. Classical closed trajectories or stationary orbits should give rise to families of resonances. This was the subject of the talk of Hitrik, who talked about resonances associated to a non-degenerate maximum of a potential in two dimensions.

In the case of a Riemannian manifold with an asymptotically hyperbolic metric, a conformal structure is induced on the boundary at infinity. Zworski and Graham have studied the relation between this conformal structure and the residues of non-spectral scattering poles in the case where the metric is asymptotically Einstein.

There are still many open problems in this area. For example, one can consider the counting function for resonances $N(\lambda)$, defined to be the number of resonances of modulus less than λ. Although upper bounds are known from the work of Melrose, Sjöstrand, Zworksi, Vodev and others, lower bounds are much harder to come by, and exact asymptotics are only known in essentially one dimensional situations.

2.4 Inverse Obstacle Problem

This problem is discussed in more detail in the previous section. A recent development in the inverse scattering problem was the introduction by Colton and Kirsch and subsequent versions by Kirsch of the *linear sampling method* which is a numerical reconstruction struction algorithm of the obstacle that relies on the blow up of the associated Green's function at the boundary of the region that one ones to determine.

A very important open theoretical and applied problem is to determine the obstacle from a single incident wave at a fixed frequency measured in every possible direction. This is a formally determined problem since the dimension of the obstacle is the same as the manifold where the scattering amplitude is given. The lectures of Colton and Kirsch dealt with this inverse problem.

2.5 Time reversal and scattering in random media

In time reversal experiments waves are emitted from a localized source recorded for an interval of time by an array of receivers-transducers, time-reversed and retransmitted. This process is repeated several times, leading in some cases to refocusing near the original source. This has several applications in nondestructive evaluation and medical techniques such as lithotripsy and hypothermia. This process works because the wave equation is invariant under reversal of time and inhomogeneities and randonmess contribute to a better refocusing. The talks by Bal and Ryzhikt with the mathematical analysis of time reversal in random media.

Scattering and inverse scattering in random media is a topic that will attract a lot of attention in the future because the only reasonable mathematical model for very complicated medium is that the medium parameters are random variables. Besides time reversal other applications of scattering in random media include propagation of radar waves through foliage and ultrasound in human tissue.

2.6 Complex geometrical optics solutions and inverse scattering at a fixed energy

The inverse scattering problem for the case of the Helmholtz equation with index of refraction one outside a compact set at a fixed energy k has been solved in dimension three or larger. This is in fact a consequence of the fact that the Dirichlet-to-Neumann map associated to this equation in any open and bounded domain uniquely determines the index of refraction. The solution of this latter problem uses complex geometrical optics solutions for the Schrödinger equation. which were developed by Calderón and Sylvester and Uhlmann. Challenges for the future include the two dimensional case and the fixed energy problem for many body scattering, that is the case of several particles interacting. Tamasan reported on progress in the two dimensional case for the two body problem.

2.7 Inverse Backscattering Problem

For the inverse backscattering problem for the case of the Helmholtz equation with variable index of refraction the medium is probed with waves in a given direction and the reflection is measured. However very little progress has been made on this very important problem very relevant to remote sensing and ultrasound except for the case that the index of refraction is small.

2.8 Optical Tomography

In this relatively new medical imaging modality one attempts to determine the absorption and scattering coefficient of an inhomogeneous medium by probing it with diffuse light. The problem is modeled as an inverse scattering problem for the stationary linear Boltzmann equation or the radiative transport equation. The information is encoded in the albedo operator, which is the analog of the Dirichlet-to-Neumann map. Although theoretical results are very encouraging there are no numerical reconstruction algorithms which deal with the severe ill-posedness of this problem.

2.9 Seismic Imaging

Seismic imaging creates images of the Earth's upper crust using seismic waves generated by artificial sources and recorded into extensive arrays of sensors (geophones or hydrophones). The heterogeneity and anisotropy of the Earth's upper crust require advanced mathematics to generate wave–equation solutions suitable for seismic imaging. In this context, microlocal analysis and Fourier integral operators have been shown to be promising because they offer the potential to manipulate a wavefield directly on its phase space. The talks of Gibson, Lamoureux and Margrave described some of this progress.

However very little is known for the case of anisotropic elastic medium which is a more accurate model of the Earth's subsurface and this is one of the challenges for the future.

2.10 Scattering in Complex Materials

When a body is exposed to an incident electromagnetic field, the resulting "scattered" field depends on the electromagnetic properties of the body. By sending controlled incident fields, and measuring the scattered fields (remote sensing), one may obtain information about the makeup of the body with respect to these properties. Alternatively, one may control, say, the electric fields imposed on an accessible boundary and record the magnetic fields on the boundary, again obtaining information about the material makeup of the body.

The material properties one may seek include the body's electric permittivity, ε, magnetic permeability, μ, conductivity, σ, and chirality, β. The relationship between the fields is modeled by Maxwell's system of equations. If we assume that the system is lossless (there is no loss of electromagnetic energy), then in greatest generality these take the form $\operatorname{curl} E = -\partial_t B$, $\operatorname{curl} H = \partial_t D$. Here, E, H are the electric and magnetic fields, and B, D are the magnetic induction and electric displacement respectively. How B and D depend on E and H, and on the material properties of the body, is described by the constituent relations; these may take a variety of forms depending on the physical assumptions one is willing to make.

Chirality has not been included in the majority of prior analysis. Chirality is an asymmetry in the molecular makeup of the material, and is physically significant and present in many applications. The inclusion of chirality enriches Maxwell's equations and yields the potential to determine practically useful properties of the body.

A chiral molecule (typically a long carbon based corkscrew) is one whose mirror image has reversed orientation. In the absence of life, chiral molecules usually appear in equal amounts in each orientation. Living organisms, on the other hand, almost always produce chiral molecules with a definite orientation.

Chiral molecules scatter light differently. When light is transmitted through chiral material as a phase-coherent wave, left– and right–circularly polarized light are absorbed differently, and propagate at different speeds (thus rotating the plane of polarization of linearly polarized light). These phenomena are collectively termed optical activity consequence of optical activity is that light scattered from chirally structured particles differs characteristically from light scattered from achiral, non-living particles. Understanding chiral materials is of basic importance.

Another fundamental scattering and inverse scattering problem is dispersive materials for which very little is known. These materials are such that the speed of propagation depends on the frequency of the wave. Most materials are dispersive although sometimes one can neglect this dispersion but this is an important effect for broadband signals. A mathematical theory for Helmholtz equations with dispersive index of refraction would have considerable theoretical and practical importance.

3 The talks

Here is a brief description of the talks presented at this workshop, presented in the order they were given. Instead of giving references to published work, we provide web addresses with the most recent work, where this is available

Rafe Mazzeo reported on his joint work with Andras Vasy on the meromorphic continuation of the resolvent for arbitrary global symmetric spaces of noncompact type. This is related to the meromorphic continuation of the scattering operator $S(k)$. The geometry at infinity for these spaces is complicated, but has an inductive structure that is similar to N-body Hamiltonians. Thus, techniques developed for the study of N-body Hamiltonians can be extended. Mazzeo's recent papers can be found at `http://math.stanford.edu/∼mazzeo/papers/papers.html`.

David Colton considered the inverse scattering problem of determining the shape and surface impedance of a partially coated perfect conductor from a knowledge of the electric far field pattern of the scattered field due to an incident time harmonic plane wave at fixed frequency. Colton's publication list is located at `http://www.math.udel.edu/∼colton/Cbib/ColtonBib.html`

Plamen Stefanov, in his talk about joint work with Gunther Uhlmann, considered the boundary rigidity problem for Riemannian manifolds - is such a manifold uniquely determined, up to an isometry, by the boundary distance function, the function that assigns to two boundary points the distance between them? Selected publications of Stefanov are available at his web page located at `http://www.math.purdue.edu/∼stefanov/`

Sönke Hansen talked about the construction of high-frequency solutions and parametrices for systems of real principal type and indicated some applications. His preprints can be found at `http://www-math.uni-paderborn.de/∼soenke/`

Lenya Ryzhik and **Guillaume Bal** each gave a talk about their work on a mathematical theory of time-reversal experiments. In such experiments a signal emitted by a localized source is recorded by an array of receivers-transducers and re-emitted into the medium reversed in time. The new signal refocuses approximately on the location of the original source despite the small size of the array. Surprisingly, medium heterogeneities improve significantly the quality of refocusing. Ryzhik explained how this phenomenon may be understood in the general framework of refocusing of re-transmitted high frequency acoustic waves recorded at a single time. He discussed in particular the self-averaging properties of the re-transmitted signal that appear due to wave mixing and related them to the self-averaging properties of the phase space energy density of waves in random media. Bal explained why refocusing is improved by random inhomogeneities and related the time-reversal theory to the phase space description of energy propagation in random media, radiative transport and random geometrical optics. He also showed

Ricardo Weder discussed recent results that he obtained on direct and inverse scattering for the forced non-linear Schroedinger equation on the half-line, namely, the construction of the scattering operator and the unique reconstruction of the potential and the non-linearity. The new technical tools that made possible to prove these results are the L^1–L^∞ estimate and the boundedness in the L^p spaces of the wave operators for the Schroedinger equation that is obtained linearizing our problem with force identically zero.

Ira Herbst reported on work with Erik Skibsted about quantum scattering for the Schrödinger equation associated with a symbol $h(x, \xi)$ that is homogeneous of degree zero in x. In classical mechanics, radial points that vary smoothly in energy have corresponding stable and unstable manifolds. Herbst showed that (under certain generic assumptions) as long as there is an unstable manifold, there are no quantum states corresponding to the stable manifold. Selected papers of Herbst are available at `http://www.math.virginia.edu/Faculty/herbst/`

Robin Graham discussed his joint work with Maciej Zworski on the "non-spectral" poles of the scattering matrix of an asymptotically hyperbolic metric. Such poles always exist and the residues are differential operators. In the case when the metric is asymptotically Einstein, these residues are invariantly associated to the induced conformal structure at infinity and this relationship can be viewed in the framework of the AdS/CFT correspondence of string theory.

Michael Hitrik reported on his work with Johannes Sjöstrand about scattering poles for a semi-classical Schrdinger operator in dimension 2, generated by a unique non-degenerate maximum of the potential. Assuming that the classical frequencies at the critical point satisfy the resonance condition, he obtained a complete asymptotic description of the poles which are at a distance h^δ, $0 < \delta < 1/2$, away from the critical energy. Hitik's web page is as `http://math.berkeley.edu/~hitrik/`

Alexandru Tamasan talked about the inverse scattering method used to prove (global) uniqueness in the 2D inverse conductivity problem. This method needs a little help when changed to a method of reconstruction. One needs a characterization of the Cauchy data in terms of the measurements. For Nachman's method (which uses a reduction to the Schroedinger equation,) this is given by the Dirichlet-to-Neumann map. In Brown and Uhlmann's method (which uses a reduction to a first order elliptic hermitian system,) it involves more then just the Dirichlet-to-Neumann map. For the particular case coming from the conductivity equation, such a characterization lead to an algorithm in EIT which allowed $W^{1+\epsilon}$ conductivities. This was joint work with K. Knudsen. Preprints are available at `http://www.math.toronto.edu/tamasan/`

Jenn-Nan Wang discussed recent joint work with Gen Nakamura on the unique continuation property for the two-dimensional inhomogeneous anisotropic elasticity system

Peter Gibson talked about the the use of pseudo-differential operators and the Gabor transform in seismic imaging.

Victor Isakov discussed uniqueness and stability of recovery of two- and three-dimensional domains and of the boundary conditions from the far field data at one or few frequencies. He reviewed old results and gave some new theorems proven by the method of singular solutions. In particular, he considered penetrable obstacles with unknown general transmission conditions Isakov's publication list is at `http://www.math.wichita.edu/~isakov/`

Antonio Sá Barreto defined radiation fields, showed how to obtain the scattering matrix from them, and used this characterization to study the problem of recovering the metric and the manifold from the scattering matrix at all energies. Sá Barreto's recent papers can be obtained from the web page `http://www.math.purdue.edu/~sabarre/papers.html`

Peter Hislop presented some new results on the spectral properties of quantum Hall Hamiltonians associated with unbounded regions in the plane. For one-edge regions that are deformations of the half-plane, he proved a lower-bound on the edge current, and the equality of the edge and bulk conductivity. He also gave a new proof of the existence of bands of absolutely continuous spectrum at energies between the Landau levels. He also studied straight, strip-like regions. This was joint work with J.-M. Combes and E. Soccorsi.

Victor Ivrii considered operators on compact closed manifolds, with coefficients, first derivatives of which are continuous with continuity modulus $O(|\log|x-y||^{-1})$. He derived semiclassical spectral asymptotics with sharp remainder estimate $O(h^{1-d})$. In the process he discussed the question "Where does microlocal analysis start?" and the answer to it "From the logarithmic uncertainty principle". These asymptotics easily yield standard asymptotics with respect to spectral parameter $\lambda \to \infty$. Ivrii's preprints are at `http://www.math.toronto.edu/ivrii/Research/Preprints.html`

Peter Perry, in joint work with Carolyn Gordon, constructed continuous families of metrics on R^n, $n \geq 8$, with the following properties: (1) The metrics are Euclidean outside a compact set, (2) the support of the perturbation may be arbitrarily small, and (3) the continuous families have the same scattering phase. The construction is based on constructions of isospectral compact manifolds by Carolyn Gordon and Dorothee Scheuth. There are examples of pairs of isophasal metrics which have very different isometry groups. Perry's preprints are located at `http://www.ms.uky.edu/~perry/papers.html`

Oliver Dorn talked about level set method for describing propagating fronts, which has become quite popular in the application of medical or geophysical tomography. The goal in these applications is to reconstruct unknown objects inside a given domain from a finite set of boundary data. Mathematically, these problems define nonlinear inverse problems, where usually iterative solution strategies are required. Starting out from some initial guess for the unknown obstacles, successive corrections to this initial shape are calculated such that the so evolving shapes eventually converge to a shape which satisfies the collected data. Since the hidden objects can have a complicated topological structure which is not known a priori, the shapes usually undergo several topology changes during this evolution before converging to the final solution. Therefore, a powerful and flexible tool for the numerical description of these propagating shapes is essential for the success of the inversion method of choice. In the talk, Dorn presented a recently developed shape reconstruction method which uses a level set representation of the shapes for this purpose. Numerical results were presented for three different practically relevant examples: cross-borehole electromagnetic tomography using a 2D Helmholtz model, surface to borehole 3D electromagnetic induction tomography (EMIT) using a model based on the full 3D system of Maxwell's equations, and diffuse optical tomography (DOT) for medical imaging using a model based on the linear transport equation in 2D. Dorn's publications can be accessed at `http://www.cs.ubc.ca/∼dorn/publications.html`

David Finch considered the problem of inversion of the spherical mean transform on Euclidean space, when the centers are restricted to a hypersurface. He gave some uniqueness results. For the case of functions supported in a ball in an odd dimensional space, he gave closed form inversion formulas analogous to the Radon inversion formula when the spherical means are known for spheres centered on the boundary of the ball. Some applications were presented. This is joint work with Rakesh and Sarah Patch.

A robust approach to seismic imaging can be derived from the Born approximation to wavefield scattering. The resulting imaging algorithm estimates subsurface reflectivity as the ratio of backward extrapolated seismic data to a forward extrapolated model of the seismic source. This places key importance on the ability to extrapolate wavefield through heterogeneous media. In the case where the wavespeed is a function only of the coordinates orthogonal to the direction of extrapolation, an exact solution to the extrapolation problem is available. **Gary Margrave** sketched this solution and compare it to two approximations. The first approximation is appears as a Fourier integral operator while the second uses the Gabor transform to approximately factorize this operator. I will discuss this approximate factorization and compare numerical results from all three methods. Selected publications of Margrave can be found at `http://www.crewes.org/AboutCREWES/Faculty/margrave/margrave.php`

One dimensional scattering theory serves as the basis for an exact wavefield extrapolator in seismic imaging, where the propagation of a seismic wave through the earth's subsurface is modeled by the acoustic wave equation. **Michael Lamoureux** showed a comparison between results of exact eigenvalue calculations on a bounded domain, with the results from the scattering theory of a (unbounded) 1D line, as well as with other standard imaging methods. Recent preprints of Lamoureux are available at `http://www.math.ucalgary.ca/∼mikel/papers.html`

Chi-Kun Lin talked about the homogenization of the Dirac type system. It generates memory effects. The memory (or nonlocal) kernel is described by the Volterra integral equation (or Fredholm integral equation depending on the initial or boundary value problem). When the coefficient is independent of time, the memory kernel can be characterized explicitly in terms of Young's measure.

Georgi Vodev discussed some recent results concerning estimates of the local energy decay of solutions to the wave equation on unbounded Riemannian manifolds with non-trapping metrics. These estimates are derived from the properties of the resolvent at high frequency. Applications to a class of asymptotically Euclidean manifolds as well as to perturbations by non-negative long-range potentials were given. Vodev's preprints appear on preprint server of the Nantes mathematics department at `http://www.math.sciences.univ-nantes.fr/prepub/liste.phtml?annee=2003`

Andreas Kirsch talked about the factorization method, a new approach to solve classes of inverse scattering problems for time harmonic acoustic or elastic waves. In the first part of the talk he explained the idea for the simplest case where one tries to recover an unknown (acoustically soft) obstacle from the far field patterns for plane wave incidence. In the second part he will presented recent extensions to problems with absorption and to mixed boundary conditions and discussed the relationship to the well known MUSIC algorithm in signal processing. Current publications of Kirsch are at `http://www.mathematik.uni-karlsruhe.de/∼ma2ki/page.php?id=kirsch&pid=publication`