BIRS Workshop
Mathematical Image Analysis and Processing
October 23-28, 2004

SCHEDULE

	Oct 23	Oct 24	Oct 25	Oct 26	Oct 27	Oct 28
	Saturday	**Sunday**	**Monday**	**Tuesday**	**Wednesday**	**Thursday**
7:00-8:15	X	7:00-8:15 Continental Breakfast, 2nd floor lounge, Corbett Hall				
8:15-9:00	X	Orientation[1]	Whitaker	Siddiqi		X
9:00-9:45	X	Vemuri	Vese	Vogel	Lucier	X
9:45-10:30	X	Levine	Tai	Chan	Malgouyres	X
10:30-11:00	X	10:30-11:00 Coffee Break, 2nd floor lounge, Corbett Hall				X
11:00-11:45	X	Scherzer	Allard	Metaxas[2]	Santosa	X
11:45-12:30	X	Spira	Vixie	Group Photo[3]	Gunturk	X
12:30-1:30	X	12:30-1:30 Buffet Lunch, Donald Cameron Hall				X
1:30-2:15	X	Rumpf	Shah	free afternoon	Bertozzi	X
2:15-3:00	X	Wolf	March	free afternoon	Greer	X
3:00-3:30	X	3:00-3:30 Coffee Break, 2nd floor lounge, Corbett Hall (except Tues.)				X
3:30-4:15	X	Nikolova	Yezzi	free afternoon	Boutin	X
4:15-5:00	X	Buades Capo	Zhou	free afternoon	García Almeida	X
5:00-5:45	X	Majava	Tsai	free afternoon	Schmidt	X
5:45-7:30	5:45-7:30 Buffet Dinner, Donald Cameron Hall					X
8:00-10:00	X	discussions	discussions			X

MEALS

Breakfast (Continental): 7:00 - 9:00 am, 2nd floor lounge, Corbett Hall, Sunday - Thursday
*Lunch (Buffet): 11:30 am - 1:30 pm, Donald Cameron Hall, Sunday - Thursday
*Dinner (Buffet): 5:30 - 7:30 pm, Donald Cameron Hall, Saturday - Wednesday
Coffee Breaks: As per daily schedule, 2nd floor lounge, Corbett Hall
***Please remember to scan your meal card at the host/hostess station in the dining room for each lunch and dinner.**

[1] At 8:45 am.
[2] Until 12:00 pm.
[3] A group photo will be taken on Tuesday at 12:00 pm, directly after the last lecture of the morning. Please meet on the front steps of Corbett Hall.

MEETING ROOMS

All lectures are held in the main lecture hall, Max Bell 159. *Please note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **William K. Allard**(Mathematics, Duke University)
Title: *On the regularity of level sets of minimizers of denoising models based on total variation regularization*
Abstract: Let Ω be an open subset of $\mathbf{R}^n$, $n \geq 2$; let $\beta, \gamma : [0, \infty) \to [0, \infty)$ be increasing, Lipschitzian on bounded sets and zero at zero; and let f be a real valued Lebesgue measurable function on Ω such that $\int_\Omega \gamma(|f(x)|)\, d\mathcal{L}^n$ where $\mathcal{L}^n$ is Lebesgue measure on $\mathbf{R}^n$. For each $\epsilon > 0$ we let

$$\mathbf{U}_\epsilon(u) = \epsilon \mathbf{TV}(u) + \beta \left(\int_\Omega \gamma(|u(x) - f(x)|)\, d\mathcal{L}^n x \right)$$

for each real valued Lebesgue measurable function u on Ω; here $\mathbf{TV}(u)$ is the total variation of u.

Rudin, Osher and Fatemi studied minimizers of this functional with $\beta(y) = y$ and $\gamma(y) = y^2$, $y \in [0, \infty)$, and Chan and Esedoglu studied minimizers of this functional with $\beta(y) = y$ and $\gamma(y) = y$, $y \in [0, \infty)$; in both cases the functional was used as a denoising model.

Let us now assume that $n \leq 7$; in case $n > 7$ much can be said but we will not treat that case here. Our main result, in which we have to assume f is essentially bounded if γ grows more than linearly at infinity, is that if u is a minimizer of $\mathbf{U}_\epsilon$ and $y \in \mathbf{R}$ then $\{u \geq y\}$ is a set of finite perimeter whose reduced boundary is a C^1 hypersurface in Ω whose normal is locally Hölder continuous with respect to any exponent $\mu \in (0, 1)$; in case $n = 2$ this normal is locally Lipschitzian. In addition, given a bit more regularity of β and γ, we obtain a very useful Euler-Lagrange type equation which minimizers must satisfy which in turn gives detailed local information on the structure of u, particularly in case $n = 2$.

Speaker: **Andrea Bertozzi** (Mathematics, University of California at Los Angeles and Duke University)
Title: *Fourth Order PDE in image processing*
Abstract:

Speaker: **Mireille Boutin** (Mathematics and Electrical & Computer Engineering, Purdue University)
Title: *Scene Reconstruction from Images: a novel approach based on moving frames*
Abstract: The problem of reconstructing a scene from a set of images taken from unknown view points involves a lot of "superflous" unknowns. For example, the camera parameters used for taking each picture are irrelevant since we are only interested in the structure of the scene. However, with the traditional approaches, we still need to solve for these parameters because they are included in the intermediate steps of the solution process. In this talk, we will show that many of these superflous parameters can be seen as group parameters acting on the other unknowns of the problems. Using the moving frame method, we obtain a set of invariants of this group action. The invariants provide a new formulation to the reconstruction problem where the superfluous unknowns do not appear anymore. In particular, the camera angles, which are difficult to solve for, do not appear in the equations. As a result, a much simpler/robust formulation is obtained.

Speaker: **Toni Buades-Capo** (Ecole Normale Superiore de Cachan)
Title: *On Image denoising methods*
Abstract: The search for efficient image denoising methods still is a valid challenge, at the crossing point

of functional analysis and statistics. In spite of the sophisticated methods recently proposed (empirical Wiener filters, total variation minimization, anisotropic diffusion, wavelet thresholdings, image + texture + noise models), most algorithms have not yet attained a desirable level of applicability. All show an outstanding performance when the image smoothness model corresponds to the algorithm assumption, but fail at other locations and create artifacts or remove image fine structures. The main focus of the talk is to define a general mathematical and experimental methodology to compare and classify classical image denoising algorithms. The mathematical analysis is based on the structure analysis of what we call the "method noise", namely the difference between the (always slightly noisy) digital image and its denoised version. A denoising algorithm is consistent when it has a low, or even a zero method noise for functions with the right regularity. We also introduce a new kind of algorithm, which seems to separate better noise from image details, the non local means algorithm (NL-means). We show why this algorithm inherits of most consistency properties of the former methods and prove its asymptotic optimality under a rather generic statistical image model.

Speaker: **Tony Chan** (Mathematics, University of California at Los Angeles)
Title: *A logic framework fo multi-channel image segmentation and application to tracking in videos*
Abstract: I'll present a general framework for defining meaningful segmentation of multi-channel images by allowing arbitrary logical combination of object information from the different channels. Under this framework, I'll develop specific active contour and segmentation models and algorithms based on extensions of the scalar region-based Chan-Vese segmentation model. Finally, I'll show an application of this logic framework to tracking of objects in low frame-rate video sequences. This is joint work with Mark Moelich and Berta Sandberg.

Speaker: **Gerardo Emilio García Almeida** (Mathematics, Universidad Autonoma de Yucatan)
Title: *Estimates for convolutions in function spaces with fractional order of smoothness*
Abstract: In this talk an integral equation of the first kind of convolution type is considered. The Tikhonov regularisation method is used to construct a sequence of approximate solutions that converges to the exact solution of this ill-posed problem. The elements of this sequence are called regularised solutions.

In the Tikhonov regularisation method it is assumed that the exact solution, the right hand side of the integral equation, and the error belong to suitable function spaces.

It is known that the choice of the particular spaces is very important. On one hand too strong a priori smoothness assumptions on the exact solution and the error which are far from their actual smoothness properties cause great difficulties in the numerical implementation of the algorithm. On the other hand too weak smoothness assumptions lead to a very slow convergence.

In some cases the assumption, which is often used, that the exact solution belongs to the Sobolev space of order of smoothness one is too strong. In investigations of V. I. Burenkov, I. F. Dorofeev and A. S. Pankratov, related to the isotropic case, it was shown that the application of the isotropic Nikol'skiĭ–Besov spaces of functions possessing some common smoothness of fractional order gave more flexibility in characterising the smoothness properties of the exact solution and the error and better fitted to the applied problems. In particular, the smoothness parameter may be chosen to be sufficiently small, thus imposing considerably weak smoothness assumptions on the exact solution which may be possessed even by unbounded solutions with power growth, whilst the convergence of the regularised solutions to the exact one is still reasonably quick.

The main aim of the present talk is to obtain similar results in the anisotropic case by using anisotropic Nikol'skiĭ–Besov spaces, thus giving more possibilities for the application of this method.

Speaker: **John Greer** (Courant Institute, New York University)
Title: *Fourth order equations for image processing*
Abstract: A number of fourth order diffusion equations have recently been introduced for image smoothing and denoising. Although numerical implementations of these methods produce impressive results, very little is known about the mathematical properties of the equations themselves. I will discuss some of the

first results regarding a few of these nonlinear diffusions. In particular, I will describe the use of energy methods to prove the well-posedness of a class of H^1 diffusions for image processing, including the 'Low Curvature Image Simplifier' (LCIS) equation of Tumblin and Turk (SIGGRAPH, August, 1999). I will demonstrate implementations of a new finite difference discretization of the LCIS equation that ensures the discrete Laplacian of the image intensity remains bounded. These results will be compared to second order methods such as the Perona-Malik equation and Total Variation flow.

Speaker: **Sinan Gunturk** (Courant Institute, New York University)
Title: *Some old and new ideas on digital halftoning*
Abstract: In the first part of this talk, we will revisit a well-known class of digital halftoning algorithms called error diffusion. We will argue in a theoretical setting that existing error diffusion schemes have low order of approximation and present a way to improve this.

In the second part of the talk, a completely different halftoning algorithm, which is based on multiscale ideas, will be introduced. The approximation order of this new algorithm, along with some generalizations of it, will follow.

Speaker: **Stacey Levine** (Math and Computer Science, Duquesne University)
Title: *Noise removal and texture extraction using nonstandard growth functionals*
Abstract: We present a variational formulation for processing images using functionals with $p(x)$ growth ($p(x) \geq 1$). The minimization problem provides a model for removing noise while preserving and enhancing edges. In particular, it significantly reduces the 'staircasing effect' which can result in the detection of false artifacts. Furthermore, based on the novel image decomposition models recently introduced by Meyer and modified by Vese and Osher, the functional can be modified to address the problem of image denoising with texture extraction; this is particularly useful in remote sensing images where textures are obscuring the boundary of an object of interest. In this talk, the mathematical validity of the model is established and numerical results demonstrate its effectiveness in both noise removal, and noise removal with texture extraction. Direct applications to boundary detection in remote sensing images are also presented.

Speaker: **Brad Lucier** (Mathematics and Computer Science, Purdue University)
Title: *YAWTSI: Yet Another Way To Smooth Images (and keep edges)*
Abstract: We show the results of two algorithms for $B^1_\infty(L_1)$ variational smoothing, one based on wavelets (similar to Wavelet Shrinkage for $B^1_1(L_1)$ smoothing) and one based on pixel values (similar to Chambolle's method for BV smoothing). Experiments show that the qualitative properties of the smoothed images depend critically on the form of the $B^1_\infty(L_1)$ seminorm that one uses in the variational problem. This is joint work with Antonin Chambolle.

Speaker: **Francois Malgouyres** (Universite Paris 13, France)
Title: *Image compression through a projection on a polyhedral set*
Abstract: Few years ago a new image restoration model has been proposed. It consists in minimizing a regularity criterion (in practice the total variation) among points of a polyhedron. This paper propose to adapt this model for image compression. The results are rather convincing while a lot of work still needs to be performed to make the model usable. This adaptation shows how to translate the usual "wavelet coefficient modulus decay rate" argument, for such optimization models.

Speaker: **Kirsi Majava** (Mathematical Information Technology, University of Jyväskylä)
Title: *Active-set algorithms for solving nonsmooth image denoising problems*
Abstract: In this presentation, we discuss active-set methods for solving nonsmooth optimization problems appearing in image denoising. The basic optimization-based formulation for image denoising problem can be given in the following form,

$$\min_u \int_\Omega |u - z|^s \, dx + \beta \int_\Omega |\nabla u|^r \, dx, \tag{1}$$

where z denotes the noisy data and β is a regularization parameter. Obviously, $r = 2$ corresponds to the classical smooth regularization and $r = 1$ yields the total variation (TV) regularization. Moreover, the usual choice $s = 2$ yields L^2-fitting, which assumes Gaussian (normally distributed) noise with zero mean, whereas $s = 1$ allows heavy tails (e.g. outliers) in the noise distribution.

The basic difficulty concerning the image denoising problems with $s = 1$ or $r = 1$ is their nondifferentiability in the classical sense, which excludes the usage of common gradient-based solution methods, such as the conjugate gradient method, for solving these problems. Ito and Kunisch (1999) first proposed a so-called active-set method (based on the augmented Lagrangian regularization of the nonsmooth optimization problem) for solving problem (**??**) with $s = 2, r = 1$. Based on these ideas, we presented (Kärkkäinen & Majava, 2000) efficient implementation of active-set algorithms to solve the same problem. Encouraged by the good performance of these algorithms, we have developed active-set algorithms also for other combinations of s, r ($s = 1, r = 2$ and $s = r = 1$). In this presentation, we describe the basic ideas of an active-set method and discuss the algorithms developed. Numerical experiments are presented to test the efficiency of the presented algorithms and to illustrate the restoration capability of the formulations. This is joint work with Tommi Kärkkäinen (University of Jyväskylä) and Karl Kunisch (Karl-Franzens University of Graz, Austria).

Speaker: **Riccardo March** (Istituto per le Applicazioni del Calcolo, Roma)
Title: *Variational approximation of a curvature depending functional*
Abstract: We consider a functional for image segmentation which is defined on families of curves. The functional is of the Mumford-Shah type and it penalizes curvature, length, and number of endpoints of the curves. A Γ-convergence theorem is presented for the approximation of such a functional by means of elliptic functionals. The approximation is close in spirit to the Ambrosio and Tortorelli approximation of the Mumford-Shah functional. Joint work with A. Braides.

Speaker: **Dimitris Metaxas** (Computer Science, Rutgers University)
Title: *Animation and Control of Breaking Waves* Abstract: Controlling fluids is still an open and challenging problem in fluid animation. We have recently developed a novel fluid animation control approach and we present its application to controlling breaking waves. In our Slice Method framework an animator defines the shape of a breaking wave at a desired moment in its evolution based on a library of breaking waves. Our system computes then the subsequent dynamics with the aid of a 3D Navier-Stokes solver. The wave dynamics previous to the moment the animator exerts control can also be generated based on the wave library. The animator is thus enabled to obtain a full animation of a breaking wave while controlling the shape and the timing of the breaking. An additional advantage of the method is that it provides a significantly faster method for obtaining the full 3D breaking wave evolution compared to starting the simulation at an early stage and using solely the 3D Navier-Stokes equations. We present a series of 2D and 3D breaking wave animations to demonstrate the power of the method.

Speaker: **Dimitris Metaxas** (Computer Science, Rutgers University)
Title: *Metamorphs: Deformable Shape and Texture Models*
Abstract: We present a new class of deformable models termed Metamorphs whose formulation integrates both shape and interior texture. The model deformations are derived from both boundary and region information based on a variational framework. This framework represents a generalization of previous parametric and implicit geometric deformable models, by incorporating model interior texture information. The shape of the new model is represented implicitly as an "image" in the higher dimensional space of distance transforms. The interior texture is captured using a nonparametric kernel-based approximation of the intensity probability density function (p.d.f.) inside the model. The deformations that the model can undergo are defined using a space warping technique - the cubic B-spline based Free Form Deformations (FFD). When using the models for boundary finding in images, we derive the model dynamics from an energy functional consisting of both edge energy terms and texture energy terms. This way, the models deform under the influence of forces derived from both boundary and region information. A MetaMorph

model can be initialized far-away from the object boundary and effi- ciently converge to an optimal solution. The proposed energy functional enables the model to pass small spurious edges and prevents it from leaking through large boundary gaps, hence makes the boundary finding robust to image noise and inhomogeneity. We demonstrate the power of our new models to segmentation applications, and various examples on finding object boundaries in noisy images with complex textures demonstrate the potential of the proposed technique.

Speaker: **Mila Nikolova**(Ecole Normale Superiore de Cachan)
Title: *Recovery of edges in signals and images by minimizing nonconvex regularized least-squares*
Abstract: We analyze the properties of images and signals restored by minimizing regularized least-squares according to the shape of the regularization function. This question is of paramount importance for a relevant choice of regularization term. We give bounds which characterize the smoothing incurred by (local) minimizers. The main point of interest is the restoration of edges. We show that under nonconvex regularization, the differences between neighboring samples at a minimizer are either shrinked, or enhanced. This naturally entails a neat classification of differences as smooth or as edges. (This effect is essentially different from edge-preservation using convex regularization functions.) We also give conditions for exact restoration of edges. Explicit expressions are derived for the truncated quadratic and the "0-1" regularization function. These results are illustrated using numerical examples.

Speaker: **Martin Rumpf** (Mathematics,Gerhard-Mercator-Universität - Gesamthochschule Duisburg)
Title: *On higher order geometric flows in image and surface processing*
Abstract: Willmore flow is well established as am important tool in many image and surface processing applications. After a general discussion of the perspectives of such higher order flows and in particular of the corresponding anisotropic versions, we will focus on their appropriate formulation in level set form. Thus, a general approach for the integration of geometric gradient flows over level sets ensembles is presented. It enables to derive a variational formulation for the level set solution of various second and fourthed order evolution problems, in particular the above fourth order flows. Furthermore, spatial and temporal discretization are discussed and numerical simulations are presented. Finally, we discuss a general implicit narrow band method for second and fourth order geometric flows.

Speaker: **Fadil Santosa** (Mathematics, University of Minnesota at Minneapolis)
Title: *An inverse problem in nondestructive evaluation of spotwelds*
Abstract: Spotwelds are used in attaching metal sheets together. This method of joining sheet metal is especially common in the auto industry. There are more than 20 thousand spotwelds in a typical car, and they play a crucial role in the structural integrity of the vehicle. A thermal imaging method for nondestructive evaluation of spotwelds have been proposed. In this method, a transducer is employed to generate heat near the weld while skin temperature of the metal sheet is measured. The inverse problem is to access the quality of the weld from the temperature reading.

In this presentation, we develop a simple model for the thermal diffusion problem. The inverse problem we seek to solve amounts to finding a heat source in a 2-D domain given temperature as a function of space and time. We solve this classically illposed problem by devising a time-stepping algorithm which solves a regularized problem in each time step. Several regularization strategies are considered. We illustrate the main ideas of our work in numerical examples.

Speaker: **Otmar Scherzer** (Computer Science, University of Innsbruck)
Title: *Linear, Non-linear, Non-differentiable, Non-convex regularization involving Unbounded Operators*
Abstract: Tikhonov initiated the research on stable method for the numerical solution of inverse and ill-posed problems. Tikhonov's approach consists in approximating a solution of an operator equation

$$F(x) = y$$

by a minimizer of the penalized functional

$$\|F(x) - y\|^2 + \alpha\|x\|^2 \quad (\alpha > 0) .$$

In the beginning mainly linear ill–posed problems (i.e. F is linear) such as computerized tomography have been solved with these methods. The theory of **Tikhonov regularization** methods developed systematically. Until around 1980 there has been success in a rigorous and rather complete analysis of regularization methods for linear ill-posed problems. We mention the books of Tikhonov & Arsenin, Nashed, Engl & Groetsch, Groetsch, Morozov, Louis Natterer, Bertero & Boccacci, Kirsch, Colton & Kress... In 1989 Engl & Kunisch & Neubauer and Seidman & Vogel developed a regularization theory for *non–linear* inverse problems where F is a non-linear, differentiable operator. About the same time Osher & Rudin used **bounded variation regularization** for denoising and deblurring, which consists in minimization of the functional

$$\|F(x) - y\|^2 + \alpha \int |\nabla x| .$$

This method is highly successful in restoring discontinuities. The analysis of bounded variation regularization is significantly more involved since the penalization functional is not differentiable. Over the past years this concept has attracted many mathematical research. The next step toward generalization of regularization methods is **non-convex regularization**. Here the general goal is to minimize functionals of the form

$$\int g(F(x) - y, x, \nabla x) ,$$

which may be nonconvex with respect to the third component ∇x.

Another complications is introduced in the analysis of regularization functionals if for instance the operator F can be decomposed into a continuous and a discontinuous operator. Such models have become popular for level set regularization recently.

Speaker: **Volker Schmidt** (Abteilung Stochastik,Universität Ulm)
Title: *Simultaneous nonparametric estimation of the specific intrinsic volumes of stationary random sets*
Abstract: An actual question in statistical analysis of image data is the development of methods by means of which one can automatically distinguish between two (or more) images of similar structure. Examples, where this type of decision problems appear, range from testing the spatial structure of biological cells or tissues in computer-aided cancer diagnostics, via space-time analysis of coverage and connectivity properties in mobile communication systems, to intelligent management of complex transportation systems.

In this talk, we present a new method for statistical analysis of the spatial structure of binary image data, which can contribute to solve the problems mentioned above. The idea behind this method is a new approach to (indirect) statistical estimation of morphological image characteristics, using tools of convex and stochastic geometry.

More precisely, we interpret binary images in the d-dimensional Euclidean space as realizations of spatially homogeneous random closed sets, assuming that they belong to the extended convex ring. Then, we construct nonparametric joint estimators for the d+1 specific intrinsic volumes (or, equivalently, the specific Minkowski functionals) of these random closed sets, including estimators for the specific Euler-Poincar characteristic (or, equivalently, the specific connectivity number), the specific surface area, and the volume fraction itself. The estimators are based on an explicit extension of the classical Steiner formula to the convex ring and they can be represented by integrals of some stationary random fields. This implies in particular that the estimators are unbiased. Moreover, conditions are derived under which they are mean-square consistent, and a positive-definite and consistent estimator for their asymptotic covariance matrix is given. Some issues of their efficient numerical implementation are also discussed.
References:
S. Klenk, V. Schmidt, E. Spodarev (2004) A new algorithmic approach to the computation of Minkowski functionals of polyconvex sets. Preprint (available at http://www.geostoch.de)

V. Schmidt, E. Spodarev (2004) Joint estimators for the specific intrinsic volumes of stationary random sets. Preprint (available at http://www.geostoch.de)

E. Spodarev, V. Schmidt (2004) On the local connectivity number of stationary random closed sets. Working paper (under preparation)

Speaker: **Jayant Shah** (Mathematics, Northestern University)
Title: *Gray skeletons and segmentation of shapes*
Abstract:

Speaker: **Kaleem Siddiqi** (Computer Science & Centre For Intelligent Machines, McGill University)
Title: *Medial integrals for shape analysis*
Abstract: In this talk we discuss a very simple type of integral performed on a vector field defined as the gradient of the Euclidean distance function to the bounding curve (or surface) of a binary object. The limiting behavior of this integral as the enclosed area (or volume) shrinks to zero reveals a very useful invariant which can be used to compute the Blum skeleton as well as to reveal the geometry of the object that it describes. Joint work with Pavel Dimitrov and James N. Damon.

Speaker: **Alon Spira** (Computer Science, Technion)
Title: *Geometric image and curve evolution on parametric manifolds*
Abstract: The motion of images and curves in R^2 has been researched extensively. Applications in image processing and computer vision include image enhancement through anisotropic diffusion, image segmentation by active contours, and many others. Extending these motions to manifolds embedded in spaces of higher dimensions can be most beneficial.

In this talk we present numerical schemes for implementing geometric flows on parametric manifolds. We consider a 2D parameterization plane that is mapped to an N-dimensional space. Our approach in devising the schemes is to implement them on the uniform Cartesian grid of the parameterization plane instead of doing so in the N-dimensional space. This enhances the efficiency and robustness of the resulting numerical schemes.

The first numerical scheme is an efficient solution to the eikonal equation on parametric manifolds. The scheme is based on Kimmel and Sethian's solution for triangulated manifolds, but uses the metric tensor of the parametric manifold in order to implement the scheme on the parameterization plane. The scheme is used to devise a short time kernel for the Beltrami image enhancing flow. The kernel enables an arbitrary time step for the flow for regular images as well as images painted on manifolds, such as face images. The numerical scheme is further used for face recognition by constructing an invariant face signature from distances calculated on the face manifold.

Another numerical scheme implements curve evolution by geodesic curvature flow on parametric manifolds. The flow is implemented by back projecting the curve from the manifold to the parameterization plane, calculating the flow on the plane by the level sets method and then mapping it back to the manifold. Combining this flow with geodesic constant flow enables the implementation of geodesic active contours for images painted on parametric manifolds.

The numerical schemes presented in this talk enable a proper implementation of image processing, computer vision, and computer graphics applications for images painted on parametric manifolds. Taking into account the geometry of the manifolds promises superior results to the conventional processing of the images as regular 2D images.

This is joint work with Ron Kimmel.

Speaker: **Xue-Cheng Tai** (Mathematics, University of Bergen)
Title: *Piecewise constant level set methods and their fast solutions for image segmentation*
Abstract: In this work we discuss variants of the PDE based level set method proposed earlier by Osher and Sethian. Traditionally interfaces are represented by the zero level set of continuous functions. We instead use piecewise constant level set (PCLS) functions, i.e. the level set function equal s to a constant in each

of the regions that we want to identify. Using the methods for interface problems, we need to minimize a smooth convex functional under a constraint. The level set functions are discontinuous at convergence, but the minimization functional is smooth and locally convex. The methods are truly variational, i.e. all the equations we need to solve are the Euler-Lagrangian equations from the minimization functionals. Thus, the fast Newton type of method can be easily used. We show numerical results using the methods for segmentation of digital images. Application to inverse problems will be briefly discussed.

We shall present two variants of the piecewise constant level set methods (PCLSM). One of them is able to use just one level set function for identifying multiphase problems with arbitrary number of phases. Another variant, which we call the binary level set method, only requires the level set function equals 1 or -1. The geometrical quantities like the boundary length and area of the subdomain can be easily expressed as functions of the new level set functions.

This is a joint work with J. Lie and M. Lysaker. The papers can be downloaded from the UCLA cam-report web page: http://www.math.ucla.edu/applied/cam/

Speaker: **Richard Tsai** (Mathematics, University of Texas at Austin)
Title: *Threshold Dynamics for the Mumford-Shah Functional*
Abstract: We proposed a fast algorithm for constructing minimizing sequences for the Mumford-Shah functional for image segmentation applications. The proposed method is motivated by the threshold dynamics of the Merriman-Bence-Osher scheme that is proposed for mean curvature motion. We present several numerical examples that are carefully designed and tested to reflect the behavior of the proposed scheme. The complexity of our scheme is $O(N)$, where N is the number of pixels in a given image.

This is a joint work with Selim Esedoglu.

Speaker: **Baba Vemuri** (Computer and Information Science and Engineering, Univ. of Florida)
Title: *An Affine Invariant "Distance" Measure for Diffusion Tensor MRI Segmentation*
Abstract: Diffusion tensor images(DTI), which are matrix valued data sets, have recently attracted increased attention in the fields of medical imaging and visualization. In this talk, I will present a novel definition of tensor "distance" grounded in concepts from information theory and incorporate it in the segmentation of DTI. In a DTI, the symmetric positive definite (SPD) diffusion tensor at each voxel can be interpreted as the covariance matrix of a local Gaussian distribution. Thus, a natural measure of dissimilarity between SPD tensors would be the KL divergence or its relative. The square root of the J-divergence (symmetrized KL) between two Gaussian distributions corresponding to the tensors being compared is proposed and this leads to a novel **closed form expression** for the "distance" as well as the mean value of a DTI. Unlike the traditional Frobenius norm-based tensor distance, our "distance" is affine invariant, a desirable property in many applications. This new tensor "distance" is then incorporated in a *region based active contour model* for DTI segmentation. Synthetic and real data experiments are shown to depict the performance of the proposed model.

Speaker: **Luminata Vese** (Mathematics, University of California at Los Angeles)
Title: *Decomposition of images into cartoon and texture using the total variation and div(BMO)*
Abstract: An important problem in image analysis is the separation of large scales (cartoon features) from smaller periodic scales (texture) in images. Yves Meyer suggested that models such as Mumford-Shah or Rudin-Osher-Fatemi can be viewed as decomposition models into cartoon and texture, and not only as image segmentation and restoration models. In these two models, the texture component is modeled by a square-integrable function. Following Y. Meyer, we propose and analyze a model where the textured component belongs to the space div(BMO) instead of L^2, while the cartoon component is a function of bounded variation. Theoretical, approximations and numerical results of image decomposition will be presented.

This is joint work with Triet Le, UCLA.

Speaker: **Kevin Vixie** (Mathematical Modeling and Analysis (T-7), Los Alamos National Lab)
Title: *Exact solutions for the Total Variation Minimization with an L1 Data Fidelity Term: Containment,*

Convexity, and Curvature
Abstract: The ROF model, which has seen quite a bit of development and study has, as a data fidelity term, the square of the L_2 distance between the measured image and the proposed solution image. While this term is easy to deal with analytically, it also has less desirable effects such as loss of contrast. If the data fidelity is changed to the L_1 distance, we obtain a different, L_1 TV functional.

In this talk I will present joint work with Selim Esedoglu on exact solutions to L_1 TV functional minimization when that initial "measured" image is a characteristic function of some bounded set (actually a Caccioppoli set). Our three result show that in fact the some exact solutions can be characterized in terms of disks with radii $1/\lambda$ and $2/\lambda$

Speaker: **Curt Vogel** (Mathematical Sciences, Montana State University)
Title: *Tracking Eye Motion from Retinal Scan Data*
Abstract: This talk deals with several different aspects of image analysis. The problem being addressed—tracking the motion of the retina of the eye—is important in understanding how the human visual system works. Our data comes from an adaptive optics scanning laser opthalomoscope (AOSLO), a device which yields high resolution scans of the living retina. Difficulties arise when the eye moves as the scans are taken. Our problem becomes the following: Given a series of scans of roughly the same object, determine both the motion of the object and the optical properties of the object. To solve this problem, we apply a variant of David Arathorn's map-seeking circuit (MSC) algorithm. In addition to being of mathematical interest in its own right, MSC provides a plausible mechanism for biological vision.

This is joint work with David Arathorn, Center for Computational Biology at Montana State University and Austin Roorda, School of Optometry at the University of Houston, the inventor of the AOSLO.

Speaker: **Ross Whitaker** (Computer Science, University of Utah)
Title: *UINTA: Unsupervised, Information-Theoretic, Adaptive Filtering*
Abstract: The problem of denoising images is one of the most important and widely studied problems in image processing and computer vision. Various image filtering strategies based on linear systems, statistics, information theory, and variational calculus, have been effective, but invariably make strong assumptions about the properties of the signal and/or noise. Therefore they lack the generality to be easily applied to new applications or diverse image collections. This paper describes a novel unsupervised, information-theoretic, adaptive filter (UINTA) which improves the predictability of image pixels from their neighborhoods by minimizing an information-theoretic measure of goodness. In this way UINTA automatically discovers the statistical properties of the signal and can thereby reduce image noise in a wide spectrum of images and applications. The talk describes the formulation required to minimize the joint entropy measure, presents several important practical considerations in estimating image-region statistics, and then presents a series of results and comparisons on both real and synthetic data.

Speaker: **Lior Wolf** (Center for Biological and Computational Learning, MIT)
Title: *Learning using the Born Rule*
Abstract: In Quantum Mechanics the transition from a deterministic description to a probabilistic one is done using a simple rule termed the Born rule. This rule states that the probability of an outcome (a) given a state (Ψ) is the square of their inner products ($(a^\top \Psi)^2$).

In this talk, I will explore the use of the Born-rule-based probabilities for clustering and image segmentation, feature selection, classification and object recognition, and for image retrieval. We show how these probabilities lead to existing and new algebraic algorithms for which no other complete probabilistic justification is known, forming a connection between spectral theory and probability theory.

Speaker: **Anthony Yezzi** (Systems & Control and Bioengineering, Georgia Tech)
Title: *Conformal H^0 Metrics on the Space of Curves*
Abstract: Ever since the introduction of snakes by Kass, Witkin, and Terzopoulos, active contours have

played a prominent role in a variety of image processing and computer vision tasks, most notably segmentation. Early reasearch on active contours saw the tansition from parameterization dependend models to geometric models independent of the parameterization of the evolving curve. Next, there were many efforts to incorporated region based image information to make the active contour depend upon global information about the image rather than just the traditional locally computed edge descriptors. In recent years, the latest trend in active contour research seems to be that of incorporating global shape priors into the active contour paradigm. This has brought up non-trivial questions such as how to define an "average shape" or how to characterize "variations in shape". All of these questions ultimately lead to a more basic and fundamental question of how to measure the distance between two given shapes. Going ever deeper, one might ask when a scheme for measuring distances between shapes is associated with a true Riemannian metric on the manifold of all possible smooth shapes and, if so, what are the interesting properties of this metric (for example, what are the geodesics).

In this talk, we will follow this top-down approach, starting with a discussion of shape priors in the active contour framework, notions of average shape, distances between shapes, and ultimately Riemannian metrics on the manifold of shapes. We will discuss the surprisingly pathologies associated with what seems to be the most natural metric (H^0) on the space of curves and show how it is possible to address these pathologies without drastically changing the metric structure by the introduction of a well-chosen conformal factor.

Speaker: **Hao Min Zhou** (Mathematics, Georgia Tech)
Title: *The PDE and Variational Techniques in Wavelet Transforms and Their Applications in Image Processing*
Abstract: Standard wavelet linear approximations (truncating high frequency coefficients) generate oscillations (Gibbs' phenomenon) near singularities in piecewise smooth functions. Nonlinear and data dependent methods are often used to overcome this problem. Recently, partial differential equation (PDE) and variational techniques have been introduced into wavelet transforms for the same purpose.

This talk will include our work on two different approaches in this direction. One is to use PDE ideas to directly change wavelet transform algorithms so as to generate wavelet coefficients which can avoid oscillations in reconstructions when the high frequency coefficients are truncated. We have designed an adaptive ENO wavelet transform by using ideas from Essentially Non-Oscillatory (ENO) schemes for numerical shock capturing. ENO-wavelet transforms retains the essential properties and advantages of standard wavelet transforms without any edge artifacts. We have shown the stability and a rigorous error bound which depends only on the size of the derivative of the function away from the discontinuities.

The second one is to stay with standard wavelet transforms and use variational PDE techniques to modify the coefficients in the truncation process so that the oscillations are reduced in the reconstruction processes. In particular, we use minimization of total variation (TV), to select and modify the retained standard wavelet coefficients so that the reconstructed images have fewer oscillations near edges. We have also proposed new TV based wavelet models for image reconstructions.

Examples in applications including image compression, denoising, inpainting are presented.