MSRI Hot Topic: Floer homology for 3-manifolds.

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Introduction

This was a lively and productive conference, as befits a "Hot Topic". Perhaps the highlight, described in more detail below, was the discovery (through discussions among the participants) that Eliashberg could prove a theorem on capping off a symplectic 4-manifold with convex boundary, which was then used by Kronheimer and Mrowka to prove Property P for knots, and was also used by Ozsváth and Szabó to determine the genus of a knot by its Floer homology.

Property P refers to the 40 year old conjecture that Dehn surgery on a knot in S^3 never gives a homotopy 3-sphere unless the knot is the unknot. It was already known that Dehn surgery did not give S^3 [15], so the Kronheimer-Mrowka result would also follow from Perelman's work once it is approved.

The genus of a knot K in S^3 is the minimal genus of a spanning "Seifert" surface $F \in S^3$, $\partial F = K$. Arguably this has been the most impmortant invariant of a knot for 80+ year, but is has veen very difficult to calculate. Now it is determined by the "highest" spin$_C$ structure for which the Heegaard Floer homology is non-trivial; this is reasonably calculable.

History and Heegaard Floer Homology

This accounts begins with the historical background to the use of gauge theoretic methods in solving topological problems about low-dimensional manifolds.

In spring 1982, Simon Donaldson announced his spectacular applications of gauge theory to the differential topology of 4-manifolds [3]. Using work of Taubes and Uhlenbeck, Donaldson showed that the moduli space of almost self dual connections on a certain C^2 bundle over a smooth 4-manifold X^4 provided invariants which ruled out the existence of 4-manifolds with definite intersection forms. Together with Freedman's classification of simply connected topological 4-manifolds [10], this showed the existence of exotic, smooth structures on R^4.

The subject was difficult and results came only after considerable work. Donaldson discovered what are now called Donaldson polynomials, and later *basic classes* were found.

In fall 1994 Nathan Seiberg and Edward Witten announced a new set of partial differential equations for a connection on the bundle of spinors on S^4. The equations were technically much simpler than in Donaldson's case, and applications were quick to follow, e.g. the Thom Conjecture [17].

Beginning in 2001, Peter Ozsváth and Zoltán Szabó introduced a new version of Floer homology – the *Heegaard Floer homology* – based on Heegaard splittings of genus g of an oriented 3-manifold Y^3. The methods are almost purely combinatorial except for the crucial use of pseudoholomorphic discs in the g-fold symmetric product of a Heegaard surface for Y.

More precisely, Y is presented by a Heegaard diagram (Σ, α, β) where Σ is an oriented two-manifold and $\alpha = \{\alpha_1, ..., \alpha_g\}$ and $\beta = \{\beta_1, ..., \beta_g\}$ are attaching circles for two handlebodies which bound Σ. A choice of complex structure on Σ induces one on its g-fold symmetric product. Moreover, the products

$$\mathbb{T}_\alpha = \alpha_1 \times ... \times \alpha_g \quad \text{and} \quad \mathbb{T}_\beta = \beta_1 \times ... \times \beta_g$$

are tori embedded in $\mathrm{Sym}^g(\Sigma)$, which are totally real with respect to the induced complex structure on $\mathrm{Sym}^g(\Sigma)$. One can now set up a variant of Lagrangian Floer homology [8] in this setting – that is, the homology of a chain complex whose generators are intersection points of $\mathbb{T}_\alpha \cap \mathbb{T}_\beta$, and whose boundary operator counts pseudo-holomorphic disks in $\mathrm{Sym}^g(\Sigma)$ whose boundary lies in $\mathbb{T}_\alpha \cup \mathbb{T}_\beta$. Indeed, in order to get non-trivial information about the three-manifold, we need another piece of data, a choice of reference point $z \in \Sigma - \alpha_1 - ... - \alpha_g - \beta_1 - ...\beta_g$. The data $(\Sigma, \alpha, \beta, z)$ is called a *pointed Heegaard diagram.* This point z induces a subvariety $\{z\} \times \mathrm{Sym}^{g-1}(\Sigma) \subset \mathrm{Sym}^g(\Sigma)$, and various variants of Heegaard Floer homology are obtained by using this subvariety in various ways.

For example, the simplest non-trivial version of Heegaard Floer homology, $\widehat{\mathrm{H}F}(Y)$, counts pseudo-holomorphic disks in $\mathrm{Sym}^g(\Sigma)$ which are disjoint from $\{z\} \times \mathrm{Sym}^{g-1}(\Sigma)$. In all, there are four versions of this Floer homology $\widehat{\mathrm{H}F}(Y)$, $\mathrm{H}F^-(Y)$, $\mathrm{H}F^+(Y)$, and $\mathrm{H}F^\infty(Y)$.

Although the definition of these groups depends on a great deal of auxiliary information – a Heegaard diagram for Y, a choice of complex structure on Σ (and indeed a small perturbation of the induced almost-complex structure on $\mathrm{Sym}^g(\Sigma)$) – it is proved in [22] that the homology of the complex is in fact a topological invariant of Y. Indeed, in [23], it is shown that Heegaard Floer homology is natural under cobordisms between three-manifolds; i.e. if W is a cobordism from Y_1 to Y_2, there is an induced map on the four variants of Floer homology, which is a diffeomorphism invariant of W. These maps are then used in [23] to construct a four-manifold invariant whose formal properties suggest a close connection to the Seiberg-Witten invariant for four-manifolds. Indeed, based on this, and an overwhelming amount of calculational evidence, it is conjectured in [22] that the two theories are isomorphic.

But each theory has its own advantages. Heegaard Floer homology is more combinatorial in flavor than Seiberg-Witten theory. For example, the generators of the Heegaard Floer complex are purely combinatorial. Thanks to this concrete nature, several technical devices to facilitate the calculation of Heegaard Floer homology groups were obtained in [24]. A key device is a surgery *exact triangle* which relates the Floer homology groups of three-manifolds which are related by certain Dehn surgeries. (Surgery exact triangles first appeared in the work of Andreas Floer for his version of instanton Floer homology [9].)

Another device is provided by a Heegaard Floer invariant for knots which is not difficult to construct once the Heegaard Floer package is constructed, see [25] and also [30]. Specifically, there is an invariant associated to a knot $K \subset S^3$ (or more generally an oriented link) which is a bigraded Abelian group $\widehat{\mathrm{HFK}}_d(K, s)$, with $d, s \in \mathbb{Z}$ (we call d the Maslov grading and s the Spin^c grading; in the case where the oriented link has an even number of components, $d \in \frac{1}{2} + \mathbb{Z}$). The Euler characteristic in the d direction gives the Alexander polynomial of K, i.e. if T is a formal variable, then the sum

$$\sum_{s \in \mathbb{Z}} \chi(\widehat{\mathrm{HFK}}_*(K, s)) \cdot T^s$$

calculates the symmetrized Alexander polynomial of K. This invariant satisfies a skein exact sequence, where the three terms appearing in the sequence are the invariants associated to the three links obtained by changing any given crossing or alternatively forming the oriented resolution of that crossing. As such, it provides a theory which is similar in spirit to work of Khovanov [19], who constructs a bigraded homology theory associated to links, whose Euler characteristic is the Jones polynomial. It should be noted, though, that unlike the knot invariant from Heegaard Floer homology, the differentials for Khovanov's complex are purely combinatorial in their definition. However, knot Floer homology can be calculated for a large family of knots, and it is related to the Heegaard Floer homology of the three-manifolds obtained by surgeries on the knot. Like the Alexander polynomial, Heegaard Floer homology gives a lower bound on the Seifert genus of a knot,

$$\max\{s \in \mathbb{Z} \big| \widehat{\mathrm{HFK}}_*(K, s)\} \leq g(K). \tag{1}$$

Generators for the knot Floer complex have a concise Morse-theoretic interpretation. Fix a knot $K \subset S^3$. A perfect Morse function is said to be *compatible with* K, if K is realized as a union of two of the flows which connect the index three and zero critical points (for some choice of generic Riemannian metric μ on S^3). Thus, the knot K is specified by a Heegaard diagram for S^3, equipped with two distinguished points w and z where the knot K meets the Heegaard surface. In this case, a *simultaneous trajectory* is a collection $\mathbf{x}$ of gradient flowlines for the Morse function which connect all the remaining (index two and one) critical points of f. From the point of view of Heegaard diagrams, a simultaneous trajectory is an intersection point in the g-fold symmetric product of Σ $\mathrm{Sym}^g(\Sigma)$ (where g is the genus of Σ) of two g-dimensional tori

$$\mathbb{T}_\alpha = \alpha_1 \times ... \times \alpha_g \quad \text{and} \quad \mathbb{T}_\beta = \beta_1 \times ... \times \beta_g,$$

where here $\{\alpha_i\}_{i=1}^g$ resp. $\{\beta_i\}_{i=1}^g$ denote the attaching circles the two handlebodies. Let $X = X(f, \mu)$ denote the set of simultaneous trajectories. Any two simultaneous trajectories differ by a one-cycle in the knot complement M and hence, if we fix an identification $H_1(M; \mathbb{Z}) \cong \mathbb{Z}$, we obtain a difference map

$$\epsilon \colon X \times X \longrightarrow \mathbb{Z}.$$

The Spin^c grading of a simltaneous trajectory is determined as follows. There is a unique map (defined up to an overall sign)

$$s \colon X \longrightarrow \mathbb{Z}$$

with the property that $s(\mathbf{x}) - s(\mathbf{y}) = \epsilon(\mathbf{x}, \mathbf{y})$, which satisfies the property that $\sum_{\mathbf{x} \in X} T^{s(\mathbf{x})}$ is symmetric, as a Laurent polynomial in $\mathbb{Z}/2\mathbb{Z}[T, T^{-1}]$.

Simultaneous trajectories can be viewed as a generalization of some very familiar objects from knot theory. To this end, note that a knot projection, together with a distinguished edge, induces in a natural way a compatible Heegaard diagram. The simultaneous trajectories for this Heegaard diagram can be identified with the "Kauffman states" for the knot projection; see [18] for an account of Kauffman states, and [26] for their relationship with simultaneous trajectories.

On the other hand, Seiberg-Witten theory has some advantages over Heegaard Floer homology. Foremost amongst these advantages is its close connection with the geometry (as opposed to the topology) of the underlying manifold. For example, in [32], Taubes shows that a symplectic four-manifold has non-vanishing Seiberg-Witten invariant, by using the symplectic form as a perturbation for the equations.

In principle, the shortcomings of the two theories can be bridged. Short of proving that the two theories are isomorphic, one could either come up with more combinatorial methods for calculating Seiberg-Witten invariants, or alternatively, one could try to translate geometric input on a manifold into more combinatorial data which are reflected in Heegaard diagrams and can be detected by Heegaard Floer homology. For example, seminal work of Donaldson [4] gives a nearly combinatorial formulation of the symplectic condition, showing that a symplectic manifold always admits a compatible Lefschetz fibration. The induced two-handle decomposition of the four-manifold can then be used to prove that Heegaard Floer invariant of a symplectic four-manifold is non-trivial [27].

And building such bridges is clearly important for topological applications. For example, in [12], Gordon conjectured that if a knot $K \subset S^3$ has the property that $r = p/q$ Dehn surgery on K is orientation-preserving diffeomorphic to p/q Dehn surgery on the unknot, then K is the unknot. To illustrate the role of orientations here, note that $+5$ surgery on the right-handed-trefoil is a lens space which is orientation-preserving diffeomorphic to -5 surgery on the unknot, i.e. it is orientation reversing diffeomorphic to $+5$ surgery on the unknot. The case where $r = 2$ provides an example where this orientation issue becomes irrelevant. In this case, one obtains a simpler statement of the conjecture (c.f. [13]) that no surgery on a non-trivial knot in S^3 gives real projective three-space.

Many cases of Gordon's conjecture have been known for some time. For example, the case where $p = 0$, the conjecture is known to hold by celebrated work of Gabai [11]. The case where $q \neq 1$, the theorem is confirmed by the cyclic surgery theorem of Culler-Gordon-Luecke-Shalen [14]. In the case where $p/q = \pm 1$, the theorem was established by Gordon and Luecke [15]. But the case where r is integral (and $|r| > 1$) – the case with the most immediate four-dimensional interpretation (since Y is obtained as an integral surgery on a knot in S^3 if and only if it bounds a four-manifold which admits a Morse function with exactly two critical points: one of index zero, the other of index one) – this case remained open until this year.

It was proved in [28] using the surgery long exact sequence for Heegaard Floer homology (which had been missing from Seiberg-Witten theory) that if a knot in S^3 has the property that $S^3_p(K) = S^3_p(U)$ (where U is the unknot), then the Heegaard Floer homology of $S^3_0(K)$ is isomorphic to that of $S^3_0(U)$. But the construction of Kronheimer and Mrowka [21] (building upon work of Gabai [11], Eliashberg and Thurston [5]) shows that the Seiberg-Witten Floer homology distinguishes $S^3_0(K)$ from $S^3_0(U)$ (a result which had been missing from Heegaard Floer homology). In sum: the remaining part of Gordon's conjecture could be proved either by establishing a surgery exact triangle for Seiberg-Witten Floer homology, or by proving the analogues of Kronheimer and Mrowka's genus bounds in Heegaard Floer homology.

In the Fall of 2003 (shortly before the conference), Kronheimer, Mrowka, Ozsváth, and Szabó verified Gordon's conjecture, by establishing a surgery long exact sequence for the Seiberg-Witten monopole equations.

History and Seiberg-Witten Floer Homology

Beginning with seminal works of Mikhail Gromov (see [16]) and Daniel Bennequin (see [1]), the symplectic topology of 4-manifolds and the contact topology of 3-manifolds have firmly established themselves as an integral part of low-dimensional topology. The theory of J-holomorphic curves developed by Gromov in [16] was linked with Seiberg-Witten (SW) theory by Clifford Taubes (see [32, 33]) who proved that for symplectic manifolds SW-invariants coincide with certain kinds of Gromov enumerative invariants for holomorphic curves. Together with Taubes' non-vanishing theorem for SW-invariants of symplectic manifolds this for the first time showed the existence of J-holomorphic curves in certain closed symplectic 4-manifolds.

It turns out that for the extension of these results to 3-dimensional topology it is important to understand the interaction between contact 3-manifolds which bound symplectic 4-manifolds. Note that although a symplectic structure on a 4-manifold does not induce any contact structure on its boundary, it is useful to consider certain compatibility conditions between symplectic and contact structures. Suppose we are given a symplectic manifold (W, ω) with boundary $V = \partial W$ which carries a contact structure ξ. First of all, both the contact structure ξ on V and the symplectic structure ω on W define an orientation of V. These two orientations may coincide: in this case the boundary is called *convex*, or be opposite in the concave case. Second, we may ask if there exists a contact form α such that $\omega|_V = d\alpha$, or at least $\omega|_\xi = d\alpha|_\xi$. In the first case (W, ω) is called a *strong symplectic filling*, in the second case a *weak symplectic filling*. One can consider even a stronger filling condition which requires that α extends to the whole W and that the Liouville vector field X defined by the equation $i(X)\omega = \alpha$ is gradient-like for a Morse function on W which is constant on the boundary. In this case (W, ω) is called a *Stein filling* of (V, ξ).

In their paper [21] Kronheimer and Mrowka developed a relative version of SW-theory and defined an invariant of a contact structure on a 3-manifold which takes its value in SW-Floer homology of the manifold. As an analogue of Taubes' non-vanishing theorem from [34] they proved that for weakly fillable (or even a seemingly weaker notion of weakly semi-fillable) contact structures their invariant does not vanish.

On the other hand, Ozsváth and Szabó defined in [27] a similar contact invariant in the context of Heegaard Floer homology theory. Their invariant was seemingly easier to compute but the analogue of the Kronheimer-Mrowka non-vanishing result was established only for Stein-fillable contact structures.

During the conference there was an active discussion of how the non-vanishing result in Heegaard Floer homology can be proven in the same generality as the the corresponding result in SW Floer homology. One of the questions asked by Olga Plamenevskaya led Eliashberg to realize that the answer depends on a problem of finding a symplectic manifold bounding a 3-manifold fibered over S^1 with symplectic fibers (which arises as a 0-surgery on the binding of an open book decomposition associated with the contact structure). He then realized that the answer to this question was essentially known and filled in the details of the argument. During the continuing discussion Ozsváth and Szabó realized that the same argument allowed several other advances in Heegard homology theory. Furthermore Peter Kronheimer pointed out that Eliashberg's observation together with the recent work of Feehan and Leness (see [7]) about the relation between Donaldson and SW-invariants, allowed him to complete his joint program with Tom Mrowka for proving Property P.

Symplectic Fillings

Here are more mathematical details of Eliashberg's argument.

Theorem 1 *Let (V,ξ) be a contact manifold and ω a closed 2-form on V such that $\omega|_\xi > 0$. Suppose that we are given an open book decomposition of V with a binding B. Let V' be obtained from V by a Morse surgery along B with a canonical 0-framing, so that V' is fibered over S^1. Let W be the corresponding cobordism, $\partial W = (-V) \cup V'$. Then W admits a symplectic form Ω such that $\Omega|_V = \omega$ and Ω is positive on fibers of the fibration $V' \to S^1$.*

Theorem 2 *Let (V,ξ) and ω be as in Theorem 1. Then there exists a symplectic manifold (W', Ω') such that $\partial W' = -V$ and $\Omega'|_V = \omega$. Moreover, one can arrange that (W', Ω') contains the symplectic cobordism (W, Ω) constructed in Theorem 1 as a subdomain, adjacent to the boundary. In particular, any symplectic manifold which weakly fills the contact manifold (V,ξ), can be symplectically embedded as a subdomain into a closed symplectic manifold.*

Eliashberg's theorem, together with previously known properties of Heegaard Floer homology (see esp. [27]) now lead quickly to the non-vanishing theorem for the Heegaard Floer invariant of a symplectically semi-fillable contact structure. Specifically, suppose that W_1 is a symplectic filling of a three-manifold Y, Eliashberg constructs a four-manifold W_2 with the property that $V = W_1 \cup_Y W_2$ is symplectic. By [27], we know that the invariant of V is non-trivial, and hence the Heegaard Floer homology of Y must be non-trivial, as well.

Thus, we have the missing piece required to verify Gordon's conjecture purely within the framework of Heegaard Floer homology. But there are applications of this non-vanishing theorem in Heegaard Floer homology which go beyond reproofs of gauge theoretic results. It now follows [29] that if g is a knot in S^3, then $\widehat{\mathrm{HFK}}(K, g)$ is always non-trivial, i.e. the lower bound from Inequality (1) is sharp. In turn, this result admits a restatement which is independent of Heegaard Floer homology:

Corollary 3 *The Seifert genus of a knot K is the minimum over all compatible Heegaard diagrams for K of the maximum of $s(\mathbf{x})$ over all the simultaneous trajectories.*

Other Lectures and Results

One of the subjects which was actively discussed during the conference is the relation of Heegaard Floer homology theory of Ozsváth-Szabó, periodic Floer homology theory of Michael Hutchings and Michael Thaddeus, and the project of embedded contact homology theory which is under construction by Michael Hutchings, Yakov Eliashberg, Michael Sullivan and others. Several students are also involved in this project. In particular, Stanford student Robert Lipshitz gave an informal talk during one of the evening sessions where he sketched the construction of Heegard Floer homology using holomorphic curves in 4-manifolds, rather than g-folded symmetric products of surfaces.

David Gay spoke on his work (with Kirby) in which they show how to construct harmonic 2-forms on 4-manifolds in terms of handlebody decompositions. The 2-forms vanish transversely along a collection of circles and are symplectic in the complement of these circles. He discussed the extent to which he can prescribe spin^C structures and J-holomorphicity of certain surfaces and he worked through some explicit examples.

Mikhail Khovanov talked about his construction of a bigraded homology theory of links with the quantum $sl(3)$ invariant as the Euler characteristic.

Ciprian Manolescu described an associated suspension spectrum $SWF(Y)$ whose homology is the Seiberg-Witten Floer homology starting with a 3-manifold Y with $b_1(Y) = 0$ and a spin^C structure on Y. Given a cobordism between 3-manifolds, there is an associated morphism between their spectra, and a gluing theorem holds: composition of cobordisms corresponds to composition of morphisms. For 3-manifolds with $b_1 > 0$, assuming the vanishing of a certain K-theoretic obstruction, there is a pro-spectrum analogue of SWF.

Andras Nemethi spoke about calculating the Ozsváth-Szabó invariant of negative definite plumbed 3-manifolds with a given spin^C-structure starting with the plumbing graph. He gave precise combinatorial algorithms for cases including all rational and weakly elliptic singularities, which in the case of Seifert fibered 3-manifolds is expressed in terms of the Seifert invariants.

Brendan Owens and Saso Strle discussed an application of Ozsváth and Szabó's Froyshov-type invariant to cobordisms of rational homology spheres. Computations in the case of Seifert fibered spaces were used to obtain bounds on the 4-ball genus of Montesinos links.

Jake Rasmussen described a strange correlation between the Khovanov and knot Floer homologies which works in a striking number of cases.

Andras Stipsicz uses Heegard Floer Homology theory to distinguish contact structures and gives examples of tight non-fillable contact 3-manifolds.

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