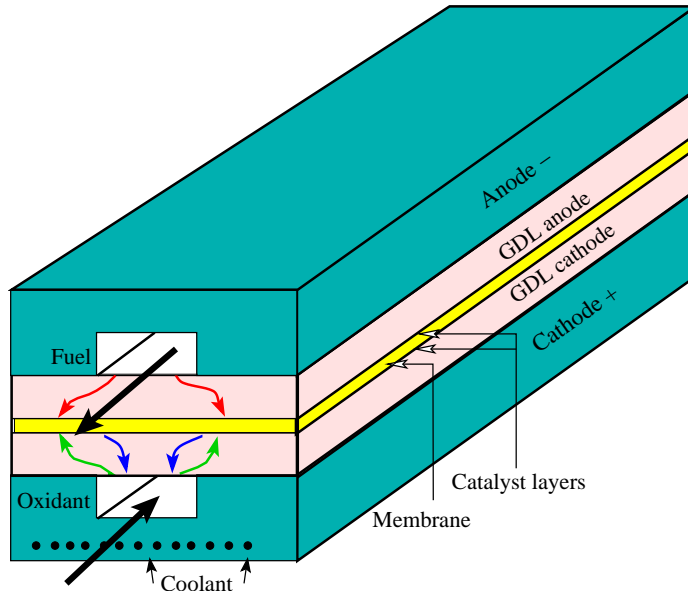


# Overview of PEM Fuel Cells

Computational Fuel Cell Dynamics  
Banff International Research Center  
April 19-24, 2003

# The PEM Unit Cell

## 3D View



## Principal Processes

Consumption of  $O_2$  and fuel along channels

Gas diffusion in GDL

Production of  $H_2O$  and heat at cathode catalyst layers

Build-up of double-layer charge at catalyst interface

Ion and water motion in membrane

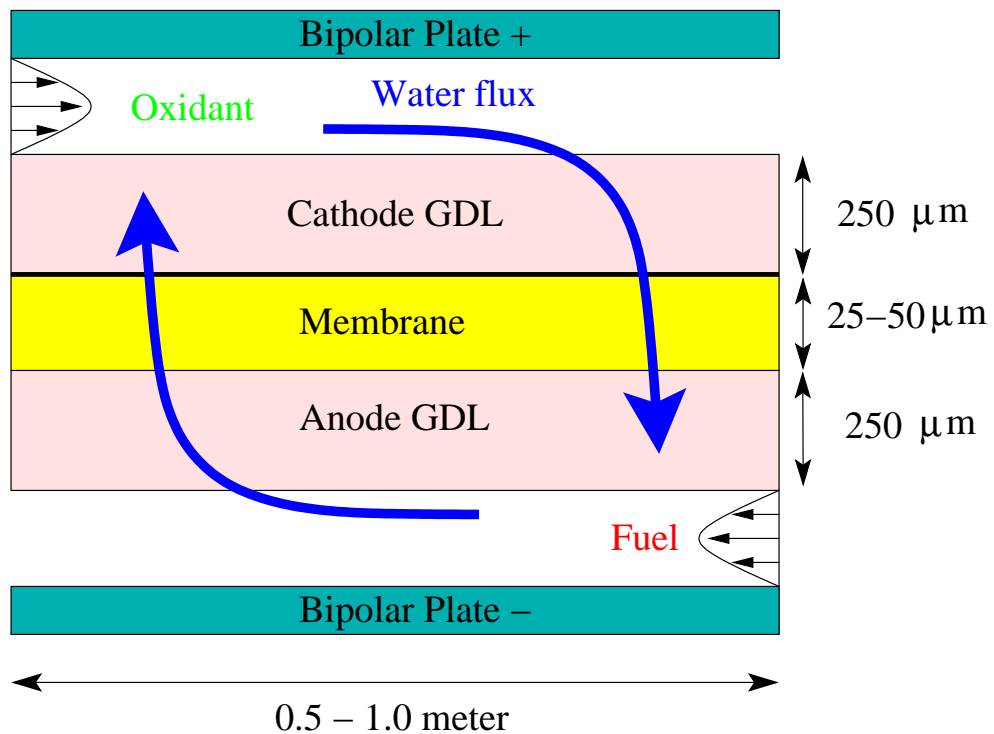
Forms barrier for fuel, oxidant,  $e^-$

Conducts protons as  $H_3O^+$

Condensation/Evaporation of water

Heat removal by coolant

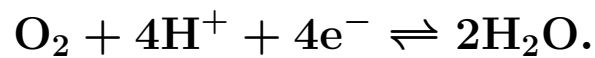
## Along-the-Channel Slice



The anode reaction



The cathode reaction



## Dimensionless Quantities

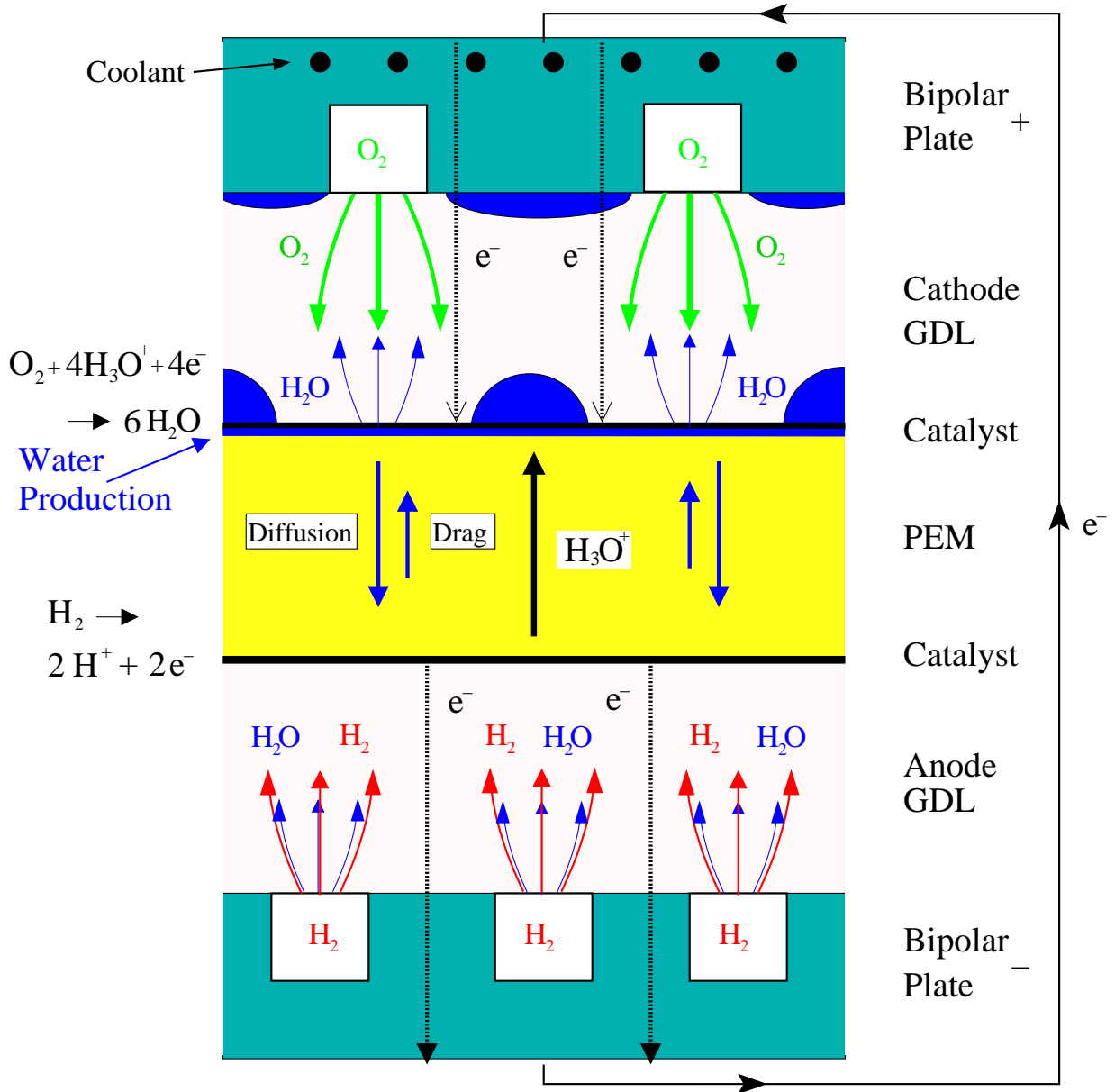
$$\text{Cathode (Air) Stoich } S_c = \frac{Q_{\text{O}_2}}{I_T/(4F)}$$

$$\text{Anode (Fuel) Stoich } S_a = \frac{Q_{\text{H}_2}}{I_T/(2F)}$$

$$\text{Local Water transfer } \alpha = \frac{J_w}{I/(2F)}$$

$$\text{Aspect ratio } 2000 : 1$$

# Cross-sectional Slice



## Multiphase Flow

Phase change and two-phase flow in hydrophobic carbon fiber paper

Gas diffusion through the carbon fiber paper (GDL)

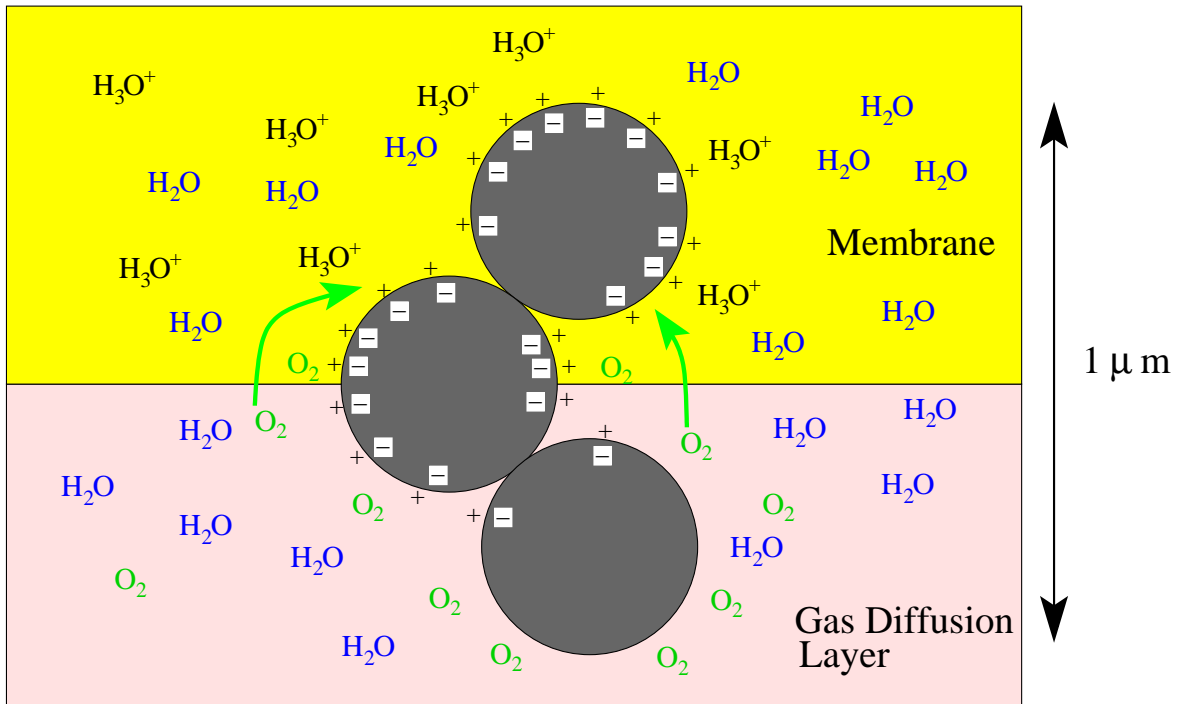
Liquid water exchange between:

membrane and GDL

GDL and Channel

Rivulet flow of water in channel

# Polarization Curves



The catalyst/membrane/GDL triple junction points are the active catalyst sites

$\text{O}_2$  diffusion from GDL and through membrane water

$\text{H}_3\text{O}^+$  transport through membrane

proton surface diffusion on Pt

electrical conductivity of carbon support

The double layer at the catalyst surface acts as a *capacitor*

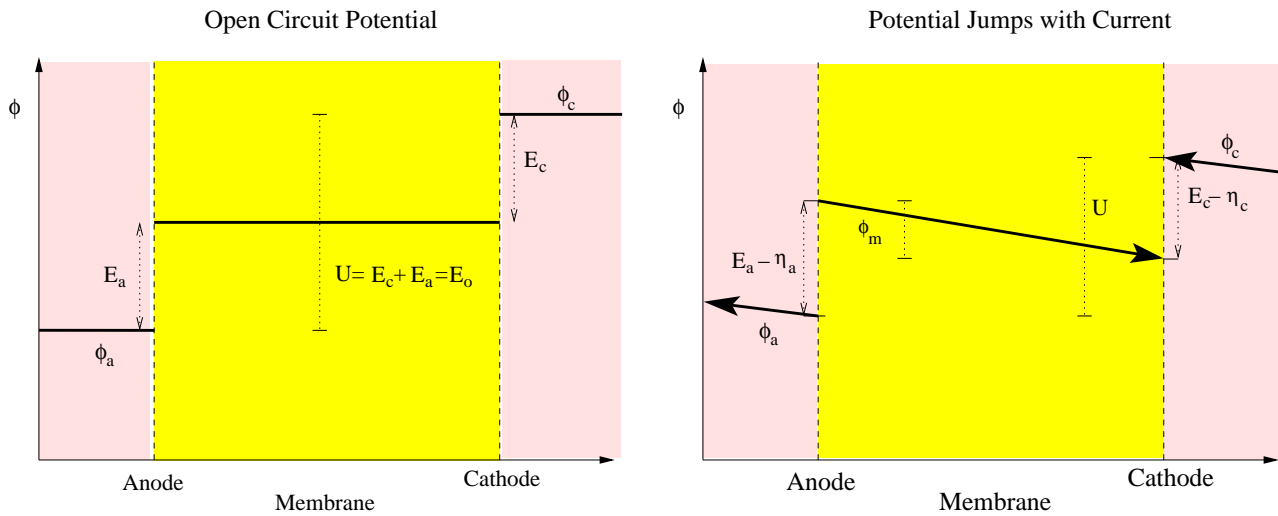
Voltage jump proportional to charge  $V = QC$

Charge  $Q$  depends upon

supply of reactants

discharge rate (current)

reaction kinetics



The electric potential pushes electrons anode to cathode  
protons cathode to anode

### Butler-Volmer Equations

$$I_c = i_{o,c} \left( \frac{c_O}{c_{O,ref}} \right)^{e_c} \left[ \exp \left( \frac{\alpha_c F}{RT} \eta_c \right) - \exp \left( -\frac{(1 - \alpha_c) F}{RT} \eta_c \right) \right]$$

$$I_a = i_{o,a} \left( \frac{c_H}{c_{H,ref}} \right)^{e_a} \left[ \exp \left( \frac{\alpha_a F}{RT} \eta_a \right) - \exp \left( -\frac{(1 - \alpha_a) F}{RT} \eta_a \right) \right]$$

describe non-equilibrium kinetics of current generation

### Local Potential balance

$$U = E_o - \eta_c(I) - \eta_a(I) - \phi_m(I)$$

Under the simple approximation

$$c_O = c_O(\text{channel}) - \delta I,$$

the cathode overpotential

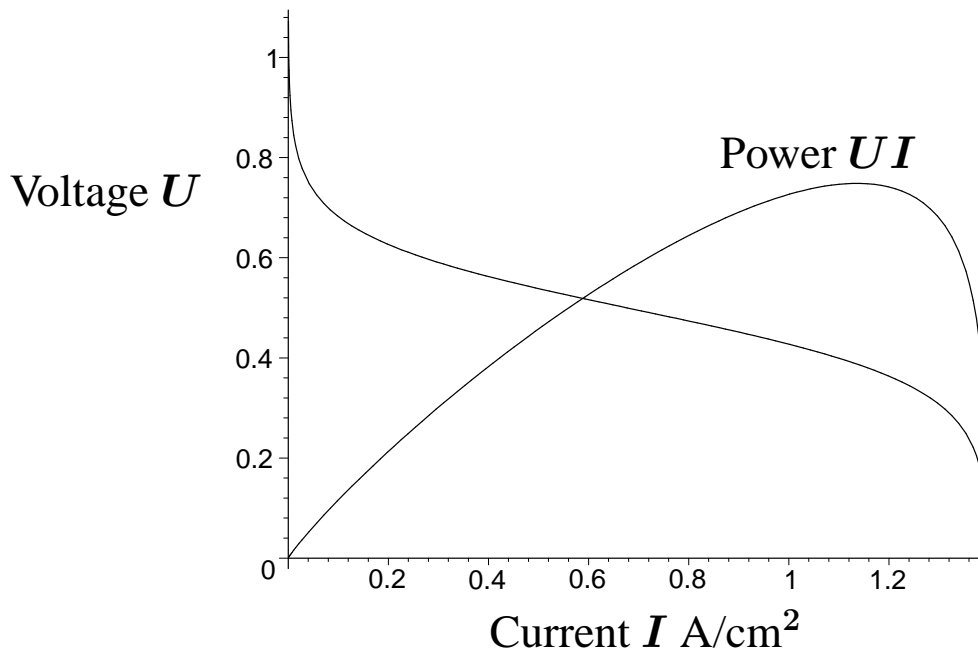
$$\eta_c \approx \frac{\overbrace{RT}^{\text{TafelSlope}}}{\alpha_c F} \ln \left( \frac{I}{i_{o,c}} \left( \frac{c_{O,\text{ref}}}{c_{O}(\text{channel}) - \delta I} \right)^{e_c} \right)$$

the membrane potential losses

$$\phi_m \approx I/\sigma$$

the polarization curve relates potential  $U$  to current  $I$ .

$$U = E_o - \frac{RT}{\alpha_c F} \ln \left( \frac{I}{i_{o,c}} \left( \frac{c_{O,\text{ref}}}{c_{O}(\text{channel}) - \delta I} \right)^{e_c} \right) - I/\sigma_m.$$



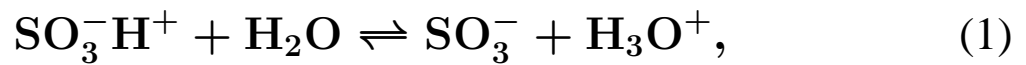
# Membrane Issues

## Unknowns

$c_w = [\text{H}_2\text{O}]/a$	Water Molar Conc.	$T$	Temperature
$c_+ = [\text{H}_3\text{O}^+]/a$	Free Proton Conc.	$\phi$	Electric Potential
$c_b = [\text{SO}_3^- \text{H}^+]/a$	Bound Proton Conc.	$a$	Acid Weight

## Ion Balance

The ion disassociation process

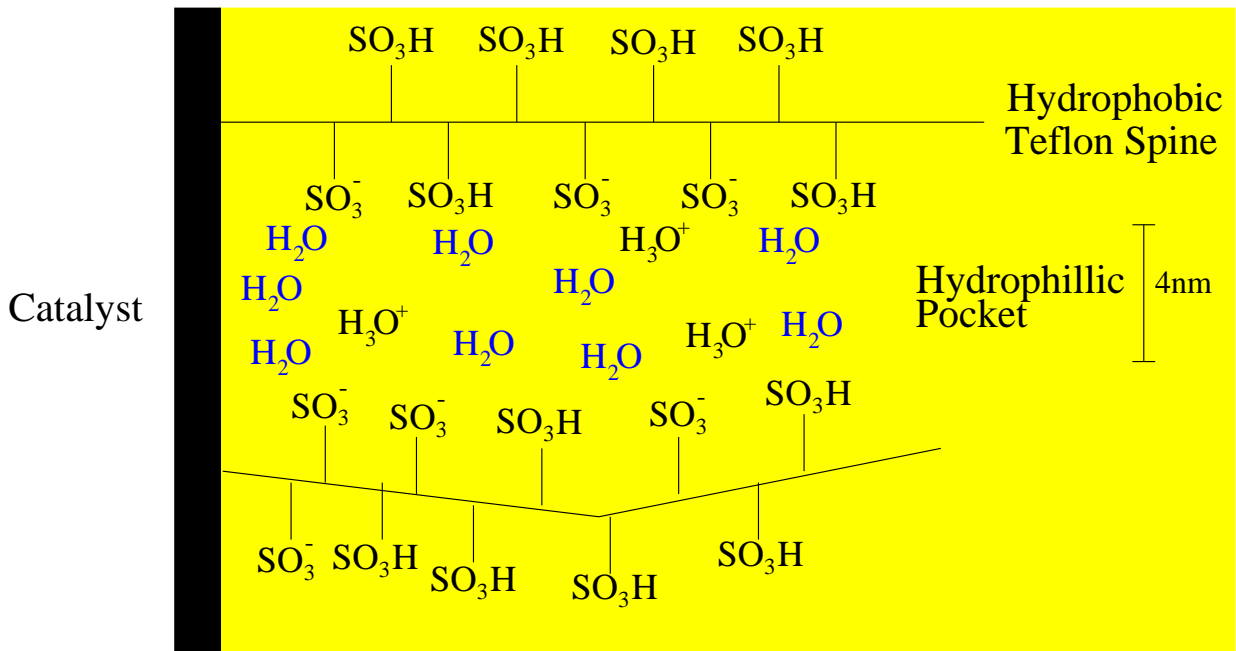


Hydronium production rate

$$S = a^2 (K_1 c_b c_w - K_2 (1 - c_b) c_+), \quad (2)$$

with rate constants

$$K_i = K_{i0} \exp\left[\frac{-H_{i0}}{RT}\right] \gg 1. \quad (3)$$



# Governing Equations

## Poisson's equation

$$\epsilon^2 \Delta \phi = -(\overbrace{c_+}^{\rho_+} - \overbrace{(1 - c_b)}^{\rho_-}), \quad \epsilon^2 = d/(4a\pi F), \quad (4)$$

## Conservation of Mass

$$\frac{\partial}{\partial t} c_w + \nabla \cdot (J_w + U_l c_w) = -\bar{S}/\delta, \quad (5)$$

$$\frac{\partial}{\partial t} c_+ + \nabla \cdot (J_+ + U_l c_+) = \bar{S}/\delta, \quad (6)$$

$$\frac{\partial}{\partial t} c_b + \nabla \cdot (J_b + U_b c_b) = -\bar{S}/\delta. \quad (7)$$

## Outer Approximation

- Bound ions do not flux:  $\nabla \cdot (J_b + U_b c_b) = 0$ ,
- $\bar{S} = 0 + \delta S_1 + O(\delta^2) \quad \delta \ll 1$
- $\rho = 0 + \epsilon^2 \rho_1 + O(\epsilon^4) \quad \epsilon \ll 1$

## Outer Equations: Parabolic-Elliptic-Algebraic

$$c_b c_w - \frac{K_2}{K_1} (1 - c_b) c_+ = 0, \quad \text{Ion Equilibrium}$$

$$c_+ - (1 - c_b) = 0, \quad \text{LEN}$$

$$\frac{\partial}{\partial t} (c_T) + \nabla \cdot (J_T + U_l(c_T)) = 0, \quad \text{Cons. Total water}$$

$$\nabla \cdot J_+ = 0, \quad (6) + (7) + \text{LEN}$$

## Ion Balance

LEN and Ion Equilibrium yield

$$c_+ = c_+(c_w, T) = -\frac{K(T)c_w}{2} + \sqrt{\left(\frac{K(T)c_w}{2}\right)^2 + Kc_w}, \quad (8)$$

where

$$K(T) = K_1(T)/K_2(T) = O(1).$$

Parabolic for  $c_w$  and elliptic for  $\phi$ .

## Maxwell-Stefan Equations

Water, hydronium, methanol:

$$\begin{pmatrix} \mathbf{J}_+ \\ \mathbf{J}_w \\ \mathbf{J}_{\text{meth}} \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{pmatrix} \begin{pmatrix} \nabla \mu_+ \\ \nabla \mu_w \\ \nabla \mu_{\text{meth}} \end{pmatrix}$$

Off diagonal terms represent interspecies friction

The chemical potential of species  $i$

$$\nabla \mu_i = RT \nabla \ln c_i + \widehat{z}_i^{\text{charge}} F \nabla \phi.$$

The average velocity  $U_l$  is given by a Darcy type relation

$$U_l = -\frac{K}{\mu} \nabla P$$

where the Pressure gradient arises from

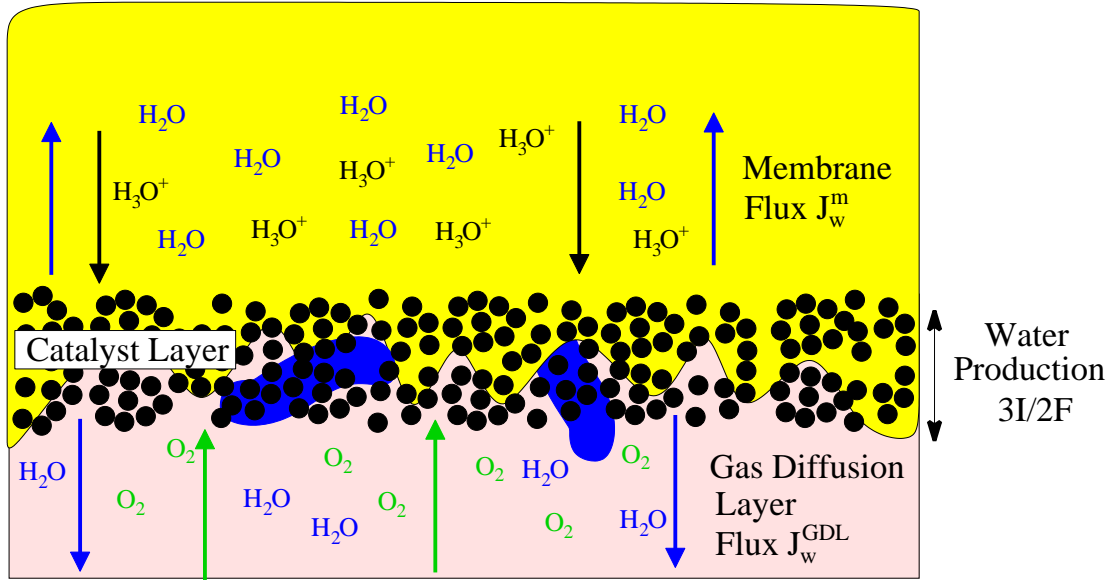
Capillary effects: hydrophilic/hydrophobic  
phase separation

Membrane swelling

External pressure differences

# Membrane-GDL Coupling

## Membrane-Catalyst Layer water transfer



Membrane equilibrium hydration levels function of GDL RH level  $r$ ,

$$c^*(r) = 0.043 + 17.81r - 39.85r^2 + 36.0r^3.$$

Flux out of membrane proportional to disequilibrium

$$\begin{aligned} \gamma(c_w) \overbrace{(c_{w,a}^* - c_{w,a})}^{\text{disequilibrium}} &= \overbrace{J_{w,a}^{GDL}}^{\text{Flux}} / a = (J_{w,a}^m + I/F) / a, \\ \gamma(c_w) (c_{w,c} - c_{w,c}^*) &= J_{w,c}^{GDL} / a = (J_{w,c}^m + 3I/2F) / a. \end{aligned}$$

$\gamma \ll 1$  controls membrane water loss.

## Heat Production in Catalyst

$$Q_{\text{heat}} = \left( \frac{Th_{rc}}{4F} + \eta_c \right) I_c - h_v \gamma (c_T - c_T^*(r)).$$

# Numerical Issues: GDL Multiphase flow

Degenerate transport of liquid water

Disparate length scales 1000 : 1

Disparate time scales:

$10^{-6}$  s for pressure

$10^3$  s for liquid flow

## Degenerate Multiphase Flow

### Unknowns

$C_o$	Oxygen Molar Conc.	$T$	Temperature
$C_v$	Water Vapour Molar Conc.	$P_l$	Liquid Pressure
$C_n$	Nitrogen Molar Conc.	$\beta$	Liquid Water Volume Frac.
$C$	Total Gas Conc.	$\alpha$	Gas Volume Frac.

### Conservation of Energy and Mass

$$\frac{\partial}{\partial t}(\alpha C) + \nabla \cdot (C U_g) = -\Gamma, \quad (9)$$

$$\frac{\partial}{\partial t}(\alpha C_o) + \nabla \cdot (C_o U_g + J_o) = 0, \quad (10)$$

$$\frac{\partial}{\partial t}(\alpha C_v) + \nabla \cdot (C_v U_g + J_v) = -\Gamma \quad (11)$$

$$\frac{\partial}{\partial t}(\beta) + \nabla \cdot (\beta U_l) = \Gamma/c_l \quad (12)$$

$$\frac{\partial}{\partial t}(\tilde{\rho} c T) + \nabla \cdot ((\tilde{\rho} c \tilde{U}) T - \tilde{\kappa} \nabla T) = h_{lg} \Gamma. \quad (13)$$

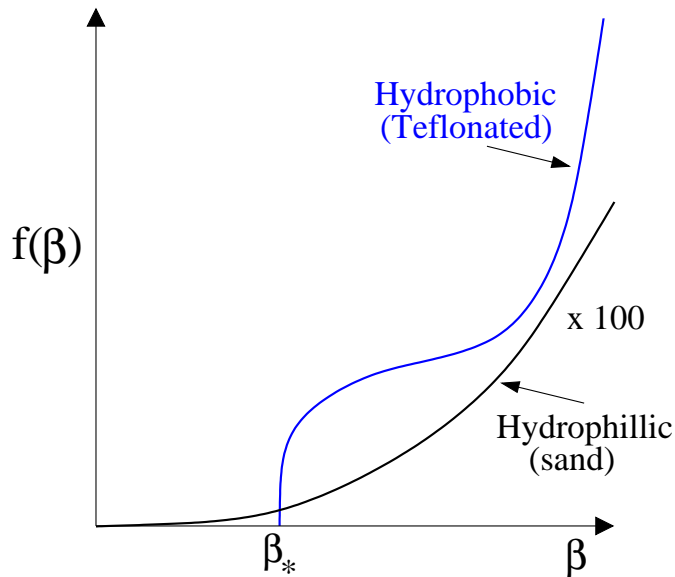
## Constitutive Relations

$\alpha + \beta + \epsilon = 1,$	Volume Fraction relation
$C = C_o + C_v + C_n,$	Total Concentration
$P_g = CRT,$	Ideal Gas Law
$U_g = -\frac{Kk_{rg}(\beta)}{\mu_g} \nabla P_g,$	Darcy's Law-Gas
$U_l = -\frac{Kk_{rl}(\beta)}{\mu_l} \nabla P_l,$	Darcy's Law-Liquid
$[J_i] = M^{-1}[\nabla C_i]$	Maxwell Stefan Flux
$P_c = P_g - P_l = L(\beta),$	Leveret-like Capillary Pressure.
$\Gamma = H(\beta)(C_v - C_{\text{sat}}(T)),$	Condensation-Saturation

Capillary pressure and relative permeability form nonlinear diffusivity

$$f(\beta) = \beta k_{rl}(\beta) L'(\beta).$$

$$\frac{\partial}{\partial t}(\beta) + D \nabla \cdot (f(\beta) \nabla \beta) = \Gamma / c_l$$



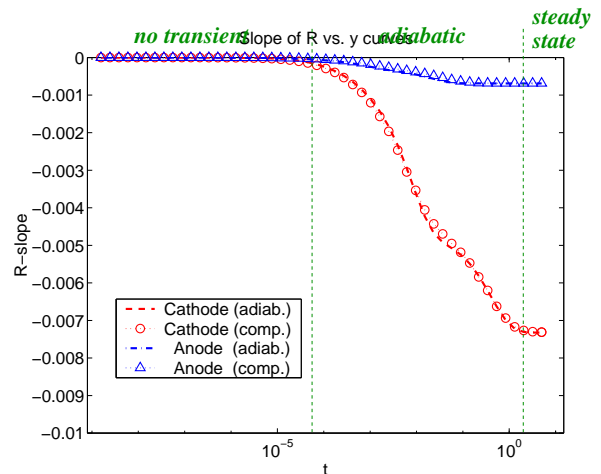
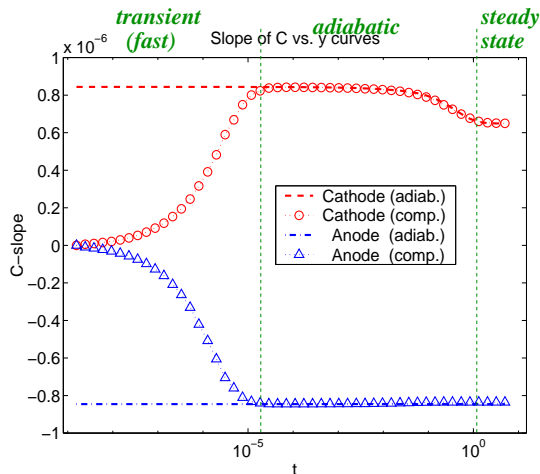
# Non-dimensional Forms

$$\begin{aligned} \frac{\partial}{\partial \bar{t}}(\alpha \bar{C}_v) + \bar{\nabla} \cdot (\text{Sc} \bar{C}_v \bar{U}_g + \bar{J}_v) &= -\bar{\Gamma}, \\ \frac{\partial}{\partial \bar{t}}(\alpha \bar{C}_o) + \bar{\nabla} \cdot (\text{Sc} \bar{C}_o \bar{U}_g + \bar{J}_o) &= 0, \\ \frac{\partial \beta}{\partial \bar{t}} + \delta_\beta (\bar{\nabla} \cdot (f(\beta) \nabla \beta) - \bar{\Gamma}) &= 0, \\ \frac{\partial \bar{T}}{\partial \bar{t}} + \bar{\nabla} \cdot (\text{Ra} \bar{U} \bar{T} - \bar{\nabla} \bar{T}) &= \delta_T \bar{\Gamma} \end{aligned}$$

Schmidt	$\text{Sc} = O(10^2 - 10^3)$
Raleigh	$\text{Ra} = O(10^2 - 10^3)$
Heat of Vaporization	$\delta_T = O(10^{-1} - 10^{-2})$ .
Water time-scale	$\delta_\beta = O(10^{-2} - 10^{-3})$ .

Convection dominates the gas flow, but  
balances diffusion at equilibrium

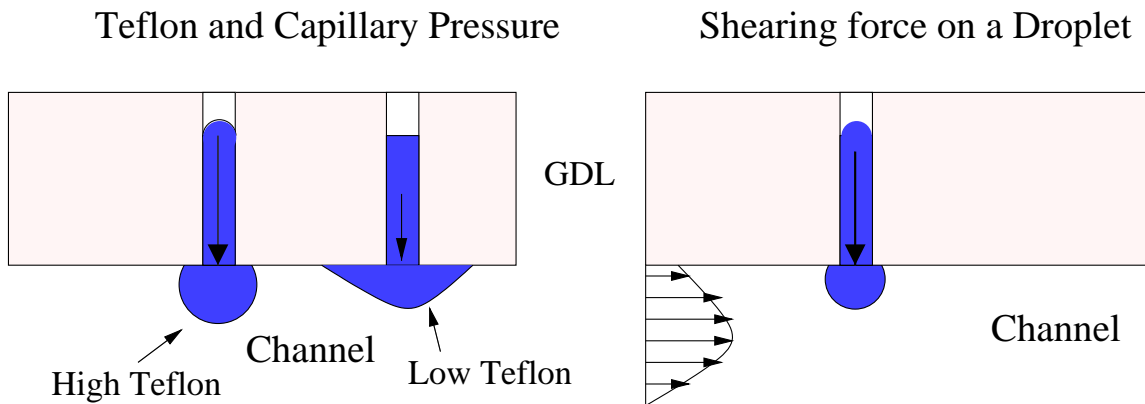
Liquid on slow time scale  $\tau = \delta_\beta t$



## GDL-Channel Liquid exchange

Proper modeling of liquid water exchange between porous carbon fiber paper and open channel is non-trivial

Water transport in channel flow truly multi-phase flow



# Open Problems in Electro-Chemistry + Ion Transport

## Electrocatalysis

- Modeling of mechanisms and kinetics of electrocatalysis
  - Proton surface diffusion on Pt
  - Percolation of conducting phase in catalysis layer
  - Liquid water inhibition of reaction
  - CO poisoning of catalyst
- Reaction kinetics for methanol

## Polymer Membranes

- Hydrophilic-hydrophobic phase separation in membrane
  - Impact on proton and water transport (osmotic drag)
- Dielectric effects: Bound and free water
- Membrane swelling and water transport

## Computational Issues

- Model stiffness
- Moving fronts
- Temporal and spatial boundary layers